两角和差的正切公式

$$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$$

$\Rightarrow$

$$\tan(\alpha+\beta) = \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta}$$

分子分母同除 $\cos\alpha \cdot \cos\beta$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

公式
$$\boxed{\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}}$$

再推 $\tan(\alpha-\beta)$ 公式：

$$\sin(\alpha-\beta) = \sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta$$

$$\tan(\alpha-\beta) = \frac{\sin\alpha \cdot \cos\beta - \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta}$$

同除 $\cos\alpha \cdot \cos\beta$

$$= \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \cdot \tan\beta}$$

公式
$$\boxed{\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha \tan\beta}}$$

记忆口诀：子同同异

例: 求 $\dfrac{1+\tan 45^\circ}{1-\tan 15^\circ}$

解: $\dfrac{1+\tan 15^\circ}{1-\tan 15^\circ} = \dfrac{\tan 45^\circ + \tan 15^\circ}{1-\tan 45^\circ \tan 15^\circ}$

$\qquad\qquad = \tan(45^\circ + 15^\circ)$

$\qquad\qquad = \tan 60^\circ$

$\qquad\qquad = \sqrt{3}$

例: $\tan\alpha \cdot \tan\beta$ 为 $X^2 - 3X + 2 = 0$ 的两根

则 $\tan(\alpha + \beta) = $ ____

解: 对于 $ax^2 + bx + c = 0$ $(a \neq 0)$, 由韦达定理可知:

$X_1 + X_2 = -\dfrac{b}{a}$

$X_1 \cdot X_2 = \dfrac{c}{a}$

$\therefore \tan\alpha + \tan\beta = -\dfrac{-3}{1} = 3$

$\tan\alpha \cdot \tan\beta = \dfrac{2}{1} = 2$

$\therefore \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} = \dfrac{3}{1-2} = -3$