Question 10.13.3.23

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Question: Two dice are numbered 1,2,3,4,5,6 and 1,1,2,2,3,3 respectively. They are thrown and the sum of then numbers on them is noted. Find the probability of getting each sum from 2 to 9 seperately.

Solution:

The Z-transform of a die is defined as

$$M_X(z) = z^{-X} = \sum_{k=-\infty}^{\infty} p_X(k) z^{-k} \tag{1}$$

The Z-transform of the first die X_1 is given by

$$M_{X_1}(z) = \frac{1}{6}\sum_{n=1}^{6} z^{-n} = \frac{z^{-1}(1-z^{-6})}{6(1-z^{-1})}, |z| > 1 \tag{2}$$

The Z-transform of the second die X_2 is given by

$$M_{X_2}(z) = \frac{1}{3}\sum_{n=1}^{3} z^{-n} = \frac{z^{-1}(1-z^{-3})}{3(1-z^{-1})}, |z| > 1 \tag{3}$$

The Z-transform of X is given as:

$$M_X(z) = \frac{z^{-1}(1-z^{-6})}{6(1-z^{-1})} \times \frac{z^{-1}(1-z^{-3})}{3(1-z^{-1})} \tag{4}$$

$$M_X(z) = \frac{1}{18}\left[\frac{z^{-2}(1-z^{-3}-z^{-6}-z^{-9})}{(1-z^{-1})^2}\right] \tag{5}$$

We also know that,

$$p_X(n-k) \overset{Z}{\longleftrightarrow} M_X(z) z^{-k}; \tag{6}$$

$$nu(n) \overset{Z}{\longleftrightarrow} \frac{z^{-1}}{(1-z^{-1})^2} \tag{7}$$

Hence, after some algebra, it can be shown that,

$$\frac{1}{18}[n-1u(n-1)-n-4u(n-4) \\ -(n-7)u(n-7)-(n-10)u(n-10)] \\ \overset{Z}{\longleftrightarrow} \\ \frac{1}{18}\left[\frac{z^{-2}1-z^{-3}-z^{-6}-z^{-9}}{(1-z^{-1})^2}\right] \tag{8}$$

where,

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{9}$$

hence,

$$p_X(n) = \frac{1}{18}[n-1u(n-1)-n-4u(n-4)- \\ (n-7)u(n-7)-(n-10)u(n-10)] \tag{10}$$

$$p_X(n) = \begin{cases} 0 & n \leq 1 \\ \frac{n-1}{18} & 2 \leq n \leq 4 \\ \frac{1}{6} & 5 \leq n \leq 7 \\ \frac{10-n}{18} & 8 \leq n \leq 9 \\ 0 & n \geq 10 \end{cases} \tag{11}$$

hence, the probabilities are,

$$p_X(n) = \begin{cases} \frac{1}{18} & n = 2 \\ \frac{1}{9} & n = 3 \\ \frac{1}{6} & n = 4 \\ \frac{1}{6} & n = 5 \\ \frac{1}{6} & n = 6 \\ \frac{1}{6} & n = 7 \\ \frac{1}{9} & n = 8 \\ \frac{1}{18} & n = 9 \end{cases} \tag{12}$$

The experiment of rolling the dice was simulated using Python for 10000 samples.

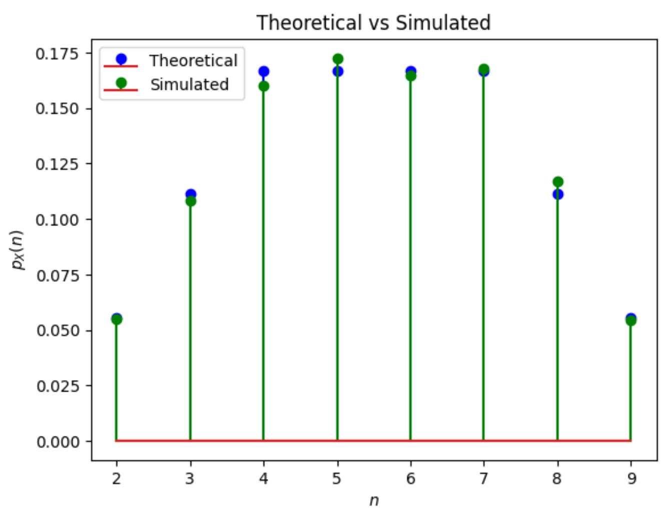

Fig. 0. Plot of $p_X(n)$.Simulations are close to the analysis.