Q:Without repetition of the numbers, four digit numbers are formed with the numbers 0,2,3,5.The probability of such a number divisible by 5 is

(A)$\frac{1}{5}$ (B)$\frac{4}{5}$ (C)$\frac{1}{30}$ (D)$\frac{5}{9}$

Solution: Number of four digit numbers possible are $3 \times 3 \times 2 \times 1 = 18$ because zero cannot be in the first place.

Random Variable	Values	Description
X	1	first digit
	2	second digit
	3	third digit
	4	fourth digit
Y	0	0 as digit
	1	5 as digit

TABLE 0
TABLE 1

As number of four digit numbers with fourth digit being 0 is $3 \times 2 \times 1 \times 1 = 6$

$$p\left(Y = 0, X = 4\right) = \frac{3 \times 2 \times 1 \times 1}{3 \times 3 \times 2 \times 1 \times 1} \tag{1}$$

$$= \frac{1}{3} \tag{2}$$

$$\tag{3}$$

As number of four digit numbers with fourth digit being 5 and second digit being 0 is $2 \times 1 \times 1 \times 1 = 2$

$$p\left(Y = 1, X = 4 | Y = 0, X = 2\right) = \frac{2 \times 1 \times 1 \times 1}{3 \times 3 \times 2 \times 1} \tag{4}$$

$$= \frac{1}{9} \tag{5}$$

$$\tag{6}$$

As number of four digit numbers with fourth digit being 5 and third digit being 0 is $2 \times 1 \times 1 \times 1 = 2$

$$p\left(Y = 1, X = 4 | Y = 0, X = 3\right) = \frac{2 \times 1 \times 1 \times 1}{3 \times 3 \times 2 \times 1} \tag{7}$$

$$= \frac{1}{9} \tag{8}$$

$$\tag{9}$$

Probability of forming four digit number divisible by 5, without repetition,

$$p = p\left(Y = 0, X = 4\right) + p\left(Y = 1, X = 4 | Y = 0, X = 2\right) + p\left(Y = 1, X = 4 | Y = 0, X = 3\right) \tag{10}$$

$$= \frac{5}{9} \tag{11}$$

Hence, option (D)$\frac{5}{9}$ is the correct option.