# Assignment-8

EE22BTECH11012-A.Chhatrapati

**Question 9.3.4)**In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

**Solution:**

TABLE 0
VARIABLES

| Variable | Value | Description |
|---|---|---|
| n | 20 | Number of questions |
| p | 0.5 | probability of question being correct |
| $\mu = np$ | 10 | mean of distribution |
| $\sigma = \sqrt{npq}$ | $\sqrt{5}$ | variance of distribution |
| $X$ | $0 \leq X \leq 20$ | Number of correct questions |

**Gaussian:**

$$Y \sim \mathcal{N}\left(\mu, \sigma^2\right) \tag{1}$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y \leq x) \tag{2}$$
$$= \Pr\left(\frac{Y-\mu}{\sigma} \leq \frac{X-\mu}{\sigma}\right) \tag{3}$$
$$\frac{Y-\mu}{\sigma} \sim \mathcal{N}(0,1) \tag{4}$$
$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{X-\mu}{\sigma}\right) \tag{5}$$
$$= 1 - Q\left(\frac{X-\mu}{\sigma}\right) \tag{6}$$

1) Without correction:

$$\Pr(Y > 11) = 1 - \Pr(Y \leq 11) \tag{7}$$
$$= 1 - F_Y(11) \tag{8}$$
$$\implies \Pr(Y > 11) = Q\left(\frac{X-\mu}{\sigma}\right) \tag{9}$$
$$= Q(0.894) \tag{10}$$
$$\Pr(Y > 11) = 0.1855 \tag{11}$$

2) With a 0.5 correction:

$$\Pr(Y > 11) = Q\left(\frac{X-\mu+0.5}{\sigma}\right) \tag{12}$$
$$= Q(0.67) \tag{13}$$
$$\implies \Pr(Y > 11) = 0.2511 \tag{14}$$

**Binomial:**

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \tag{15}$$
$$= \sum_{k=12}^{20} {}^{n}C_k p^k (1-p)^{n-k} \tag{16}$$
$$= 0.2517 \tag{17}$$

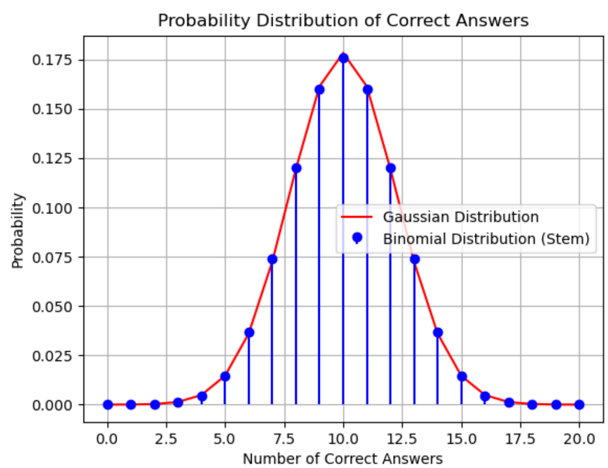


Fig. 2. Binomial vs Gaussian