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***SOIL MECHANICS***

# MESH-LESS SMOOTHED PARTICLE HYDRODYNAMICS IN MODELING OF SOIL BEHAVIOR

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**H. Niroumand[1], M. E. M. Mehrizi[2], and M. Saaly[3]**
[1]Buein Zahra Technical University, Qazvin, Iran; [2]Islamic Azad University of Central Tehran Branch, Tehram, Iran; [3]Amirkabir University of Technology, Tehran, Iran.

*In some cases there is no possibility for modelling of the soil behavior by traditional finite element method or other mesh-based techniques. It has been found that as a completely Lagrangian and mesh-free technique, smoothed particle hydrodynamics (SPH) provides advanced approaches for simulation of soil materials. The advantages of SPH are its high power, simplicity of concept, relative simplicity in combination with modern physics, and particularly its potential in the study of large deformations and failures.*

## Introduction

The Finite Element Method (FEM) is among the standard numerical methods for simulation of material deformations, but Mesh-based techniques cannot deal with some cases. According to the findings, as a completely Lagrangian and mesh-free technique, smoothed particle hydrodynamics (SPH) can present a cogent solution to the aforementioned problem [1, 2]. The authors have conducted another review research into the application of the SPH method in the simulation of failure and soil and rocks exposed to great pressure, in which the principles of the SPH method have been demonstrated explicitly.

## Simulation of Soil Material Behavior by SPH

The SPH model was proposed by Bui et al [3]. An ideal elasto-plastic model based on the Mohr-Coulomb failure criterion was applied for describing soil behavior in the form of SPH. Also, dry soil and saturated soil modeling was introduced through the simulation of dry soil collapse while erosion occurred during the drilling process by a water jet. As mentioned in [3], the outstanding hindrance to the application of SPH, which is particle deficiency around the solid boundary, has been resolved by ghost particles that share all properties of real particles except the velocity. However, ghost particles are not a proper method for simulation of the characteristics of soil material due to the friction force between particles. In this regard, a new approach for estimating the frictional boundary has been presented in this research. It is mainly based on the assumption that on the solid boundary of the model, a line of virtual particles have been located that will not contribute their characteristics to the SPH summations, and the premise that real particles and virtual ones have a virtual radius equal to half of the smoothing length. Thus, the interactions between each real particle and the virtual one can be determined by a radial force and a tangential force. The radial force

$$F_n = k_n \delta_n^{3/2}, \tag{1}$$

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where $\delta_n$ is the overlap distance between two particles in the radial direction, and $k_n$ is a constant:

$$k_n = \frac{4}{3}\frac{1}{((1-\theta_1^2)E_1 + (1-\theta_2^2)E_2)}\sqrt{\frac{r_1 r_2}{r_1 + r_2}}, \tag{2}$$

where $r$ is the radius of each particle, $\theta$ is Poisson's ratio, $E$ is Young's modulus.

The tangential force

$$F_t = k_t \Delta\delta_t, \tag{3}$$

where $k_t$ is a constant that can be approximately be taken equal to $k_n$, and $\Delta\delta_t$ is the relative displacement in the tangential direction.

The model consists of almost 2500 soil particles arranged in square volume, and frictional boundry condition for solid boundaries. After implementation of the soil collapse, it can be observed that the reposed angle of soil collapse conform pretty well to the Mohr-Coulomb failure criterion. The numerical results obtained of this work revealed that SPH can simulate the large deformations without any problem and present reasonable results of soil behavior simulation. Although the results of this paper are not quantitatively compared with those of empirical data, the proposed design is stable and offers acceptable results. The preliminary results of this article are promising; however, to enhance its applicability, it is required to improve constitutive modeling of the soil material.

Bui et al. [4] simulated large deformations and post-failure of the geo-materials in an SPH framework. The Drucker-Prager model was implemented for describing the behavior of elastic-plastic soil using the relevant and irrelevant plastic flow laws in the SPH code. The Drucker-Prager yield criterion applied in this study can be presented based on the following equation:

$$f(I_1, \mathrm{J}_2) = \sqrt{J_2} + \alpha_\varphi I_1 - k_c = 0; \tag{4}$$

$$\alpha_\varphi = \frac{\tan\varphi}{\sqrt{9 + 12\tan^2\varphi}}; \tag{5}$$

$$k_c = \frac{3c}{\sqrt{9 + 12\tan^2\varphi}}; \tag{6}$$

$$I_1 = \sigma^{xx} + \sigma^{yy} + \sigma^{zz}; \tag{7}$$

$$J_2 = \frac{1}{2} S^{\alpha\beta} S^{\alpha\beta}, \tag{8}$$

where $\alpha_\varphi$ and $k_c$ are the Drucker-Prager constants, which can be determined by the cohesion $c$ and friction angle $\varphi$, $I_1$, and $J_2$ are the first and second invariants of the stress tensor.

One of the main problems of SPH simulation is particle deficiency near the boundary. The prevailing method for addressing this problem is virtual particles. Bui et al. [2] implemented three layers of virtual particles, called boundary particles in the boundary regions parallel to the solid wall. The results obtained in this work shows that the main SPH method, which was successfully applied for a wide range of problems, is not able to solve elastic-plastic flows of soils due to the elastic instability of the SPH. These numerical instabilities might result in unrealistic failures and classification of the particles in SPH simulation. For noncohesive soils, this instability is not serious and can be dealt with by removing the tensile cracking induced by the constitutive model of the soil and then representing the real behavior of the soil. Nevertheless, a serious elastic instability was observed for application of SPH in cohesive soils, which calls for a specific treatment. An artificial stress method was utilized for elimination of numerical instabilities of SPH in cohesive soils. A number of numerical tests were carried out and compared using the FEM modeling and laboratory tests. Shear band development within the failure, which is generally

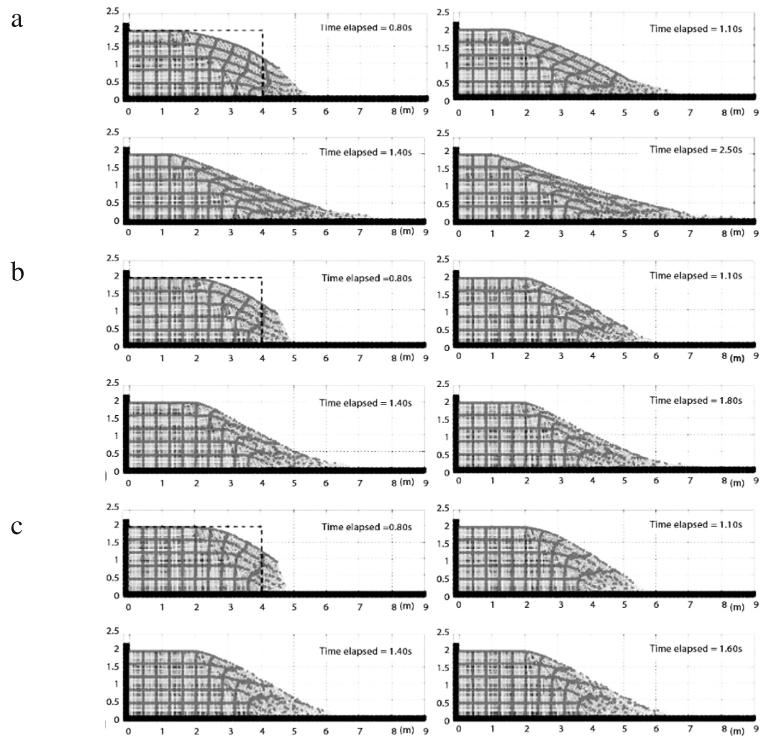


**Fig. 1.** Failure process of noncohesive soil by SPH simulation at representative times: a) $\varphi = 25^{o}$; b) $\varphi = 45^{o}$; and c) $\varphi = 65^{o}$.

difficult for FEM simulations, can be clearly conducted and easily tolerate large deformations and failure, using the SPH. These results show that SPH can be a potential method for solving large deformations and post-failures of geomaterials. Figure 1 illustrates the failure process of cohesionless soils by SPH simulation at representative times for different friction angles. To enhance the application of SPH for geomechanics, it is necessary to extend this method for general constituting models of soil such as elastic-plastic with hardening/stiffening, the viscose elastic-plastic model, Cap models, etc.

Yuu et al [5] computed the 3D stress and strain and bulk density of cohesionless granular materials using the distinct element method (DEM). Based on these data, the computed 3D stress-strain relations and constitutive relations were derived for granular materials. The proposed 3D constitutive relations cover the elastic, plastic, and flow zones. The nonlinear parts are considered using the functions of shear and normal strain sizes, so that it is not required to refer to the performance conditions. The friction coefficient and modulus of elasticity and rigidity were automatically computed for granular materials. Fig.1a presents the nonlinear parts of constitutive relations using functions in which the independent variables are only equal to the strain size. Assuming Hooke's law in the elastic zone, the modulus of elasticity of glass bead granular material is $E = 4.3 \times 10^{5}$ Pa. It must be noted that for the matrix of glass bead granular material is $E = 7.5 \times 10^{10}$ Pa. These values represent the difference in mechanical properties of granular materials and their matrix. Dilation occurs in the small initial deformations, while densification takes place in large successive deformations (see Fig. 1a). The coefficient of friction rises quickly from zero to a constant value of 0.31, which is a reasonable amount for the static coefficient of friction of glass bead granular materials. Simulation of real flow fields (the dynamics for collapse of granular layer and heaps) is carried out using the constitutive relations obtained from the SPH method and laboratory verifications. Figure 2 compares computation results with laboratory data. The obtained results efficiently describe the details of laboratory measured granular layer and heap, particularly the initial steps of collapse and formation of the ultimate body.

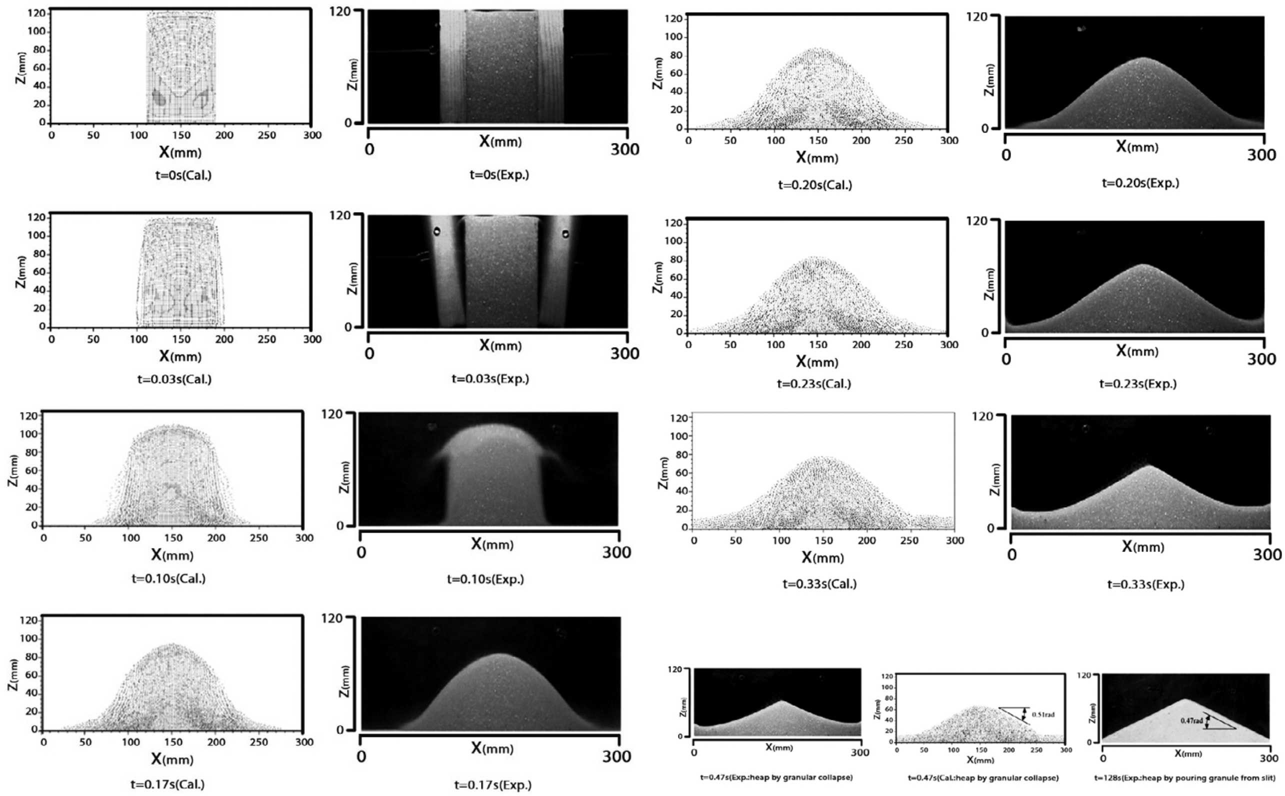


**Fig. 2.** Comparison of the calculated and experimental collapse of the granular layer and the heap formation process.

Chen and Qiu [6] simulated granular materials using the SPH method for large deformations. The constitutive Drucker-Prager model with the nonrelated flow law in SPH formulation was utilized for modelling the fine grain flow in the continuous framework. Their model was verified by some tests conducted on 2D grain column collapse, where excellent agreement was observed between model simulation and the laboratory observations. Simulation of 3D sand column collapse was also carried out for different dimensions. Figure 3 presents a comparison between 2D and 3D simulations. The final deviation distance, final deposition height, and the zone without deformation obtained from the numerical and empirical results indicated good agreement. Simulation of the unchanged zone for a sand column during the collapse indicated smaller values as compared to the observed ones. The main cause for such observation might be the fact that the numerical modeling is able to extract small deformations through the direct evaluation of plastic strain inside the sand column. However, the small deformations or particle rearrangement might not be visually detected through the tests. Figs. 3 and 4 compare SPH simulation and laboratory specimen. Having a continuous-scale model, the SPH model can be effectively applied for simulation of large deformations, compact material flow in the fine grains, and geomaterials, in particular, in the case of selecting proper constitutive models as SPH is a grain-based meshless method. In the SPH method, the variables of the SPH particle field are computed through an interpolation process on the neighboring particles placed in the impact zone and using the smoothing length. The greater smoothing length constantly results in a smoother or more continuous behavior, resulting in the interdependent behavior of SPH particles. On the other hand, the shorter lengths of the smoothing length lead to more discrete behaviors, so that the SPH particles would be more independent. By selecting a proper smoothing length, the developed SPH model would be able to conserve many separate properties of dense grains flow for materials with fine grain subject to large deformations. The developed model can find applications related to dense grain flow and large deformations such as sliding and avalanching flows.

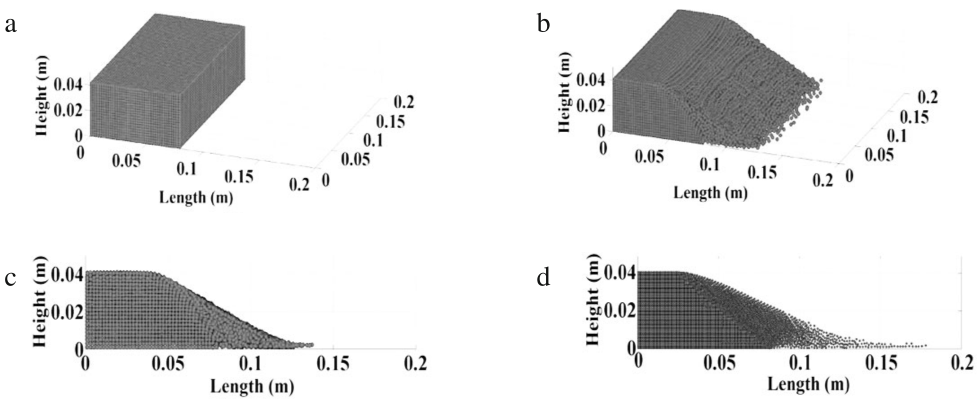


**Fig. 3.** Comparison of SPH simulations for unidirectional collapse in 3D and 2D conditions: a) initial configuration; b) isometric view of final profile in 3D simulation; c) side view of final profile in 3D simulation; d) final profile in 2D simulation.

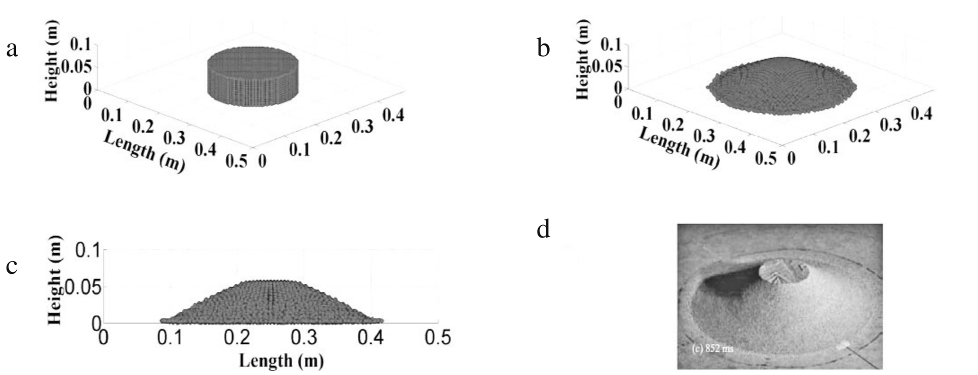


**Fig. 4.** Comparison of SPH simulation and experiment with $a$ = 0.55: a) initial configuration of sand column; b) simulated final profile after collapse; c) side view of simulated final profile; d) experimental final profile.

Ma et al [7] developed a new numerical model based on the SPH for simulation of brittle heterogeneous materials by tracing the microscopic crack propagation as well as the macro-mechanical behaviors. The developed SPH code was applied for simulation of compressive failure of rock-like heterogeneous materials under uniaxial and triaxial loading conditions. Results show that material heterogeneity has a considerable impact on the macroscopic mechanical behavior of materials. For samples with different homogeneity levels, the predicted uniaxial compressive strength (UCS) increases with increased homogeneity; however, the macro-mechanical response is rather the same. Furthermore, the simulation results show that the failure progress has common features in brittle materials. Firstly, the shear failures occurred near to the peak stress and mainly parallel to the loading direction. Secondly, for post-failure zones, axial failure or faulting occurs and leads to sample failure. The predicted failure processes indicate the impact of heterogeneity. Even for samples with the same heterogeneity level, the initial small cracks and their propagation is highly controlled by the different levels of local stresses because of their heterogeneous microscopic structures. The results of biaxial compressive test under the confinement pressure show that the failure pattern of the samples is affected by the confinement stress. Through the increase in lateral pressure, the axial failure is controlled and the shear failure is more obviously developed and has a shear fault form. Moreover, application of confining pressure results in the development of a wider fault plane. The developed SPH code can record the failure evolution and show the real cracks and places with larger deformations. Hence, it is seen that the SPH has clear advantages for simulation of failure in brittle rock-like materials.

Using the refined SPH method, Ma et al. [8] simulated the failure process in heterogeneous materials. The elastic-plastic damage model was extracted in this work based on a unified twin shear strength (UTSS) criterion. The microstructural modelling was proposed based on the spatial variation of elements distribution and distribution of elemental strength in the multiphase materials. This method was successfully applied for producing artificial granite specimens, where the dynamic failure of these samples was further elaborated using this method. Polycrystalline modelling was proposed for the creation of artificial microstructures for dynamic simulation of Brazilian splitting test and uniaxial compression test, and the effect of strain rate on predicted dynamic tension and compressive strength was discussed.

The macro-mechanical behaviors of the specimens indicates pronounced strain-rate sensitivity. By increasing the strain rate, both bearing capacity and the maximum sustained displacement of the sample become greater. Finally, the effect of strain rate on the strength was analyzed, and the results were compared with empirical data. Such behavior is attributed to the variations of internal pressure considering the inertial impacts and activities of samples split under different strain rates. The results revealed that the polycrystalline modeling approach using SPH simulation is promising for modeling of the failure process.

**Conclusion**

As mentioned in the first sections of the current article, the SPH method is useful for the simulation of large deformations and failure problems known as geo-disasters. The SPH is able to deal with behavior and interaction of various (liquid and solid) materials and is capable of modelling multiphase soils. Also, the saturated and submerged soils can be simulated using the SPH method. In this regard, SPH can be used in a wide range of geotechnical applications. It is worth noting that the effect of pore water pressure can be properly presented using SPH simulation. SPH can predict the maximum post-failure displacement of soil in a given slope, where the rise of multiphase (layer-by-layer) failure of the slope triggered by heavy rainfalls is possible. Also, prediction of path, rate, and depth of flow-like catastrophic landslides is possible using the SPH method. SPH is able to analyze water-induced deformation, which is not an easy task using FDM and FEM methods due to the mesh distortion and other computational problems. However, the method requires further development to expand its applications in different geotechnical fields.

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