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Background/Brief

Our task was to break down a C4 note into its approximate Fourier series, calculating all coefficients and modelling this note using our choice of MATLAB script. Part of the challenge with breaking down this C4 note is that it contained harmonics. To help us identify all parts of the C4 note, we had to break this down and analyze a select few harmonics.

Methods Summary

Our methods included the following:

- Hand calculations
- Basic MATLAB analysis of the C4 sample provided
- Advanced MATLAB analysis (aided by ChatGPT)
- Analysis of FFT and frequency graphs

All code will be provided in a .zip file as instructed, the MATLAB file named team_return_main_code. All figures and .wav files will also be found in this file. Any time we use AI, we will notate it.

Figures/Required Steps Summary

Part 1

In this part, we loaded in the sample audio file given of a C4 note into MATLAB. We gained the following figure:

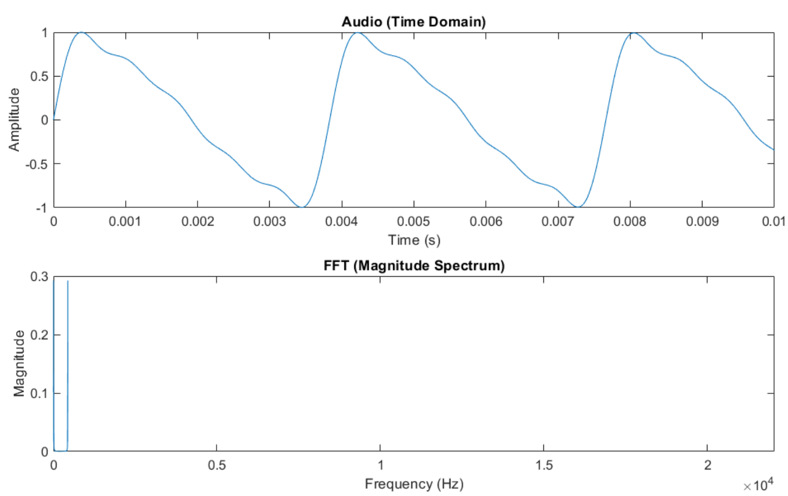


Figure 1 – Basic C4 Note in Time and Frequency Domain

We approximated the first peak to be about 260.11 Hz. Based on a frequency chart we used in our own assignment creation, we know this is the approximate frequency for a pure toned C4 note, the actual frequency being 261.63 Hz. Knowing this, we can approximate the harmonics to be at intervals of about 261 Hz, meaning the second harmonic is at 522 Hz, and the third harmonic should be at 783 Hz.

Our assignment, however, asks us to take this a step further by specifically applying a Hann window to a centered 1.0 second slice, creating a graph of the FFT from 0 to 4000 Hz. We used ChatGPT to help us create a Hann window, and the specified 1 second slice to get the following figure:

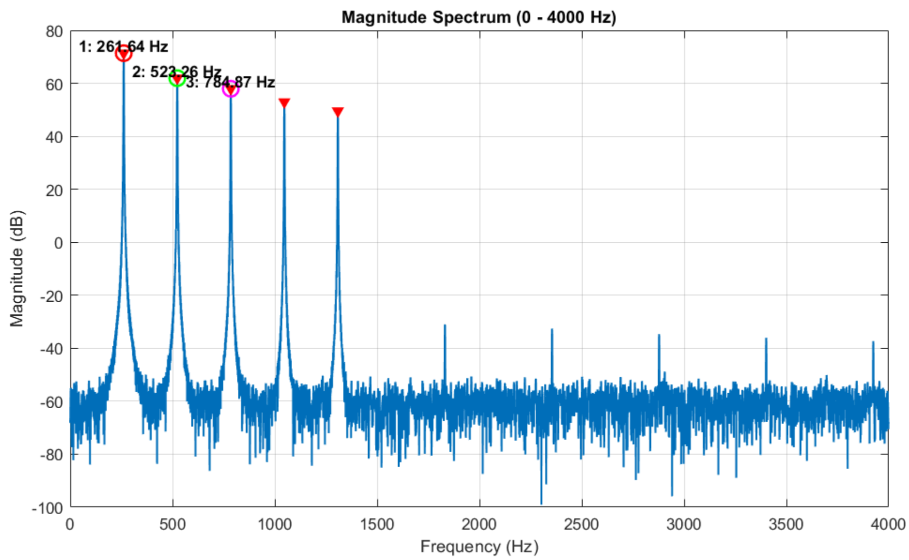


Figure 2 – FFT from 0 to 4000 Hz of C4 Note

Here, our exact approximations of the harmonic frequencies are 261.64 Hz, 523.26 Hz, and 784.87 Hz which agrees with our initial approximations.

Part 2

We then wanted to compute $T_0 = \frac{1}{f_0}$, where f_0 are the harmonic fundamental frequencies acquired in part 1, to find the following: 0.0038 s, 0.0019 s, 0.0013 s.

Then, we could find the $N_{per} = round(T_0 * f_s)$, being

We needed this information to be able to loop the slice of the audio we had extracted.

We can confirm that the loop sounds periodic because of the consistent tone and pitch that is being repeated over consistent intervals of time. A non periodic tone would sound “inconsistent” in tone, perhaps with a wobbling or wave sound that is not evenly spread between time intervals.

Part 3

Over a single period, we could compute all of the coefficients of the signal by using the following equations:

$$x(t) = a_0 + \sum_{k=1}^{\infty} a_k cos(k\omega_0 t) + \sum_{k=1}^{\infty} b_k sin(k\omega_0 t)$$

$$a_0 = \frac{1}{T_0} \int_{t_0}^{t_0+T_0} x(t)dt,\ a_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} x(t)cos(k\omega_0 t)dt,\ b_k = \frac{2}{T_0} \int_{t_0}^{t_0+T_0} x(t)sin(k\omega_0 t)dt$$

Solving each by hand, we can find that a_0 is approximately -0.000012, the dc component of the signal.

Finding the coefficients of a_k and b_k is much harder because they are dependent on K. Our assignment tasks us to find K=12. To do this, we used MATLAB to aid us by using a for loop to compute all 24 coefficients. We got the following values:

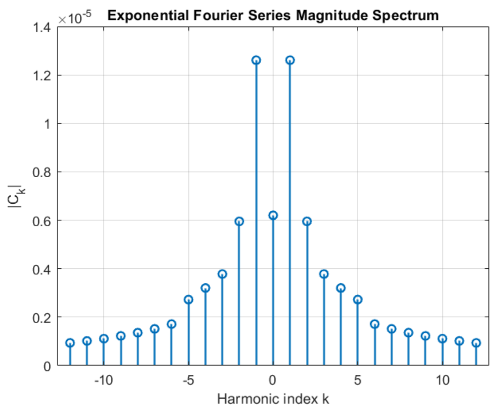
```
ak values:
  -0.000022  -0.000003  -0.000000  -0.000000  0.000000  0.000001  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000
bk values:
  0.000013  0.000012  0.000008  0.000006  0.000005  0.000003  0.000003  0.000003  0.000002  0.000002  0.000002  0.000002
```

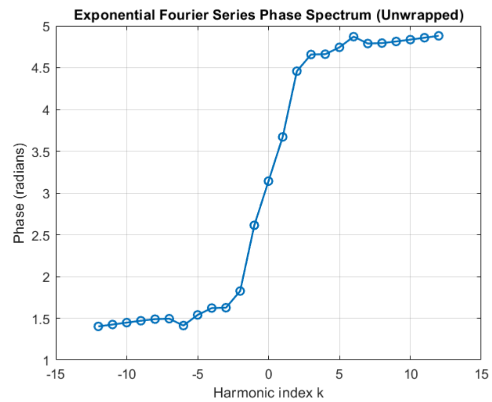
If you notice, many of the AK values are zero, meaning that cosine waves are much less prevalent in the make up of the entire wave form in comparison to the sine values, which are all nonzero.

Using the TSF, we could convert to an ESF by the following equation, using MATLAB to save time and manually do the calculations for us:

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

Using this information, we plotted them as a magnitude plot versus K. We got the following figures:

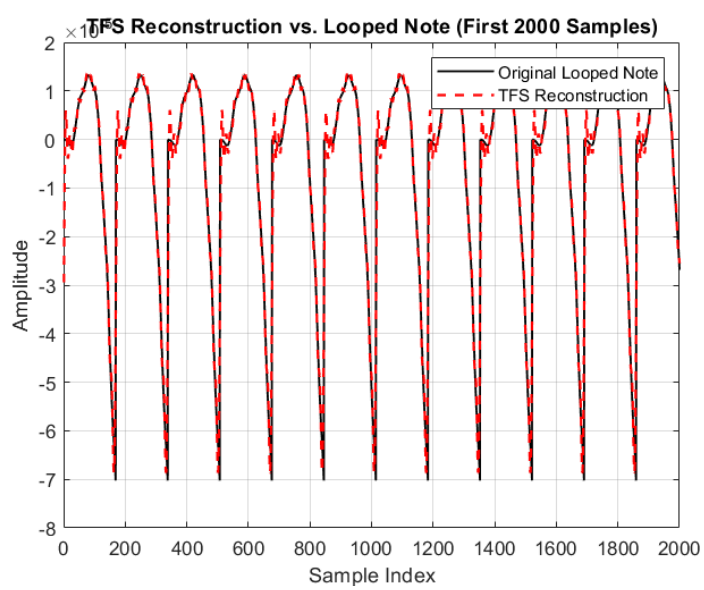




We can see by magnitude that the first harmonic is by far dominating the part of the whole signal. This can be confirmed where we can see the highest peak in our FFT graph being the first harmonic, closely followed by the second and third, which are depicted in the Exponential Fourier Series Magnitude Spectrum graph.

Part 4

Here we returned to a TFS to reconstruct y(t) from the partial sum up to N=12 over the time axis of the looped signal. Using AI as an aid, we were able to overlay the reconstruction of the looped note for a short window of samples (about 2000) and saved the figure:



We can see that the quality of the construction is actually pretty good for the first 2000 samples, and playing around with our MATLAB script, we can find that the error we see actually goes down when we increase the number of samples. Misalignments due to estimation and rounding can be seen within this graph as there are parts that do not overlay exactly. However, this is still a relatively good approximation to the signal.

Part 5

The last part of this assignment had us create audio outputs of the K partials. We noticed that lower K partials, such as K=1 and K=2 produced a lower octave tone than the higher K partials such as K=4 and K=5. Additionally, we noticed that the sum of K=1 to K=5 produced a very rich sounding tone that sounded much more natural to what a normal instrument would produce, almost sounding like a synthesized clarinet.

Thinking Questions

1. Estimates in the fundamental frequency will change the FFT size because a larger FFT will provide better frequency resolution, meaning that error in the fundamental frequency will result in poor frequency resolution in the final signal.
2. A small N is only good for the lower octaves/harmonics, as can be seen in Part 5. Increasing the N will reconstruct a wave that is much closer to the original one, and the tone will become richer sounding.
3. The spectral leakage is not really visible with the steps we took to capture the frequencies and magnitudes. However, if we were to not apply the Hann window, the leakage is more pronounced.
4. Shortening the slice reduces the frequency resolution, which makes the peaks less defined and noisier.
5. Different instruments will have different magnitude spectrums, and can be demonstrated in Part 5. We noticed the sound produced by the sum of the partials was clarinet-like, with a rich tone that had many partials put together. Perhaps an instrument that had more of a “pure tone” (unlike the richness of a reed instrument) would be even more uniform in EFS magnitude because there is less timbre.