EE23010 NCERT Exemplar

Vishal A - EE22BTECH11057

Question 63.2022
Consider a channel over which either symbol x_A or symbol x_B is transmitted. Let the output of the channel Y be the input to a maximum likelihood (ML) detector at the receiver. The conditional probability density for Y given x_A and x_B are:

$$f_{Y|x_A}(y) = e^{-(y+1)}u(y+1)$$
$$f_{Y|x_B}(y) = e^{(y-1)}(1-u(y-1))$$

where $u(.)$ is the standard unit step function. the probability of symbol error for this system is (GATE EC 2022)
Solution:
Decision in favour of x_A when

$$f_{Y|x_A}(y) > f_{Y|x_B}(y) \tag{1}$$

Decision in favour of x_B when

$$f_{Y|x_A}(y) < f_{Y|x_B}(y) \tag{2}$$

From the figure,

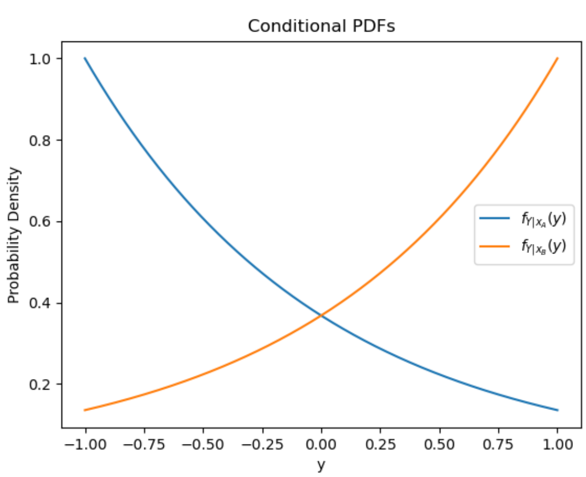

Fig. 1. Conditional pdf

$$\begin{cases} f_{Y|x_A}(y) < f_{Y|x_B}(y) & , y < -1 \\ f_{Y|x_A}(y) > f_{Y|x_B}(y) & , -1 < y < 0 \\ f_{Y|x_A}(y) < f_{Y|x_B}(y) & , 0 < y < 1 \\ f_{Y|x_A}(y) > f_{Y|x_B}(y) & , y > 1 \end{cases} \tag{3}$$

1) When $0 < y < 1$.
   In this interval, when x_A is transmitted, error occurs because the likelihood of observing Y given x_A is lower than the likelihood of observing Y given x_B, Therefore,

$$P_{e_{x_A}} = \int_0^1 f_{Y|x_A}(y)\,dy \tag{4}$$
$$= \int_0^1 e^{-(y+1)}u(y+1)\,dy \tag{5}$$
$$= \int_0^1 e^{-(y+1)}dy \tag{6}$$
$$= e^{-1} - e^{-2} \tag{7}$$
$$P_{e_{x_A}} = 0.23 \tag{8}$$

2) When $-1 < y < 0$.
   In this interval, when x_B is transmitted, error occurs because the likelihood of observing Y given x_A is higher than the likelihood of observing Y given x_B, Therefore,

$$P_{e_{x_B}} = \int_{-1}^0 f_{Y|x_B}(y)\,dy \tag{9}$$
$$= \int_{-1}^0 e^{(y-1)}(1-u(y-1)) \tag{10}$$
$$= \int_{-1}^0 e^{(y-1)} \tag{11}$$
$$= e^{-1} - e^{-2} \tag{12}$$
$$P_{e_{x_B}} = 0.23 \tag{13}$$

3) When $y < -1$ and $y > 1$.
   There are no errors in these intevals as the ML detectors can more reliably to make a decison. Therefore,

$$P_e = 0 \tag{14}$$

Hence, the total probability of error for this system can be given as,

$$P_e = \Pr(x_A)P_{e_{x_A}} + \Pr(x_B)P_{e_{x_B}} \tag{15}$$
$$= 0.23 \times (\Pr(x_A) + \Pr(x_B)) \tag{16}$$
$$= 0.23 \tag{17}$$