

Assignment-8

EE22BTECH11012-A.Chhatrapati

Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution:

TABLE 0
VARIABLES

Variable	Value	Description
n	20	Number of questions
p	0.5	probability of question being correct
$\mu = np$	10	mean of distribution
$\sigma = \sqrt{npq}$	$\sqrt{5}$	variance of distribution
X	$0 \leq X \leq 20$	Number of correct questions

Gaussian:

$$Y \sim \mathcal{N}(\mu, \sigma^2) \quad (1)$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y \leq x) \quad (2)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} \leq \frac{X - \mu}{\sigma}\right) \quad (3)$$

$$\frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1) \quad (4)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{X - \mu}{\sigma}\right) \quad (5)$$

$$= 1 - Q\left(\frac{X - \mu}{\sigma}\right) \quad (6)$$

1) Without correction:

$$\Pr(Y > 11) = 1 - \Pr(Y \leq 11) \quad (7)$$

$$= 1 - F_Y(11) \quad (8)$$

$$\Rightarrow \Pr(Y > 11) = Q\left(\frac{X - \mu}{\sigma}\right) \quad (9)$$

$$= Q(0.894) \quad (10)$$

$$\Pr(Y > 11) = 0.1855 \quad (11)$$

2) With a 0.5 correction:

$$\Pr(Y > 11) = Q\left(\frac{X - \mu + 0.5}{\sigma}\right) \quad (12)$$

$$= Q(0.67) \quad (13)$$

$$\Rightarrow \Pr(Y > 11) = 0.2511 \quad (14)$$

Binomial:

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \quad (15)$$

$$= \sum_{k=12}^{20} {}^nC_k p^k (1-p)^{n-k} \quad (16)$$

$$= 0.2517 \quad (17)$$

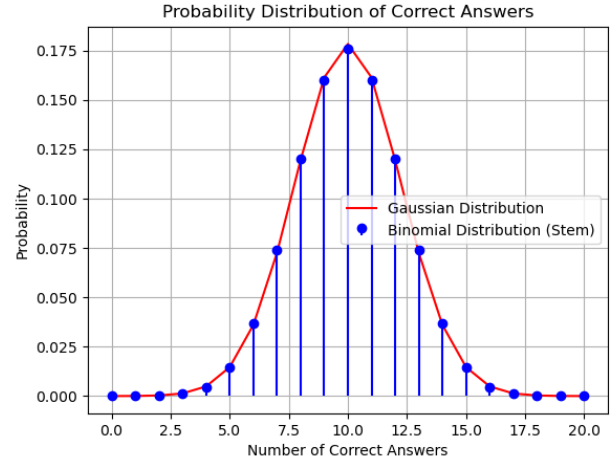


Fig. 2. Binomial vs Gaussian