

## ECE 595: Homework 1

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### Exercise 2

(a) For a gaussian distribution:

$$\begin{aligned} E[x] &= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\ &= \int_{-\infty}^{\infty} (x + \mu) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx \\ &= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx + \int_{-\infty}^{\infty} \mu \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx \\ &= 0 + \mu \frac{1}{\sqrt{2\pi\sigma^2}} \times \sigma\sqrt{2\pi} \\ &= \mu \end{aligned} \tag{1}$$

$$\begin{aligned} Var[x] &= \int_{-\infty}^{\infty} (x - \mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\ &= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}} dx \\ &\text{let } y = \frac{x}{\sigma}, \text{ then } dy = \frac{1}{\sigma} dx \\ &= \frac{\sigma^3}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}} dy \\ &= \sigma^2 \end{aligned} \tag{2}$$

(b) Data generated and plotted as follows.

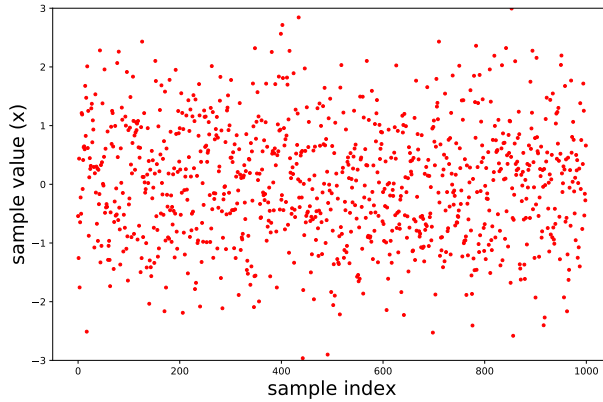
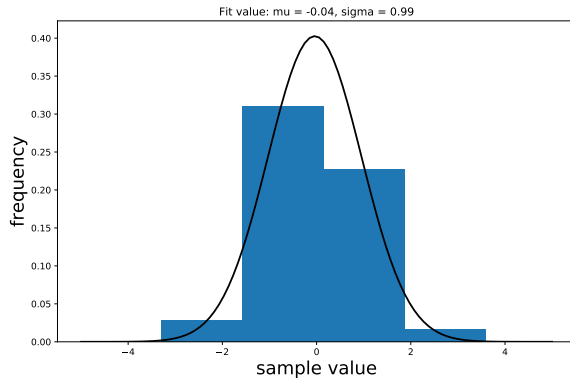


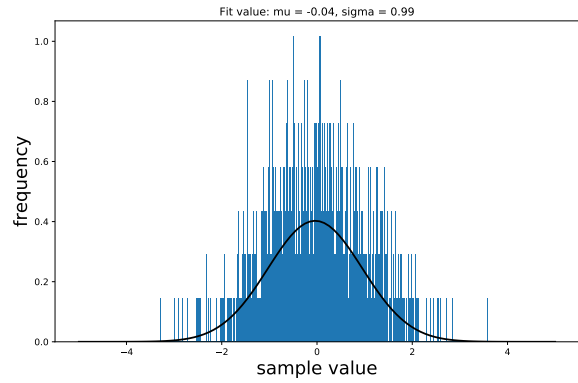
Figure 1: Gaussian random data.

(c)

(i)..(iv) plots shown below



(a) 4 bins



(b) 1000 bins

(v) TODO: fill this

### Exercise 3

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{2\pi^2|\mathbf{\Sigma}|}} \exp\left\{-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^T \mathbf{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu})\right\}$$

(i) plug in

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \boldsymbol{\mu} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \text{ and } \mathbf{\Sigma} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

we get

$$\begin{aligned} f \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \left( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) &= \frac{1}{\sqrt{2\pi^2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}}} \exp \left\{ -\frac{1}{2} \begin{bmatrix} x_1 - 2 \\ x_2 - 6 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} x_1 - 2 \\ x_2 - 6 \end{bmatrix} \right\} \\ &= \frac{1}{\pi\sqrt{6}} \exp \left\{ -\frac{1}{3} ((x_1 - 2)^2 - (x_1 - 2)(x_2 - 6) + (x_2 - 6)^2) \right\} \end{aligned} \quad (3)$$

### Exercise 4

Type your third problem here.