Probability and Random Processes

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Q)A die is loaded in such a way that each odd number is twice as likely to occur as each even number. Find P(G), where G is the event that a number greater than 3 occurs on a single roll of the die.

Solution: Let the random variable be X and p is the probability that rolled number is odd.

The PMF of X in terms of p is given by:

$$p_X(k) = \begin{cases} 2p & \text{if } k \text{ is odd} \\ p & \text{if } k \text{ is even} \end{cases}$$

To find Probability that rolled number is odd: We know,

$$\sum_{k=1}^{6} p_X(k) = 1 \tag{1}$$

$$\Rightarrow \quad p + \frac{p}{2} + p + \frac{p}{2} + p + \frac{p}{2} = 1 \tag{2}$$

$$\Rightarrow \quad \frac{9p}{2} = 1 \tag{3}$$

$$\Rightarrow \quad p = \frac{2}{9} \tag{4}$$

$$p_X(k) = \begin{cases} \frac{2}{9} & \text{if } k \text{ is odd} \\ \frac{1}{9} & \text{if } k \text{ is even} \end{cases} \tag{5}$$

The cdf of X is given by

$$F_X(k) = \Pr(X \le k) \tag{6}$$

The cdf of X in terms of k is given by:

$$F_X(k) = \begin{cases} \frac{k+1}{9} & \text{if } k \text{ is odd} \\ \frac{k}{6} & \text{if } k \text{ is even} \end{cases} \tag{7}$$

1) We require $\Pr(X > 3)$.

$$\Pr(X > 3) = 1 - \Pr(X \le 3) \tag{8}$$

$$= 1 - F_X(3) \tag{9}$$

$$= 1 - \frac{5}{9} \tag{10}$$

$$= \frac{4}{9} \tag{11}$$

$$= 0.44 \tag{12}$$

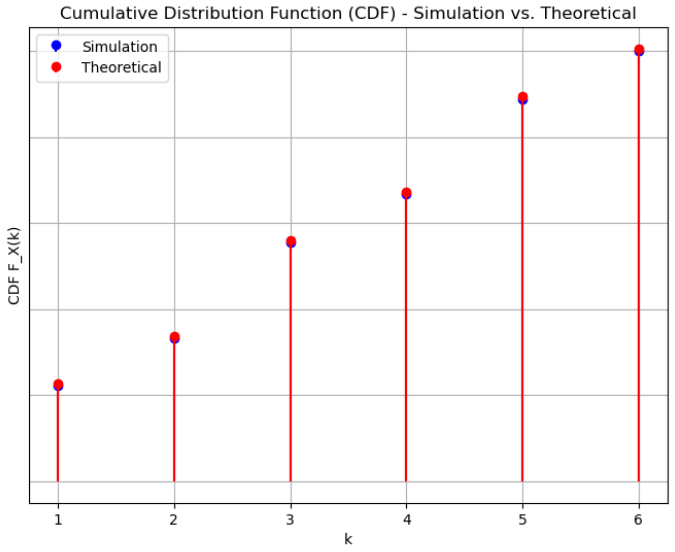

Fig. 1: CDF plot - Simulation and theoretical.