Exercise 1

$$(\lambda+\mu)\pi(n) = \lambda\pi(n-1) + \mu\pi(n+1)$$

for $n \geq 1$, $\lambda\pi(0) = \mu\pi(1) \Rightarrow \boxed{\frac{\lambda}{\mu}\pi(0) = \pi(1)}$

$$(\lambda+\mu)\pi(1) = \lambda\pi(0) + \mu\pi(2)$$

$$(\lambda+\mu)(\lambda\pi(0)) = \mu\lambda\pi(0) + \mu^2\pi(2)$$

$$\lambda^2\pi(0) \cancel{+\mu\lambda\pi(0)} = \cancel{\mu\lambda\pi(0)} + \mu^2\pi(2)$$

$$\Rightarrow \boxed{\frac{\lambda^2}{\mu^2}\pi(0) = \pi(2)}$$

$$\Rightarrow \boxed{\frac{\lambda^n}{\mu^n}\pi(0) = \pi(n)}$$

$$\sum_{n=0}^{\infty}\pi(n) = 1 \Rightarrow \left(1 + \left(\frac{\lambda}{\mu}\right) + \frac{\lambda^2}{\mu^2} + \cdots \infty\right)\pi(0) = 1$$

$$\Rightarrow \left(\frac{1}{1-\left(\frac{\lambda}{\mu}\right)}\right)\pi(0) = 1 \Rightarrow \boxed{\pi(0) = \frac{\mu-\lambda}{\mu}}$$

$$\Rightarrow \boxed{\pi(n) = \frac{\lambda^n}{\mu^n}\left(1-\frac{\lambda}{\mu}\right)}$$

$$\pi(0) = 1 - \frac{\lambda}{\mu} = 1 - \frac{1}{4} = \frac{3}{4} = \boxed{0.75}$$

Exercise 3

$$\pi(1) = \frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right) = \frac{1}{4}\left(\frac{3}{4}\right) = \boxed{\frac{3}{16}}$$

Exercise 4

using $\sum_{n=1}^{\infty} n r^n = \frac{r}{(1-r)^2}$

$$L = \sum_{n=0}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n\left(1-\frac{\lambda}{\mu}\right) = \left(1-\frac{\lambda}{\mu}\right)\sum_{n=0}^{\infty} n\left(\frac{\lambda}{\mu}\right)^n$$

$$L = \cancel{\left(1-\frac{\lambda}{\mu}\right)}\left(\frac{\left(\frac{\lambda}{\mu}\right)}{\left(1-\frac{\lambda}{\mu}\right)^{\cancel{2}}}\right) = \boxed{\frac{\lambda}{\mu-\lambda}}$$

$$L = \frac{1}{4-1} = \boxed{\frac{1}{3}}$$ = average expected queue length

Exercise 6

$$W = \frac{L}{\lambda} = \frac{\lambda}{(\mu-\lambda)\cancel{\lambda}} = \boxed{\frac{1}{\mu-\lambda}}$$

$\Rightarrow$ "Little's Law" $\Rightarrow$ $\boxed{W = \frac{1}{\mu-\lambda}}$