Probability and Random Processes

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Q)One million random numbers are generated from a statistically stationary process with a Gaussian distribution with mean zero and standard deviation σ_0.

The σ_0 is estimated by randomly drawing out 10,000 number of samples (x_n). The estimates $\hat{\sigma}_1^2$ and $\hat{\sigma}_2^2$ are computed in the following two ways:

$$\hat{\sigma}_1^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2$$

$$\hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2$$

Which of the following statements is true?

1) $E(\hat{\sigma}_2^2) = \sigma_0^2$
2) $E(\hat{\sigma}_2) = \sigma_0$
3) $E(\hat{\sigma}_1^2) = \sigma_0^2$
4) $E(\hat{\sigma}_1) = E(\hat{\sigma}_2)$

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Solution:

$$\hat{\sigma}_1^2 = \frac{1}{10000} \sum_{n=1}^{10000} x_n^2 \tag{1}$$

$$\hat{\sigma}_2^2 = \frac{1}{9999} \sum_{n=1}^{10000} x_n^2 \tag{2}$$

We know,

$$\text{Var}(x_n) = E(x_n^2) - [E(x_n)]^2 \tag{3}$$

$$= E(x_n^2) - 0 \tag{4}$$

$$= E(x_n^2) \tag{5}$$

$$= \sigma_0^2 \tag{6}$$

$$\implies E(x_n^2) = \sigma_0^2 \tag{7}$$

$$E[\hat{\sigma}_1^2] = \frac{1}{10000} \sum_{n=1}^{10000} E[x_n^2] \tag{8}$$

$$E[\hat{\sigma}_2^2] = \frac{1}{9999} \sum_{n=1}^{10000} E[x_n^2] \tag{9}$$

Using Equation 7, we get:

$$E[\hat{\sigma}_1^2] = \sigma_0^2 \tag{10}$$

$$E[\hat{\sigma}_2^2] = \frac{10000}{9999} \sigma_0^2 \tag{11}$$

Hence, the correct answer is 3