

# Assignment

## EE23010: Probability and Random Processes

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Question: Let  $X$  be a random variable with probability density function

$$f(x; \lambda) = \begin{cases} \frac{1}{\lambda} e^{-\frac{x}{\lambda}} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

where  $\lambda > 0$  is an unknown parameter. Let  $Y_1, Y_2, \dots, Y_n$  be a random sample of size  $n$  from a population having the same distribution as  $X^2$ . If

$$\bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i \quad (2)$$

then which of the following statements is true?

- 1)  $\sqrt{\frac{\bar{Y}}{2}}$  is a method of moments estimator of  $\lambda$
- 2)  $\sqrt{\bar{Y}}$  is a method of moments estimator of  $\lambda$
- 3)  $\frac{1}{2} \sqrt{\bar{Y}}$  is a method of moments estimator of  $\lambda$
- 4)  $2\sqrt{\bar{Y}}$  is a method of moments estimator of  $\lambda$

(GATE ST 2023)

**Solution:**

- 1) Using PDF in (1) we need to find an estimator for the unknown parameter  $\lambda$  in terms of sample mean  $\bar{Y}$   
we know  $Y_i = X_i^2$  then,

$$E(Y_i) = E(X_i^2) \quad (3)$$

$$= \int_0^{\infty} x^2 \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \quad (4)$$

$$= 2\lambda^2 \quad (5)$$

Method of moment is defined by (2) which gives,

$$\bar{Y} = E(Y_i) \quad (6)$$

$$= 2\lambda^2 \quad (7)$$

where

$$\lambda = \sqrt{\frac{\bar{Y}}{2}} \quad (8)$$

$\therefore$  Option (1) is correct.

- 2) The simulation steps to estimate  $\lambda$  using method of moment estimator in python.
  - a) Generate a random value of  $\lambda$  within the specified range using **np.random.uniform(1,10)**
  - b) Use the generated  $\lambda$  to create a random sample of  $X$  values following the given PDF using **np.random.exponential()**
  - c) Then, generate  $Y$  as  $Y = X^2$
  - d) calculate the mean ( $\bar{Y}$ ) as **np.mean(Y)**
  - e) Hence, the estimated value of  $\lambda$  is **np.sqrt( $\frac{\bar{Y}}{2}$ )**

Graph of simulated CDF vs Theoretical CDF

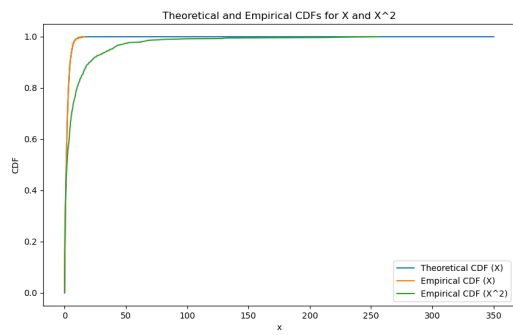


Fig. 2. Figure1