Assignment-8

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Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution:

TABLE 0
VARIABLES

Variable	Value	Description
X	$0 \leq X \leq 20$	Number of correct questions
n	20	Number of questions
p	0.5	probability of question being correct
μ	10	$n \times p$
σ	$\sqrt{5}$	$\sqrt{n \times p \times (1-p)}$

Gaussian

In the table μ and σ are mean and variance respectively.

Central limit theorm:

$$Y \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$Y \sim \mathcal{N}\left(10, \frac{1}{2}\right) \tag{2}$$

$$Z \approx \frac{X-\mu}{\sigma}, \mathcal{N}(0,1) \tag{3}$$

Here, Z is a random variable

The CDF of Y:

$$F_Y(k) = \int_{-\infty}^{x} f(x)\,dx \tag{4}$$

$$= 1 - \int_{x}^{\infty} f(x)\,dx \tag{5}$$

$$= 1 - Q(x) \tag{6}$$

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \tag{7}$$

The Q-function from the Normal-Distribution

$$Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{8}$$

$$\Pr(X \geq 12) = 1 - Q(0.67) \tag{9}$$

$$\Pr(X \geq 12) = 0.2511 \tag{10}$$

Binomial

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \tag{11}$$

$$= \sum_{k=12}^{20} {}^{n}C_k p^k (1-p)^{n-k} \tag{12}$$

$$= 0.2517 \tag{13}$$

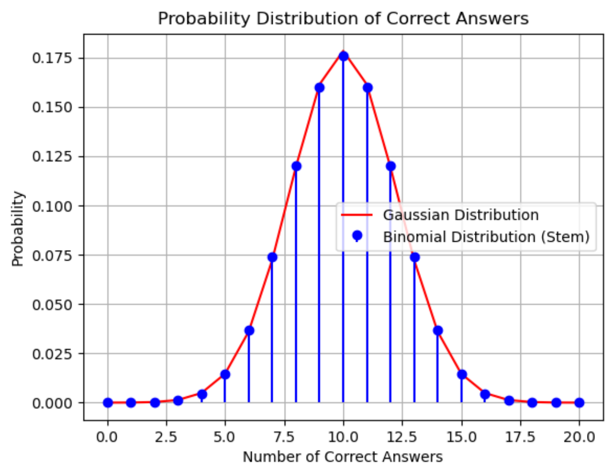

Fig. 0. Binomial vs Gaussian