Assignment-8

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Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution:

Gaussian

Let X be a Binomial random variable

$$X = \text{Bin}(n, p) \tag{1}$$
$$= \text{Bin}(20, 0.5) \tag{2}$$

The mean μ of X

$$\mu = n \times p \tag{3}$$
$$= 10 \tag{4}$$

The variance σ^2 of X

$$\sigma^2 = n \times p \times (1-p) \tag{5}$$
$$= 5 \tag{6}$$

Let

$$Z = \frac{X - \mu}{\sigma} \tag{7}$$

Here, Z is a random variable with $\mu = 0$ and $\sigma^2 = 1$ Normal-Distribution $f(x)$

$$f(x) = \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{8}$$
$$\tag{9}$$

The Q-function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \tag{10}$$
$$= \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{11}$$

Since

$$X \geq 12 \tag{12}$$
$$\Pr(X \geq 12) = 1 - \Pr(X < 11.5) \tag{13}$$

Note: An additional of 0.5 correction term is present.
As

$$X < 11.5 \tag{14}$$
$$\implies Z < \frac{11.5 - \mu}{\sigma} \tag{15}$$
$$Z < \frac{1.5}{\sqrt{5}} \tag{16}$$
$$Z < 0.67082 \tag{17}$$
$$\Pr(X \geq 12) = 1 - \Pr(Z < 0.67) \tag{18}$$

On compution,

$$\Pr(Z < 0.67) = 0.74883 \tag{19}$$
$$\implies \Pr(X \geq 12) = 0.2511 \tag{20}$$

Binomial

$$X \sim \text{Bin}(n, p) \tag{21}$$

The cdf of X is given by

$$F_X(k) = \Pr(X \leq k) \tag{22}$$
$$= \begin{cases} 0 & k < 0 \\ \sum_{i=1}^{k} \binom{n}{i} p^i (1-p)^{n-i} & 1 \leq k \leq n \\ 1 & k \geq n \end{cases} \tag{23}$$

In this case,

$$p = \frac{1}{2}, \; n = 20 \tag{24}$$

We require $\Pr(X \geq 12)$. Since $n = 20$,

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \tag{25}$$
$$= F_X(20) - F_X(11) \tag{26}$$
$$= \sum_{k=12}^{20} p_X(k) \tag{27}$$
$$= \sum_{k=12}^{20} \binom{n}{k} p^k (1-p)^{n-k} \tag{28}$$
$$= 0.2517 \tag{29}$$

The graph

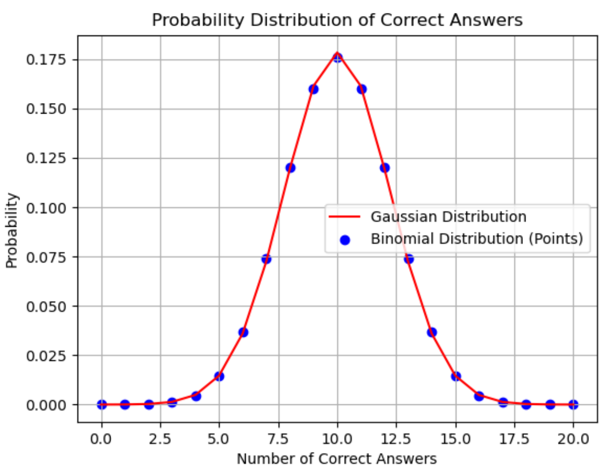

Fig. 0. Binomial vs guassian