ECE 595: Homework 1

Yi Qiao, Class ID

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Exercise 2

(a) For a guassian distribution:

$$\begin{aligned}
E[x] &= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\
&= \int_{-\infty}^{\infty} (x+\mu) \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx \\
&= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx + \int_{-\infty}^{\infty} \mu \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}} dx \\
&= 0 + \mu \frac{1}{\sqrt{2\pi\sigma^2}} \times \sigma\sqrt{2\pi} \\
&= \mu
\end{aligned} \tag{1}$$

$$\begin{aligned}
Var[x] &= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} x^2 e^{-\frac{x^2}{2\sigma^2}} dx \\
&let\ y = \frac{x}{\sigma},\ then\ dy = \frac{1}{\sigma} dx \\
&= \frac{\sigma^3}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} y^2 e^{-\frac{y^2}{2}} dy \\
&= \sigma^2
\end{aligned} \tag{2}$$

(b) Data generated and plotted as follows.

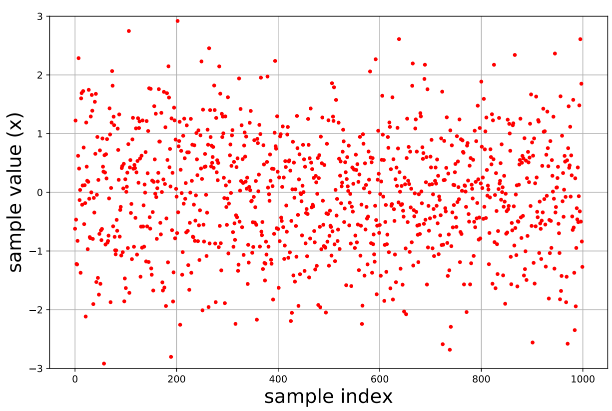
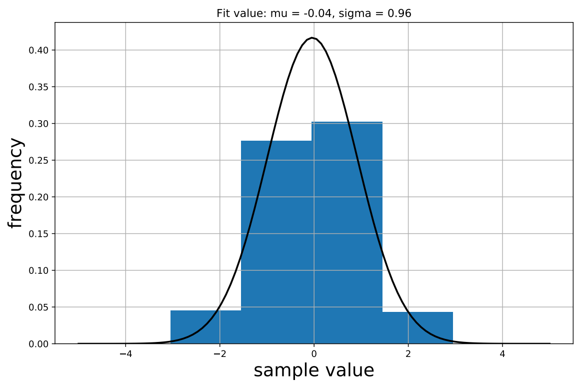
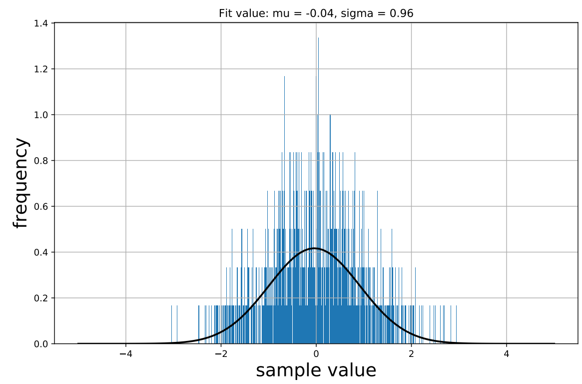


Figure 1: Gussian random data.

(c)
(i)..(iv) plots shown below



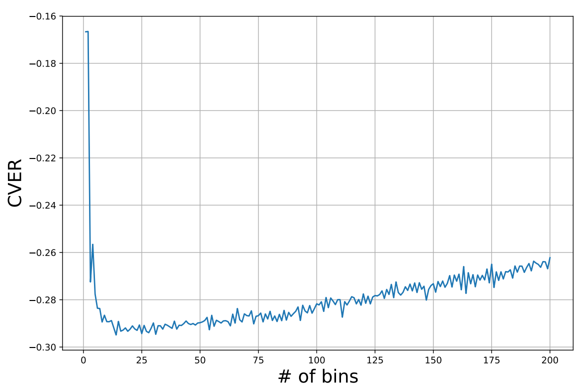
(a) 4 bins



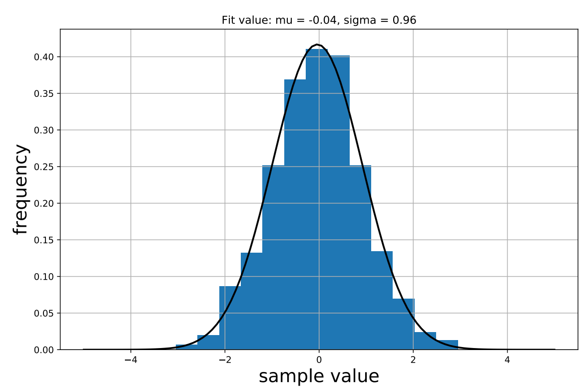
(b) 1000 bins

(v) TODO: fill this

(d) compare to part (c) the histogram fits a lot better with the PDF plots shown below



(c) Cross validation estimator of risk vs. # of bins



(d) Histogram and PDF overlayed with optimized number of bins

Exercise 3

(a)

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{1}{\sqrt{2\pi^2|\mathbf{\Sigma}|}} exp\{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\}$$

(i) plug in

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \boldsymbol{\mu} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \text{ and } \mathbf{\Sigma} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

we get

$$
\begin{aligned}
f_{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}}\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) &= \frac{1}{\sqrt{2\pi^2 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}}} exp\left\{-\frac{1}{2}\begin{bmatrix} x_1 - 2 \\ x_2 - 6 \end{bmatrix}^T \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} x_1 - 2 \\ x_2 - 6 \end{bmatrix}\right\} \\
&= \frac{1}{\pi\sqrt{6}} exp\left\{-\frac{1}{3}\left((x_1 - 2)^2 - (x_1 - 2)(x_2 - 6) + (x_2 - 6)^2\right)\right\}
\end{aligned} \tag{3}
$$

(ii) plot shown below

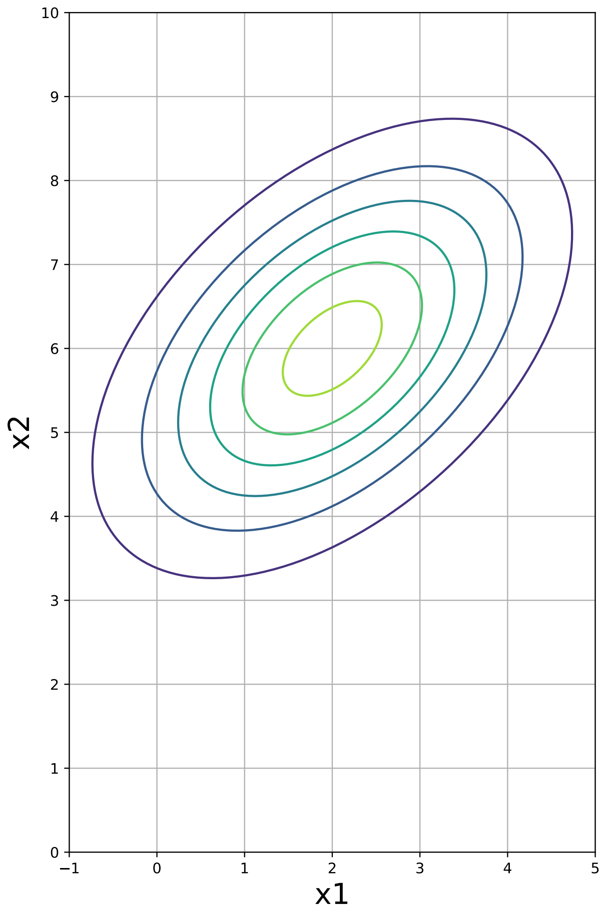


Figure 2: Gussian random data.

Exercise 4

Type your third problem here.