

# Assignment 2

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## **Problem 11.16.3.5(exemplar):-**

A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find  $P(G)$ , where  $G$  is the event that a number greater than 3 occurs on a single roll of the die.

## **Solution:-**

Parameter	Value	Description
$X$	$\{1,2,3,4,5,6\}$	Number obtained on the die

TABLE 0  
PARAMETERS AND THEIR DESCRIPTION

Let  $m$  be any natural number such that  $m \in \{1, 2, 3\}$ .

$$\Pr(X = k) = \begin{cases} 2p, & \text{if } k = 2m - 1 \\ p, & \text{if } k = 2m \end{cases} \quad (1)$$

Since  $1 \leq X \leq 6$ ,

$$\sum_{i=1}^6 \Pr(X = i) = 1 \quad (2)$$

$$\Rightarrow \sum_{i=1}^3 \Pr(X = 2i - 1) + \sum_{i=1}^3 \Pr(X = 2i) = 1 \quad (3)$$

$$\Rightarrow \sum_{i=1}^3 2p + \sum_{i=1}^3 p = 1 \quad (4)$$

$$\Rightarrow 6p + 3p = 1 \quad (5)$$

$$\Rightarrow 9p = 1 \quad (6)$$

$$\Rightarrow p = \frac{1}{9} \quad (7)$$

Let  $F_X(k)$  be the cumulative distribution function such that,

$$F_X(k) = \Pr(X \leq k) \quad (8)$$

If  $k = 2m - 1$ ,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \quad (9)$$

$$= \sum_{i=1}^m \Pr(X = 2i - 1) + \sum_{i=1}^{m-1} \Pr(X = 2i) \quad (10)$$

$$= \sum_{i=1}^m 2\left(\frac{1}{9}\right) + \sum_{i=1}^{m-1} \left(\frac{1}{9}\right) \quad (11)$$

$$= \frac{2m}{9} + \frac{m-1}{9} \quad (12)$$

$$= \frac{3m-1}{9} \quad (13)$$

$$= \frac{3k+1}{18} \quad (14)$$

If  $k = 2m$ ,

$$F_X(k) = \sum_{i=1}^{2m-1} \Pr(X = i) \quad (15)$$

$$= \sum_{i=1}^m \Pr(X = 2i - 1) + \sum_{i=1}^m \Pr(X = 2i) \quad (16)$$

$$= \sum_{i=1}^m 2\left(\frac{1}{9}\right) + \sum_{i=1}^m \left(\frac{1}{9}\right) \quad (17)$$

$$= \frac{2m}{9} + \frac{m}{9} \quad (18)$$

$$= \frac{m}{3} \quad (19)$$

$$= \frac{k}{6} \quad (20)$$

So,

$$F_X(k) = \begin{cases} \frac{3k+1}{18}, & \text{if } k = 2m - 1 \\ \frac{k}{6}, & \text{if } k = 2m \end{cases} \quad (21)$$

$$\Pr(G) = \Pr(X > 3) \quad (22)$$

$$= F_X(6) - F_X(3) \quad (23)$$

$$= 1 - \frac{3(3) + 1}{18} \quad (24)$$

$$= 1 - \frac{5}{9} \quad (25)$$

$$= \frac{4}{9} \quad (26)$$