Assignment

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Question 9.3.9

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

1) none
2) not more than one
3) more than one
4) at least one

will fuse after 150 days of use.

Solution:

Guassian :

let Y be a gaussian Random variable

Parameter	Value	Description
X	0,1,2,3,4,5	No. Of bulbs fused
n	5	Total no. Of bulbs
p	0.05	bulb fusing
q	0.95	not fusing
$\mu = np$	0.25	Mean of Binomial Distribution
$\sigma^2 = npq$	0.2375	Varience of binomial Distribution

TABLE I
RANDOM VARIABLE AND PARAMETER

$$Y \sim N(\mu, \sigma) \tag{1}$$
$$\sim N(1.25, 0.9375) \tag{2}$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$p_Y(x) \approx \Pr(x - 0.5 < Y < x + 0.5) \tag{3}$$
$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \tag{4}$$
$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \tag{5}$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y < x) \tag{6}$$

$$= \Pr\left(\frac{Y-\mu}{\sigma} < \frac{x-\mu}{\sigma}\right) \tag{7}$$

$$\implies \frac{Y-\mu}{\sigma} \sim N(0,1) \tag{8}$$

$$= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) \tag{9}$$

$$= \begin{cases} 1 - Q\left(\frac{x-\mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu-x}{\sigma}\right) & x < \mu \end{cases} \tag{10}$$

Then probability in terms of Q funtion is

$$\implies p_Y(x) \approx Q\left(\frac{(x-0.5)-\mu}{\sigma}\right) - Q\left(\frac{(x+0.5)-\mu}{\sigma}\right) \tag{11}$$

Binomial :

$$\Pr(X = k) = {}^nC_k p^k (1-p)^{n-k} \tag{12}$$

$$= {}^5C_k (0.05)^k (0.95)^{5-k} \tag{13}$$

CDF of X

$$F_X(k) = \Pr(X \leq k) \tag{14}$$

$$= \sum_{i=0}^{k} {}^{10}C_i (0.05)^i (0.95)^{5-i} \tag{15}$$

The solution
The graph

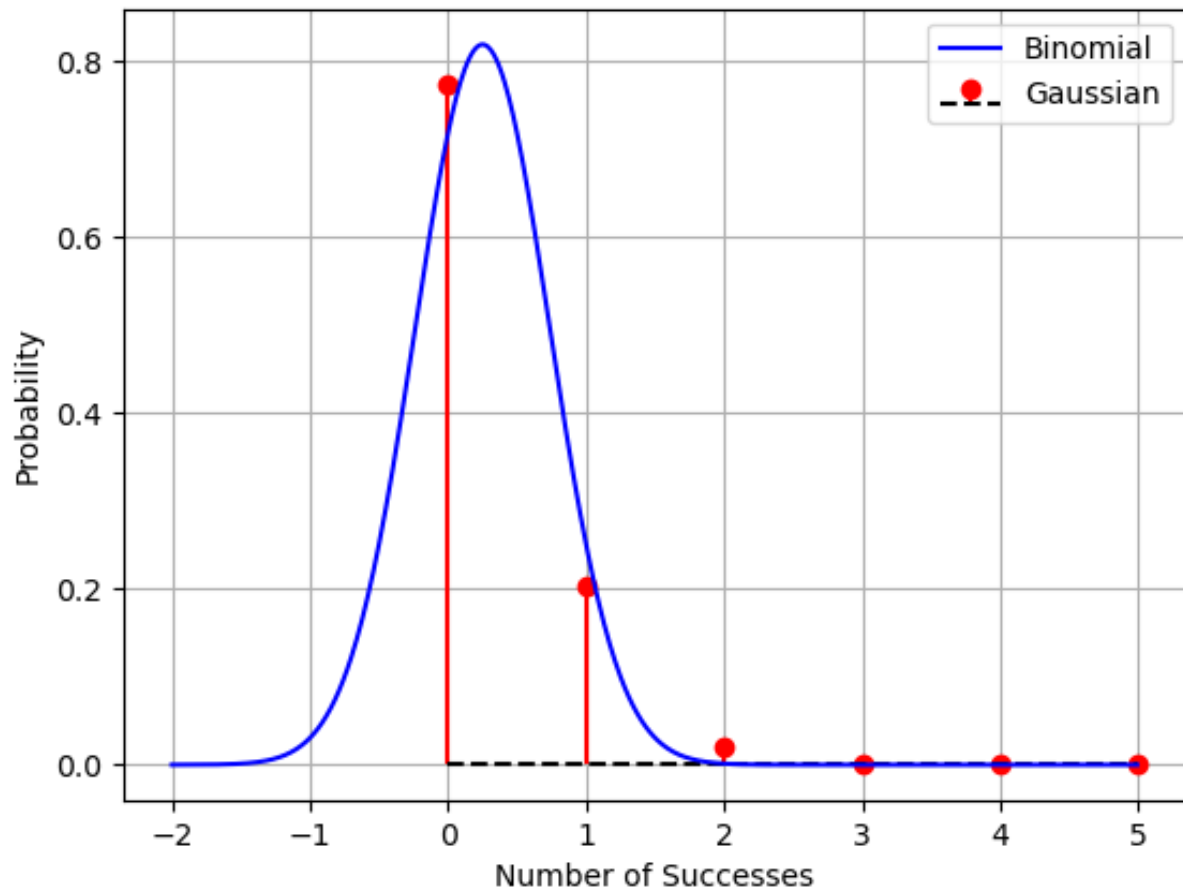| $\Pr(X = x)$	in term of Q	Numercal value	Binomial solution
$\Pr(X = 0)$	$Q(1.5389) - Q(0.512)$	0.6960	0.773
$\Pr(X \leq 1)$	$Q(1.5896)$	0.9948	0.9774075
$1 - \Pr(X = 0)$	$1 - (Q(1.5389) - Q(0.512))$	0.304	0.227
$1 - \Pr(X \leq 1)$	$1 - Q(1.5896)$	0.006	0.0226

TABLE II
RANDOM VARIABLE AND PARAMETER

Fig. 1. Binomial vs guassian