

Brain Teasers Battle

Total Time: 35 minutes ONLY

INSTRUCTIONS for PARTICIPANTS:

1. You have 35 minutes to answer as many questions as possible.
 2. There are two sections:
 - **Section A (Brainteasers):** Designed to test logical thinking, pattern recognition, and problem-solving skills.
 - **Section B (Mental Math & Quick Calculations):** Focuses on numerical accuracy and speed under pressure.
 - i. *Answer within a certain reasonable range will be accepted for any question that is labeled with **
 3. You may not use a calculator, computer, or any external aids.
 4. Show all reasoning for the brainteasers. For the mental math questions, write down your final answer clearly.
 5. Your final score will be based on the accuracy and completeness of your responses.
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RUBRIC (Total: 100 Points)

Section A (15 Questions, 60 points total):

- Each correct answer: 4 points.
- If partial reasoning is correct but the final answer is incorrect, up to 2 points may be awarded.
- No points for blank or entirely incorrect reasoning.

Section B (80 Questions, 40 points total):

- Each correct numerical answer: 0.5 points.
- Each incorrect numerical answer: -1 point.

Tie-Breakers:

- If multiple participants tie, consider the speed of completion or the quality of reasoning steps in Section A will be considered by judges.
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SECTION A: Brainteasers (15 Questions)

1. Random Walk on a Line:

You have a fair coin. You start at position 0 on the number line. Each time you flip a coin:

- If heads, you move +1 step
- If tails, you move -1 step

After 10 independent coin flips, what is the probability that you end up exactly at position 0?
(Hint: Consider the number of heads needed.)

2. Painting a Cube:

A large cube is painted on all six faces and then cut into 125 smaller, equally sized cubes (arranged as $5 \times 5 \times 5$). If you pick one small cube at random, what is the probability that exactly one of its faces is painted?

3. Bonds and Interest (Logic Puzzle):

You have three zero-coupon bonds that mature in 1, 2, and 3 years respectively. Each bond, upon maturity, returns \$100. Suppose one bond is priced at \$96.15, another at \$92.45, and another at \$88.90, each corresponding to a different maturity. Without calculating exact yields, arrange the bonds in order of shortest to longest maturity based on their prices.

4. Randomized Market Indicator:

In a hypothetical scenario, the "Market Index" can either go up by 10% or down by 10% each day with equal probability. After two days, what's the expected value of the index as a multiple of the original value? Is it greater than, equal to, or less than 1.00 times the original value?

5. Counting Common Factors:

How many of the first million positive integers share no common factors greater than 1 with 2020?

6. Counting Potential Sum of Powers:

How many positive integers less than or equal to 1000 can be written as the sum of an even number of distinct powers of 3?

7. Zeros and Factorials:

How many zeros are there at the end of $100!$?

8. Dice Rolls:

If two dice are rolled at random, one after the other, what is the probability that the first number is greater than the second?

9. Rope-Triangle:

You cut a rope into three pieces (at random). What is the probability that you can make a triangle out of these pieces? *Hint:* triangle inequality!

10. Uphill Numbers:

How many three digit numbers have the property that the digits are in strictly increasing order? (e.g. 249 works, but 117 or 260 don't).

11. Non-Crossing Handshakes:

Ten people are seated around a circular table. They want to shake hands in pairs such that no two handshakes cross each other. How many such handshake arrangements are possible?

12. Expected Coin Flips for Two Consecutive Heads

When flipping a fair coin repeatedly, what is the expected number of flips required to achieve two consecutive heads?

13. Noodles Slurp:

Imagine a bowl with 100 noodles, each noodle having two ends dangling outside. You repeatedly pick two random ends and tie them together until no free ends remain. This process creates one or more loops, where a loop is a cycle of noodles joined end-to-end into a closed chain. Find a 0.5-wide interval that contains the expected number of loops formed when the process is complete.

14. Which is Greater?

Which is greater $2^{(1/3)}$ or $10^{(1/10)}$?

15. Future Minimizer :)

What is the earliest (valid) future date that has all different digits in YYYY MM DD format?

SECTION B: Mental Math & Quick Calculations (80 Questions: +0.5/-0.5 point each)

(No partial credit here; just provide the final numeric answers.)

Note: Answer within 5% of the actual value will be accepted for any question that is labeled as *

- (1) $2468 \div 9$ has a remainder of _____
- (2) $3\frac{1}{2} \times 5\frac{6}{7} =$ _____ (improper fraction)
- (3) $36 + 72 + 24 + 58 =$ _____
- (4) $2008 + 2009 =$ _____
- (5) $34^2 =$ _____
- (6) $743 - 347 =$ _____
- (7) $\frac{4}{7} - \frac{7}{8} =$ _____ (proper fraction)
- (8) $25 \times 20 - 15 + 10 \div 5 =$ _____
- (9) $2014 \times 4 + 6 =$ _____
- *(10) $374 - 1056 + 1916 =$ _____
- (11) 16% of 20 = _____
- (12) $32 - 16 \div 8 + 4 \times 2 =$ _____
- (13) $44 \times 36 =$ _____
- (14) 9 is what % of 180? _____ %
- (15) $14443 \times 21 =$ _____
- (16) $15 \times 38 =$ _____
- (17) $(34 + 65 + 96) \div 3$ has a remainder of _____
- (18) The product of the first 4 prime numbers is _____
- (19) $1\frac{7}{9} \times 2\frac{1}{4} =$ _____
- *(20) $85858 \div 585 =$ _____
- (21) $3212015 \div 11$ has a remainder of _____
- (22) $200_6 =$ _____ 10
- (23) If 6 oz. of candy costs \$0.96, then one pound of candy costs _____
- (24) $\{s, l, o, p, e\} \cap \{l, i, n, e\}$ has _____ distinct elements
- (25) $131_5 =$ _____
- (26) If $A = 1$, $B = -A$, and $C = A - B$,
then $ABC =$ _____
- (27) If $\frac{3}{4} = \frac{3x}{5}$, then $x =$ _____
- (28) $4^5 \div 11$ has a remainder of _____
- (29) $6\frac{2}{3} \times 3\frac{1}{3} =$ _____ (mixed number)
- *(30) $97531 \div 209 =$ _____
- (31) $111 \times 345 =$ _____
- (32) The smallest root of $2x^2 + 13x + 20 = 0$ is _____
- (33) $(5! \div 4!)+(3! \div 2!)-1! =$ _____
- (34) If Universal set $U = \{2, 3, 5, 7, 9, 11, 13, 17, 19\}$ and
set $A = \{3, 7, 13, 17\}$, then $A^{\text{complement}}$ contains how many
distinct elements? _____
- (35) The sum of the positive integral divisors of 108 is _____
- (36) $21 \times \frac{23}{25} =$ _____ (mixed number)
- (37) 12% of 200 is _____ % of 50
- (38) $\frac{5}{11} - \frac{11}{21} =$ _____
- (39) $63^2 + 24^2 =$ _____
- *(40) $\sqrt{7152023} =$ _____
- (41) $\frac{1}{4}(64^2 - 36^2) =$ _____
- (42) $13 \times 15 + 1 =$ _____
- (43) The sum of the roots of
 $f(x) = (2x - 5)^3(x - 5) =$ _____
- (44) Find k if the product of the roots of $x^2 + 2x + k = 0$
is 8. $k =$ _____
- (45) The positive geometric mean of 4 and 36 is _____

- (46) If $4^{(x+2)} = 48$, then $4^x =$ _____
- (47) If $x + y = \frac{1}{3}$ and $xy = 3$, then $x^3 + y^3 =$ _____
- (48) If $14x + 5 = 23$, then $14x - 5 =$ _____
- (49) $45_7 - 26_7 =$ _____ $_7$
- *(50) $\sqrt{1062017} =$ _____
- (51) If $\log_x 36 = 2$, then $x =$ _____
- (52) $57 \times 57 =$ _____
- (53) The odds of rolling a composite number on a single die is _____ (proper fraction)
- (54) $12^2 \div 6^2 \times 3^2 =$ _____
- (55) Let $|2 + 3x| \leq 4$. The greatest value of x is _____
- (56) If A is 20 more than B and C is 10 less than A , then C is how much more than B ? _____
- (57) If $\frac{15 + 30i}{-5i} = a + bi$, then $a + b =$ _____
- (58) If $\log_x 2744 = 3$, then $x =$ _____
- (59) If $852k$ is divisible by 6 then the largest units digit value for k is _____
- *(60) $8^3 \div 4^6 \times 2^{10} =$ _____
- (61) 630° equals $k\pi$ radians. Find k . _____
- (62) $\left[2 \sin\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{6}\right)\right] \times \left[\tan\left(\frac{\pi}{6}\right)\right] =$ _____
- (63) The simplified sum of the coefficients of the expansion of $(4x + 3y)^3 =$ _____
- (64) 3.25% of 24 is _____
- (65) How many ways can 3 people be seated in a row of 5 chairs? _____
- (66) The number of distinct diagonals of a 5 sided regular polygon is _____
- (67) If $f(x) = x^2 + 3x - 1$, then $f[f^{-1}(4)] =$ _____
- (68) The shortest distance between $(0, -2)$ and $5x + 12y = 11$ is _____
- (69) $222_3 \times 2_3 =$ _____ $_3$
- *(70) $142857 \times 21 =$ _____
- (71) The 1st triangular number times the 2nd hexagonal number times the 3rd pentagonal number is _____
- (72) If $\ln(50) = \ln(2) + k \ln(5)$, then $k =$ _____
- (73) $6^8 \div 8$ has a remainder of _____
- (74) If $N \div 8$ has a remainder of 5, then $3N \div 8$ has a remainder of _____
- (75) $|3x - 1| = 17$. Find x if $x < 0$. _____
- (76) The set $\{a, b, c\}$ has _____ 2-element subsets
- (77) The remainder, in base 8, when 153 base 8 is divided by 7 is _____
- (78) $\int_0^2 x^2 dx =$ _____
- (79) $1 + 2^2 + 3^3 + 4^4 =$ _____
- *(80) $\sqrt[3]{1234567} =$ _____