@ 1 player game. Your strategy to maximize earning 20 sided dice. 100 rounds. Either roll OR take money in a round

EV of the game?

Sol Chances of rolling a 20... will take me around 20 rolls to achieve that

One strategy (first-pass) ⇒ roll until you get a 20, and then every round take the money

⇒ $80(20) = \underline{1600}$

Now, I am thinking of balancing this and comparing to taking anything 18 or greater.

Get to	20 & take	(19 or 20) & take	(18 or 19 or 20) & take
	///	///	$\frac{20}{3}$ rounds to get there
	$(20)(80)$ $= 1600$	~~(90)~~$(90)(19.5)$ $= 1755$	/// $\approx 93 \times 19 \cong$ 1767

Further,	(17 or 18 or 19 or 20) & take
	↳ 5 rounds to get here
	⇒ ~~95~~ $95 \times 18.5 \cong \underline{1757}$

Continued...

continued...

(6) Now, if you take the money, then you have to roll again & take dice off the table.

My first inclination is that I would be willing to accept a much lower number to take the money (and re-roll)

Let's say, if I have a strategy to accept the top half of numbers (11-20), I am thinking through how that would look like.

⇒ $\frac{1}{2}$ times take money

⇒ 2 rolls & 1 more round to take money ⇒ 15.5 avg ⇓ 33 times ⇒ (511) slightly worse

VS

Baseline strategy: ⇒ 10.5 avg ⇒ 50 times ≃ (525)
(1 roll & 1 collect)

⇒ Now, I think it is matter of finding above which number, I would be willing to take the money.

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I will take anything in top x% from 1-20

⇓

will take $(20-20x)$ or higher

$\#\text{turns used} = \left(\frac{1}{x}+1\right) \Rightarrow \#\text{games} = \frac{100}{\left(\frac{1}{x}+1\right)} = \left(\frac{100x}{x+1}\right)$

also, average payout per game $= \frac{(20-2x)+20}{2} = (20-10x)$

$\text{Payout total}(x) = \left[\frac{100x(20-10x)}{(x+1)}\right]$ → derivative $= 0$

↳ do a sanity check to make sure it is max

$\Rightarrow x = 0.732 \Rightarrow$ Willing to accept anything ≥ 6

Makes sense since I was expecting somewhere between 1 to 10.

$\Rightarrow$ Sanity: calculated that going to make around $500 in total using earlier strategy naïvely. So roughly making $5 using that strategy in each round, so I should spend a round taking $7 or $6 if there, hence answer makes sense, and is SATISFYING!!

continued...

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© one more variant: [getting back to @]
This time casino gets to play. [Oh wow!!]
Once you take money, casino gets to choose whether or not to re-roll the die [once every time you take the money]

⇒ Note that this is complicated enough that we cant exactly solve by hand, but I am curious to hear how you think this version would work?

My first guess about how this would go is if I play with a strategy is that I will accept anything n or higher, and casino plays with a strategy of re-rolling any x or higher, perhaps at some point (say 10) we both will accept 10 & continue without asking and wanting to re-roll kind of like an equillibrium condition.

Where do you think this number is going to be?
well, if I play a strat of taking 18 or higher, casino will definitely re-roll me on that

continued...

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Let's say you rolled a 19, you take it, then obviously Casino re-rolls, now what happens?

damn, my strategy has been failed, I don't get to keep 19 in future rounds.

Then, I would probably have to accept a lower number. My strategy would become to accept 15 or higher. Casino sees 15 and thinks "if I re-roll expectation is 10.5, so I should reroll."

I would guess it should settle at 10 or 11 where we both don't have significant incentive to re-roll and try

let's say we take you playing strategy of 10 or higher OR 11 or higher, tell me something concrete. (10 or higher strategy for me)

What do you think casino is going to do?

If it sees 15 (or something > 10.5) it will still re-roll me and try to bring me down.

[If it lands on 10, then casino might let me just keep it since 10 < 10.5]

continued...

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Why don't we imagine playing this out and seeing how much money you will make? You are playing 10 or higher & Casino playing re-roll if 11 or higher.

⇒ we both agree ≥ 11 is too good for me

⇒ I will want it, but Casino will re-roll me whenever I get it.

⇒ seems like an oscillation between us.

$\frac{1}{2}$ times (# is < 10) ⇒ Casino doesn't want to re-roll

$\frac{1}{2}$ times (# is ≥ 10) ⇒ I keep it, and casino re-roll me on it.

I make \$15.5 $\left(= \frac{1}{2}(2\times 10 + 11)\right)$ every 2 rolls

⇒ 50 times repeated ⇒ \$755

each roll is worth around 7-8 \$

⇒ in this case I would like to change my threshold.

continued...

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Aha! what do you want to do with it?

If I get a 8 on a roll, then I should say Hmm... not bad & just take the money.

⇓

calculate that one

[8-20] ⇒ 14 on average

⇓

$\frac{13}{20}$ times ⇒ $\left[\frac{100}{\left(1/\left(\frac{13}{20}\right)\right)}\right]$ games

⇓

65 games

⇓

$\approx 65 \times 14 = \boxed{\$910}$ ⇒ each ~~round~~ is worth around $9.

⇓

strategy optimal answer is between 8 & 11.
parameter

- Loved iterative & collaborative problem-solving
- liked to see diff. strategies, spurred ideas.
- clarifying is good.
- communication ⇒ help us give you hints!!
- quantitative breaking down of the problem
- Perceptive and able to grasp hints leads to more exciting discussions.