If no-reroll $\Rightarrow \frac{1}{100}(1+2+3+\dots+100) = \frac{100\times 101}{2\times \cancel{100}} = \boxed{50.5}$

If 1-re-roll, we will only re-roll if we obtain less than 50.5...

$$EV = \frac{1}{100}(51+52+\dots+100) + \frac{50}{100}(50.5-1)$$

$$= \frac{1}{100}\left(\frac{100\times 101}{2} - \frac{50\times 51}{2}\right) + \frac{49.5}{2} = \frac{1}{2}\left[101 - \frac{51}{2}\right] + \frac{49.5}{2}$$

$$= \frac{75.5+49.5}{2} = \frac{125}{2} = \boxed{62.5}$$

If 2-re-rolls, we will only re-roll if we obtain less than 62.5...

$$EV = \frac{1}{100}(63+64+\dots+100) + \frac{62}{100}[62.5-1]$$

$$= \frac{1}{100}\left(\frac{100\times 101}{2} - \frac{62\times 63}{2}\right) + \frac{62\times 61.5}{100} = \frac{(10100 - 62\times 63)+(62\times 123)}{200}$$

$$= \frac{\overset{505}{\cancel{10100}} + (62\times \overset{3}{\cancel{60}})}{\underset{10}{\cancel{200}}} = \boxed{69.1}$$

If 3-rerolls, $\Rightarrow EV = 73.359$
If 4-rerolls $\Rightarrow EV = 76.31207$
If 5-rerolls $\Rightarrow EV = 78.477$
If 6-re-rolls $\Rightarrow EV = 80.1221951$
If 7-rerolls $\Rightarrow EV = 81.3977561$
If 8-re-rolls $\Rightarrow EV = 82.4121824$

...

To gain intuition from smaller subproblems

✓ You get the idea!!

Optimal Strategy to roll again if we roll $< (E-1)$.

$$\mathbb{P}(\text{rolling} < E-1) = \frac{E-1}{100}$$

If we roll $> (E-1)$, we stop & take the money

$$E[\text{given that we roll} > E-1] = \left(\frac{(E-1)+100}{2}\right)$$

$$\Rightarrow E = \left(\frac{E-1}{100}\right)(E-1) + \left(\frac{(E-1)+100}{2}\right)\left(1 - \frac{E-1}{100}\right)$$

$$\Rightarrow \boxed{E = 86.85}$$

If we get any thing less than E, then re-roll, else take & go.

$$E = \left(\frac{E-1}{100}\right)(E-1) + \frac{1}{100}\left(E + (E+1) + \dots + 100\right)$$

$$\Rightarrow E^2 - 203E + 10102 = 0 \Rightarrow E = \frac{203 \pm \sqrt{801}}{2}$$

$$\Rightarrow E \approx \frac{203 \pm 20\sqrt{2}}{2}$$

$$\Rightarrow \approx 101.5 \pm 10\sqrt{2}$$

$$\approx 101.5 - 14.142$$

$$\approx \boxed{87.358} \text{ (good estimate)}$$

Collect if $X > a-1$ and roll again if $X \leq a-1$

$$a = \frac{1}{100}\left(\lfloor a-1 \rfloor \cdot a + \sum_{k=\lfloor a-1\rfloor+1}^{100} k\right)$$

$$= \frac{1}{100}\left(\lfloor a-1 \rfloor a + \frac{100\cdot 101}{2} - \frac{\lfloor a-1\rfloor \lfloor a \rfloor}{2}\right)$$

$$\Rightarrow a = 87.3571 = \boxed{87\tfrac{5}{14}} \Rightarrow$$ continue rolling until > 87.

$$\Rightarrow \boxed{E = \frac{1}{100}\sum_{x=1}^{100} \max(x, E-1)}$$