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Stability and vibrations of an all-terrain vehicle subjected to nonlinear structural deformation and resistance

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Abstract

This study intends to investigate the dynamic behaviour of an all-terrain vehicle travelling on rough terrains. A nonlinear analytical model is established for quantifying the response of the vehicle with spring and damping nonlinearities to various operation and terrain conditions. Focus of the study is on the motion characteristics of the vehicle operating on rough terrains. Stability analyses are performed for the all-terrain vehicles under the operation of surmounting large obstacles and the operation of the vehicle on rough terrain surfaces. Stability conditions are provided and stable and unstable region diagrams are plotted and analyzed with the analytical model developed. Analytical solutions are provided for weakly nonlinear dynamic systems. Numerical simulations for the motion of the all-terrain vehicle are also presented.

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Keywords: Vehicle dynamics; ATV; Numerical simulation; Vehicle dynamic stability; Nonlinear vibration; Vehicle modelling; Mathieu equation

1. Introduction

All-terrain vehicles (ATV) are widely used off-road vehicles. It is largely applied not only for entertainment but also for transportation procession, especially in the areas of military, agriculture and forestry. The ability that the vehicles can run on extremely complex and diverse road states is the most prominent character of the all-terrain vehicles [1]. The modern all-terrain vehicles are required to provide higher flexibility and suitability for driving in various operational and environmental conditions. These in turn demands a better understanding of the stability and dynamic response of the ATV subjected to the loading conditions of high nonlinearity and complexity. Since 1960s, systematic investigations on ATV motions are found in the literature with focus

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Nomenclature

a	constant system parameter
A	amplitude of the surface, matrix
c	damping coefficient
$f(k)$	effective nonlinear resistance
$f(c)$	effective nonlinear force
k	spring constant
M	effective mass of the vehicle
$p(t)$	exponential function of time
q_a	absolute vehicle motion
q_e	variation of the terrain surface amplitude
q_r	relative motion of $y - y_1$
$\bar{q}_r$	original periodic solution
$w(0)$, $\dot{w}(0)$	initial conditions
W	displacement matrix
x	perturbation value
y	displacement of the vehicle
α	coefficient relating to the nonlinearities of the effective spring force
β	coefficient relating to the nonlinearities of the damping resistance
ε	small quantity comparing to unit value
φ	phase angle
θ	phase angle
ω	frequency

on the aspects of general vehicle motion and the interactions between terrain and vehicles. Recent investigations are found on ATV's performance and handling characteristics, track force distribution, steering ability as well as ride properties [2–5]. Based on the research works available in the current literature, one may find that static analyses are commonly employed in this field, especially in ATV design practice. Linearization and simplification are also the common practices in the dynamic studies on the motion of ATV and dynamical response of ATV structures. Thorough and systematic analysis on the nonlinear motion of ATV and nonlinear dynamics response of the vehicle structures subjected to the loading and operating conditions of the real world is still in need.

It is the diversity of the road states that makes the nonlinearity become one of the most important factors that is needed to be highly taken into account during the analysis process. There are many factors that lead the ATV to display the highly nonlinear dynamic responses during operation, such as high irregularities of the ground, the nonlinear deformation of the terrain under the vehicle loading and the corresponding nonlinear response of the vehicle suspension system and undercarriage system.

The present research is to establish a methodology for investigating the stability and nonlinear response of all-terrain vehicles with spring and damping nonlinearities. A nonlinear model is to be established for nonlinear vibration and stability analyses on the basis of Lyapunov stability theory and Mathieu equation. Analytical and numerical approaches will be performed for investigating the stability and the nonlinear dynamic response of an all-terrain vehicle to the operation and terrain conditions.

2. Model development

The all-terrain vehicle studied in this research is assumed to be a symmetric body and its flexibility can be represented by a nonlinear spring and the energy absorption of the vehicle structure can be represented by a nonlinear damping system. The layout of the vehicle considered is illustrated in Fig. 1.

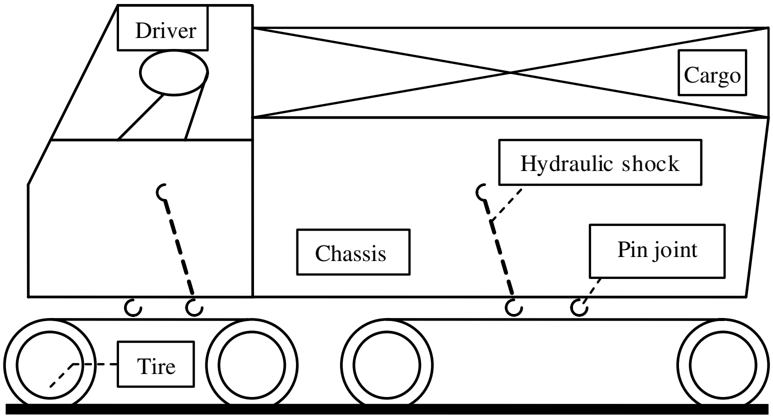

Fig. 1. Sketch of the all-terrain vehicle.

For such a vehicle system, the main components of the vehicle can be considered as solid bodies fixed with the chassis. The major portion of the vehicle mass including the mass of the solid cargo carried by the vehicle is located above the suspension system of the vehicle and there is no relative motion between the vehicle structure and the solid cargo carried. The vehicle is travelling on the terrain with a speed less than 35 km/h as it usually is. From the layout of this vehicle, it can be realized that all of the major components of the vehicle such as cab, vehicle body and solid cargo can be considered as a massive whole construction and the relative motion between the components are negligible. The chassis can be treated as a rigid frame system, which includes the engine, transmission and the other corresponding components. All these cause the considerably even distribution of the masses. The unique wheels and track system design of the vehicle makes the fairly even distribution of the ground reaction, therefore, the whole suspension system and undercarriage system is represented by a nonlinear spring and damping system. Due to the scale of the vehicle and its low driving speed, the deformation of the vehicle structure is as negligible in comparing with the vibratory motion of the vehicle. It is also noted that the angular displacement of the vehicle in pitch plane is small with respect to the vehicle's vertical displacement. The major vehicle mass vibrating on the top of the suspension system is therefore reasonably representative of the motion and stability of the vehicle. Additionally, the main concern of the present research is on the stability and nonlinear vibration of the vehicle with high nonlinearities of spring and damping. To reach the goal of the present research with a theoretical and numerical approach, and to describe the approach clearly and systematically, the nonlinear vehicle system is modelled as a vibration system governed by the following equations:

$$\begin{cases} M\ddot{y} = -f(k) - f(c) \\ f(k) = ky + \alpha k y^3 \\ f(c) = c\dot{y} + \beta c \dot{y}^2 \mathrm{sign}(\dot{y}) \end{cases} \tag{1}$$

where M designates the effective mass of the vehicle (including all of the component masses of the vehicle and the maximum cargo), $f(k)$ represents the effective nonlinear force (including the forces due to the deformations of the suspension system, tyres, track system and terrain), $f(c)$ denotes the effective nonlinear resistance generated by the damping of the vehicle (including the damping of the vehicle structure, tyres, tyres, and the suspension system), k designates the spring constant, y is the vertical displacement of the vehicle, c represents the damping coefficient, and α and β are the coefficients relating to the nonlinearities of the effective spring force and damping resistance respectively. For the convenience of theoretical analysis, Eq. (1) is transferred into the following form:

$$\begin{cases} \dot{y} = Z \\ \dot{Z} = -\frac{k}{m}(y + \alpha y^3) - \frac{c}{m}(Z + \beta Z^2 \mathrm{sign}(Z)) \end{cases} \tag{2}$$

3. Stability considerations

For determining the balance points, the following equations from Eq. (2) can be considered:

$$\begin{cases} \dot{y}=0 \\ \dot{Z}=0 \end{cases} \quad \text{or} \quad \begin{cases} Z=0 \\ \left[-\frac{k}{m}(y+\alpha y^3)-\frac{c}{m}(Z+\beta Z^2\mathrm{sign}(Z))\right]=0 \end{cases} \tag{3}$$

From this equation, the corresponding balance points can be obtained with the analyses for the following cases:

Case 1: $\alpha > 0$

The spring in this case is hardening (the effective spring is modelled as a nonlinear hardening spring). The balance point is at

$$\begin{cases} y=0 \\ Z=0 \end{cases} \tag{4}$$

Obviously this is the final balance point when the vibration is decayed down.

Case 2: $\alpha < 0$

The effective spring in this case is a softening spring. The balance point can be obtained for this case as at

$$\begin{cases} y=0 \\ Z=0 \end{cases} \tag{5}$$

or

$$\begin{cases} y=0 \\ Z=\pm\sqrt{\frac{1}{|\alpha|}} \end{cases} \tag{6}$$

According to the characteristics of the extreme value points of the corresponding conservative system's potential energy [6,7], the stable balance point can be attained if the extreme value is the local minimum value whereas the unstable point responding to the local maximum value. For determining the extreme values, let

$$\frac{\partial^2 U(y,\alpha)}{\partial y^2}=\frac{\partial f(y,\alpha)}{\partial y}=k(1+3\alpha y^2) \tag{7}$$

If $y=0$, such that $k(1+3\alpha y^2)=k>0$, the corresponding balance point is a stable centre point.

If $y=\pm\sqrt{\frac{1}{|\alpha|}}$ and $\alpha<0$, such that $k(1+3\alpha y^2)=-2k<0$. The corresponding balance point is then an unstable saddle point.

The system of Eq. (2) is a dissipative system. However, for the conservative portion of the system, the governing equation can be expressed as

$$\begin{cases} \dot{y}=Z \\ \dot{Z}=-\frac{k}{m}(y+\alpha y^3) \end{cases} \tag{8}$$

Integrating Eqs. (2) and (8) respectively, one may obtain the corresponding energy distribution equation.

For the dissipative system, one may obtain

$$\begin{aligned}\frac{1}{2}mZ^2+\int K(y+\alpha y^3)\mathrm{d}y &= h-c\int(Z+\beta Z^2\mathrm{sign}(Z))\mathrm{d}y = h-c\int(Z+\beta Z^2\mathrm{sign}(Z))Z\frac{\mathrm{d}x}{\mathrm{d}y}\mathrm{d}x \\ &= h-c\int_0^t(Z+\beta Z^2\mathrm{sign}(Z))\mathrm{d}t \end{aligned} \tag{9}$$

For the conservative system, one may have

$$\frac{1}{2}mZ^2+\int K(y+\alpha y^3)\mathrm{d}y=h \tag{10}$$

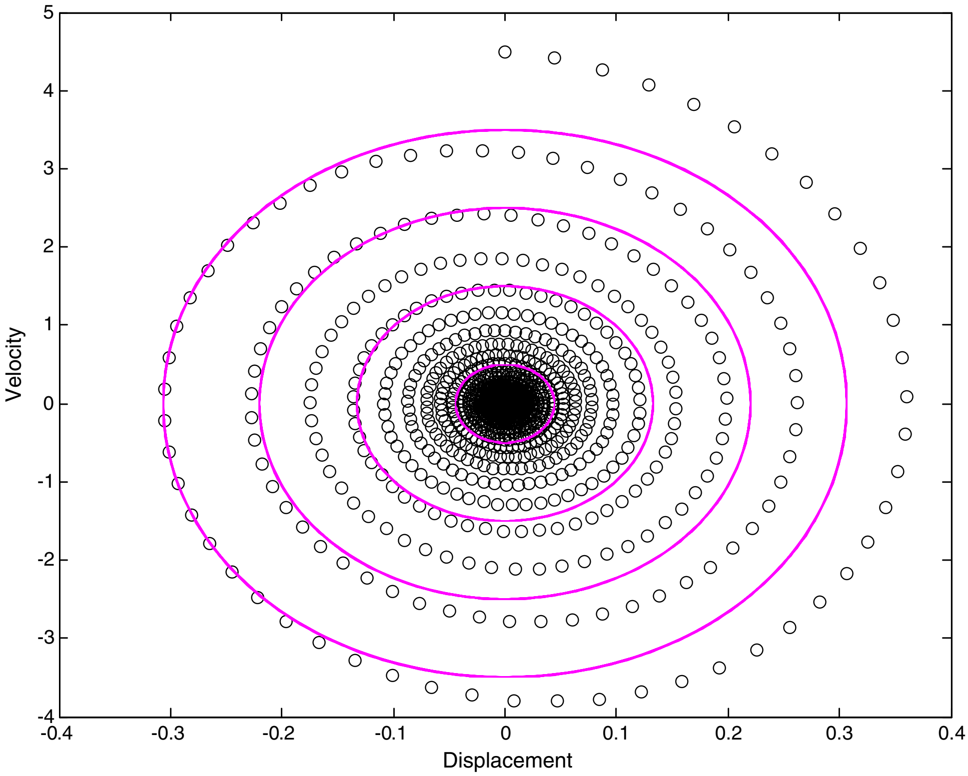

Fig. 2. Phase diagrams of conservative and dissipative systems.

The phase diagrams of the above two systems are illustrated in Fig. 2, where the elliptical curves of solid line express the equal-potential lines of the conservative system corresponding to the different energy levels, whereas the spiral curve formed by the small circles expresses the convergence process of the dissipation system to the stable state point, when no external excitations are taken into consideration.

4. Analytical solution of a weakly nonlinear system

An analytical approximate solution of free vibration of the vehicle can be reached based on the assumption of weakly nonlinearity of the system. In this case, the governing equation can be given as

$$M\ddot{y} + c[\dot{y} + \beta\dot{y}^2\mathrm{sign}(\dot{y})] + ky + \alpha k y^3 = 0 \tag{11}$$

or

$$\ddot{y} + \frac{c}{M}[\dot{y} + \beta\dot{y}|\dot{y}|] + \frac{k}{M}y + \alpha\frac{k}{M}y^3 = 0 \tag{12}$$

With weakly nonlinearity, Eq. (12) can also be expressed as

$$\ddot{y} + \omega_0^2 y + \varepsilon(2u_1\dot{y} + u_2\dot{y}|\dot{y} + \alpha\gamma y^3) = 0 \tag{13}$$

In this equation, $\varepsilon \ll 1$ and $2u_1\varepsilon = \frac{c}{M}$, $u_2\varepsilon = \beta\frac{c}{M}$, $\omega_0^2 = \frac{k}{M}$, $\varepsilon\gamma = 2\omega_0^2$. Conditions of u_1, $u_2 > 0$ are also satisfied. For this weakly nonlinear case, the perturbation method [6] can be applied. The straightforward perturbation of expansion is expressed as

$$\begin{aligned} x &= \alpha\cos\varphi + \varepsilon x_1(\alpha,\varphi) + \varepsilon^2 x_1(\alpha,\varphi) + \ldots\ldots + \varepsilon^m x_m(\alpha,\varphi) + 0(\varepsilon^{m+1}) \\ \dot{\alpha} &= \varepsilon\alpha_1(\alpha) + \varepsilon^2\alpha_2(\alpha) + \ldots\ldots \\ \dot{\theta} &= \omega_0 + \varepsilon\omega_1(\alpha) + \varepsilon^2\omega_2(\alpha) + \ldots\ldots + 0(\varepsilon^{m+1}) \end{aligned} \tag{14}$$

Submitting Eq. (14) into Eq. (13) and arranging the equation according to the coefficients of powers of ε independently, a set of hierarchical equations can be attained. The first order approximate solution can be expressed as

$$\begin{cases} x = \alpha\cos(\omega_0 t + \theta) + 0(\varepsilon) \\ \dot{\alpha} = -\varepsilon\alpha(u_1 + \frac{4}{3\pi}u_2\omega_0\alpha) \\ \dot{\theta} = \dfrac{3\varepsilon\gamma}{8\omega_0}\alpha^2 \end{cases} \tag{15}$$

From Eq. (15), it can be obtained that

$$t + C = -\frac{1}{\varepsilon}\left[-\frac{1}{u_1}\ln\frac{\left(\frac{4}{3\pi}u_2\omega_0\alpha + u_1\right)}{\alpha}\right] \tag{16}$$

$$\left(\frac{4}{3\pi}u_2\omega_0 + \frac{u_1}{\alpha}\right) = \mathrm{e}^{\varepsilon u_1(t+C)} \tag{17}$$

Finally, the solution in the following form can be obtained:

$$x = \alpha\cos(\omega_0 t + \theta) + 0(\varepsilon) \tag{18}$$

$$\begin{cases} \alpha = \dfrac{u_1}{\frac{4}{3\pi}u_2\omega_0 - \mathrm{e}^{\varepsilon u_1(t+C)}} \\ \dot{\theta} = \dfrac{3\varepsilon\gamma}{8\omega_0}\alpha^2 \end{cases} \tag{19}$$

It should be noted that the above equation is true only if the systems is weakly nonlinear and the assumptions presented previously are satisfied. As an example, let $M = 1800$ kg, $k = 160{,}000$ N/m, and

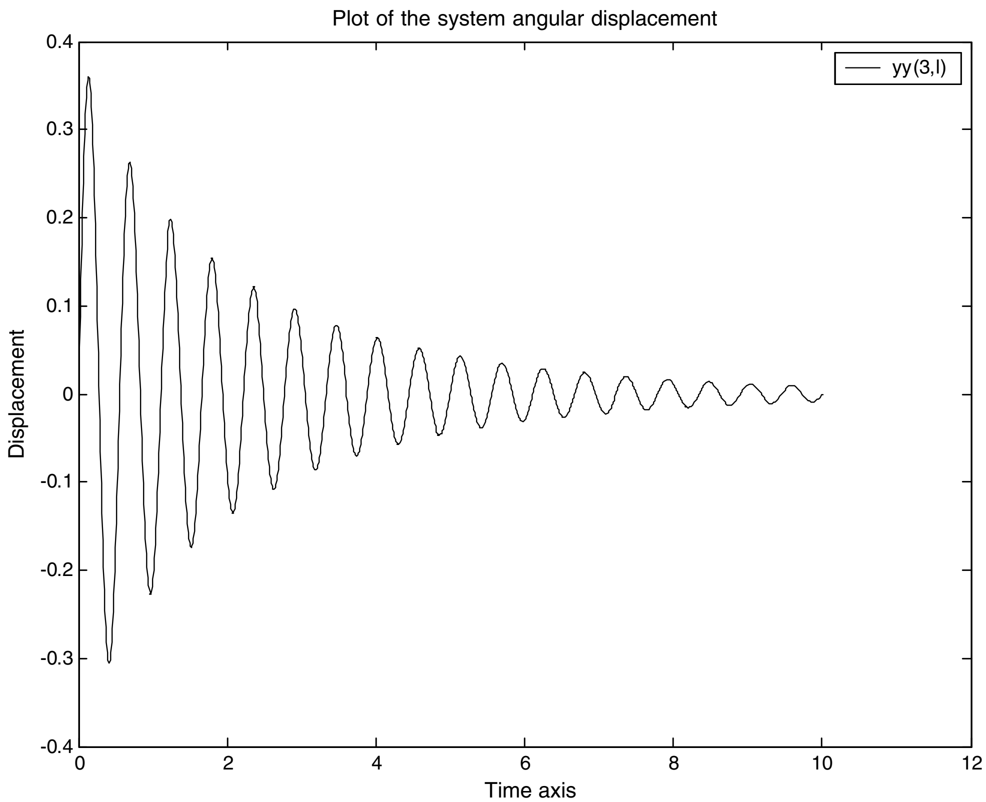

Fig. 3. Numerical simulation of a weakly nonlinear system.

$c = 6788$ N s/m, the response of the vehicle's displacement can be quantified numerically on the basis of the solution derived. With given initial conditions, a wave curve in the form of Fig. 3 can be plotted.

5. Motion of ATV on a highly rough terrain surface

The motion of the vehicle under the operations of real world is complicated and highly nonlinear, especially for the all-terrain vehicle considered. In order to reveal the dynamic responses of the nonlinear system to the external excitations generated by the complexity of the terrain surface, a reliable approach is to represent the real world terrain surface as a set of Fourier series in trigonometric functions. Assume that the nonlinear terrain surface on which the vehicle is travelling in a horizontal direction can be expressed by a sine function $y_1 = A\sin(\omega t + \varphi)$, where A is the amplitude of the surface, ω is the frequency and φ designates the phase angle.

The governing equation of the vehicle system subjected to the nonlinear excitation is then taken the following form:

$$\begin{aligned}&\{M\ddot{y} + c\lfloor(\dot{y} - \dot{y}_1) + \beta(\dot{y} - \dot{y}_1)^2\mathrm{sign}(\dot{y} - \dot{y}_1)\rfloor + k(y - y_1) + \alpha k(y - y_1)^3 = 0\\ &\{y_1 = A\sin(\omega t + \varphi) = q_e\end{aligned} \tag{20}$$

Assuming that the variation of the terrain surface amplitude is q_e, the absolute vehicle motion is q_a, and the relative motion of $y - y_1$ is q_r, the governing equation can be rewritten as

$$\ddot{q}_r + 2u(\dot{q}_r + \beta\dot{q}_r\lfloor\dot{q}_r\rfloor) + \omega_0^2 q_r + \varepsilon q_r^3 = A\omega^2\sin(\omega t + \varphi) \tag{21}$$

where $2u = \frac{c}{m}$, $\omega_0^2 = \frac{k}{m}$, $\varepsilon = \alpha\frac{k}{m}$ (ε is the small value comparing to unit value). If linear damping is further considered, the governing equation can be expressed as the standard damping Duffing equation as

$$\ddot{q}_r + 2u\dot{q}_r + \omega_0^2 q_r + \varepsilon q_r^3 = A\omega^2\sin(\omega t + \varphi) \tag{22}$$

Applying the perturbation method, the first order approximate solution can be assumed as

$$q_r = a\cos\omega t \tag{23}$$

Submitting Eq. (23) into Eq. (22) and utilizing trigonometry formulas to obtain

$$\begin{cases} y = a\cos\omega t + A\sin\omega t\\ -a(\omega^2 - \omega_0^2) + \frac{3}{4}\varepsilon a^3 = A\omega^2\sin\varphi\\ 2ua\omega = -A\omega^2\cos\varphi \end{cases} \tag{24}$$

Add the square of the second and third equation of Eq. (24); the first-order frequency response equation can be determined as the following:

$$\left[-a(\omega^2 - \omega_0^2) + \frac{3}{4}\varepsilon a^3\right]^2 + 4u^2a^2\omega^2 = A^2\omega^4 \tag{25}$$

Let $M = 1800$ kg, $k = 160{,}000$ N/m, $c = 6788$ N s/m as an example, the first order response of the vehicle's displacement can be numerically quantified. A plot with the parameters given is exhibited in Fig. 4.

For obtaining more accurate results, more terms in an assumptive solution of the form of Fourier series $q_r = \sum_{m=1}^{\infty} a_m\sin(m\omega t + m\varphi_0)$ can be employed to replace Eq. (22). With a similar procedure, the solutions and frequency response equation of higher accuracy can be acquired. It should also be noted that more terms of a general form terrain profile, $q_e = \sum_{n=1}^{\infty} a_n\sin(\omega_n t + \phi_n)$, can be used to express a terrain surface of complex profile or a terrain surface of any form as desired. As such, accurate solutions for the motion of an ATV travelling on a more complex terrain surface can be developed through a similar procedure as described for obtaining the solutions of Eqs. (24) and (25).

The resonance vibrations may occur for the vehicle moving on the simplified sinusoidal terrain surface. When the frequency of the terrain profile is close to that of the natural frequency of the vehicle system. Com-

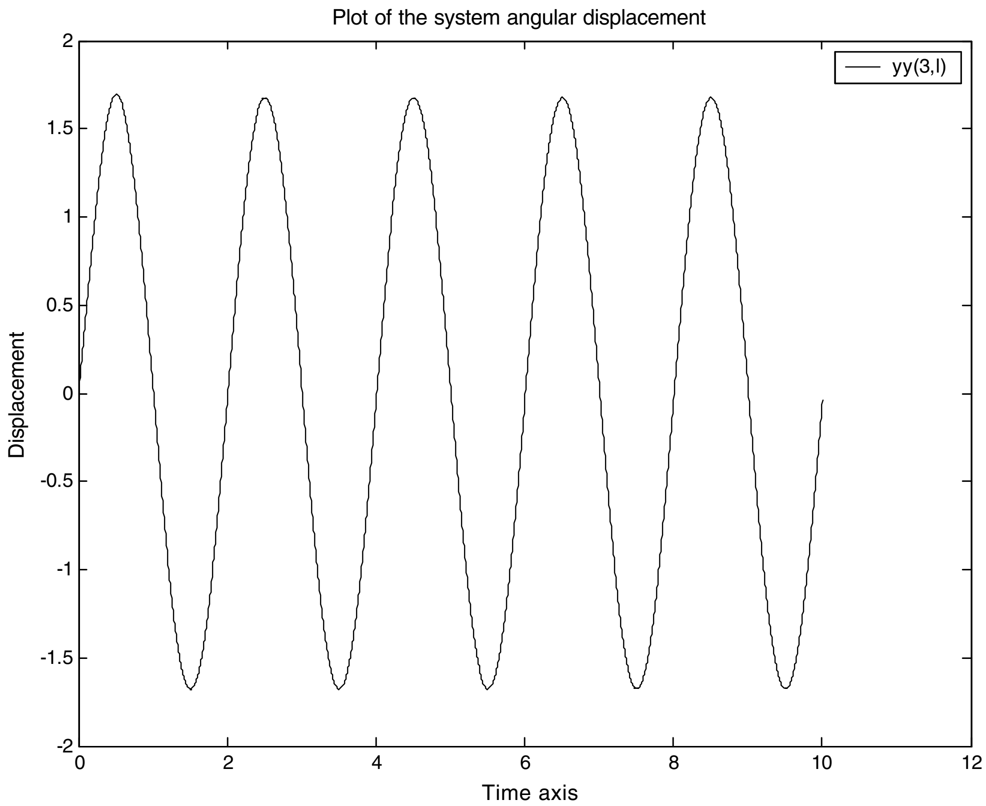

Fig. 4. Displacement vs. time of an ATV on rough terrain.

bination resonance may also occur when $\omega_0 \approx |\pm 2\omega_m \pm 2\omega_n|$, or $\omega_0 \approx \frac{1}{2}(\omega_m \pm \omega_n)$. For these cases, the vibration will be sustained in the absence of an external excitation and any additional energy input may result in growth of the free vibration and consequently bring the corresponding resonance.

6. Stability of the ATV on a rough terrain

Stability of the all-terrain vehicle plays a very important rule on the proper operation of the vehicle that moves on rough terrains. Based on Lyapunov stability theory [6], given the periodic solution a small perturbation, the perturbation equation can be developed. The stability of the 'origin point' solution is equivalent to that of the periodic solution. Setting the perturbation value as x, one may have

$$q_r = \bar{q}_r + x \tag{26}$$

where $\bar{q}_r$ is the original periodic solution, q_r represents the perturbed solution. Submitting Eq. (26) into Eq. (22), the perturbation equation can be constructed as

$$(\ddot{\bar{q}}_r + \ddot{x}) + 2u(\dot{\bar{q}}_r + \dot{x}) + \omega_0^2(\bar{q}_r + x) + \varepsilon(\bar{q}_r + x)^3 = A\omega^2 \sin(\omega t + \varphi) \tag{27}$$

This equation leads to

$$\ddot{x} + 2u\dot{x} + \omega_0^2 x + \varepsilon(3\bar{q}_r^2 x + 3\bar{q}_r x^2 + x^3) = 0 \tag{28}$$

Neglecting the high-order terms of the small value x, one obtains

$$\ddot{x} + 2u\dot{x} + \lfloor \omega_0^2 + 3\varepsilon\bar{q}_r^2 \rfloor x = 0 \tag{29}$$

As $\bar{q}_r$ is the periodic solution, Eq. (29) is actually the differential equation with linear periodic coefficients.

Setting $p_1(t) = 2u$, $p_2(t) = \lfloor \omega_0^2 + 3\varepsilon\bar{q}_r^2 \rfloor$ and let $x = we^{\int -\frac{1}{2}p_1(t)\mathrm{d}t_1}$, Eq. (29) can be rewritten as

$$\ddot{w} + p(t)w = 0 \tag{30}$$

where

$$p(t) = p_2(t) - \frac{1}{4}p_1^2(t) - \frac{1}{2}p_1(t) = \omega_0^2 - u^2 + 3\varepsilon q_r^2$$

such that

$$\ddot{w} + (\omega_0^2 - u^2 + 3\varepsilon q_r^2)w = 0 \tag{31}$$

Assuming $w_1(t)$ and $w_2(t)$ as the two particular solutions of the equation, the solution can be expressed as

$$w(t) = a_1 w_1(t) + a_2 w_2(t) \tag{32}$$

where a_1 and a_2 are constant coefficients. For periodic solutions, $w(t + T)$ is a solution of Eq. (31), or $\ddot{w}(t + T) + p(t)w(t + T) = 0$, and $w_1(t)$ and $w_2(t)$ are the prime solutions of the equation.

Assume

$$\begin{cases} w_1(t+T) = a_{11}w_1(t) + a_{12}w_2(t) \\ w_2(t+T) = a_{21}w_1(t) + a_{22}w_2(t) \end{cases} \tag{33}$$

where a_{ij} are the system parameters. This equation can also be written in a matrix form as $w(t + T) = Au(t)$, where A is a nonsingular constant matrix, named state transform matrix.

From Eq. (31), let

$$\begin{cases} \dfrac{\mathrm{d}w_1}{\mathrm{d}t} = w_2 \\ \dfrac{\mathrm{d}w_2}{\mathrm{d}t} = -(\omega_0^2 - \mu^2 + 3\varepsilon q_r^2)w_1 \end{cases} \quad \Rightarrow \quad \dot{w} = Aw \tag{34}$$

According to Lyapunov stability theory, if the nontrivial solution of Eq. (34) has a boundary, the solution is then stable. Any solution of Eq. (34) can be expressed as $w(t) = W(t)w(0)$, where $w(0)$ is the initial condition, the characteristic of stability corresponding to Eq. (34) is equivalent to the stability of the following equation:

$$\dot{W} = AW, \quad \text{where} \quad W(0) = I \tag{35}$$

Set the following initial conditions:

$$w_1(0) = 1, \quad \dot{w}_1(0) = 0 \quad \text{and} \quad w_2(0) = 0, \quad \dot{w}_2(0) = 1 \tag{36}$$

From Eq. (33), at $t = 0$, the following constant coefficients can be obtained:

$$a_{11} = w_1(T), \quad a_{21} = w_2(T), \quad a_{12} = \dot{w}_1(T), \quad a_{22} = \dot{w}_2(T) \tag{37}$$

Submitting Eqs. (36) and (37) into the characteristic equation of matrix A to obtain the following expression:

$$D(\lambda) = |A - \lambda I| = 0 \;\; \Rightarrow \;\; \lambda^2 - 2a\lambda + \varDelta = 0 \tag{38}$$

where

$$a = (a_{11} + a_{12})/2 = [w_1(T) + \dot{w}_2(T)]/2,$$
$$\varDelta = a_{11}a_{22} - a_{12}a_{21} = w_1(T)\dot{w}_2(T) - \dot{w}_1(T)w_2(T)$$

From an algebra transformation, one may have $\varDelta \equiv 1$, as such

$$\lambda_{1,2} = a \pm \sqrt{a^2 - 1} \tag{39}$$

The values of $u_i(T)$ and a_{ij} can be determined by a numerical approach.

The P–T method recently developed by Dai and Sign [8] is applied for this research. Stability of the vehicle's motion can then be analysed for the following cases:

- If $|a| > 1$, either of the λ is unbound, the corresponding periodic solution is therefore unstable.
- If $|a| < 1$, and $|\lambda_1| = |\lambda_2| = 1$, the corresponding solution is stable.
- If $a = 1$, and $\lambda_1 = \lambda_2 = 1$, Eq. (35) has a solution with period of T.
- If $a = -1$, and $\lambda_1 = \lambda_2 = -1$, Eq. (35) has a solution with period of $2T$.

Eq. (31) with the periodic solution $q_r = a\cos\omega t$ may take the following form:

$$\ddot{w} + \left(\omega_0^2 - u^2 + \frac{3}{2}\varepsilon a^2 + \frac{3}{2}\varepsilon a^2 \cos 2\omega t\right) w = 0 \tag{40}$$

For periodic solutions, set $\omega t = T$, thus $w(t) = w\left(\frac{T}{\omega}\right) = W(T)$, Eq. (40) will then take the form

$$\omega^2 W + \left(\omega_0^2 - u^2 + \frac{3}{2}\varepsilon a^2 + \frac{3}{2}\varepsilon a^2 \cos 2T\right) W(T) = 0 \tag{41}$$

Eq. (41) can be expressed in the simpler form as

$$\ddot{W} + (\delta + 2\varepsilon' \cos 2T) W = 0 \tag{42}$$

where $\delta = \frac{\left(\omega_0^2 - u^2 + \frac{3}{2}\varepsilon a^2\right)}{\omega^2}$ and $\varepsilon' = \frac{3\varepsilon a^2}{4\omega^2}$.

The above equation can be rearranged to form the standard linear Mathieu equation [9,10]. Assuming the parameter δ is fixed near the value 0, 1, 4,... in the parameter plane (δ, ε'), based on the results of Mathieu equation stability region, the boundary of the stability region can be determined by the following equations:

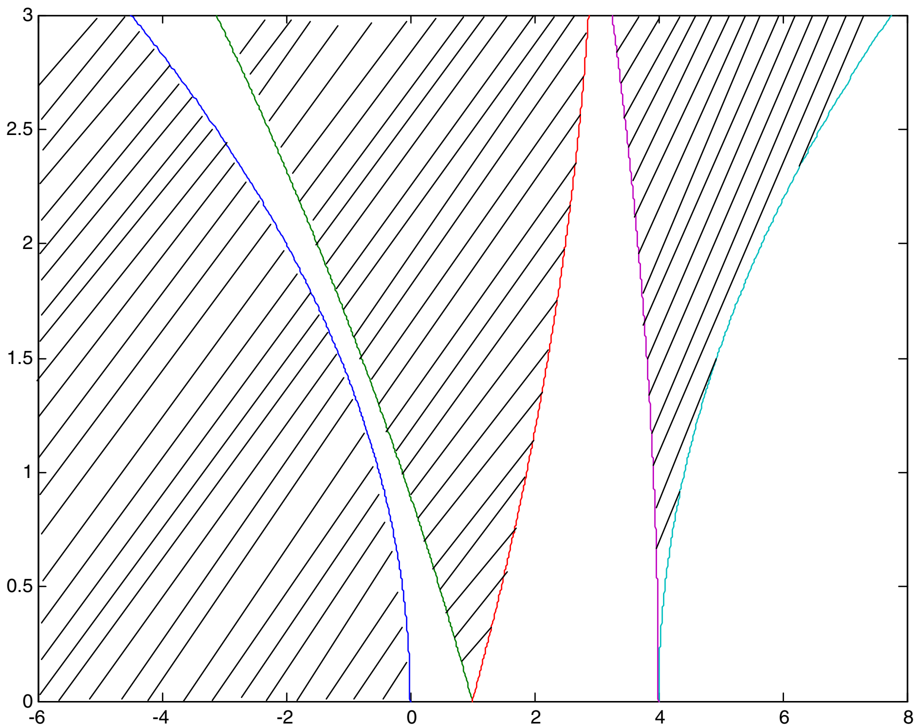

Fig. 5. Stable and unstable regions diagram.

$$\begin{cases} \delta = -\frac{1}{2}\varepsilon'^2 + 0(\varepsilon'^2) \\ \delta = 1 - \varepsilon' - \frac{1}{8}\varepsilon'^2 + 0(\varepsilon'^3) \quad \text{or} \quad \delta = 1 + \varepsilon' - \frac{1}{8}\varepsilon'^2 + 0(\varepsilon'^3) \\ \delta = 4 + \frac{5}{12}\varepsilon'^2 + 0(\varepsilon'^3) \quad \text{or} \quad \delta = 4 - \frac{1}{12}\varepsilon'^2 + 0(\varepsilon'^3) \\ \ldots \end{cases} \tag{43}$$

With the equations developed, stable and unstable regions can be plotted. Fig. 5 depicts the outline between the stable and unstable regions, where the hatched regions indicates the unstable regions.

7. Conclusive remarks

As can be seen from the discussions in the previous sections, the dynamic response of the all-terrain vehicle considered is very complex. To describe the nonlinearity and complexity of the vehicle motion, an analytical model with spring and damping nonlinearities is established. The analytical model has shown effectiveness in the stability and nonlinear motion analyses for the global motion of the ATV travelling on a highly rough terrain surface. The methodology established in the present research provides a foundation for accurately investigating the stability and nonlinear response of all-terrain vehicles moving on complex terrain profiles with any terrain surface profile desired.

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