Part 1: $p_1 = 0$ (since 1st number can't be a repeat)

$p_2 = \frac{1}{6}$ (2nd roll must be the same as the first)

$p_3 = P(\text{first roll} = x) \cdot P(\text{second roll} = y \neq x) \cdot P\left(\begin{matrix}\text{3rd roll either} \\ x \text{ or } y\end{matrix}\right)$

$= \left(\frac{6}{6}\right)\left(\frac{5}{6}\right)\left(\frac{2}{6}\right) = \boxed{\frac{5}{18}}$

Part 2 ~~Trick question!!~~

$$\underbrace{p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7}_{=1} + \underbrace{p_8 + p_9 + p_{10}}_{=0 \text{ (each)}} = 1$$

since there will be atleast one repeat in first 7 because there is a (d6) in hand

Part 3 $P(p_5 \mid 3 \text{ rolls taken})\ ? = P(\text{taking (ending) in 5 rolls} \mid 3 \text{ rolls taken})$

~~Using Bayes rule~~ $\left[P(A|B) = \left(\frac{P(A \cap B)}{P(B)}\right) = \frac{P(A)}{P(B)}\right.$

We know $P(A) = \frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{4}{6} = \frac{240}{6^4} = \frac{20}{108} = \boxed{\frac{5}{27}}$

$P(B) = p_3 + p_4 + p_5 + p_6 + p_7 = 1 - P(\text{two rolls}) = 1 - p_2 = 1 - \left(\frac{6}{6} \cdot \frac{1}{6}\right)$

$= \boxed{\frac{5}{6}}$

$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{48}{216} = \boxed{0.\overline{22}}$ Ans