Problem 1

restart

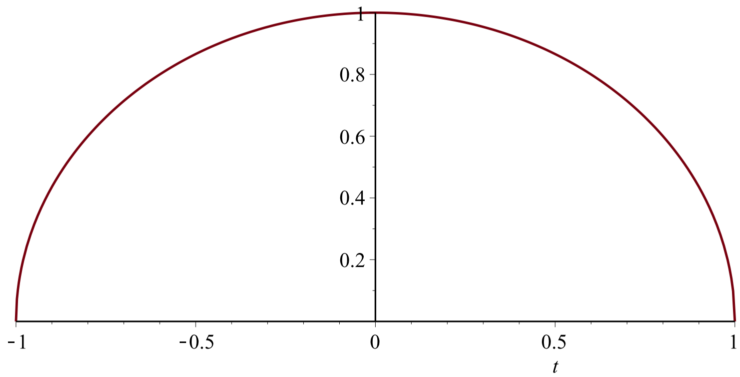
a)

Below is the formula for a semicircle that has a radius 1.

$sc := t \rightarrow \mathrm{sqrt}(1 - t^2)$

$$t \rightarrow \sqrt{1 - t^2} \tag{1}$$

$plot(sc(t), t = -1..1)$



I plot it to make sure my formula is right and it is the same plot as in the picture of exam

Also, just in case I let maple know tha n is a positive integer

$assume(n :: integer, n > 0)$

I choose L to be 1, half of the period

$L := 1:$

I calculate coeffifiencs a0, an and bn. I show both, mathematical expressions and numerical results for n=1.

$a0 := \frac{1}{2 \cdot L} \cdot int(sc(t), t = -L..L)$

$$\frac{1}{4}\pi \tag{2}$$

$evalf(a0)$

$$0.7853981635 \tag{3}$$

$an := n \rightarrow \frac{1}{L} \cdot int\left(sc(t) \cdot \cos\left(\frac{\mathrm{Pi} \cdot n \cdot t}{L}\right), t = -L..L\right)$

$$n \rightarrow \frac{\int_{-L}^{L} sc(t) \cos\left(\frac{\pi n t}{L}\right) \mathrm{d}t}{L} \tag{4}$$

$an(n)$

$$\frac{\text{BesselJ}(1, \pi\, n\sim)}{n\sim} \tag{5}$$

Here we get BesselJ function, which is a valid finction that is build in in maple. We can still plot is and evaluate

$evalf(an(1))$

$$0.2846153430 \tag{6}$$

$bn := \frac{1}{L} \cdot int\left(sc(t) \cdot \sin\left(\frac{\text{Pi} \cdot n \cdot t}{L}\right), t = -L..L\right)$

$$0 \tag{7}$$

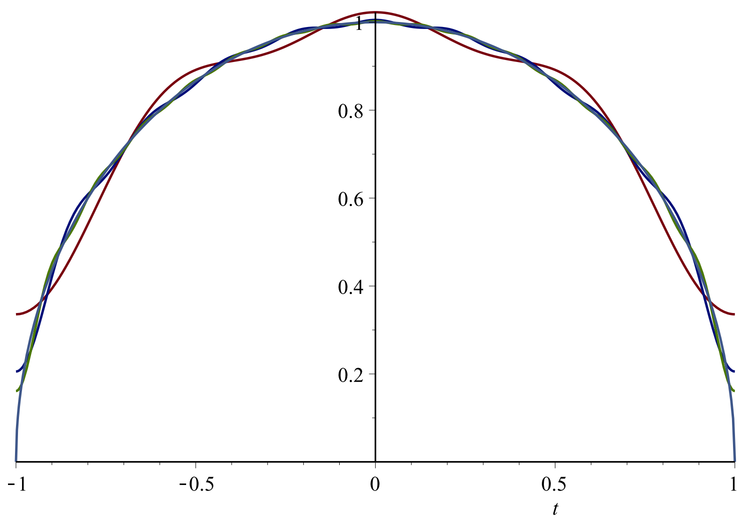
b)

$f3 := t \rightarrow sum\left(an(n) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right) + bn \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..3\right) + a0:$

$f9 := t \rightarrow sum\left(an(n) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right) + bn \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..9\right) + a0:$

$f15 := t \rightarrow sum\left(an(n) \cdot \cos\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right) + bn \cdot \sin\left(\frac{n \cdot \text{Pi} \cdot t}{L}\right), n = 1..15\right) + a0:$

$plot([f3(t), f9(t), f15(t), sc(t)], t = -1..1)$



As we expect, higher number of terms in the sum gives us a better approximation for the real plot. Higher number sum approximation is less waivy and better approaches y=0 at x=1 and x=-1. I was playing with it and observed that if we keep more terms then we can see how y actually gets to 0 at x=1, -1. After x=1 and before x=-1 the approximation just repeats itself with same period.

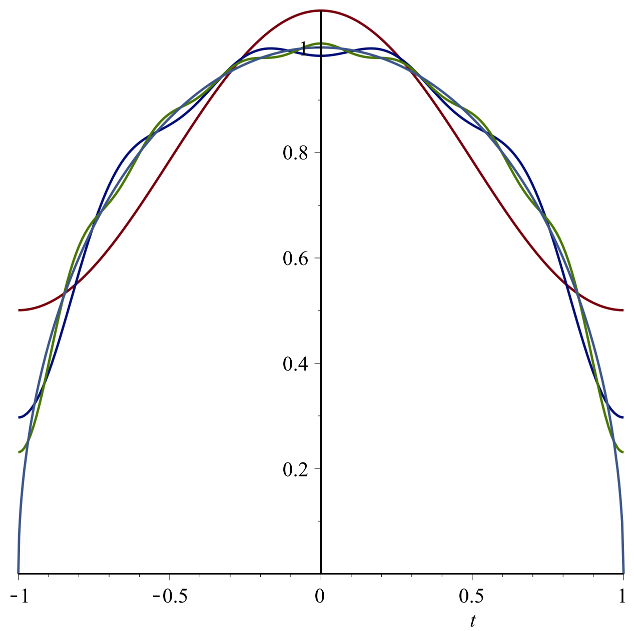
c)

Here I choose L to be 2 (the period). The whole lenght of x from -1 to 1.

$L := 2:$
for our formula we need angular frequency that we can calculate from the period
$w := \frac{2 \cdot \mathrm{Pi}}{L}:$
$gj := \frac{1}{L} \cdot int(sc(t) \cdot \exp(I \cdot j \cdot w \cdot t), t = -1..1)$

$$\frac{1}{2} \int_{-1}^{1} sc(t)\, e^{I j \pi t}\, dt \qquad \textbf{(8)}$$

I found my gj which also happen to be bessel fucntion. Again we can use this for all our purposes as calculation, plotting and etc. BesselJ is a mathematical expression.
$f3e := t \rightarrow sum(gj \cdot \exp(-I \cdot j \cdot w \cdot t), j = -1..1):$
$f9e := t \rightarrow sum(gj \cdot \exp(-I \cdot j \cdot w \cdot t), j = -4..4):$
$f15e := t \rightarrow sum(gj \cdot \exp(-I \cdot j \cdot w \cdot t), j = -7..7):$
$plot([f3e(t), f9e(t), f15e(t), sc(t)], t = -1..1)$



We see a similar picture to the plot ir part b. But less precise. If we use the same amount of terms (for example, 3 - j =-1,0,1) then we get less precise graph that with 3 terms of the graph in part b, where I go from 0 to 3. To get the same graph as in part b I need to go from -3 to 3 which involves 7 total terms.

Problem 2

I load the plots comand becasue I know I will need it later. And upload the data points.

with(*plots*) :

pp ≔ [[0., 1.108], [0.2, 1.696], [0.4, 2.278], [0.6, 1.079], [0.8, 1.471], [1., 1.208], [1.2, 0.14], [1.4, 0.378], [1.6, −0.785], [1.8, −1.271], [2., −0.788], [2.2, 0.143], [2.4, 1.789], [2.6, 0.868], [2.8, 1.976], [3., 0.157], [3.2, 0.538], [3.4, −0.132], [3.6, −0.224], [3.8, 0.251], [4., 2.216], [4.2, 1.487], [4.4, 2.167], [4.6, 2.769], [4.8, 1.006], [5., 1.936], [5.2, 0.006], [5.4, 0.111], [5.6, 1.046], [5.8, 0.377], [6., 0.665], [6.2, 1.596], [6.4, 2.2], [6.6, 0.77], [6.8, 0.888], [7., 0.483], [7.2, −1.078], [7.4, −0.067], [7.6, −1.64], [7.8, −0.444], [8., −0.668], [8.2, −0.145], [8.4, 0.942], [8.6, 0.243], [8.8, 1.047], [9., 0.508], [9.2, 0.059], [9.4, −0.208], [9.6, −1.201], [9.8, −1.229], [10., 1.165], [10.2, 1.046], [10.4, 0.583], [10.6, 1.44], [10.8, 0.503], [11., −1.182], [11.2, −2.093], [11.4, −1.244], [11.6, −2.956], [11.8, −2.368], [12., −1.043], [12.2, −0.313], [12.4, −0.6], [12.6, −0.514], [12.8, −0.948], [13., −1.724], [13.2, −1.811], [13.4, −0.715], [13.6, −1.914], [13.8, −1.85], [14., 0.318], [14.2, −0.547], [14.4, 0.421], [14.6, −0.699], [14.8, −0.913], [15., 0.612], [15.2, −0.073], [15.4, −1.123], [15.6, −0.839], [15.8, −0.191], [16., −0.94], [16.2, 0.946], [16.4, −0.101], [16.6, 1.003], [16.8, −0.195], [17., 0.541], [17.2, −1.311], [17.4, −0.444], [17.6, −1.759], [17.8, 0.198], [18., −0.095], [18.2, −0.219], [18.4, 0.239], [18.6, 0.165], [18.8, 0.291], [19., 1.421], [19.2, 0.33], [19.4, −0.411], [19.6, 1.046], [19.8, 1.714], [20., 1.096], [20.2, 3.154], [20.4, 1.505], [20.6, 2.861], [20.8, 1.64], [21., 1.383], [21.2, −0.161], [21.4, −0.052], [21.6, −1.252], [21.8, 0.482], [22., 0.617], [22.2, 0.957], [22.4, 0.862], [22.6, 1.278], [22.8, 0.351], [23., 0.051], [23.2, −0.321], [23.4, −1.081], [23.6, 0.216], [23.8, −0.53], [24., 1.077], [24.2, 0.242], [24.4, 2.192], [24.6, 1.898], [24.8, 0.877], [25., 0.432], [25.2, 0.525], [25.4, −0.963], [25.6, 0.223], [25.8, 0.096], [26., −0.911], [26.2, −0.021], [26.4, 0.59], [26.6, 1.108], [26.8, 0.346], [27., −0.09], [27.2, −1.051], [27.4, −2.459], [27.6, −1.652], [27.8, −1.305], [28., −1.571], [28.2, −0.179], [28.4, −1.696], [28.6, 0.313], [28.8, −0.887], [29., −1.865], [29.2, −0.21], [29.4, −0.61], [29.6, −1.061], [29.8, −0.215], [30., −0.652], [30.2, 0.013], [30.4, −0.115], [30.6, −0.201], [30.8, −1.054], [31., −1.071], [31.2, −1.77], [31.4, −1.517], [31.6, −0.982], [31.8, −2.313], [32., −0.871], [32.2, −1.28], [32.4, 0.587], [32.6, 0.969], [32.8, 0.049], [33., 0.056], [33.2, −1.165], [33.4, 0.006], [33.6, 0.039], [33.8, −0.477], [34., 0.554], [34.2, 0.429], [34.4, 2.308], [34.6, 1.729], [34.8, 1.322], [35., 0.557], [35.2, 0.093], [35.4, −0.024], [35.6, −0.666], [35.8, −0.075], [36., 1.103], [36.2, 1.649], [36.4, 2.461], [36.6, 2.731], [36.8, 1.563], [37., 0.529], [37.2, 0.869], [37.4, 0.593], [37.6, −1.163], [37.8, 0.693], [38., 0.835], [38.2, 1.189], [38.4, 1.809], [38.6, 1.976], [38.8, 0.203], [39., 0.642], [39.2, −0.195], [39.4, 1.315], [39.6, 0.034], [39.8, 1.083], [40., 0.789], [40.2, 1.294], [40.4, 2.198], [40.6, 0.754], [40.8, 0.509], [41., 0.107], [41.2, −0.236], [41.4, −1.734], [41.6, −1.662], [41.8, −0.924], [42., 0.194], [42.2, 0.137], [42.4, 0.286], [42.6, 0.957], [42.8, −0.624], [43., −1.59], [43.2, −0.603], [43.4, −0.378], [43.6, −0.428], [43.8, 0.01], [44., −1.226], [44.2, −0.161], [44.4, −0.554], [44.6, 0.726], [44.8, −0.123], [45., 0.064], [45.2, −0.308], [45.4, −2.072], [45.6, −2.291], [45.8, −0.223], [46., −1.033], [46.2, −0.73], [46.4, 0.949], [46.6, −0.46], [46.8, −0.963], [47., −1.341], [47.2, −2.514], [47.4, −2.417], [47.6, −2.67], [47.8, −2.599], [48., −1.874], [48.2, −1.109], [48.4, −1.174], [48.6, 0.898], [48.8, 0.678], [49., 0.437], [49.2, −0.292], [49.4, −0.812], [49.6, −0.266], [49.8, 0.017], [50., 0.65]] :

N ≔ *nops*(*pp*)

$$251 \tag{9}$$

I counted how many points I have total in my data.

Out of the data I need the second part of each given point. Which is our fk. To get the sequance of just these poitns, without the first part, I create a new sequence that consists just form our fk.

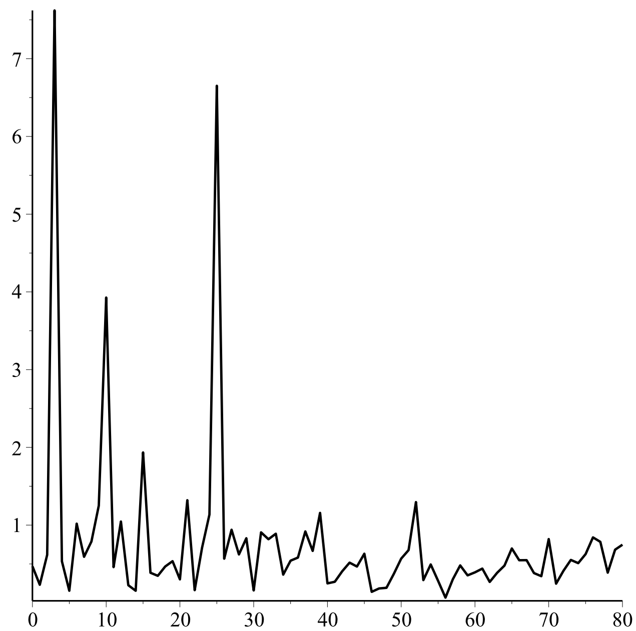
$p := seq(pp[n+1, 2], n = 0..N-1)$:

Then I initilize my gj, where j is the frequency.

$$gj := j \rightarrow \frac{1}{\mathrm{sqrt}(N)} \cdot sum\left(p[k+1] \cdot \exp\left(-\frac{I \cdot 2 \cdot \mathrm{Pi} \cdot j \cdot k}{N}\right), k = 0..N-1\right) :$$

$G := seq([j, \mathrm{abs}(evalf(gj(j)))], j = 0..80)$:

$pointplot([G], connect = true)$



My plot looks good. I choose to plot up to 80 because later there are no more peaks. I see 4 clear peaks(4 underlying frequencies) -> The strongest one is the first one which is j=3, Second strongest - j=25, Third strongest - j=9 and fourth strongest is j=15. There also 2 not as clear peaks, could have been the noise but I want to point them out. They are at j=21 and j=52.

Problem 3

a,b)

In my loop I inicialize two identical matrices. One stays unchanged and other one becomes a new matrix An. I do it, so later when we compare them I have both matrices to look at.

$A := \langle\langle 1,-2,-1,2\rangle|\langle 3,1,2,3\rangle|\langle 3,2,-2,1\rangle|\langle 4,2,1,-1\rangle\rangle;$

$An := \langle\langle 1,-2,-1,2\rangle|\langle 3,1,2,3\rangle|\langle 3,2,-2,1\rangle|\langle 4,2,1,-1\rangle\rangle :$

$n := 4 :$

$c := 3 :$

for j **from** 1 **to** n **do**

  for i **from** 1 **to** 2 **do**

  $An[i,j] := c \cdot A[i,j] :$

  end do

end do:

An

$$\begin{bmatrix} 1 & 3 & 3 & 4 \\ -2 & 1 & 2 & 2 \\ -1 & 2 & -2 & 1 \\ 2 & 3 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 9 & 9 & 12 \\ -6 & 3 & 6 & 6 \\ -1 & 2 & -2 & 1 \\ 2 & 3 & 1 & -1 \end{bmatrix} \quad \textbf{(10)}$$

Indeed my loop works and I got my first two rows multiplied by 3.

c)

```
with(LinearAlgebra) :
AD := Determinant(A) :
AnD := Determinant(An) :
```

$$\frac{AnD}{AD}$$

$$9 \quad \textbf{(11)}$$

We multiplyed by 3 only two rows -> our determinant of a new matrix is 9 times bigger than determinant of an old matrix. That is c^2. If we multipied 3 rows by c, new determinant would be c^3 times bigger.

d)

The idea behind my loop is that I create a random matrix nxn and then i substitute all the elemets with the number I need. I use if statement to see when I have the diagonal element (i=j) when this element is equal to 1, the rest of the elements are equal to 0.

```
n := 4 :
 In := RandomMatrix(n, n) :
 for i from 1 to n do
   for j from 1 to n do
   if i = j then
   In[i, j] := 1;
   else
   In[i, j] := 0;
   end if
   end do
 end do:
In
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \textbf{(12)}$$

Problem 4

I start with restarting my maple, to get rid of all the variables in the memory.
restart
I do the maple in build plot first because if I do it after then variable t has something assign to it. So it is better to do it before the loop. Important thing - ln(cos(t)) does not always gove us real numbers. So we want to have the absolute value of ln(cos(t)) to get rid of not real part.

$$nn := dsolve\left(\left\{\frac{\mathrm{d}}{\mathrm{d}t}\,y(t) = \mathrm{abs}(\ln(\cos(t))) - y(t), y(0) = 1\right\}, type = numeric\right)$$

$$\mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \tag{13}$$

$with(plots):$
$RP := odeplot(nn, [t, y(t)], t = 0..15)$

$$PLOT(\ldots) \tag{14}$$

I start with inicillizing what are my c1, c2, c3 and c4. If I initilize them after I give value to y0 and th and say what is g, my loop does not do what I expect.

$c1 := th\cdot g(y0, t):$

$c2 := th\cdot g\left(y0 + \frac{c1}{2}, t + \frac{th}{2}\right):$

$c3 := th\cdot g\left(y0 + \frac{c2}{2}, t + \frac{th}{2}\right):$

$c4 := th\cdot g(y0 + c3, t + th):$

Then I give my equation, initial condition and th step. I also create a sequence N and give it its first term.

In the loop I calculate y1, add it to N sequence. Importat - I have to make the absolute value of y1 when I add it to the sequence becasue again ln(cos(t) I do not always get real numbers. Then I update my y0 with y1 and repeat the loop.

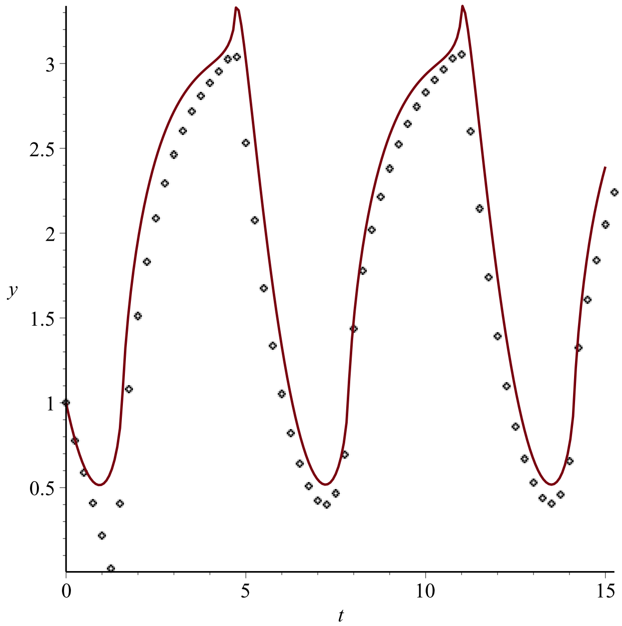
$y0 := 1:$
$th := 0.25:$
$g := (y, t) \rightarrow (\ln(\cos(t))) - y:$
$N := \{[0, y0]\}:$
for t **from** 0 **by** th **to** 15 **do**

$y1 := y0 + \frac{1}{6}\cdot(c1 + 2\cdot c2 + 2\cdot c3 + c4);$

$N := N \cup \{[t + th, \mathrm{abs}(y1)]\};$

$y0 := y1;$

end do:
$PP := pointplot(N):$
$display(PP, RP)$



I display my N sequence point plot together with the maple in build comand solution plot and they look pretty close. 2 diffrences - at around 1.2 I see how my values in pointplot drop to almost 0, which is not the case in the actual plot. Also sequence plot does not have these interesting peaks as the actual plot does at the maximums. However, decreasing th step would solve this problem.