

QUESTION : 12.13.3.7

ROLL NO:EE22BTECH11027

NAME: KATARI SIRI VARSHINI

12.13.3.7.A and B are two events such that 4)
 $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$ and $\Pr(AB) = \frac{1}{4}$.

Find:

- i $\Pr(A|B)$
- ii $\Pr(B|A)$
- iii $\Pr(A'|B)$
- iv $\Pr(A'|B')$

Solution: : Given, $\Pr(A) = \frac{1}{2}$, $\Pr(B) = \frac{1}{3}$ and $\Pr(AB) = \frac{1}{4}$. Then,

$$\begin{aligned}\Pr(A') &= 1 - \Pr(A) = \frac{1}{2} \\ \Pr(B') &= 1 - \Pr(B) = \frac{2}{3}\end{aligned}\quad (1)$$

$$\begin{aligned}\Pr(A + B) &= \Pr(A) + \Pr(B) - \Pr(AB) \\ &= \frac{7}{12}\end{aligned}$$

1)

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} = \frac{3}{4} \quad (2)$$

2)

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} = \frac{1}{2} \quad (3)$$

3)

$$\Pr(A'|B) = \frac{\Pr(A'B)}{\Pr(B)}$$

We have,

$$B = AB + A'B$$

Applying probabilities on both sides,

$$\begin{aligned}\Pr(B) &= \Pr(AB) + \Pr(A'B) \\ \Pr(A'B) &= \Pr(B) - \Pr(AB) \\ &= \frac{1}{12} \\ \therefore \Pr(A'|B) &= \frac{1}{4}\end{aligned}\quad (4)$$

$$\begin{aligned}\Pr(A'|B') &= \frac{\Pr(A'B')}{\Pr(B')} \\ \therefore \Pr(A'B') &= \Pr(A + B)' \\ &= 1 - \Pr(A + B)\end{aligned}\quad (5)$$

using the conclusions from (1)

$$\begin{aligned}\Pr(A'B') &= \frac{5}{12} \\ \therefore \Pr(A'|B') &= \frac{5}{8}\end{aligned}\quad (6)$$