

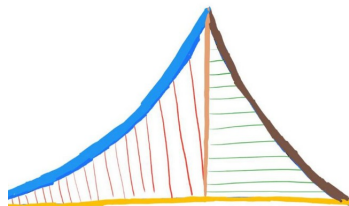
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# GATE PROBABILITY

## Through Simulations

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# Introduction

This book solves probability problems in GATE question papers.



# Chapter 1

## Axioms





## Chapter 2

# Distributions

1. Let  $\phi(\cdot)$  denote the cumulative distribution function of a standard normal random variable. If the random variable  $X$  has the cumulative distribution function

$$F(x) = \begin{cases} \phi(x), & x < -1 \\ \phi(x+1), & x \geq -1 \end{cases} \quad (2.1)$$

then which one of the following statements is true?

- (a)  $P(X \leq -1) = \frac{1}{2}$
- (b)  $P(X = -1) = \frac{1}{2}$
- (c)  $P(X < -1) = \frac{1}{2}$
- (d)  $P(X \leq 0) = \frac{1}{2}$

(GATE ST 2023)

**Solution: Gaussian**

Q function is defined

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{u^2}{2}} du \quad (2.2)$$

From question and (2.2);

$$F_X(x) = \begin{cases} Q(-x), & x < -1 \\ 1 - Q(x+1), & x \geq -1 \end{cases} \quad (2.3)$$

From (2.3);

(a)

$$\Pr(X \leq -1) = F_X(-1) = 1 - Q(0) \quad (2.4)$$

$$= 0.5 \quad (2.5)$$

So Option A i.e.,  $P(X < -1) = \frac{1}{2}$  is correct

(b) The pdf of X can be defined in terms of cdf as

$$\Pr(X = b) = F_X(b) - \lim_{x \rightarrow b^-} F_X(x) \quad (2.6)$$

From (2.6);

$$\Pr(X = -1) = F_X(-1) - \lim_{x \rightarrow -1^-} F_X(x) \quad (2.7)$$

$$= 1 - Q(0) - Q(-(-1)) \quad (2.8)$$

$$= 0.341 \quad (2.9)$$

So Option B i.e.,  $P(X = -1) = \frac{1}{2}$  is incorrect

(c)

$$\Pr (X < -1) = \lim_{x \rightarrow -1^-} F_X(x) = F_X(-1) \quad (2.10)$$

$$= Q(-(-1)) \quad (2.11)$$

$$= 0.159 \quad (2.12)$$

So Option C i.e.,  $P(X < -1) = \frac{1}{2}$  is incorrect

(d)

$$\Pr (X \leq 0) = F_X(0) = 1 - Q(1) \quad (2.13)$$

$$= 0.8413 \quad (2.14)$$

So Option D i.e.,  $P(X \leq 0) = \frac{1}{2}$  is incorrect

Gaussian CDF plot of X is given in fig2.1

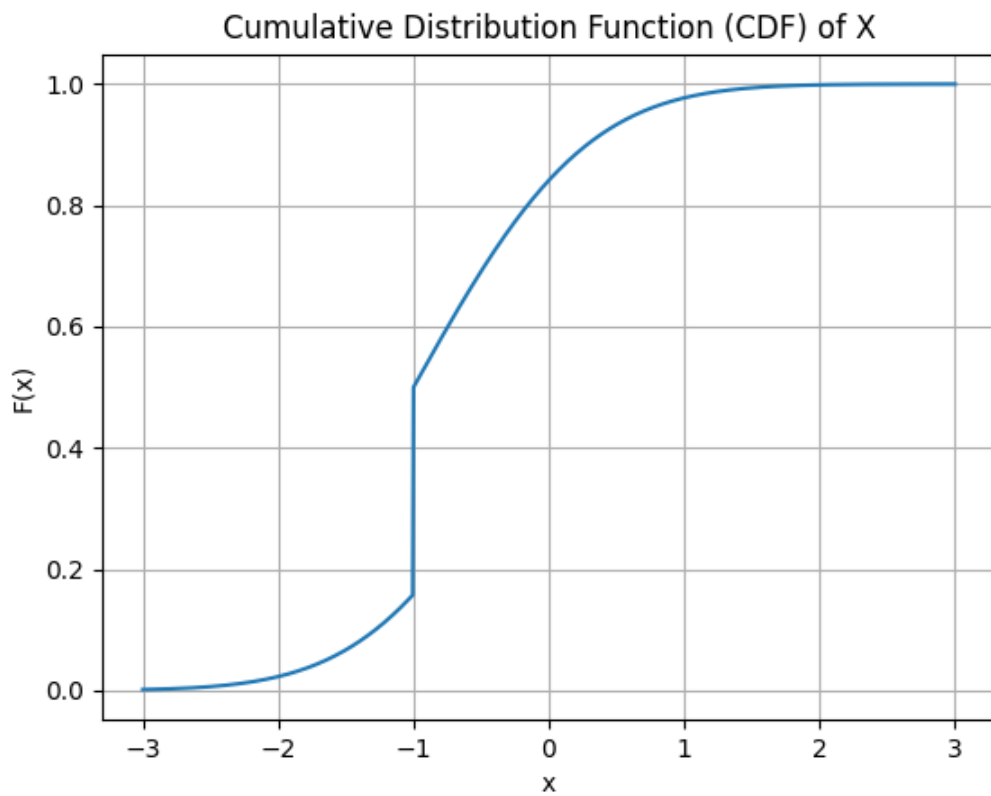


Figure 2.1:

2. Let  $X$  be a random variable with the probability density function  $f(x)$  such that

$$f(x) = \begin{cases} \frac{1}{2\sqrt{3}}, & -\sqrt{3} \leq x \leq \sqrt{3} \\ 0, & \text{otherwise} \end{cases} \quad (2.15)$$

Then the variance of  $X$  is?

(GATE XH-C1 2023)

**Solution:**

The mean of  $X$

$$\mu_X = \int_{-\infty}^{\infty} x f(x) dx \quad (2.16)$$

As the integrand is odd

$$\implies \mu_X = 0 \quad (2.17)$$

The variance of  $X$  is:

$$\sigma_X^2 = \mathbb{E} (X - \mu_X)^2 \quad (2.18)$$

From (2.17)

$$\implies \sigma_X^2 = \mathbb{E} (X^2) \quad (2.19)$$

$$= \frac{1}{2\sqrt{3}} \int_{-\sqrt{3}}^{\sqrt{3}} x^2 dx \quad (2.20)$$

$$= 1 \quad (2.21)$$



## Chapter 3

# Conditional Probability





## Chapter 4

# Moments



## Chapter 5

# Random Algebra



## Chapter 6

# Hypothesis Testing



## Chapter 7

# Bivariate Random Variables





## Chapter 8

# Random Processes



## Chapter 9

# Information Theory

1. The frequency of occurrence of 8 symbols (a-h) is shown in the table below. A symbol is chosen and it is determined by asking a series of “yes/no” questions which are assumed to be truthfully answered. The average number of questions when asked in the most efficient sequence, to determine the chosen symbol, is

Symbols	Frequency of occurrence
a	$\frac{1}{2}$
b	$\frac{1}{4}$
c	$\frac{1}{8}$
d	$\frac{1}{16}$
e	$\frac{1}{32}$
f	$\frac{1}{64}$
g	$\frac{1}{128}$
h	$\frac{1}{128}$

**Solution:**

Parameter	Value	Description
$X$	$1 \leq X \leq 8$	number of symbols
$l$	2	base of algorithm
$H(X)$	$\sum_i p_X(i) \log_l \left( \frac{1}{p_X(i)} \right)$	average number of question

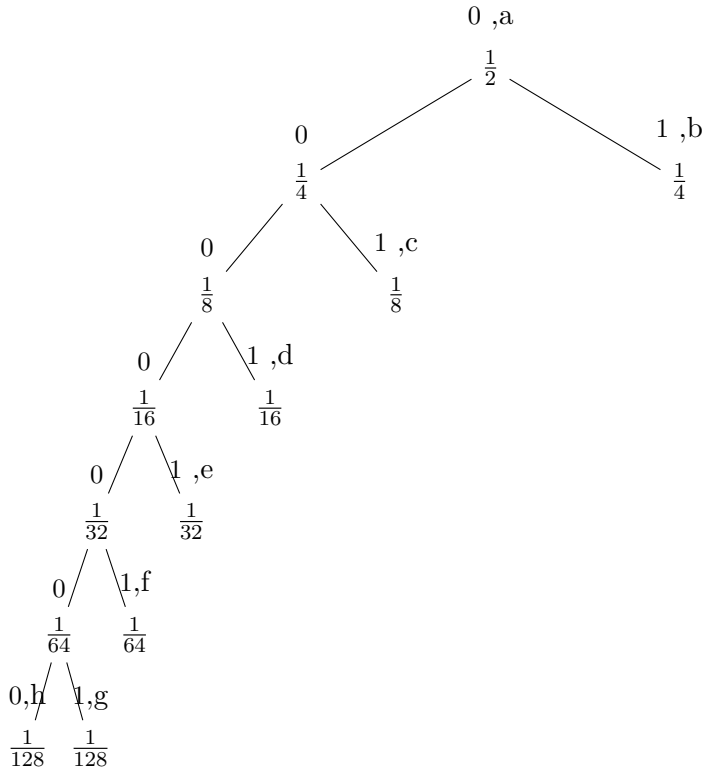
$$H(X) = \sum_i p_X(i) \log_b \left( \frac{1}{p_X(i)} \right) \quad (9.1)$$

$$= \frac{1}{2} \log_2 (2) + \frac{1}{4} \log_2 (4) + \dots + \frac{1}{128} \log_2 (128) \quad (9.2)$$

$$= 0.5 + 0.5 + 0.375 + \dots + 0.0078125 \quad (9.3)$$

$$= 1.984375 \quad (9.4)$$

Now, finding the average using Huffman code,



Using the above binary table following code is generated;

Symbols	Frequency	Code	Size
$a$	$\frac{1}{2}$	1	0.5
$b$	$\frac{1}{4}$	01	0.25
$c$	$\frac{1}{8}$	001	0.125
$d$	$\frac{1}{16}$	0001	0.0625
$e$	$\frac{1}{32}$	00001	0.03125
$f$	$\frac{1}{64}$	000001	0.015625
$g$	$\frac{1}{128}$	0000001	0.0078125
$h$	$\frac{1}{128}$	0000000	0.0078125

Table 9.1: Huffman table

The average number of question = Weighted path length = 1.9844

