Lab session 1: Signals and Systems

Problem 1: Cartesian coordinates to Polar coordinates
Write a program that reads a Cartesian coordinate (x, y) from the input and converts it into the corresponding Polar coordinate (r, θ). The input of this problem consists of two integers: x and y. The output of your program should be two floating point numbers (use `doubles` in your programs): r (length, so a non-negative number) and the principal value of θ (in radians). The output values should be rounded and have two digits after the decimal dot.

Example 1:
input:
```
2 0
```
output:
```
2.00 0.00
```

Example 2:
input:
```
0 2
```
output:
```
2.00 1.57
```

Example 3:
input:
```
-2 0
```
output:
```
2.00 3.14
```

Problem 2: Polar coordinates to Cartesian coordinates
Write a program that reads a Polar coordinate (r, θ) from the input and converts it into the corresponding Cartesian coordinate (x, y). The input consists of two floating point numbers: r (a non-negative number) and θ (in radians). The output of your program should be two floating point numbers (rounded to two digits after the decimal dot):

Example 1:
input:
```
1.00 1.57
```
output:
```
0.00 1.00
```

Example 2:
input:
```
1.00 0.00
```
output:
```
1.00 0.00
```

Example 3:
input:
```
1.00 -1.57
```
output:
```
0.00 -1.00
```

Problem 3: Sum of sinusoids of the same frequency
In this problem, you are constructing the sum of a number of sinusoids with the same frequency. The input of this problem consists of several lines. The first line contains two integers: the frequency f (in Hz) and the number n of sinusoids to sum.

The following n lines each contain two floating point numbers, representing the amplitude A_i and phase φ_i of the i-th sinusoid $A_i \cos(2\pi f t + \varphi_i)$. Your program should calculate the resulting signal $x(t) = \sum_i A_i \cos(2\pi f t + \varphi_i) = A \cos(2\pi f t + \varphi)$. The values of A and φ should be rounded to two digits after the decimal dot. Note that if $A = 0$, the output should be `x(t)=0.00`.

Example 1:
input:
```
42 2
1 1.047198
1 0.523599
```
output:
```
x(t)=1.93cos(2*pi*42*t+0.79)
```

Example 2:
input:
```
100 3
5 3.665191
5 1.570796
5 -0.5235988
```
output:
```
x(t)=0.00
```

Example 3:
input:
```
10 2
3 1.570796
4 0
```
output:
```
x(t)=5.00cos(2*pi*10*t+0.64)
```

Problem 4: Aliasing

The input for this problem is a single line containing two integers representing signal frequency f_0 (in Hz) and sampling frequency f_s (in Hz). The output should be a single integer that represents the frequency (in Hz) of the reconstructed sinusoid after a sinusoid with frequency f_0 has been sampled with frequency f_s.

Example 1:

input:
```
100 500
```
output:
```
100
```

Example 2:

input:
```
100 100
```
output:
```
0
```

Example 3:

input:
```
100 90
```
output:
```
10
```

Problem 5: Products of sinusoids

A beat note can be represented as a product of two sinusoids. In this problem, we consider the product $\prod_i \cos(2\pi f_i t)$ of arbitrary numbers of sinusoids $\cos(2\pi f_i t)$ and determine what (non-negative) frequencies are present in the spectrum.

The input for this problem consists of several lines. Each line contains a single integer that represents the frequency f_i (in Hz) of one of the sinusoids in the product. The integer 0 signals the end of the input. The output should be a list of all the non-negative frequencies (in Hz) in the spectrum, in order from smallest to largest. Note that the spectrum does not contain the same frequency twice.

Example 1:

input:
```
2000
515
27
0
```
output:
```
1458
1512
2488
2542
```

Example 2:

input:
```
300
20
0
```
output:
```
280
320
```

Example 3:

input:
```
100
100
100
0
```
output:
```
100
300
```

Problem 6: Nyquist frequency

A beat note can be represented as a product of two sinusoids. In this problem, we consider the product $\prod_i \cos(2\pi f_i t)$ of arbitrary numbers of sinusoids $\cos(2\pi f_i t)$ and determine what the Nyquist frequency would be for sampling the signal.

The input for this problem consists of several lines. Each line contains a single integer that represents the frequency f_i (in Hz) of one of the sinusoids in the product. The integer 0 signals the end of the input. The output should be a single integer representing the Nyquist frequency (in Hz) needed to accurate reproduce the signal.

Example 1:

input:
```
2000
515
27
0
```
output:
```
5084
```

Example 2:

input:
```
300
20
0
```
output:
```
640
```

Example 3:

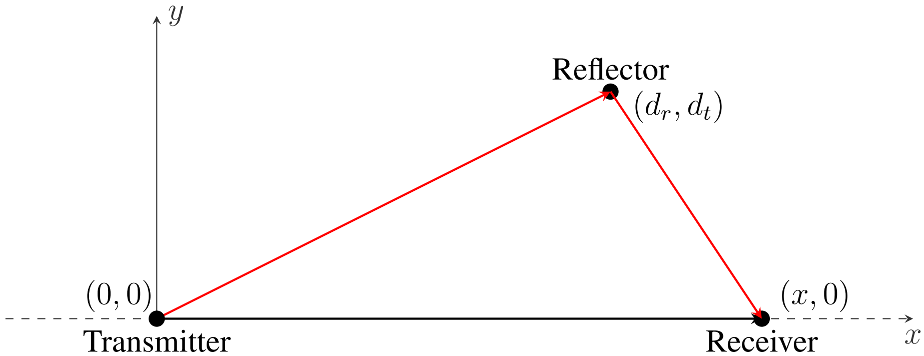
input:
```
100
100
100
0
```
output:
```
600
```

Problem 7: Multipath fading

When radio waves waves reflect off smooth surfaces, these reflections may interfere with the original signal. In such reflection cases, the received signal is a sum of the original signal and the reflected signal. Since these signals travel different distances, they differ in their time delay. Given a transmitted signal $s(t)$, the received signal $r(t)$ is therefore given by

$$r(t) = s(t - t_1) + s(t - t_2)$$

Consider the case where a receiver located at $(x, 0)$ receives a signal from a transmitter located at $(0, 0)$, which is also reflected off a reflecter at (d_r, d_t), as depicted below.



In this case, the transmitter sends out the signal $s(t) = \cos(2\pi(150 \times 10^6)t)$, which travels at a speed of 3×10^8 m/s. For simplicity, we ignore propagation losses. That is, the amplitude is not affected by distance or reflection.

Write a program that reads a single line containing the three integers d_r, d_t, and x (in m). The output of your program should be floating point number (rounded to two digits after the decimal dot) that represents the amplitude of the received signal r.

Example 1:

input:
```
30 40 30
```
output:
```
2.00
```

Example 2:

input:
```
30 40 31
```
output:
```
0.04
```

Example 3:

input:
```
10 100 50
```
output:
```
1.90
```