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Magnetic Torque

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Abstract

The experiments in this lab are dedicated towards measuring the magnetic moment μ of the embedded dipole in the cue ball. Each experiment uses different techniques and physical properties of how magnet interacts with uniform or gradient magnetic field. First experiment deals with balancing magnetic torque and gravitational torque. In the second experiment we studies the harmonic oscillation of the cue ball depends on the uniform magnetic field. During the third experiment investigated precessional motion of a spinning cue ball. In the last experiment we explored the net force in a magnetic field gradient. We did weighted average on the values of μ obtained from the independent experiment and we got 0.424±0.009 J/T as our experimentally measured μ for the magnet in the cue ball.

Introduction

Everyone knows a concept of a regular permanent magnet. However, a permanent magnet can be used to represent a current loop to study the interactions between the fields. Current loop creates a dipole magnetic field and an associated magnetic moment that would interact with external magnetic fields. We can study the interactions under different circumstances using uniform and gradient fields [1]. Through studying the interactions between the fields, we can measure the magnetic moment of the dipole. In this lab, we did four experiments to measure the magnetic moment μ, where each experiment uses different physics theory and measuring techniques.

In the first experiment, we balanced the torque of the magnetic field and gravitational field. Magnetic field torque depends on the magnetic dipole and the external magnetic field. We set the magnet in the magnetic field that is pointing up, so the magnetic torque pushed the magnet to rotate and point upwards. The gravitational field is pointing down, therefore the gravitational torque pushes the magnet to point downwards. When both fields have the same torque, they cancel each other and we achieve the balance. We set two equations for the torque equal to each other and we derived he relationship between distance r and strength of magnetic field B as shown in the Equation 1 [2]. B and r are the variables that we measure and m is the mass of the weight that causes gravitational torque.

$$r = \frac{\mu}{mg} B \qquad [1]$$

For the second experiment, we use the principle that when the object is rotating, the net torque causes the change in angular momentum of that object. The rotational motion in this experiment is the oscillation of the magnet around the upward direction of the magnetic field. We set the magnetic torque equal to the change of the angular momentum and derived the Equation 2 [2], where T is the period of the oscillations, I – moment of inertia of the magnet and B represents the strength of the uniform magnetic field. In this derivation, we used the small angle approximation; therefore, this equation is valid only for small angular displacements in the oscillation.

$$T^2 = \frac{4\pi^2 I}{\mu B} \qquad [2]$$

The third experiment studies the precessional motion of the magnet that has a spin. We use the same relationships as in experiment 2, however now the magnet is spinning it has additional angular momentum from the spin. We used geometrical relations and derived the Equation 3 , where Ω_p is the frequency of the precession, L_s is the spin angular momentum and B is the strength of the magnetic field [2].

$$\Omega_p = \frac{\mu}{L_s} B \qquad [3]$$

In the last experiment, we were dealing with the gradient magnetic field. It is important to mention, that in the uniform magnetic field magnetic dipole has no net force acting on it, only the net torque. However, in the presence of the gradient magnetic field there is a net force acting on the magnetic dipole. We used the Maxwell second equation and equation of the force for every infinitesimal section of the loop to derive the equation for the magnitude of the force acting on the magnetic dipole in the gradient magnetic field. In the experiment we were balancing the magnetic force with the gravitational force Equation 4 represents that balance [2]. For this equation, we assume that the magnetic field is parallel to the z-axis and changes only in the z-axis direction.

$$mg = \mu_z \frac{\partial B}{\partial z} \qquad [4]$$

During this lab, we used the TechSpin Mτ1-A Magnetic Torque Instrument. The main part of the equipment is the magnet that is made out of two coils where we can control the current. When the current is going the same direction on both coils, we create the uniform magnetic field between the coils. If the current is going the opposite directions, then we create a gradient magnetic field. The strength constant and gradient magnetic field can be calculated using the Biot-Savart law. We were given the those values: B=$(1.36 \pm 0.03)\times10{-}3\ i$ T/A and dB/dz=$(1.69\pm0.04)\times10{-}2\ i$ T/A·m [2]. Since we know the current that we apply to the coils, we can use the above shown relationships to calculate uniform and gradient magnetic fields strength.

For each experiment, we have an equation where we measure the variables and we have an unknown magnetic dipole moment μ. We plot the variables as they appear in the equation and from the slope we can determine the μ. Since for all the experiments we are measuring the μ of the same magnet, we used the weighted average of four experimentally determined μ to get the final value of the magnetic moment of that dipole based on four experiments.

Procedure

In the introduction section, I introduced the equipment that we used for our experiments as well as I talked about its magnet part. However, this instrument has many more useful parts and functions. Our magnetic dipole is actually a cue ball that has a cylindrical permanent magnet in its center. A handle on the ball shows the direction of the magnet's magnetic moment vector. The apparatus has a place where we can put the cue ball - the air bearing that has an opening in the bottom through which an air comes out. This air acts as a support for the ball, so the ball has minimal friction and can rotate freely. The airflow should be evenly distributed below the ball to prevent any additional torque, so there is a bulls-eye level that we can check to make sure that out apparatus is leveled. The instruments has a build in strobe light and we can control its frequency from the controller. In addition, the instrument comes with the magnetic force balance kit that is very useful for the fourth experiment. Each experiment has its own unique procedure therefore; it is convenient to look at four procedures separately.

Balancing the magnetic torque and gravitational torque

For this experiment we modify our cue ball by adding an aluminum rod with a steel end and the weight that sits on the rod. Figure 1 has a great representation on how magnetic torque and gravitational torque are acting on the ball.

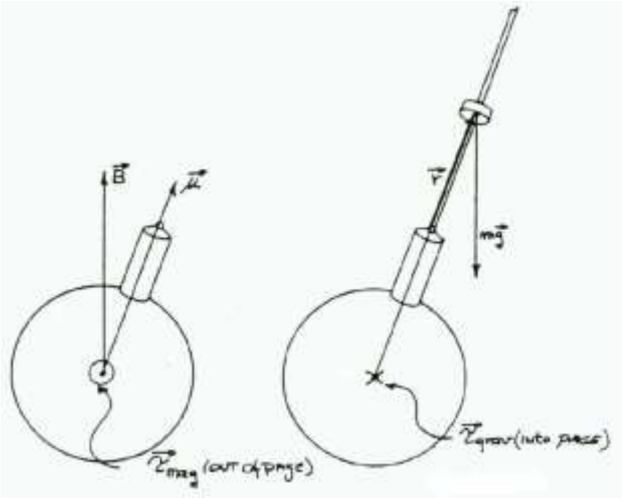

Figure 1: Representation of the magnetic and gravitational torques on the cue ball

The distance r that we are using in the Equation 1 is the distance between the weight and the center of the ball and the mass is just the mass of the weight. Before we started the experiments, we measured all the involved constants: diameter of the ball, the mass of the weight and the length of the balls handle. All the distance measurements were made with calipers that have a precision of 1 millimeter.

Once the constants are measured, we turn the air of the air bearing on, and set the field to be uniform with the direction "up". We measure the distance between the balls handle and the weight and set the ball in the air bearing. We increase the current in the coils until the ball handle is 90 degrees with respect to vertical axis. We record that current and repeat for 6 different positions of the weight. The ball tends to oscillate and drift, so we had to use our hands to help the ball to achieve the balance position.

Harmonic Oscillation of a Spherical Pendulum

This experiment involved the moment of inertia, therefore first we had to measure the ball parameters to determine the moment of inertial of our cue ball. We already measured the diameter of the ball. Now we just had to measure the mass of the ball. For this experiment, we have air on and uniform magnetic field with a direction "up". We set the ball so the handle is point up and the ball is still. Then we give it a small angular displacement and as soon as I release the handle and let the ball oscillate, we start the stopwatch. We measure how much time does it take to have 20 full cycles. We performed the measurements for current starting at 1 amp up to 4 amps with 0.5 amp increments. We also got additional data for smaller currents: 0.8 amp and 1.2 amp for a better data precision.

Precessional Motion of a Spinning Sphere

During this experiment, we were dealing with a spinning ball. To measure the frequency of the spinning ball we use a build in strobe light. We want to have the same spin frequency for each measurement a we choose it to be 5.8 Hz. We set this frequency on the controller and keep it constant. To know if the ball is spinning on the right frequency we observe the white dot on the end of the handle. When the ball is spinning at the 5.8 Hz frequency, the whit dot appears steady because the spin and strobe light are in sync. If the ball has a frequency a little above or below 5.8 Hz then the white dot is spinning with its own angular momentum. The further away we are from the desired frequency the faster the white dot is spinning. The frequency of the spin is decreasing with time. To achieve the 5.8 Hz spin we span the ball hard with the handle facing the strobe light and then used the fingertip to settle and slow down the ball to the pint when white dot is slowly moving around and then we let it go until we have a spin of the desired frequency. To prevent the ball from precessing while we are setting the desired frequency we turned the gradient field on. We already have moment of inertia calculations from the previous experiment that we can use it along with the known 5.8 Hz spin frequency to calculate the spin angular momentum of the ball. Therefore, we achieved the constant spin angular momentum of the ball for each measurement so the only variables are the strength of magnetic field and the precision frequency.

To get the value for the precession frequency we measure the period of the precession. To get the measurements first we spin the ball with air on, current off, field gradient on and upwards direction. One we achieve the desired frequency we turn the current on, gradient off and start the stopwatch. We measure the time that it takes for the ball to make a full precession. We repeated the measurements for the currents from 1 amp to 4 amps with the increments of 0.5 amps.

Net force in a magnetic field gradient using the Magnetic Force Balance

The Magnetic Force Balance is a very convenient accessory that comes with the instrument. On what side of the balance we attach the ball and on the other side we have a counterweight. We place the balance beam on the bearing block that is placed on top left of the brass magnet support. We used the metal ball nearing to adjust the weight of the counterweight. There are two sized of the ball bearings. We determined mass for both sizes of the ball bearing so we can use it later for our calculations.

For this experiment, we have the air off. We set the balance beam so that the ball is right above the center of the air bearing. Then we added some ball bearings to equalize the masses on the both sides of the balance, to make the cue ball to be right above the air bearing. We also had to use the brass thumbscrews for the continuous adjustment of the balance to achieve the best balance we can get. We used the level marker to mark the height of the counterweight against the table – this is our balance height.

The variables of this experiment are the mass of the ball bearing that we add and the strength of the gradient magnetic field. We were adding one small ball bearing to the counterweigh and adjust the current until we reach the balance. We record the current that corresponds to the amount of the ball bearings added. We kept adding weight and adjusted the current until we reached the maximum possible current value. The maximum amount of balls we could add were 6 balls.

Results

For each of the performed experiments we plotted the data points and did linear fit on them. From the slope of the linear fit, we extracted the value for the magnetic moment μ. Each experimentally determined μ has its own uncertainty. The uncertainty of the magnetic field, which is our dependent variable, comes from the current measurement that we observed on the controller and the given value of B and dB/dz. The current display was not digital, but analog. The division was 0.1 amp. However, I took my uncertainty to be 0.05amp because it was easy to observe even small current changes. The independent variables are different for each experiment and have their own sources of uncertainty.

For the first experiment, we plotted distance r versus the strength of the magnetic field. The distance r has a big uncertainty because this is the sum of three independent measurements. We measured the distances with a caliper that has divisions of 1mm, which resulted in 2.5mm uncertainty for each distance data point. The data point plot and linear fit are shown in Figure 2. The slope is equal to 29.9±1.63, and we divide this value by the weight mg of the weight and our μ value is 0.38±0.02 (J/T).

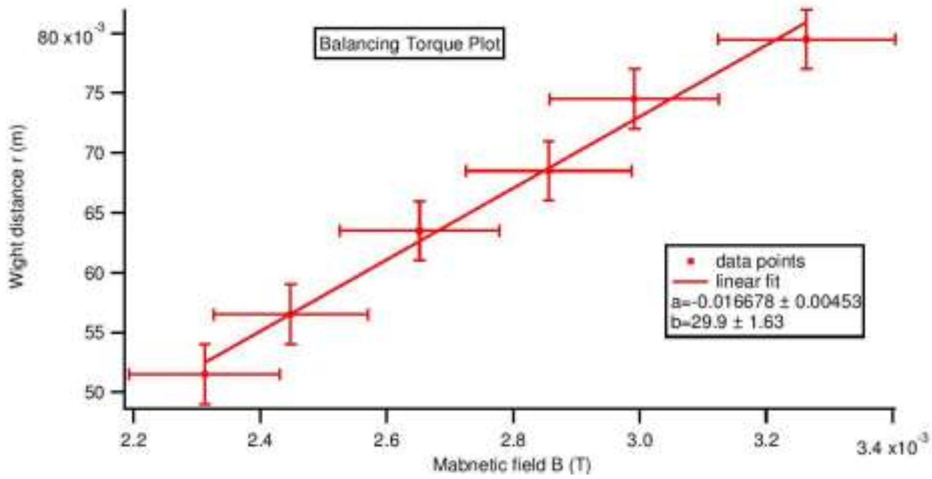

Figure 2: Plot for the balancing magnetic torque and gravitational torque experiment

In plot for the second experiment, we were studying relationship between the square of the oscillation period and the inverse of the magnetic field strength. The oscillation period does not have a big relative uncertainty because the period times are big compering to the uncertainty value. The slope of the plot shown in Figure 3 is 0.091929±0.00162. Looking at Equation 2, we can see that to find μ we had to divide the constants by the slope. One of the constants is the inertia and since our ball is roughly a sphere we just found the inertia from the measurement of the mass and radius of the ball and it is equals to ($3.976*10^{-5}$) kg*m^2±1.89%. The uncertainty in the inertia mostly comes from the radius measurement uncertainty, as describes in the first experiment. The weight of the ball was relatively big to the uncertainty of the weight measurement. The Calculation for μ resulted in (1.71±0.06)*10^{-2} J/T.

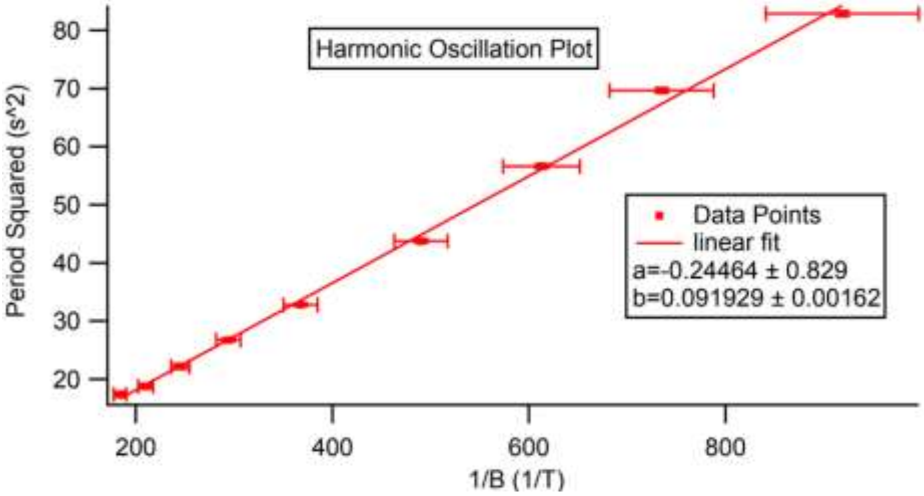

Figure 3: Plot for harmonic oscillation of a spherical pendulum experiment

The third experiment we were observing the relationship between the precession frequency and the strength of the magnetic field. To determine the precession frequency we measure the period of precession. The greater the magnetic field strength the faster the ball would precess. As we can observe on the plot in Figure 4, the uncertainty for the precession frequency is greater for greater magnetic field values. When the cue ball was spinning slowly, it was easy to stop the stopwatch exactly at the point of the full precession. However, when the ball was spinning fast the human factor uncertainty takes a big role, therefore we recorded a bigger uncertainty for these values.

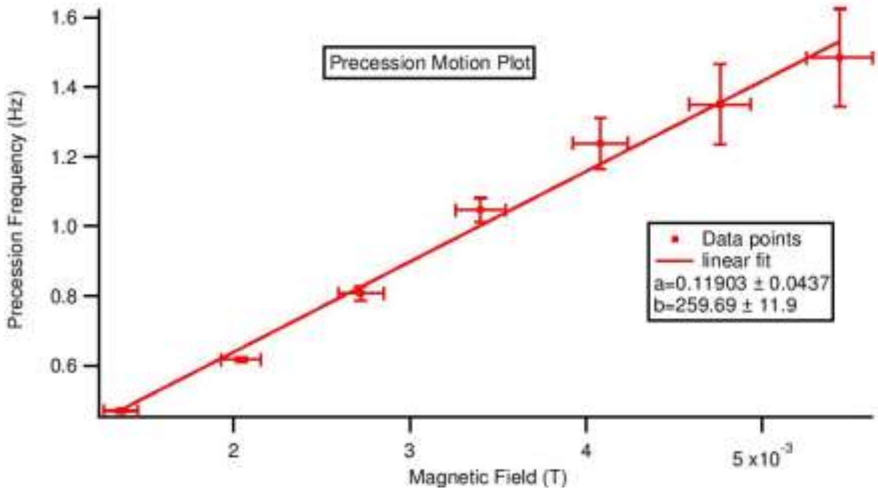

Figure 4: Plot for the Precession motion of the spinning sphere experiment

The slope of the linear fit is 259.69±11.9. To calculate μ from the slope we have to multiply the slope value by the cue ball spin angular momentum L_s, which is the product of the balls spin frequency and balls inertia that we calculated in our second experiment. We take our balls spin frequency to be 0.1 Hz, because 0.1 Hz is the smallest digit of the displayed frequency on the controller. Our calculation resulted in L_s to be 0.001446 kg*m^2 / sec ± 3.61%, which resulted in μ=0.37±0.03 (J/T).

During the last experiment, we examined the relationship between the magnetic force and gravitational force. The strength of the magnetic field gradient was our independent variable while the weight of the added mass was out dependent variable. As we can observe the uncertainties for the weights are very small because the balls have a very precise mass that we were able to measure on the digital scale with a precision of 0.01g. The uncertainty of the gradient field comes from the given value and the measured current. The slope of the plot in Figure 5 is the value of μ and it is equal to 0.447±0.012 (J/T).

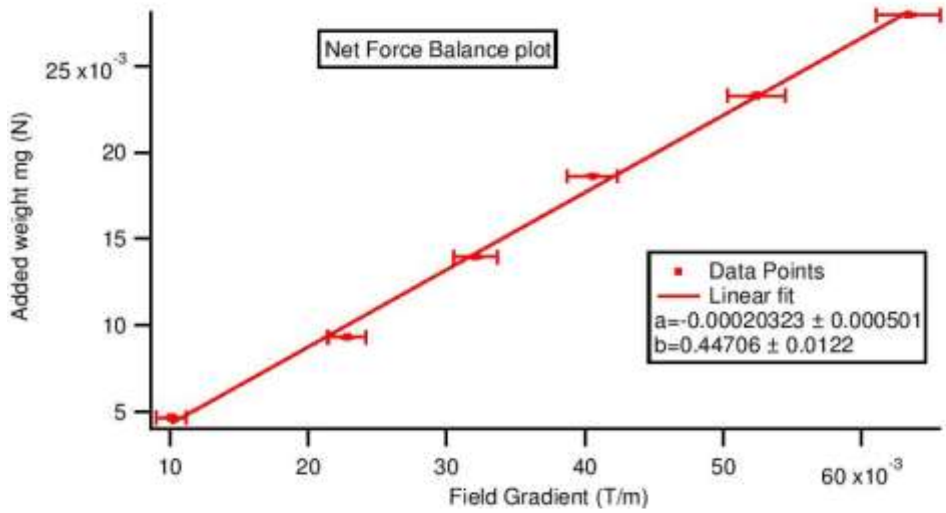

Figure 5: Plot for net force in the magnetic field using the Magnetic force balance experiment

From the four different experiments, we got four different values of μ. Since we were measuring the data for the same magnet, our μ value should be approximately the same for all four experiments. To get a more precise value of μ we take the weighted average of all four values, which takes the uncertainty of each value into account and gives more weight to the value with a smaller uncertainty.

Three of our values are close to each other however, the value for the second experiment is much smaller than the rest of the values. Since it is smaller it has small uncertainty compared to the other values, therefore when we do the weighted average this small value of the second experiment has much greater weight then other values so the weighted average results to be 0.0186 which is not reasonable because it is not close to the other 3 values. Since the second value is so much off we decided to take the weighted average only of the results of the first, third and fourth experiment and this weighted average for μ is 0.424±0.009 J/T.

Conclusion

All four experiment conducted were made to measure the magnetic moment of the embedded dipole in the cue ball. Each experiment has its own sources of uncertainty, so we choose to treat the value with the smallest uncertainty the most reliable. The values of measured μ for the four experiments are 0.38±0.02 (J/T), (1.71±0.06)*10^{-2} J/T, 0.37±0.03 (J/T) and 0.447±0.012 (J/T). The value for the second experiment is much smaller than other values so we decided to not take it into account for our final calculations. The experiment from the last experiment has the smallest values out of the three reasanoble values. We calculated the weighted average of the first, third and fourth experiment values based on their uncertainties and we got 0.424±0.009 J/T as our final result for the μ of the magnet in the cue ball.

References

[1] R. D. Knight, “Physics for Scientists and Engineers: A strategic Approach, 3e” (Pearson, 2013).

[2] D. LaFountain and J. Reichert, “Magnetic Torques Mτ1-A Instructor’s User Manual, Rev 2.0” (TeachSpin, Inc., Buffalo, 2013).