ECE 595: Homework 1

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Exercise 1

(a)

Refer to code in the back.

(b)

TODO: add a screenshot here

Exercise 2

(a)

$$\begin{bmatrix}\boldsymbol{\omega}^* \\ \omega_0^*\end{bmatrix} = \underset{\boldsymbol{\omega},\omega_0}{argmin} \sum_{j=1}^{N} (\boldsymbol{\omega}^T \boldsymbol{x}_j + \omega_0 - y_j)^2$$

$$set\ \boldsymbol{\theta} = \begin{bmatrix}\boldsymbol{\omega}^* \\ \omega_0^*\end{bmatrix} \tag{1}$$

$$\boldsymbol{\theta}^* = \underset{\boldsymbol{\theta}}{argmin} \sum_{j=1}^{N} (\begin{bmatrix}\boldsymbol{x}_j^T & 1\end{bmatrix} \boldsymbol{\theta} - y_j)^2$$

thus,

$$A = \begin{bmatrix} -\boldsymbol{x}_1 - & 1 \\ -\boldsymbol{x}_2 - & 1 \\ \cdots & \cdots \\ -\boldsymbol{x}_N - & 1 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_N \end{bmatrix}$$

(b)

by least square

$$\begin{aligned} \boldsymbol{A}^T \boldsymbol{A} \boldsymbol{\theta}^* &= \boldsymbol{A}^T \boldsymbol{b} \\ \boldsymbol{\theta}^* &= (\boldsymbol{A}^T \boldsymbol{A})^{-1} \boldsymbol{A}^T \boldsymbol{b} \end{aligned} \tag{2}$$

if $A^T A$ is invertible, A needs to be full column rank (or $null(A) = 0$). TODO: find out how to avoid this issue

(c)

(d)

Exercise 3

(a)

(i)

show a plot here

(ii)

$$\begin{aligned} \boldsymbol{\omega}^{*T}\boldsymbol{x} + \omega_0^* &= 0 \\ \omega_1^* x_1 + \omega_2^* x_2 + \omega_0^* &= 0 \\ x_2 &= -\frac{\omega_1^* x_1 + \omega_0^*}{\omega_2^*} \end{aligned} \tag{3}$$

(iii)

show a plot here

(b)