Assignment-5

EE22BTECH11012-A.Chhatrapati

Question 12.13.3.32) Suppose you have two coins which appear identical in your pocket. You know that one is fair and one is 2-headed. If you take one out, toss it and get a head, what is the probability that it was a fair coin?
Solution:

TABLE 0
Random Variables

Variable	Value	Description
X	1	Fair coin
	0	2-headed coin
Y	1	if output is heads
	0	if output is tails

Given,

$$\Pr(X = 1) = \frac{1}{2}, \tag{1}$$

$$\Pr(X = 0) = \frac{1}{2}, \tag{2}$$

$$\Pr(Y = 1 \mid X = 1) = \frac{1}{2}, \tag{3}$$

$$\Pr(Y = 1 \mid X = 0) = 1, \tag{4}$$

The probability of the coin being fair when the output comes as heads is

$$\Pr(X = 1 \mid Y = 1) = \frac{\Pr(Y = 1 \mid X = 1) \times \Pr(X = 1)}{\sum_{k=0}^{1} \Pr(Y = 1 \mid X = k) \times \Pr(X = k)} \tag{5}$$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{1 \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}} \tag{6}$$

$$= \frac{1}{3} \tag{7}$$