Assignment-8

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Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution:

TABLE 0
VARIABLES

Variable	Value	Description
n	20	Number of questions
p	0.5	probability of question being correct
$\mu = np$	10	mean of distribution
$\sigma = \sqrt{npq}$	$\sqrt{5}$	variance of distribution
X	$0 \le X \le 20$	Number of correct questions

Gaussian

$$X \approx Y \sim \mathcal{N}(10, 5) \qquad (1)$$

1) Without correction:

$$\Pr(Y > 11) = 1 - \Pr(Y \le 11) \qquad (2)$$

$$= 1 - F_Y(11) \qquad (3)$$

$$F_Y(x) = \Pr(Y \le x) \qquad (4)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} \le \frac{X - \mu}{\sigma}\right) \qquad (5)$$

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \qquad (6)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{X - \mu}{\sigma}\right) \qquad (7)$$

$$= 1 - Q\left(\frac{X - \mu}{\sigma}\right) \qquad (8)$$

$$\implies \Pr(Y > 11) = Q\left(\frac{X - \mu}{\sigma}\right) \qquad (9)$$

$$= Q(0.894) \qquad (10)$$

$$\Pr(Y > 11) = 0.1855 \qquad (11)$$

2) With a 0.5 correction:

$$\Pr(Y > 11) = Q\left(\frac{X - \mu + 0.5}{\sigma}\right) \qquad (12)$$

$$= Q(0.67) \qquad (13)$$

$$\implies \Pr(Y > 11) = 0.2511 \qquad (14)$$

Binomial

$$\Pr(X \ge 12) = 1 - \Pr(X < 12) \qquad (15)$$

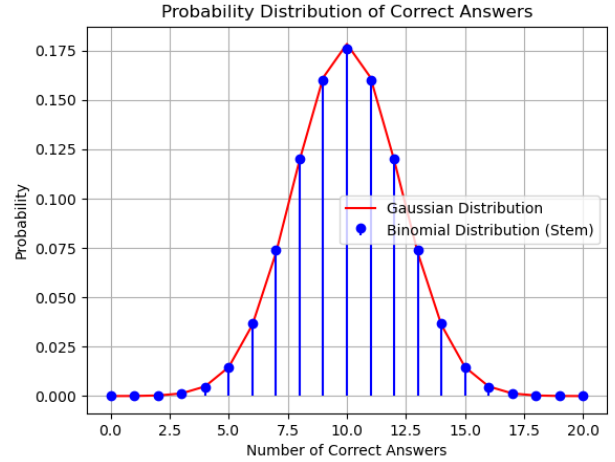$$= \sum_{k=12}^{20} {}^nC_k p^k (1 - p)^{n-k} \qquad (16)$$

$$= 0.2517 \qquad (17)$$

Fig. 2. Binomial vs Gaussian