Assignment-8

EE22BTECH11012-A.Chhatrapati

Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.
Solution: Let X be a Binomial random variable

$$X = \text{Bin}\,(n, p) \tag{1}$$
$$= \text{Bin}\,(20, 0.5) \tag{2}$$

The mean μ of X

$$\mu = n \times p \tag{3}$$
$$= 10 \tag{4}$$

The variance σ^2 of X

$$\sigma^2 = n \times p \times (1 - p) \tag{5}$$
$$= 5 \tag{6}$$

Let

$$Z = \frac{X - \mu}{\sigma} \tag{7}$$

Now, Z is a random variable with $\mu = 0$ and $\sigma^2 = 1$
Since

$$X \geq 12 \tag{8}$$
$$\implies Z \geq \frac{12 - \mu}{\sigma} \tag{9}$$
$$Z \geq \frac{2}{\sqrt{5}} \tag{10}$$
$$Z \geq 0.8944 \tag{11}$$
$$\Pr\,(X \geq 12) = \Pr\,(Z \geq 0.8944) \tag{12}$$

On compution,

$$\Pr\,(Z \geq 0.8944) = 0.2517 \tag{13}$$