Assignment-8

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Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.
Solution:

TABLE 0
VARIABLES

Variable	Value	Description
X	$0 \le X \le 20$	Number of correct questions
n	20	Number of questions
p	0.5	probability of question being correct
μ	10	$n \times p$
σ	$\sqrt{5}$	$\sqrt{n \times p \times (1-p)}$

Gaussian

In the table μ and σ are mean and variance respectively.
Central limit theorm:

$$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \tag{1}$$

$$Z \approx \frac{X-\mu}{\sigma}, \mathcal{N}(0,1) \tag{2}$$

Here, Z is a random variable
The Q-function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \tag{3}$$

Since

$$X \ge 12 \tag{4}$$

1) With a 0.5 correction:

$$\Pr(X \ge 12) = 1 - \Pr(X < 11.5) \tag{5}$$

$$\implies Z < \frac{11.5-\mu}{\sigma} \tag{6}$$

$$Z < \frac{1.5}{\sqrt{5}} \tag{7}$$

$$Z < 0.67082 \tag{8}$$

$$\Pr(X \ge 12) = 1 - \Pr(Z < 0.67) \tag{9}$$

On compution,

$$\Pr(Z < 0.67) = 0.74883 \tag{10}$$

$$\implies \Pr(X \ge 12) = 0.2511 \tag{11}$$

2) Without correction:

$$X \ge 12 \tag{12}$$

$$Z \ge \frac{12-\mu}{\sigma} \tag{13}$$

$$Z \ge \frac{2}{\sqrt{5}} \tag{14}$$

$$Z \ge 0.894 \tag{15}$$

$$\Pr(X \ge 12) = \Pr(Z \ge 0.894) \tag{16}$$

$$= 0.1855 \tag{17}$$

Binomial

$$\Pr(X \ge 12) = 1 - \Pr(X < 12) \tag{18}$$

$$= \sum_{k=12}^{20} {}^nC_k p^k (1-p)^{n-k} \tag{19}$$

$$= 0.2517 \tag{20}$$

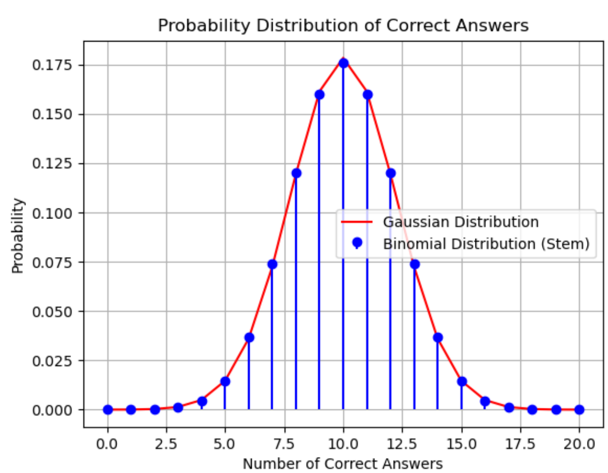

Fig. 2. Binomial vs Gaussian