If no-re-roll =)
$$\frac{1}{100} (1+2+3+...+100) = \frac{100 \times 101}{2 \times 100} = \frac{50.5}{2 \times 100}$$

If 1-re-roll, we will only re-roll if we obtain less than 50.5...

$$EV = \frac{1}{100} \left(51 + 52 + ... + 100 \right) + \frac{50}{100} \left(50.5 - 1 \right)$$

$$= \frac{1}{100} \left(\frac{100 \times 101}{2} - \frac{50 \times 51}{2} \right) + \frac{49.5}{2} = \frac{1}{2} \left(109 - \frac{51}{2} \right) + \frac{49.5}{2}$$

$$= \frac{75.5 + 49.5}{2} = \frac{125}{62.5}$$

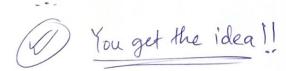
If 2-re-rolls, we will only re-roll if we obtain less than 62.5 ...

$$EV = \frac{1}{100} \left(\frac{62 + 64 + \dots + 100}{2} \right) + \frac{62}{100} \left[\frac{62.5 - 1}{100} \right]$$

$$= \frac{1}{100} \left(\frac{100 \times 101}{2} - \frac{62 \times 63}{2} \right) + \frac{62 \times 61.5}{100} = \frac{(10100 - 62 \times 63) + (62 \times 123)}{200}$$

$$= \frac{10100 + (62 \times 80)}{200} = 69.1$$

To gain intuition from smaller Subproblems



Optimal Strategy to roll again if we roll
$$< (E-1)$$
.

$$P(rolling < E-1) = \frac{E-1}{100}$$

If we roll $> (E-1)$, we stop 4 fake the money

$$E(given that we roll > E-1) = \left(\frac{(E-1)+100}{2}\right)$$

$$\Rightarrow E = \left(\frac{E-1}{100}\right)(E-1) + \left(\frac{(E-1)+100}{2}\right)\left(1 - \frac{E-1}{100}\right)$$

$$\Rightarrow E = 86.85$$

It we get any thing less than E , then re-roll, else take $(E-1) = \frac{E-1}{100}$.

$$E = \frac{E-1}{100}(E-1) + \frac{1}{100}(E+(E+1)+...+100)$$

$$E = \frac{E-1}{100}(E-1) + \frac{1}{100}(E-1) + \frac{1}{100}(E+(E+1)+...+100)$$

$$E = \frac{E-1}{100}(E-1) + \frac{1}{100}(E-1) + \frac{1}{100}(E$$