

Assignment-8

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Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.

Solution:

TABLE 0
RANDOM VARIABLES

Variable	Value	Description
X	$0 \leq X \leq 20$	Number of correct questions

Gaussian

Here $n = 20$ and $p = 0.5$

The mean μ of X

$$\mu = n \times p \quad (1)$$

$$= 10 \quad (2)$$

The variance σ^2 of X

$$\sigma^2 = n \times p \times (1 - p) \quad (3)$$

$$= 5 \quad (4)$$

Let

$$Z \approx \frac{X - \mu}{\sigma} \quad (5)$$

Here, Z is a random variable with $\mathcal{N}(0, 1)$

Normal-Distribution $f(x)$

$$f(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \quad (6)$$

The Q -function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \quad (7)$$

Since

$$X \geq 12 \quad (8)$$

1) With a 0.5 correction:

$$\Pr(X \geq 12) = 1 - \Pr(X < 11.5) \quad (9)$$

$$X < 11.5 \quad (10)$$

$$\Rightarrow Z < \frac{11.5 - \mu}{\sigma} \quad (11)$$

$$Z < \frac{1.5}{\sqrt{5}} \quad (12)$$

$$Z < 0.67082 \quad (13)$$

$$\Pr(X \geq 12) = 1 - \Pr(Z < 0.67) \quad (14)$$

On computation,

$$\Pr(Z < 0.67) = 0.74883 \quad (15)$$

$$\Rightarrow \Pr(X \geq 12) = 0.2511 \quad (16)$$

2) Without correction:

$$X \geq 12 \quad (17)$$

$$Z \geq \frac{12 - \mu}{\sigma} \quad (18)$$

$$Z \geq \frac{2}{\sqrt{5}} \quad (19)$$

$$Z \geq 0.894 \quad (20)$$

$$\Pr(X \geq 12) = \Pr(Z \geq 0.894) \quad (21)$$

$$= 0.1855 \quad (22)$$

Binomial

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \quad (23)$$

$$= \sum_{k=12}^{20} \binom{n}{k} p^k (1-p)^{n-k} \quad (24)$$

$$= 0.2517 \quad (25)$$

The graph

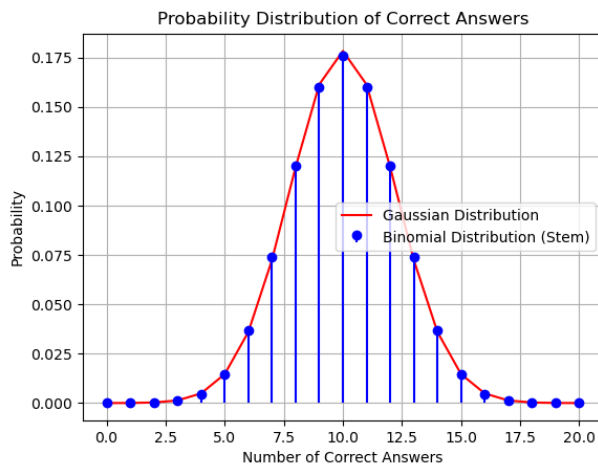


Fig. 2. Binomial vs gaussian