Assignment-6

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Question 34.2023) Let $X_{n\geq 1}$ be a Markov chain with state space { 1, 2, 3 } and transition probability matrix

$$\begin{pmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

Then $\Pr(X_2 = 1|X_1 = 1, X_3 = 2)$ equals

Solution: Consider transition matrix as:

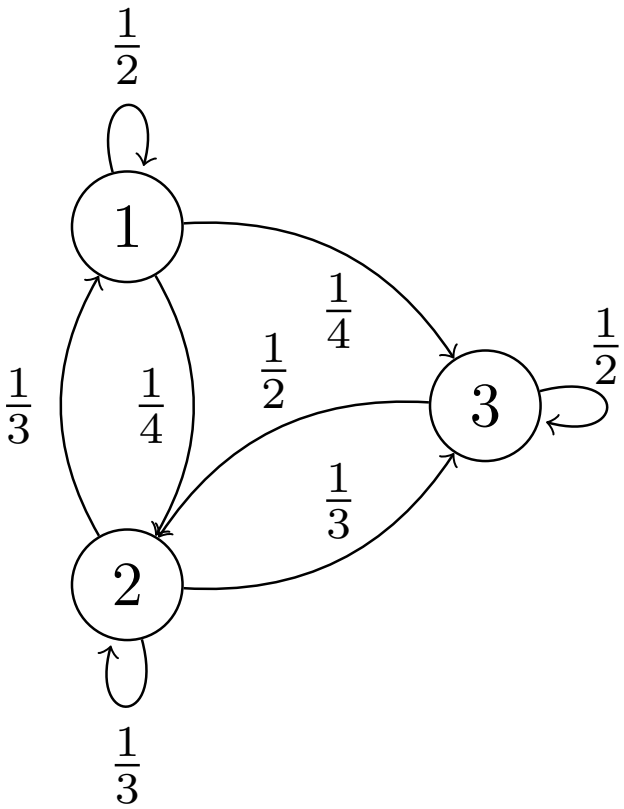

Fig. 0. Markov Chain diagram

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} \tag{1}$$

$$\Pr(X_2 = 1|X_1 = 1, X_3 = 2) = p_{X_2,X_1}(1,1) \tag{2}$$

(by markov's property)

Let's calculate $p_{X_2,X_1}(1,1)$ using transition probability matrix

$$\implies p_{X_2,X_1}(1,1) = p_{11} = \frac{1}{2} \tag{3}$$

$$\implies \Pr(X_2 = 1|X_1 = 1, X_3 = 2) = 0.5 \tag{4}$$