

ECE 595: Homework 1

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(Spring 2019)

Exercise 1

(a)

Refer to code in the back.

(b)

The data was successfully read, shown by the screen below.

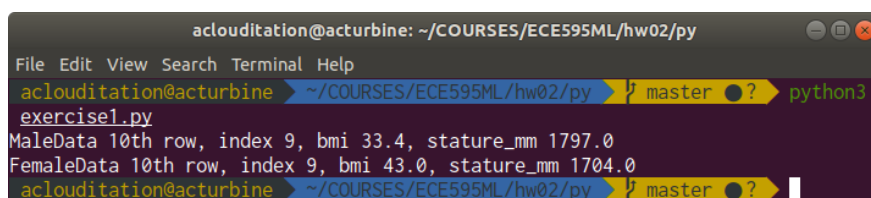


Figure 1: screenshot for reading data

Exercise 2

(a)

$$\begin{aligned} \begin{bmatrix} \boldsymbol{\omega}^* \\ \omega_0^* \end{bmatrix} &= \underset{\boldsymbol{\omega}, \omega_0}{\operatorname{argmin}} \sum_{j=1}^N (\boldsymbol{\omega}^T \mathbf{x}_j + \omega_0 - y_j)^2 \\ \text{set } \boldsymbol{\theta} &= \begin{bmatrix} \boldsymbol{\omega}^* \\ \omega_0^* \end{bmatrix} \\ \boldsymbol{\theta}^* &= \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{j=1}^N ([\mathbf{x}_j^T \quad 1] \boldsymbol{\theta} - y_j)^2 \\ &\text{thus,} \end{aligned} \tag{1}$$

$$A = \begin{bmatrix} -\mathbf{x}_1^T & 1 \\ -\mathbf{x}_2^T & 1 \\ \dots & \dots \\ -\mathbf{x}_N^T & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} y_1 \\ y_2 \\ \dots \\ y_N \end{bmatrix}$$

(b)

by least square

$$\begin{aligned} A^T A \boldsymbol{\theta}^* &= A^T \mathbf{b} \\ \boldsymbol{\theta}^* &= (A^T A)^{-1} A^T \mathbf{b} \end{aligned} \tag{2}$$

if $A^T A$ is invertible, A needs to be full column rank (or $\text{null}(A) = 0$). TODO: find out how to avoid this issue

(c)

Refer to the code in the back.

The optimal weight θ^* is:

$$\theta^* = \begin{bmatrix} -1.23396767e-2 \\ 6.67486843e-3 \\ -1.07017505e+1 \end{bmatrix}$$

(d)

Refer to the code in the back again.

The weight vector computed using CVXPY is the same as the one computed in the previous step.

$$\theta^* = \begin{bmatrix} -1.23396767e-2 \\ 6.67486843e-3 \\ -1.07017505e+1 \end{bmatrix}$$

Exercise 3

(a)

(i)

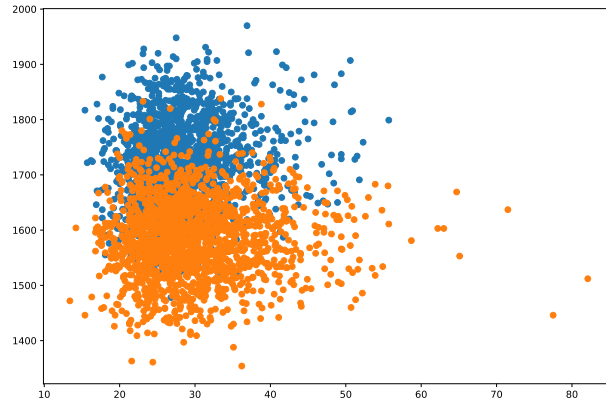


Figure 2: Training data

(ii)

$$\begin{aligned} \omega^{*T} \mathbf{x} + \omega_0^* &= 0 \\ \omega_1^* x_1 + \omega_2^* x_2 + \omega_0^* &= 0 \\ x_2 &= -\frac{\omega_1^* x_1 + \omega_0^*}{\omega_2^*} \end{aligned} \tag{3}$$

(iii)

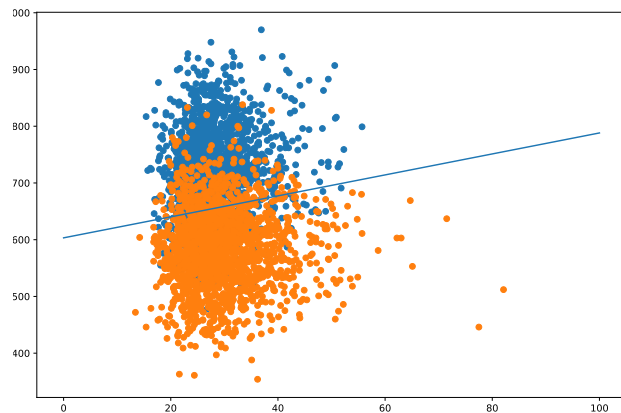


Figure 3: Training data with decision boundary

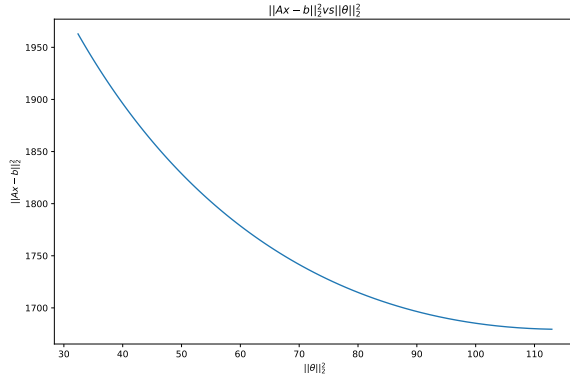
(b)

Refer to the code in the back, the success rate is:

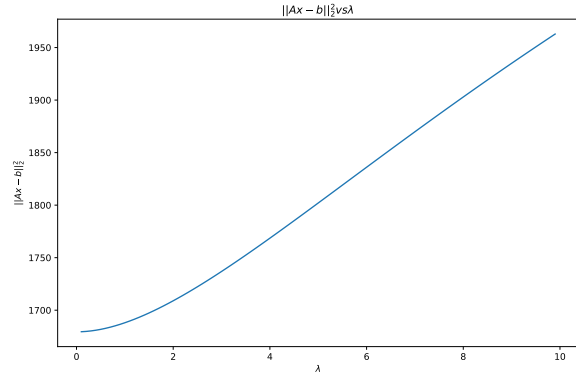
$$success\ rate = 83.93\%$$

Exercise 4

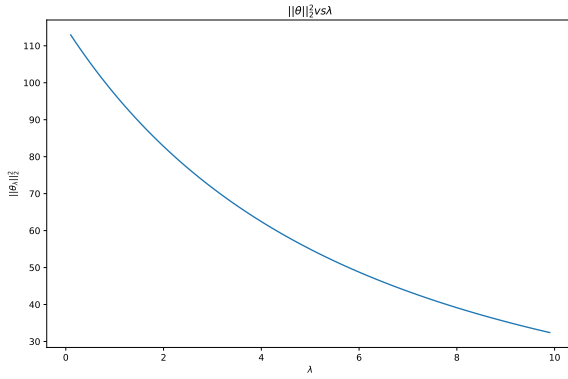
(a)



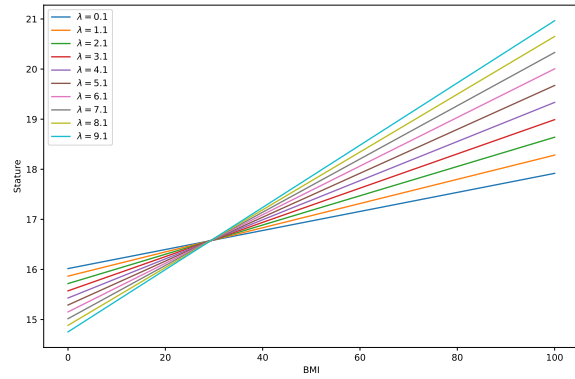
(a) 1



(b) 2



(c) 3



(d) 4

(b)

for equation (8) idk

for equation (9), $\theta_\alpha = \underset{\theta}{\operatorname{argmin}} \|A\theta - b\|_2^2$ subject to $\|\theta\|_2^2 \leq \alpha$:

$$\begin{aligned} \mathcal{L}(\theta, \lambda) &= \|A\theta - b\|_2^2 - \lambda(\|\theta\|_2^2 - \alpha) \\ \text{let } \nabla \mathcal{L}(\theta, \lambda) &= 0 \\ \nabla \|A\theta - b\|_2^2 - \lambda \nabla (\|\theta\|_2^2 - \alpha) &= 0 \end{aligned} \tag{4}$$

for equation (10), $\theta_\epsilon = \underset{\theta}{\operatorname{argmin}} \|\theta\|_2^2$ subject to $\|A\theta - b\|_2^2 \leq \epsilon$