Assignment-8

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Question 9.3.4) In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answer true; if it falls tails, he answer false. Find the probability that he answers at least 12 questions correctly.
Solution:

TABLE 0
RANDOM VARIABLES

Variable	Value	Description
X	$0 \leq X \leq 20$	Number of correct questions

Gaussian
Here $n = 20$ and $p = 0.5$
The mean μ of X

$$\mu = n \times p = 10 \tag{1}$$
$$\tag{2}$$

The variance σ^2 of X

$$\sigma^2 = n \times p \times (1 - p) = 5 \tag{3}$$
$$\tag{4}$$

Let

$$Z \approx \frac{X - \mu}{\sigma} \tag{5}$$

Here, Z is a random variable with $\mathcal{N}(0, 1)$
Normal-Distribution $f(x)$

$$f(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{6}$$

The Q-function from the Normal-Distribution

$$Q(x) = \Pr(Z > x) \tag{7}$$

Since

$$X \geq 12 \tag{8}$$

1) With a 0.5 correction:

$$\Pr(X \geq 12) = 1 - \Pr(X < 11.5) \tag{9}$$
$$X < 11.5 \tag{10}$$
$$\implies Z < \frac{11.5 - \mu}{\sigma} \tag{11}$$
$$Z < \frac{1.5}{\sqrt{5}} \tag{12}$$
$$Z < 0.67082 \tag{13}$$
$$\Pr(X \geq 12) = 1 - \Pr(Z < 0.67) \tag{14}$$

On compution,

$$\Pr(Z < 0.67) = 0.74883 \tag{15}$$
$$\implies \Pr(X \geq 12) = 0.2511 \tag{16}$$

2) Without correction:

$$X \geq 12 \tag{17}$$
$$Z \geq \frac{12 - \mu}{\sigma} \tag{18}$$
$$Z \geq \frac{2}{\sqrt{5}} \tag{19}$$
$$Z \geq 0.894 \tag{20}$$
$$\Pr(X \geq 12) = \Pr(Z \geq 0.894) \tag{21}$$
$$= 0.1855 \tag{22}$$

Binomial

$$\Pr(X \geq 12) = 1 - \Pr(X < 12) \tag{23}$$
$$= \sum_{k=12}^{20} \binom{n}{k} p^k (1 - p)^{n-k} \tag{24}$$
$$= 0.2517 \tag{25}$$

The graph

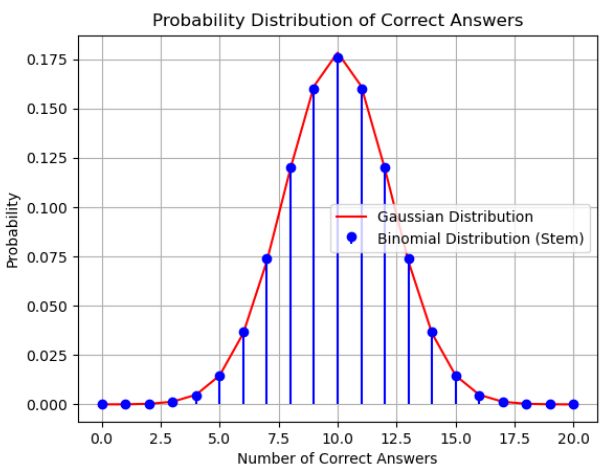

Fig. 2. Binomial vs guassian