(6.) Which number is larger? $(\log_3 2 + \log_2 3) \boxed{?} \frac{13}{6} \sim \frac{1599}{738}$

Sol ① Elementary sol^n: Comparing powers of 2 & 3 ...

$2^8 > 3^5 \Rightarrow \begin{cases} 2^{8/5} > 3 \Rightarrow \log_2 3 < 8/5 \\ 2 > 3^{5/8} \Rightarrow \log_3 2 > 5/8 \end{cases}$

$2^{11} < 3^7 \Rightarrow \begin{cases} 2^{11/7} < 3 \Rightarrow \log_2 3 > 11/7 \\ 2 < 3^{7/11} \Rightarrow \log_3 2 < 7/11 \end{cases}$

By summing: $\frac{123}{56} < \log_2 3 + \log_3 2 < \frac{123}{55} \Rightarrow \log_2 3 + \log_3 2 \in \left(\frac{1599}{728}, \frac{1599}{715}\right)$

② Straightforward sol^n $\quad a = \log_2 3 \Rightarrow a^2 + 1 \square \frac{13}{6} a$

$\Rightarrow a^2 - \frac{13a}{6} + 1 \square 0 \Rightarrow$ roots of function are $\frac{2}{3}, \frac{3}{2}$

x	$(-\infty, 2/3)$	$2/3$	$(2/3, 3/2)$	$3/2$	$(3/2, \infty)$
$f(x)$	+	0	−	0	+

we know that $a > 1 > \frac{2}{3}$, if $\log_2 3 < \frac{3}{2} \Rightarrow 9 < 8$ [Not True]

$\Rightarrow a = \log_2 3 > \frac{3}{2}$, $f(a) > 0 \Rightarrow$ $\boxed{a^2 + 1 \boxed{>} \frac{13a}{6}}$ Ans

(7.) How many cards do you expect to pull until getting an Ace?

sol

① Straightforward soln

4 Aces, n total cards $\Rightarrow f(4,n)$ = exp. no. of cards drawn until first Ace

$f(4,4) = 1$, $f(4,5) = \frac{6}{5}$, find $f(4,52)$

$f(4,n)$ — first card Ace → Draws needed: n, Prob $= \frac{4}{n}$

$f(4,n)$ — first non ace → Draws needed: $1 + f(4,n-1)$, Prob $= \frac{n-4}{n}$

$$f(4,n) = \frac{4}{n} \cdot 1 + \frac{n-4}{n} \cdot (1 + f(4,n-1)), \quad \forall n \geq 4$$

$$\Rightarrow f(4,n) = \frac{n+1}{5}, \quad \forall n \geq 4, \quad f(4,52) = \frac{53}{5} = 10.6$$ Ans

② Clever soln

4 Aces divide other cards into 5 sub-decks.

$$X_1 + X_2 + X_3 + X_4 + X_5 = 52 - 4 = 48$$

Due to linearity of expectations & symmetry $E[X_1] = \frac{48}{5}$ because $E(X_1 + X_2 + X_3 + X_4 + X_5) = 48$

$\Rightarrow$ The bounding ace added to $\frac{48}{5}$ $\Rightarrow$ 10.6 cards req. to be drawn to get an Ace.