Assignment

EE23010: Probability and Random Processes
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Question: Let X be a random variable with probability density function

$$f(x;\lambda) = \begin{cases} \frac{1}{\lambda}e^{-\frac{x}{\lambda}} & \text{if} x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $\lambda > 0$ is an unknown parameter. Let $Y_1, Y_2, ..., Y_n$ be a random sample of size n from a population having the same distribution as X^2.If

$$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \tag{2}$$

then which of the following statements is true?

1) $\sqrt{\frac{\bar{Y}}{2}}$is a method of moments estimator ofλ
2) $\sqrt{\bar{Y}}$is a method of moments estimator ofλ
3) $\frac{1}{2}\sqrt{\bar{Y}}$is a method of moments estimator of λ
4) $2\sqrt{\bar{Y}}$is a method of moments estimator ofλ

(GATE ST 2023)

Solution:

1) Using PDF in (**??**) we need to find an estimator for the unknown parameter λ in terms of sample mean $\bar{Y}$
we know $Y_i = X_i^2$ then,

$$E(Y_i) = E(X_i^2) \tag{3}$$
$$= \int_0^\infty x^2 \frac{1}{\lambda} e^{-\frac{x}{\lambda}} \tag{4}$$
$$= 2\lambda^2 \tag{5}$$

Method of moment is defined by (**??**) which gives,

$$\bar{Y} = E(Y_i) \tag{6}$$
$$= 2\lambda^2 \tag{7}$$

where

$$\lambda = \sqrt{\frac{\bar{Y}}{2}} \tag{8}$$

$\therefore$ Option (**??**) is correct.

2) The simulation steps to estimate λ using method of moment estimator in python.
   a) Generate a random value of λ within the specified range using **np.random.uniform(1,10)**
   b) Use the generated λ to create a random sample of X values following the given PDF using **np.random.exponential()**
   c) Then, generate Y as $Y = X^2$
   d) calculate the mean ($\bar{Y}$) as **np.mean(Y)**
   e) Hence, the estimated value of λ is **np.sqrt($\frac{\bar{Y}}{2}$)**

Graph of simulated CDF vs Theoretical CDF

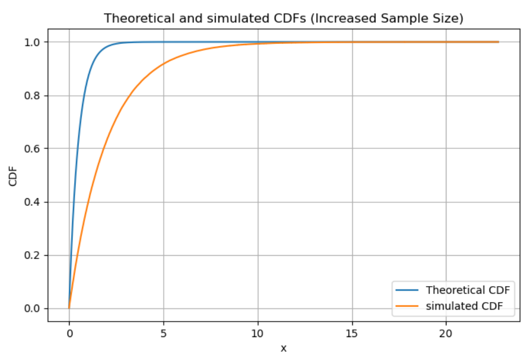

Fig. 2. Figure1