Physical Models of Loose Soils
Dynamically Marked by a Moving Object

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ABSTRACT

This paper deals with the modeling of loose soil (sandy, muddy, etc.) When an object moves on such grounds, the object's and the soil's movement both depend the mutual physical interactions, and therefore are very difficult to achieve with kinematic or geometric models.

We use a particle-based dynamic modeler and achieve a discrete model of plasticity, which accounts for the influence of the soil on objects moving on this soil, but also for the influence of the object on the movement and the shape of the soil. Thus we have simulated soil compression and piling, vehicles leaving tire traces, spinning, skidding and even sinking. This first step is the simulation of the soil-object system at a discretization scale that can be termed "intermediate". A subsequent step consists of the simulation of a finer physical soil model in order to account for smaller-scale dynamic phenomena.

1. Introduction

The physical modeling of grounds and soils is very important for locomotion as well as for objects standing, falling or sliding on the ground. In all these cases, the nature of the physical interaction is essential for the characteristics of the resulting movement : objects do not move identically on a quasi-rigid soil or on a more or less plastic soil such as dry or wet sand. In the case of the interaction between a moving object and a rigid flat surface, it is possible to determine the reaction forces exerted by the soil on the object, simply by the detection of the application point. In this simple but frequently occuring case, it is sufficient to take into account only the reaction force exerted by the soil on the object. This method is not sufficient in the equally frequent cases where the soil-object system is deformable.

This happens :

• either when the dynamics of the contacts is complex : in the case of complex soil profiles, similar to the very deformable natural soil profiles ; this is also true when the moving object is very deformable (soft tyres or paste...).

• or when the soil is deformable. In this case, at each instant, the soil acts on the moving object, but the object also acts on the soil and modifies the state of the soil. For instance, the sand pile formed in front of a wheel, rolling on a sandy soil modifies its resistance to displacement, i.e. the force applied on the wheel. This influence depends on the shape and state of the pile and thus, open loop systems are not appropriate here.

Jimenez, Luciani and Laugier have developed several models of terrain [1], which were either rigid with canonical shapes, such as regular slopes or regular steps, or rigid with complex natural profiles, or deformable constituted of a rigid substratum covered with moving stones.

Xin Li and J.M. Moshell [2] have modeled the evolution of the shape of a sandpile by considering that a pile can be divided into homogeneous continuous vertical slices. The sand on the top of each slice can slide to a neighboring slice with friction forces expressed by the Coulomb model, as well as cohesion and interlocking forces. Their method is similar to that developed in [3].

In [4], granular piles were modeled with a physically-based multi-scale model : an intermediate-scale model composed of point masses in thresholded visco-elastic interaction models the non-linear behavior of granular piles (piling, avalanches, collapses) ; a physically-based small-scale model describes linear phenomena such as flow.

In the case of terrain, the formation of large piles, the occurrence of large avalanches and internal collapses are not the main phenomena. Plasticity and marking are more important phenomena. Avalanches and collapsing still occur, but very locally in the formation of marks and traces. This is why it is possible to optimize granular models with a high number of interactions between particles :

1. on the one hand, by taking into account the specificity of the topological structure of terrain. In this way, loose soils are composed of more or less loose layers and are set on a rigid substratum.

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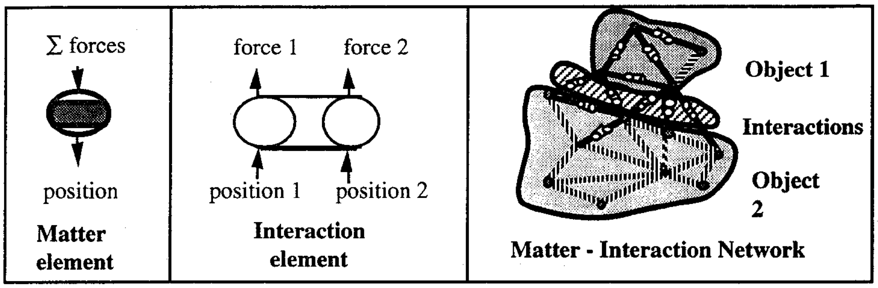


Figure 1 The principles of the physical modeler

2. on the other hand, by defining less elementary interaction functions than visco-elastic collision interactions, and more integrated interactions. The task is to directly represent the horizontal and vertical layers' plasticity.

This work tackles the modeling of loose soils of any profile, sandy or muddy, whose behavior are visco-plastic deformation, shearing and collapsing. Under the weight of a mobile, these soils may be subjected to irreversible deformation, visible as traces, whose characteristics depend on the shape, the physical characteristics and locomotion dynamics of the mobile, but also on the physical characteristics of the soil.

2. Physical characteristics of loose soils

As any object, a soil can be subject to two types of constraints : longitudinal in compression (or traction) or transverse in shearing [2], [6].

In compression, the soil packs down and the interstices between the soil particles (filled with air, water,...) are reduced and the particles move. While moving, the fluids or gas in the interstices, and especially the particles, generate reaction forces. This resistance increases in an exponential way. When the particles cannot move anymore, the particles are fully compressed.

In shearing, the matter planes move along each other. The soil resistance to the constraint is the consequence of two phenomena : particle friction and cohesion. The first one is proportional to the load, perpendicular to the shear plan, the second one is constant. The usual values range from very high for clay to "zero" for dry sand.

Moreover, as for any material, three phases in the behavior can be observed. First, for small deformations, in the elastic zone, the material returns to the initial state once the deformation force is removed. Then, in the plastic zone for a higher constraint, the material will not return to the initial position after the removal of the force. Eventually the material may break into pieces, if the force reaches a pre-defined value.

The plastic zone is obviously the zone we are interested in, since the vehicle remains on the soil.

3. Principles of the physically-based modeler-simulator *Cordis-Anima*

We briefly state the main features of the modeler-simulator, used in this work for the design and implementation of the models of soil, mobiles and mobile-soil interactions. This tool has been described and used to model different kinds of physical objects in computer animation and computer music [7], [8], [9].

The fundamental design principle of this modeler-simulator is the particle physics paradigm, based on physical interactions between point masses. In this formalism, all physical objects or set of objects are modeled and simulated as a set of point masses linked by centered interaction chains.

The most basic ones are linear elasticity and viscosity combined by finite state automata processes, allowing the description of any kind of non-linear interaction. Between masses, interactions can be put in parallel, and the combination of these effects allows the creation of very complex interactions. By these means, we can create any kind of deformable material (rigid, elastic, plastic, friable...), of complex materials (pastes, soils, wood, metal, sand, mud...) and of complex object assemblies (articulated objects, collisions, dry friction, adherence, sticking...) . With the choice of point physics, an object or a scene is calculated as a large assembly of a few types of simple algorithms that may run in parallel performing real-time simulation. Any physical object or set of objects is represented by a network (fig 1c) composed of two kinds of components : the mass component (fig 1a) and the interaction-without-mass component (fig 1b). These models are a sort of lumped constant models such as the Kirshoff networks used in electricity or electromagnetics. In principle, this modeler guarantees the best genericity with respect of what we called the "syntaxic physical consistance". In other words, if the assembly of automata in a given model is syntaxically correct, the formalism of our modeler guarantees that the corresponding behavior is physically consistent. For example, the action - reaction principle is always maintained. This allows an inexperienced person to describe physical models that will always be physically correct.

4. Modeling loose soils by variable topology particle networks

4.1. Models

Firstly we have considered that loose soils are composed of quasi-rigid grains of sand in collisions. Then, in the same way as [4] and [10], we modeled each grain by point masses linked by thresholded visco-elastic collision interactions. The plasticity of the modeled soil depends also on the interaction function used. Thus, the cohesion of the soil has to be modeled by the link element, since friction is caused by the collision interactions. Consequently, we have experimented two kinds of interactions between masses : Force profiles with one (fig 2) and two minima (fig 3).

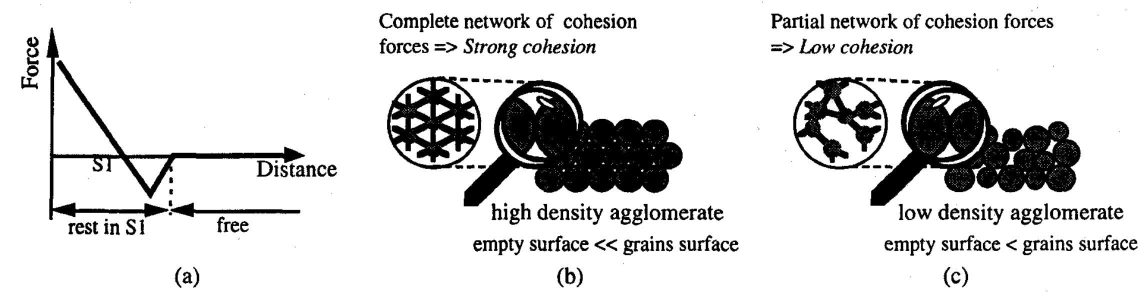


Figure 2 Cohesive collision interaction function (a) with two agglomerate densities and cohesions (b & c)

The interaction function with one minimum is given on fig 2a. Two models have been developed according to this principle. In the first (fig 2b), each pair of masses is coupled by the same cohesion interaction. In the second (fig 2c), two agglomerates are mixed, composed of masses linked by a cohesive interaction having a threshod S for the first and S' for the second. In the first case, the grains arrange regularly like atoms in a crystal, and the cohesion is strong. In the second case, the diameter irregularity creates larger interstices. The cohesion is low which makes the reorganization of the grains easier.

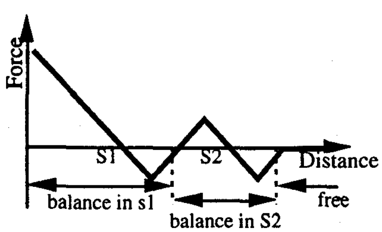


Figure 3 Cohesive collision interaction function with two states of stability

The interaction function force profile with two stable distances is displayed on fig 3. These two stable states enable us to model grains with two different virtual bulks. It is possible to adjust the stiffness of the stability corresponding to the larger radius, to obtain a more or less fragile agglomerate. Furthermore, once compressed, the grains offer a higher cohesion and thus a higher resistance. This behavior is rather realistic for pulverulent soils.

4.2. Experiments and results

To test the penetrability of these different agglomerates, a ball of various weight was put on the agglomerates. The different phases of a material behavior under a load have been observed :

• For a low weight, the agglomerate behavior is in the elastic zone. It is simply compressed under the ball. If the ball is removed the agglomerate returns to the initial state ;

• For a heavy weight, the pressure overcomes the agglomerate's breaking limit. The ball sinks totally ;

• Between both states, the ball penetrates the agglomerate, without sinking : this state is called the plastic state. Once the load is removed, the agglomerate does not return to the initial state.

At last, for the plasticity test, the load of the ball is removed. This enables us to observe the agglomerate evolution with no constraint and to determine its plasticity, i.e. its capacity to go back to the initial state.

General results are as follows :

• model 3b: low penetrability, average plasticity (quite rigid soil)

• model 3c: high penetrability, low plasticity (sandy soil)

• model 4: average penetrability, high plasticity (pulverulent soil, highly fragile and crumbly)

5. Layered plastic soils

The advantages of the previous models are simplicity and the possibility to simulate collapses avalanches and the sinking of a moving object. The drawback is the relatively high computational cost. However, if the soil does not reach the breaking threshold, it is not necessary to rearrange the soil's masses, and the plasticity can be modeled by interaction elements (figure 4). Then the model can be re-expressed as plastic interactions between particles in a constant-topology network (plastic carpet). This may be computed more efficiently.

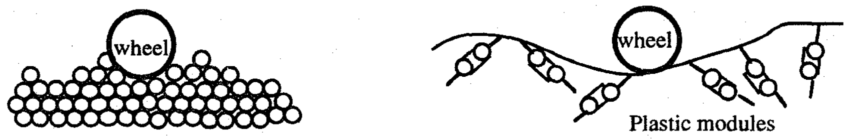


Figure 4 Granular modeling of a loose soil : with particles and with a plastic carpet

5.1. Plastic interaction model

The model used here was first developed in [11]. In plastic materials, as soon as the constaint Q exceeds a Q_m value, called *elasticlty limit* or *plasticity threshold* (figure 5.a), the loading curves OB or unloading curves BB' are not the same any more, whereas in elastic phenomena, they are identical. On figure 5, q represents deforamtion and Q represents constraints.

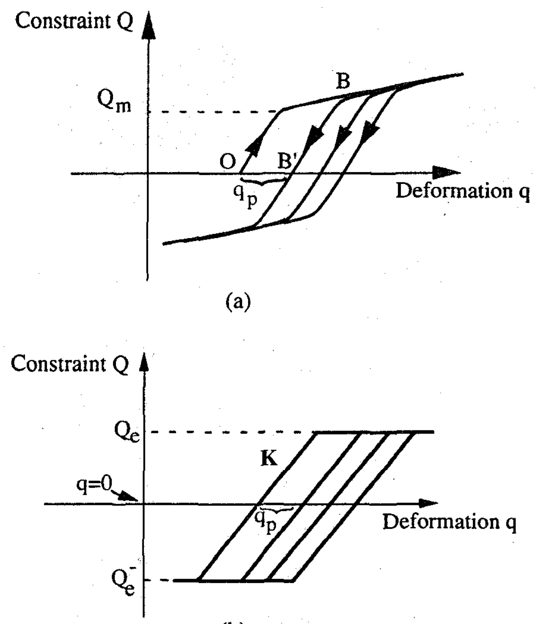


figure 5 : Plastic behavior models

The Q=f(q) characteristic presents hysteresis and after the unloading, a *permanent* or *plastic* deformation q_p still remains. As long as we perform loading/unloading cycles while staying beneath a new Q_m constraint threshold, the behavior is elastic again : the current point stays on the BB' curve (figure 5.a). If the load is increased further, the *breaking threshold* of the material is reached. In natural materials, the plasticity threshold Q_m itself may increase with q.

However, the *ideal elasto-plastic model* shown on figure 5.b is sufficient for the modeling of loose soils as long as the breaking threshold is not exceeded.

Let us associate the constraint Q with the norm of the axial interaction force : $Q = f = \| \vec{F} \|$, and the deformation q with the distance variation between both points : $q = D - L_0$. (L_0 is a constant which represents the nominal value of the distance D). The plasticity threshold Qm is represented by constant values Qe for positive constraints and Qe^- for negative constraints.

Let $S_1 = \frac{Q_e}{k}$ and $S_2 = \frac{Q_e^-}{k}$ and P_1 and P_2 be the positions of two masses. (k represents the stiffness of the interaction in its linear part)

The <LIA> algorithm, which simulates a plastic interaction between two masses, will yield two opposite forces (F, -F) such that :

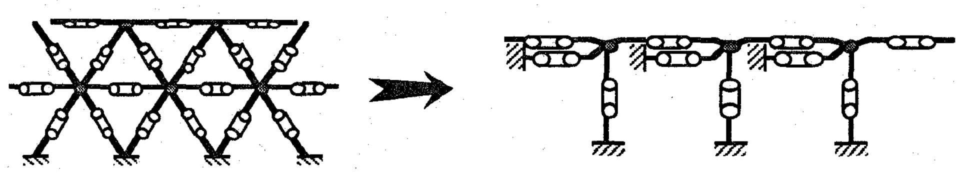
$$\overrightarrow{\Delta P} = \vec{P}_1 - \vec{P}_2, \quad D = \| \overrightarrow{\Delta P} \|, \vec{u} = \frac{1}{D} * \overrightarrow{\Delta P},$$
$$q = (D - S)$$

if ($q > S_1$), $f = Q_e$; $S = D - S_1$;
else *if* ($q < S_2$), $f = Q_e^-$; $S = D - S_2$;
else $f = k * q$;
$\vec{F}_2 = -\vec{F}_1 = f.\vec{u}$;

5.2 Topology of layered plastic soils

The first new model we tested has a triangular and regular physical topology. At the nodes, there are point masses linked according to figure 6.a by the plastic interactions described previously.

Only the interactions between neighboring masses were kept. With this type of physical topology, the



(a) (b)

Figure 6 Stratified and stitched soil

behavior of the soil is correct under impacts or rolling. However, because of the multi-layer structure, it is difficult to dissociate the transverse effects (shear) from the longitudinal ones (compression) and folds appear when mobiles are sliding on the soil. Moreover, the soil stratification is not relevant. Indeed, the complex behavior of the subsoil can be introduced in the link elements, and that leads to the removal of the soil's lower layers. Then a single layer seems to be sufficient such as is shown on (fig 6.b).

A shear and compression resistance has to exist in the subsoil and the contact surface has to resist to dislocation.

Thus, the necessary interactions are as follows :

• Cohesion of the surface layer ⇒ plasticity with the neighboring masses ;

• Shear resistance with the subsoil ⇒ horizontal plasticity with a fixed point ;

• Compression resistance with the subsoil ⇒ vertical plasticity with a fixed point.

This model enabled us to obtain correct soil behavior for any movement (impacts, rolling, sliding), as well as to optimize the calculation. This structure is very simple. It enables us to create any kind of natural profiles of grounds. It is also possible to set the subsoil more or less deeply to model a very thick loose soil, or on the contrary, a ground where the subsoil rocks come to the surface. The choice of parameters is easy, as it is easy to modify the interactions to improve the characteristics. Indeed, it is possible to determine the horizontal plasticity according to the vertical ones to improve the shear resistance as a function of the compression.

5.3. Experiments and results

The principles of the modeler-simulator allow the modeling and simulation of complex heterogeneous objects having articulate rigid structures, deformable parts and physical motors. The mobiles implemented in this work are various types of vehicles [12], having various chassis structures, deformable tires and different kinds of propulsion.

The vehicle chassis are articulated rigid objects composed of masses linked together with hard visco-elastic interactions. The joints can be constructed with two masses defining a rotation axis. The deformable tires are defined in the same way.

The performed simulations produced realistic phenomena (figures 7 and 8): wheel traces on the soil, packing of the soil under a spinning wheel. Moreover, the easy modeling of the vehicle allows fast modifications : this allows to see the influence of various factors on the obstacle-climbing ability : mass distribution, length and proportion, ground clearance, control, etc.

The models realized thanks to the plastic carpet made it possible to simulate vehicles on various profiles of soils, in several situations of obstacle-climbing. The number of samples constituting the soil is 200 masses for 2D soils (fig. 9) and 28×28 masses for 3D soils (fig. 8), and the deformation of the soil is very natural.

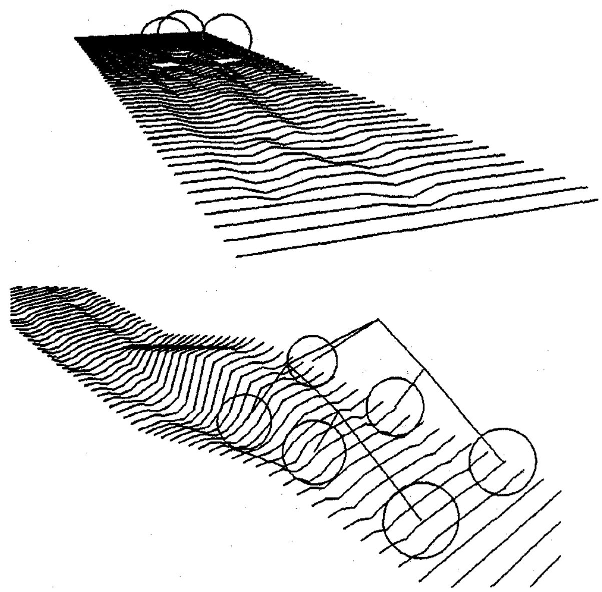


Figure 7 Examples of vehicles and profiles of simulated soils

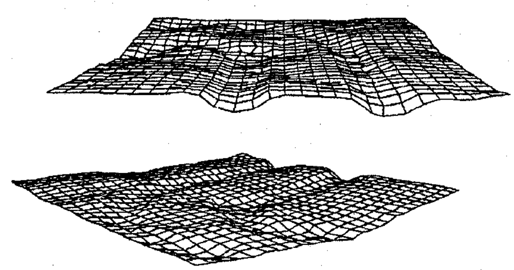


Figure 8 Examples of traces left by a four-wheels vehicle on a 3D soil

6. Refinement : principles and modeling

6.1 Physical refinement model for loose soils

The previous sections showed very plausible simulations of 3D plastic soils obtained with 28×28 masses linked by non-linear interactions. However, realistic visualization, and especially the simulation of small-scale dynamic phenomena such as local avalanches, requires the introduction of additional physical points. For the sake of optimization, we account for the large-scale and small-scale dynamic phenomena with two separate models. The large-scale phenomena have already been successfully simulated. The next step is *refinement*, which invokes the modeling of small-scale phenomena and the modeling of

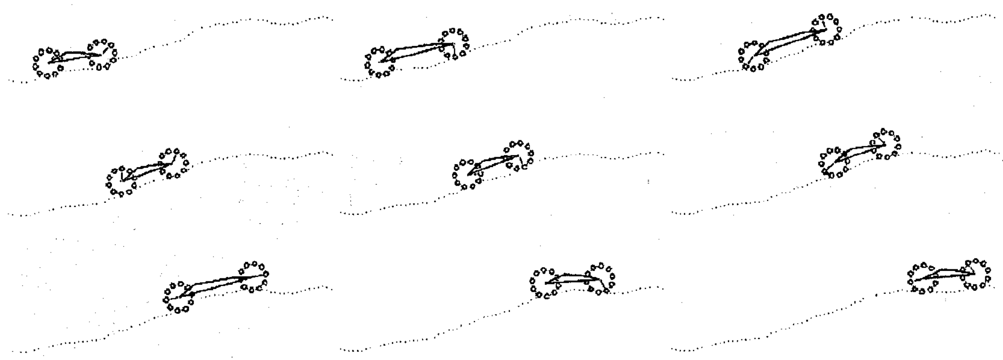


Figure 9 Progression of a climbing vehicle propelled by a peristaltic movement (one second interval)

the interaction with the large-scale model. Since refinement must account for physical phenomena such as small matter slides, it cannot be performed by geometrical or lighting models but require also physically-based modeling.

We name the soil-object physical system mentioned above, the *control network.* It is composed of all the masses of the soil and the moving objects that mark it. We also define a *refinement network,* which is a physical object in physical interaction with the control network. The former is a sparse network of 3D physical masses and the latter is a dense network of physical masses. The area of the refinement network is the same as that of the control network. The structure and the physical parameters of the refinement network will be determined in order to account only for the small-scale phenomena that were not rendered by the control network.

A few remarks :

1. For small-scale deformations, non-linear phenomena may be fairly approximated by linear models.

2. We aim at the representation of a soil considered as a surface. If we limit our concern to soils whose surface can be described by a scalar field of two variables z=f(x,y), it is possible to limit the movement of the refinement network to a 1D vertical movement and still account for horizontal movements.

Therefore the refinement network can be modeled as a surface composed of a dense network of 1D point masses linked by linear physical interactions, characterized by vertical deformations and horizontal propagation. These masses do not represent actual grains of sand, but simply the deformation of the surface at that point. Thus it is possible to render a horizontal movement of matter on the soil, even if none of these masses had moved horizontally.

6.2 The structure of our refinement model.

The refinement surface was also achieved with the Cordis-Anima modeler. It is a network composed of n×m physically simulated masses in 1D vertical motion. This surface is placed between the soil and the mobiles rolling on it.

For a given $i \in [1,n]$ and $j \in [1,m]$, mass m_{ij} is characterized by two dynamic variables. $x_{ij}(t)$ represents its 1D position, and $f_{ij}(t)$ represents the vertical component of the sum of the forces exerted on m_{ij}.

The physical properties of the refinement surface are expressed by 1D linear spring-damper interactions F_p (as in "point-to-point" interaction) linking each mass of the refinement network to its four closest neighboring masses. F_p is characterized by a stiffness k_p and a viscosity z_p.

The masses may have a common rest level, and move in a common viscous environment. This is achieved by linking each mass to a rigid substratum by another 1D linear spring-damper interaction function F_g (as in *ground* interaction) characterized by a stiffness k_g and a viscosity z_g. This rest level is associated with x=0.

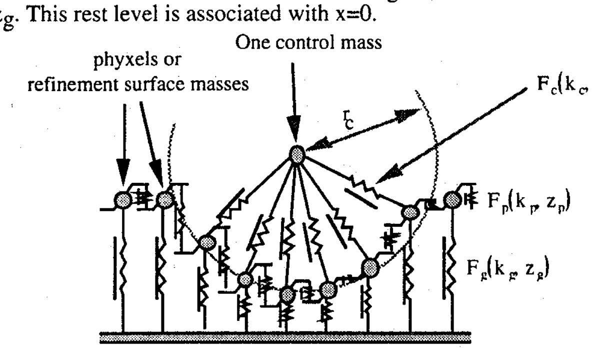


Figure 10 A cross section of a small part of the refinement network being deformed by one control mass.

The interaction between the masses of the control network and those of the refinement network is modeled by 3D non-linear visco-elastic collision interaction functions F_c (as in *control* interaction) linking all the refinement masses with those of the control network. F_c is characterized by a stiffness k_c , a viscosity z_c and a collision threshold r_c. This whole structure is summerized on figure 10.

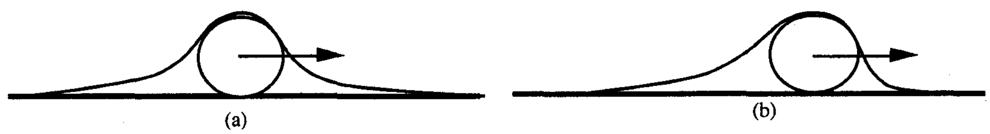


figure 11 Dynamic low-pass filtering of an object in motion. The surface has (a) a low time constante (b) a high time constant

7. The physical behavior of the refinement surface

7.1. Dynamic filtering

Physical refinement with this type of surface is an implicit physical filtering.

The first characteristic of this kind of filtering is that the filter response is a decreasing exponential function whose parameters depend on the stiffnesses of the refinement network (k_g and k_p) :

Let us assume that the external forces (F_c) interact with only one masse m_q of the refinement network. The deformation field at rest is expressed by an exponential function (1) and because the refinement network is a linear system, the global deformation field, for an arbitrary force field is a weighted sum of exponential functions (2).

$$\forall i \quad x_i(t) = x_q h_{\alpha_i}$$

$$\text{where} \quad h_{\alpha_n} = e^{-\alpha.|n|} \qquad \alpha > 0 \tag{1}$$

$$x_i(t) = C.\sum_{j=-\infty}^{+\infty} f_j.h_{\alpha_{i-j}} \tag{2}$$

$$C = \text{Constant}$$

When the soil is in contact with the refinement network, (i.e. when F_c can be considered as a mere linear interaction function) the force profile is a faithful image of the soil profile. Thus according to (2) the refinement network performs low-pass filtering. In all others cases, the non-linearities of F_c enable to model some local non-linear phenomena. (see paragraph 7.2)

The second characteristic of this type of filtering lies in the fact that it is dynamic. This means that the final rest position is reached more or less rapidly, possibly with oscillating modes (figure 11.a) The rest position may also not be reached, if such is the intention of the modeler (figure 11.b). These transient behaviors are controlled by the stiffnesses and viscosities of the refinement surface.

7.2. The approximation of granular flow phenomena

a. Reference behavior

As we mentioned above, granular materials may form piles. They are characterized by a maximum slope angle. This slope may, temporarily and locally, grow greater than the maximum value. But this is a transient state that causes small avalanch phenomena, after which the slope decreases and returns to a value that is smaller than the maximum slope. [4]

Furthermore, granular material may accumulate against a stable obstacle, and in this case, the contour of the pile is locally the same as the contour of the obstacle, and the "slope" of this contour may be arbitrarily high. These are the reference phenomena that we aim to simulate.

b. The maximum characteristic slope

The non-linear nature of F_c makes it possible to account for a few non-linear phenomena as those stated above. If the rigid substratum is placed far below the soil level, such that the variation of the soil level is small in comparison with its global level L, abrupt soil irregularities produce, on the refinement surface, irregularities that are all characterized by a characteristic slope α.L (figure 12.a). This is due to the exponential characteristic of the refinement network. If the surface is pressed on both sides by the soil and a mobile (a wheel for example), the slope may grow far greater than this maximum slope (figure

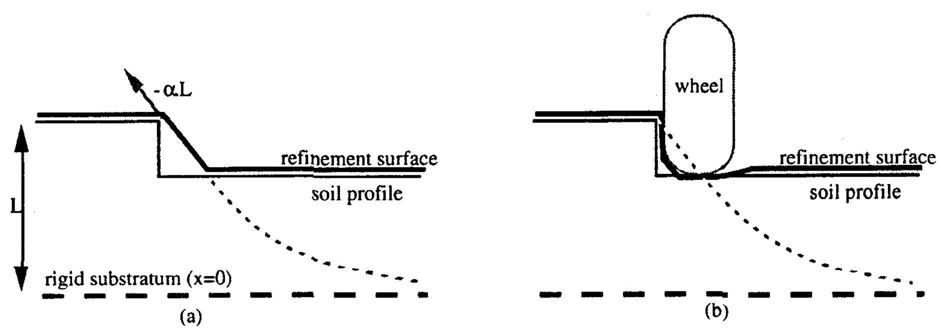


Figure 12 Soil profile on an abrupt irregularity, without (a) or with (b) a marking mobile

12.b) . But as soon as the mobile moves away, the surface recovers its initial slope, with various possible intermediate states and dynamic phrasings.

c. The dynamic deformation of the traces

The physical model of the surface in itself is very simple. However it produces very complex dynamic behavior. Therefore in order to control this behavior we can decompose the movement of the surface in the movement of N independent "deformation modes".

> **NB.** This is not a part of the model, but simply the way we analyse the produced phenomena and choose the correct parameters.

The set of positions and forces of all the masses of our refinement model form respectively two vectors X and F. The dynamic behavior of the surface can be expressed by :

$$M.\frac{\partial^2 X}{\partial t^2} + Z.\frac{\partial X}{\partial t} + K.X = F_c \qquad (3)$$

where F_c represents the external forces. M, K and Z represent respectively the mass, viscosity and stiffness matrices of the surface. If K and Z commute, (which is the case for our surface, since it is homogeneous) then M, K and Z can be diagonalized in the same basis. In this new basis, (3) can be decomposed into N independent scalar second order differential equations (E_i), where N is the number of masses in the surface. These N equations characterize each, one "deformation mode".

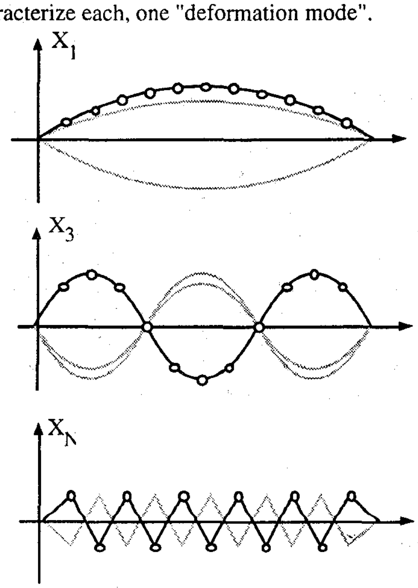


figure 13 The shape of the fundamental mode, the second and the N-th harmonic of a string

Each mode is defined by a shape of the surface. figure 13 displays the shape of the fundamental mode , the first second and the N-th harmonic in the case of a string. At each instant, the shape of our surface (vector X) is the sum of the shapes (X_i) of all the deformation modes of the surface. The higher order modes account for small deformations and the lower order modes account for global deformations.

The amplitude of each deformation mode is given by the N differential equations (E_i). This means that this movement is that of a mass attached to a fixed point by a damped spring. The parameters of this system (mass, stiffness viscosity) are given by the equations (E_i).

In our case, the i-th order mode can be characterized by a mass m (the same as that of each mass) and by stiffness k_i and viscosity z_i.

$$k_i = k_g + \lambda_i.k_p$$
$$z_i = z_g + \lambda_i.z_p \qquad (4)$$

where the (λ_i) are an increasing sequence of real numbers that range from 0 to 8.

This enables us to control the time behavior of the modes by controlling the associated physical parameters.

Figure 14 describes the soil behavior that we wish to reproduce : the soil profile before it was marked by the wheel (dashed line), immediately after it was marked (gray) and at rest (solid black). This rest profile is an approximation of the soil final profile after a small local avalanch.

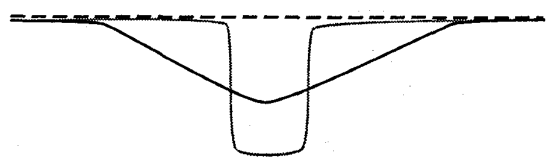


Figure 14 Soil profile before, during, and after interaction with the mobile.

The first step (marking) corresponds to the excitation of high-order and low-order deformation modes. The second phase (the deformation of the traces) corresponds to the damping of higher order modes.

In order to obtain this type of behavior, the higher-order modes must be critically damped whereas the lower order modes must by over-damped.

According to (4) this requires a low value for k_g and a high value for k_p. The value of zg is not of great importance as long as (m,kp,zp) produces a critical behavior.

Figures 15 represents the result of the refinement of the large-scale simulation described on figure 8, where the soil was marked by a four wheel vehicle. The vehicle and the control network are not displayed on figure15 but the wheels can clearly be distinguished. This simulation was achieved with a sampling rate of 150 Hz. The parameters are:

k_g = 0.0 N/m	z_g = 0.01 N/m.s^{-1}	
k_p= 3 N/m	z_p= 0.05 .N/m.s^{-1} .	
k_c = 1 N/m	z_c = 0.01 N/m.s^{-1}	r_c = 1 cm

8. Results and conclusion

Both types of simulations (large-scale and refinement) were carried out by the same physical modeler-simulator on Silicon Graphics Indigo stations. (Time and space costs of both algorithms are linear in the number of masses) The 3D soil displayed on figure 14 is composed of 28×28 soil masses and was refined by 175×175 = 30 625 masses. The large-scale runs at 3 seconds/frame and the refinement runs off-line at 25 seconds/frame.

The essential achievement of this work lies in the optimization of a complex physical model by the use of a separate specific model for each type of phenomenon.

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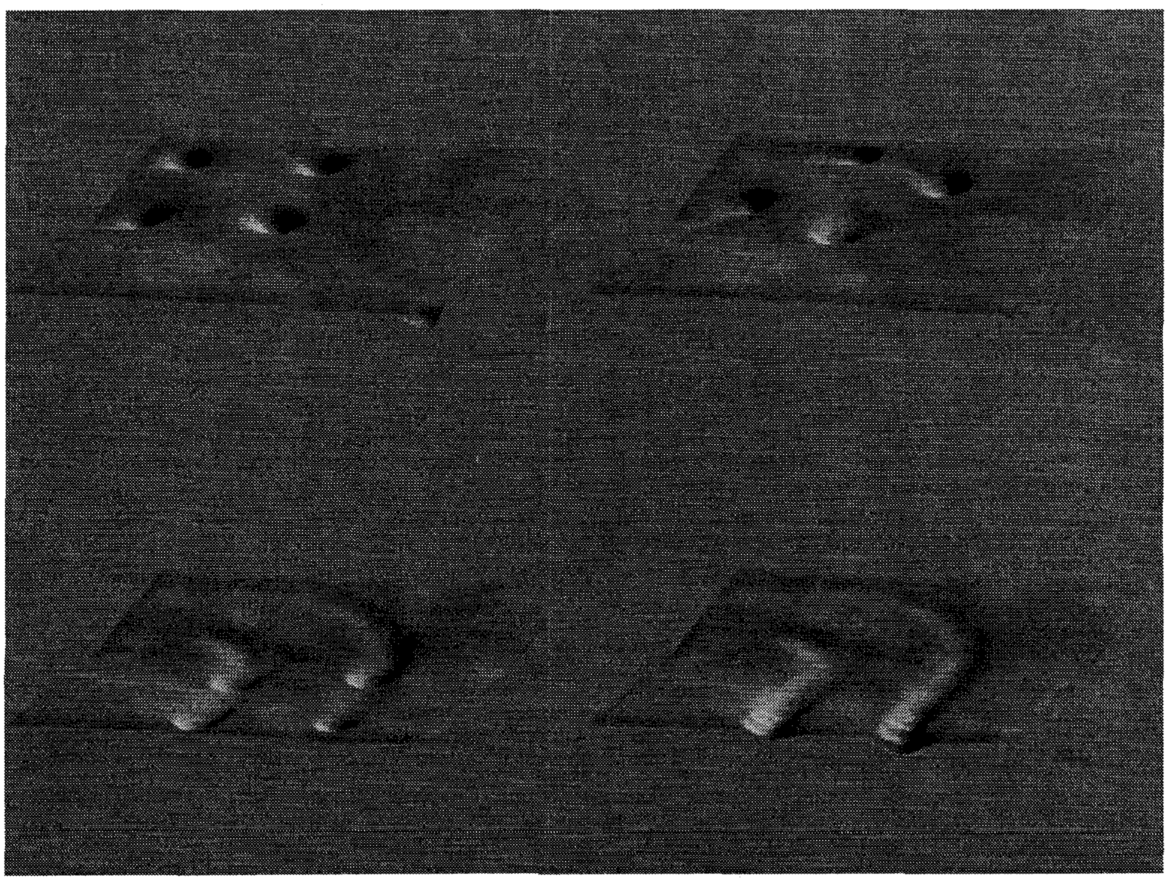

figure 15 A refined soil of 175×175 masses