

Assignment

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Question : Two natural numbers r, s are drawn one at a time, without replacement from the set $S = 1, 2, 3, \dots, n$. Find $P[r \leq p | s \leq p]$

Solution: There are two conditions,

1) Case 1: s is chosen first

Let X, Y and p be random variables as defined in Table I,

RV	Value	Description
X	$\{1, 2, 3, \dots, n\}$	First number (s)
Y	$\{1, 2, 3, \dots, n\} \setminus \{s\}$	Second number (r)
p	\mathbb{Z}	number to be compared

TABLE I
RANDOM VARIABLE X DECLARATION

We need to find the value of

$$\Pr(Y \leq p | X \leq p) = \frac{\Pr(Y \leq p, X \leq p)}{\Pr(X \leq p)} \quad (1)$$

There are 3 cases for the value of p ;

a) if $p < 1$: This case is never possible as $X, Y \geq 1$

b) if $1 \leq p \leq n$: Then we can say that,

$$\Pr(Y \leq p, X \leq p) = \frac{p(p-1)}{n(n-1)}, \quad (2)$$

$$\Pr(X \leq p) = \frac{p}{n} \quad (3)$$

From (2) and (3):

$$\Pr(Y \leq p | X \leq p) = \frac{\Pr(Y \leq p, X \leq p)}{\Pr(X \leq p)} \quad (4)$$

$$= \frac{\frac{p(p-1)}{n(n-1)}}{\frac{p}{n}} = \frac{p-1}{n-1} \quad (5)$$

c) if $p > n$: Then we can say that,

$$\Pr(Y \leq p, X \leq p) = 1, \quad (6)$$

$$\Pr(X \leq p) = 1 \quad (7)$$

From (6) and (7):

$$\Pr(Y \leq p | X \leq p) = \frac{\Pr(Y \leq p, X \leq p)}{\Pr(X \leq p)} \quad (8)$$

$$= 1 \quad (9)$$

2) Case 2: r is chosen first

Let X, Y and p be random variables as defined in Table II,

We need to find the value of

$$\Pr(X \leq p | Y \leq p) = \frac{\Pr(X \leq p, Y \leq p)}{\Pr(Y \leq p)} \quad (10)$$

There are 3 cases for the value of p ;

a) if $p < 1$: This case is never possible as $X, Y \geq 1$

RV	Value	Description
X	$\{1, 2, 3, \dots, n\}$	First number (r)
Y	$\{1, 2, 3, \dots, n\} \setminus \{s\}$	Second number (s)
p	\mathbb{Z}	number to be compared

TABLE II
RANDOM VARIABLE X DECLARATION

b) if $1 \leq p \leq n$: Then we can say that,

$$\Pr(X \leq p, Y \leq p) = \frac{p(p-1)}{n(n-1)}, \quad (11)$$

$$\Pr(Y \leq p) = \frac{p-1}{n-1} \quad (12)$$

From (11) and (12):

$$\Pr(X \leq p | Y \leq p) = \frac{\Pr(X \leq p, Y \leq p)}{\Pr(Y \leq p)} \quad (13)$$

$$= \frac{\frac{p(p-1)}{n(n-1)}}{\frac{p-1}{n-1}} = \frac{p}{n} \quad (14)$$

c) if $p > n$: Then we can say that,

$$\Pr(X \leq p, Y \leq p) = 1, \quad (15)$$

$$\Pr(Y \leq p) = 1 \quad (16)$$

From (15) and (16):

$$\Pr(X \leq p | Y \leq p) = \frac{\Pr(X \leq p, Y \leq p)}{\Pr(Y \leq p)} \quad (17)$$

$$= 1 \quad (18)$$