


7.2 A Markov chain on $\{1, 2, 3, 4\}$ has nonzero transition rates

$$q_{12} = q_{23} = q_{31} = q_{41} = 1 \quad \text{and} \quad q_{14} = q_{32} = q_{34} = q_{43} = 2.$$

- (a) Exhibit the (i) generator, (ii) holding time parameters, and (iii) transition matrix for the embedded Markov chain.
- (b) If the chain is at state 1, how long on average will it take before moving to a new state?
- (c) If the chain is at state 3, how long on average will it take before moving to state 4?
- (d) Over the long term, what proportion of visits will be to state 2?

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7.4 During lunch hour, customers arrive at a fast-food restaurant at the rate of 120 customers per hour. The restaurant has one line, with three workers taking food orders at independent service stations. Each worker takes an exponentially distributed amount of time—on average 1 minute—to service a customer. Let X_t denote the number of customers in the restaurant (in line and being serviced) at time t . The process $(X_t)_{t \geq 0}$ is a continuous-time Markov chain. Exhibit the generator matrix.



7.6 A Markov chain $(X_t)_{t \geq 0}$ on $\{1, 2, 3, 4\}$ has generator matrix

$$Q = \begin{pmatrix} -2 & 1 & 1 & 0 \\ 1 & -3 & 1 & 1 \\ 2 & 2 & -4 & 0 \\ 1 & 2 & 3 & 6 \end{pmatrix}.$$

Use technology as needed for the following:

- (a) Find the long-term proportion of time that the chain visits state 1.
- (b) For the chain started in state 2, find the long-term probability that the chain visits state 3.
- (c) Find $P(X_1 = 3 | X_0 = 1)$.
- (d) Find $P(X_5 = 1, X_2 = 4 | X_1 = 3)$.

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