Class 2: Sample Spaces

Thursday, September 5, 2019 11:39 AM

- 614 Schermerhorn — Tues Sept 10
- Optional HW 1 due Thurs, Sept 12 1pm Gradescope
- Textbooks

Sample Spaces

Next time: $\left(P : \Omega \to [0,1]\right)$ satisfying...

Probability space $(\Omega, \mathbb{P})$

sample space (Ω) — probability measure / function ($\mathbb{P}$)

Ω = set of all possible outcomes

Ex: roll a die $\Omega = \{1,2,3,4,5,6\}$

- Exactly one outcome "happens".

Ex: coin flip $\Omega = \{H, T\}$

2 coin flips $\Omega = \{HH, HT, TH, TT\} = \{H,T\}^2$

$\vdots$

n flips $\Omega = \{H,T\}^n$ = all sequences of H/T of length n

Ex: kid $\Omega = \{\text{boy}, \text{girl}\}$

Events: A subset of Ω, or a set of possible outcomes.

$E \subseteq \Omega$ $(\omega \in E \Rightarrow \omega \in \Omega)$

(cf. $\leq$) $\emptyset$ = empty set $\emptyset \subseteq E$

Ex: $\Omega = \{1, \dots, 6\}$ rolling a die

Ex: $\Omega = \{1, \ldots, 6\}$ rolling a die

$E = \{2,4,6\} =$ "roll an even number"

$E = \{1,3,5\} =$ " " odd " "

$E = \{3\} =$ "roll a 3"

$E = \Omega$ is an event

Note: $\{3\} \subset \Omega$ is an "event" but 3 is an "outcome": $3 \in \Omega$, $3 \not\subset \Omega$

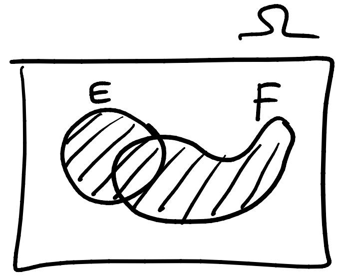
Ex: 2 coins $\Omega = \{HH, HT, TH, TT\}$

$E = \{HH, TT\} =$ "both coins show same side"

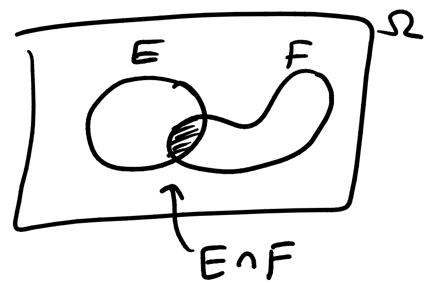
Language: When randomness is "realized", one outcome $\omega \in \Omega$ "occurs". An event $E \subseteq \Omega$ with $\omega \in E$ also "occurs".

Manipulating Events:

Let E and F be events.



Union: $E \cup F =$ "either E or F, or both"



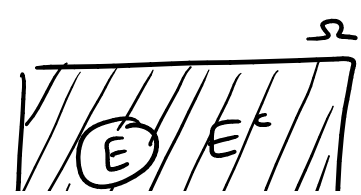
Intersection: $E \cap F =$ "E and F"

(Mathematically:

$$E \cup F = \{\omega \in \Omega : \omega \in E \text{ or } \omega \in F\}$$

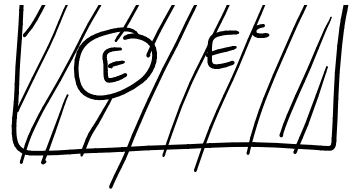
$$E \cap F = \{\omega \in \Omega : \omega \in E \text{ and } \omega \in F\}$$

)



Complement: $E^c =$ "not E"

Complement: E^c = "not E"
$= \{\omega \in \Omega : \omega \notin E\}$



Ex: Ω = roll a die $= \{1, 2, 3, 4, 5, 6\}$.

E = "even #" $= \{2, 4, 6\}$

F = "less or equal to 3"
$= \{1, 2, 3\}$

$E \cap F = \{2\}$

$E \cup F = \{1, 2, 3, 4, 6\}$

E^c = "odd #" $= \{1, 3, 5\}$

Always:
$E \cup E^c = \Omega$
$E \cap E^c = \emptyset$

Notation: If $E_1, E_2, E_3, \dots, E_n$ are events,

$\bigcup_{i=1}^{n} E_i = E_1 \cup E_2 \cup \dots \cup E_n$ = "at least one of the E_i's occurs"

$\bigcap_{i=1}^{n} E_i = E_1 \cap E_2 \cap \dots \cap E_n$ = "all of the E_i's occur"

(cf. $\sum_{i=1}^{n} a_i$)

Properties:

Associative: $(E \cup F) \cup G = E \cup (F \cup G)$

Commutative: $E \cup F = F \cup E$

same for intersection

Distributive: $E \cap (F \cup G) = (E \cap F) \cup (E \cap G)$

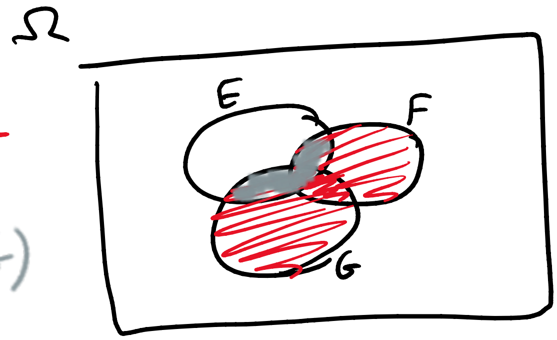
(cf. $a(b+c) = ab + ac$)

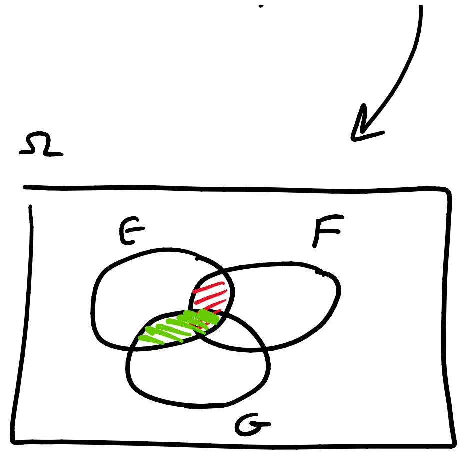
$E \cup (F \cap G) = (E \cup F) \cap (E \cup G)$

Ω

$F \cup G$

$E \cap (F \cup G)$





$E \cap F$ $E \cap G$