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EE23010 Assignment

Sayyam Palrecha* EE22BTECH11047

Question 10.13.3.19

Two dice are thrown at the same time. Find the probability of getting

- (i) same number on both dice.
- (ii) different numbers on both dice.

Solution: Let the random variables:

parameters	value	description
X	$1 \le X \le 6$	outcome of the first die
Y	$1 \le Y \le 6$	outcome of the second die

Consider a random variable W:

$$W = X - Y \tag{1}$$

W can take values ranging from {-5 to 5}.

$$p_X(k) = \begin{cases} \frac{1}{6}, & 1 \le k \le 6\\ 0, & \text{otherwise} \end{cases}$$
 (2)

$$p_X(k) = p_Y(k) \tag{3}$$

PMF of W using z-transform: applying the z-transform on both the sides

$$z\{W\} = z\{X - Y\} \tag{4}$$

$$M_W(z) = M_{X-Y}(z) \tag{5}$$

Using the expectation operator:

$$E[z^{-W}] = E[z^{-X+Y}] (6)$$

$$= E[z^{-X}] \cdot E[z^Y] \tag{7}$$

$$= \left(\sum_{i=1}^{6} p_X(i) \cdot z^{-i}\right) \cdot \left(\sum_{j=1}^{6} p_Y(j) \cdot z^j\right)$$
 (8)

Extracting the PMF by considering the defenition of z-transform

$$M_W(z) = p_W(0) + p_W(1)z + \dots + p_W(k)z^k + \dots$$
(9)
= $\frac{1}{36} (z^{-1} + \dots + z^{-6}) \cdot (z^1 + \dots + z^6)$ (10)
= $\frac{1}{36} (z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6$
+ $5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5)$ (11)

defined for all the values of $-5 \le k \le 5$

$$p_W(k) = \frac{1}{k!} \left(\frac{d^{|k|}}{dz^{|k|}} M_W(z) \right)_{z=0}$$
 (12)

$$= \frac{1}{36k!} \left(\frac{d^{|k|}}{dz^{|k|}} \left(z^{-5} + \dots + z^5 \right) \right)_{z=0}$$
 (13)

$$= \frac{1}{36k!} \left(\frac{d^{|k|}}{dz^{|k|}} \left(\sum_{m=-5}^{5} (6 - |m|) z^m \right) \right)_{z=0}$$
 (14)

$$= \frac{1}{36k!} (6 - |k|) \frac{d^{|k|}}{dz^{|k|}} z^k \tag{15}$$

$$= \frac{1}{36k!} (6 - |k|) k! \tag{16}$$

$$=\frac{1}{36}(6-|k|)\tag{17}$$

(i) Finding the probability for W = 0

From the result (17)

$$p_W(0) = \frac{1}{36} (6 - |0|) \tag{18}$$

$$\implies \Pr(W = 0) = \frac{1}{36}(6) \tag{19}$$

$$=\frac{1}{6}\tag{20}$$

(ii) Finding the probability for $W \neq 0$

$$Pr(W \neq 0) = 1 - Pr(W = 0)$$
 (21)

$$=1-\frac{1}{6}$$
 (22)

$$=\frac{5}{6}\tag{23}$$

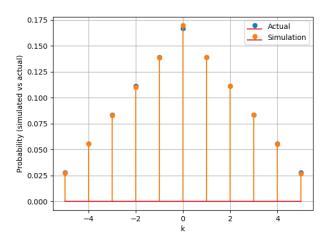


Fig. 0. PMF analysis of W $(p_W(k))$