

Probability Assignment 1

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Question : Suppose X is a binomial distribution $B\left(6, \frac{1}{2}\right)$. Show that $X = 3$ is the most likely outcome. (Hint : $P(X = 3)$ is the maximum among all $P(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$)

Solution: Given that, X is a binomial distribution with parameters

$$n = 6 \quad p = \frac{1}{2} \quad (1)$$

the probability of getting exactly k successes in n trials is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \quad (2)$$

From equations in (1), The pmf simplifies as,

$$p_X(k) = {}^nC_k \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k} \quad (3)$$

$$= {}^nC_k \left(\frac{1}{2}\right)^6 \quad (4)$$

We know that nC_k can be written as,

$${}^nC_k = \frac{n!}{(n-k)!k!} \quad (5)$$

If pmf is the greatest, then nC_k is the maximum for $k \in [0, n]$, Therefore It can be said that,

$${}^nC_k \geq {}^nC_{k-1} \quad \text{and} \quad (6)$$

$${}^nC_k \geq {}^nC_{k+1} \quad (7)$$

From (5) and (6), we can state that

$$\frac{n!}{(n-k)!k!} \geq \frac{n!}{(n-k+1)!(k-1)!} \quad (8)$$

$$\Rightarrow \frac{n!}{(n-k)!k!} \geq \frac{n!}{(n-k)!k!} \frac{k}{n-k+1} \quad (9)$$

$$\Rightarrow 1 \geq \frac{k}{n-k+1} \quad (10)$$

$$\therefore k \leq \frac{n+1}{2} \quad (11)$$

From (5) and (7), we can state that

$$\frac{n!}{(n-k)!k!} \geq \frac{n!}{(n-k-1)!(k+1)!} \quad (12)$$

$$\Rightarrow \frac{n!}{(n-k)!k!} \geq \frac{n!}{(n-k)!k!} \frac{n-k}{k+1} \quad (13)$$

$$\Rightarrow 1 \geq \frac{n-k}{k+1} \quad (14)$$

$$\therefore k \geq \frac{n-1}{2} \quad (15)$$

From (11) and (15), we can state that

$$\frac{n-1}{2} \leq k \leq \frac{n+1}{2} \quad (16)$$

We know that, $k \in \mathbb{W}$ and $k \in [0, n]$ and from (16),

$$k = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2} \text{ or } \frac{n-1}{2}, & \text{if } n \text{ is odd} \end{cases} \quad (17)$$

As,

$$n = 6 \quad (18)$$

$$\Rightarrow k = \frac{n}{2} = 3 \quad (19)$$

Hence proved that,

$$X = 3 \quad (20)$$

is the most likely outcome;

$$p_X(3) = {}^6C_3 \left(\frac{1}{2}\right)^6 \quad (21)$$

$$= \frac{5}{16} \quad (22)$$

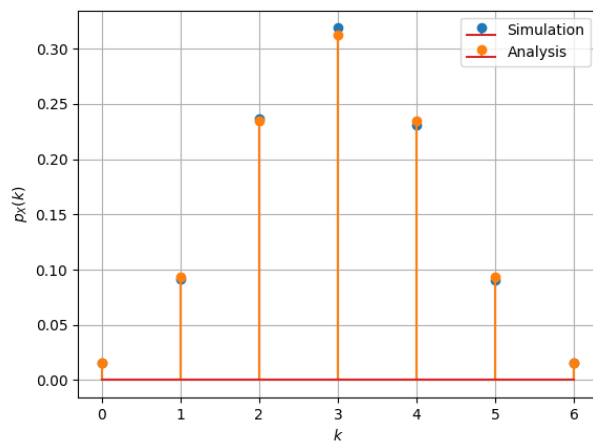


Fig. 0. Figure compares the therotical and simulation output