FINAL PROJECT

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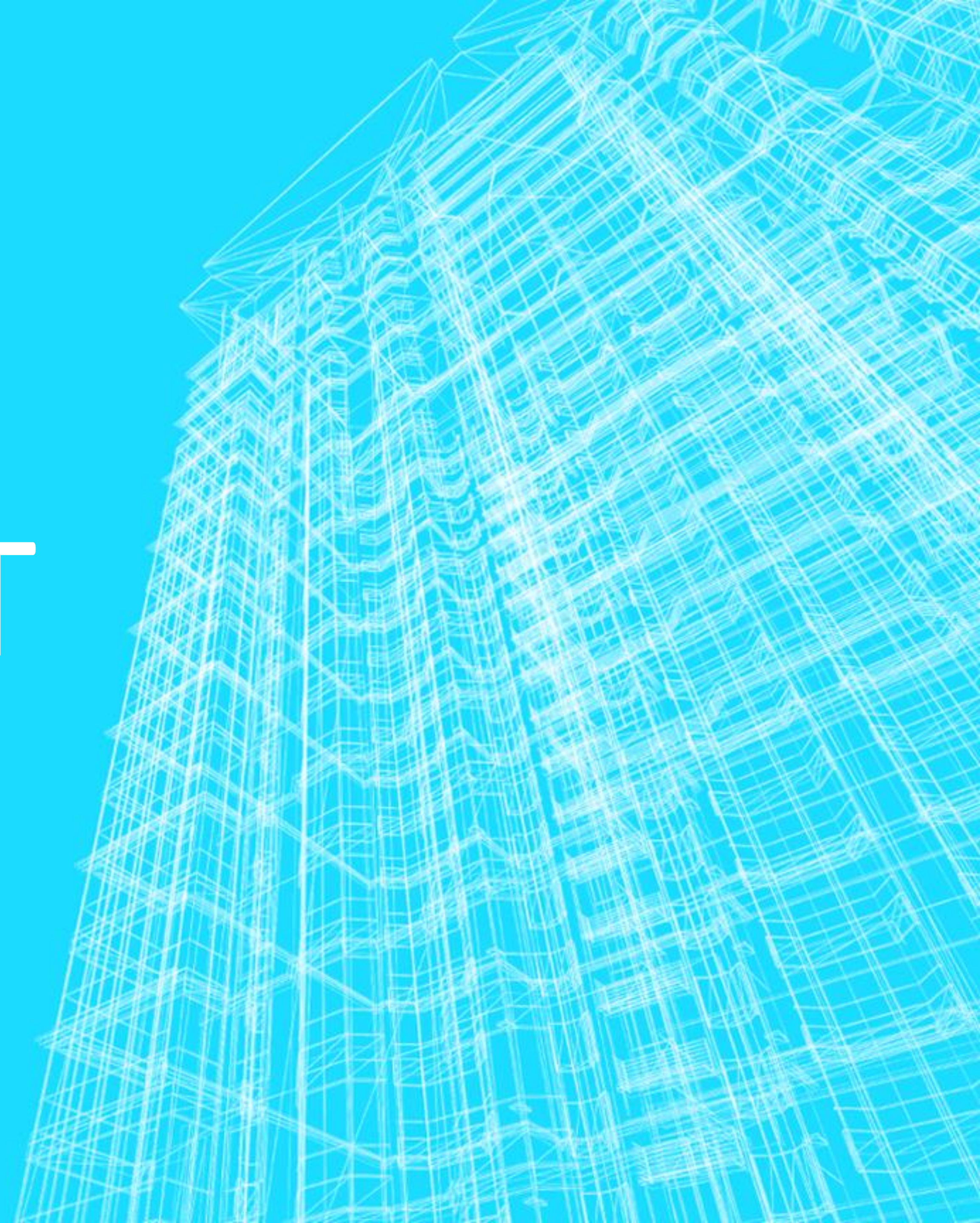
Structural analysis and design 2020



CYLINDER SHRUNK IN ANOTHER CYLINDER

Dimensions:

1. Inner cylinder: R=25..50 m
2. Outer cylinder: R=50..75 m
3. Length = 250 mm

Material – Al 7075-T6:

1. Young’s Modulus: 71.7 GPa
2. Poisson’s ratio : 0.33
3. Yield stress (for 25.43-50.8mm thickness): >= 462 MPa

Factor of Safety = 2

http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA7075T6
http://www.matweb.com/search/DataSheet.aspx?MatGUID=4f19a42be94546b686bbf43f79c51b7d&ckck=1

Find stresses and strains distribution for max allowable radial interference

MAXIMUM ALLOWABLE RADIAL INTERFERENCE

$$p_i = \frac{E\delta}{b}\left[\frac{(c^2-b^2)(b^2-a^2)}{2b^2(c^2-a^2)}\right] \quad (7.37)$$

Where:

E = Young's Modulus
δ= radial interference between the two cylinders
a = inner radius of the inner cylinder
b = outer radius of inner cylinder and inner radius of outer cylinder
c = outer radius of outer cylinder

Using tresca critria:

$$tmax := \frac{sy}{2 \cdot SF}$$

$$eq := tmax = \frac{(s1 - s2)}{2}$$

Lame's equations

Case 1: Internal pressure only (P_o=0):

At inside surface, $r = r_i$:

$$\sigma_\theta = P_i\left[\frac{r_o^2+r_i^2}{r_o^2-r_i^2}\right] \quad \sigma_r = -P_i \quad \sigma_z = \frac{P_i r_i^2}{r_o^2-r_i^2} \quad (7.32)$$

At outside surface, $r = r_o$:

$$\sigma_\theta = \left[\frac{2P_i r_i^2}{r_o^2-r_i^2}\right] \quad \sigma_r = 0 \quad \sigma_z = \frac{P_i r_i^2}{r_o^2-r_i^2} \quad (7.33)$$

Case 2: External Pressure only (P_i= 0):

At inside surface, $r = r_i$:

$$\sigma_\theta = \left[\frac{-2P_o r_o^2}{r_o^2-r_i^2}\right] \quad \sigma_r = 0 \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2-r_i^2} \quad (7.35)$$

At outside surface, $r = r_o$:

$$\sigma_\theta = -P_o\left[\frac{r_o^2+r_i^2}{r_o^2-r_i^2}\right] \quad \sigma_r = -P_o \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2-r_i^2} \quad (7.36)$$

Max allowable radial interference:

0.1909mm

Interface pressure:

64.17 MPa

ANALYTICAL RESULT OF STRESSES AND STRAINS DISTRIBUTION

Lame's equations

Case 1: Internal pressure only (P_o=0):

$$\sigma_\theta = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 + \frac{r_o^2}{r^2}\right] \quad \sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2}\left[1 - \frac{r_o^2}{r^2}\right]$$

Case 2: External Pressure only (P_i= 0):

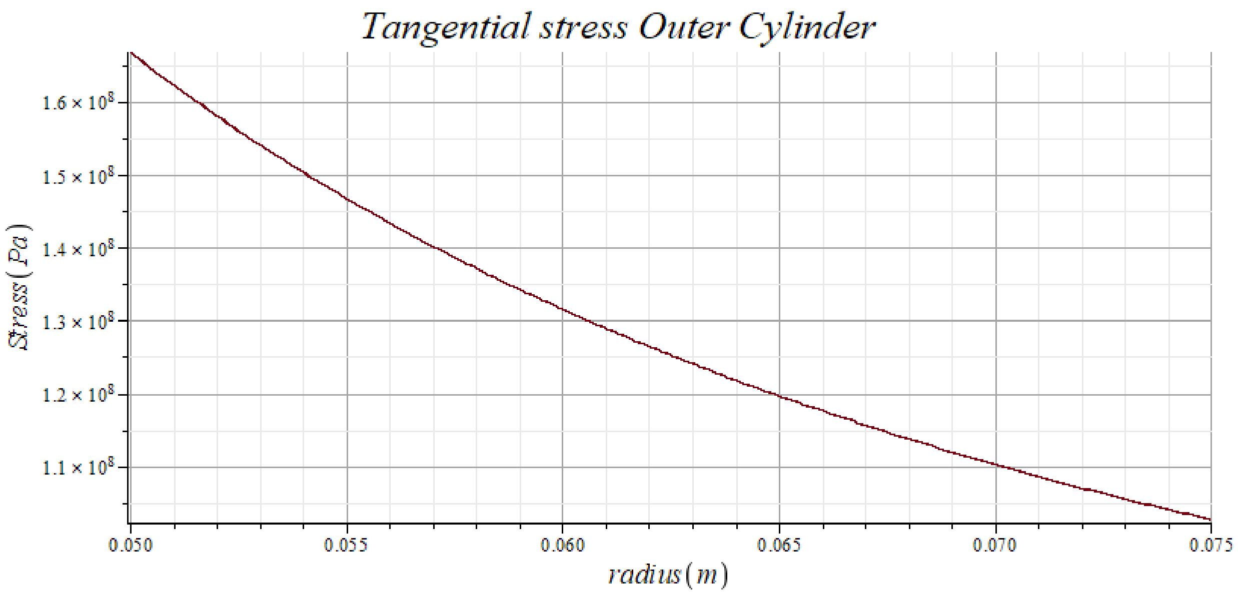
$$\sigma_\theta = \frac{-P_o r_o^2}{r_o^2 - r_i^2}\left[1 + \frac{r_i^2}{r^2}\right] \quad \sigma_r = \frac{-P_o r_o^2}{r_o^2 - r_i^2}\left[1 - \frac{r_i^2}{r^2}\right]$$

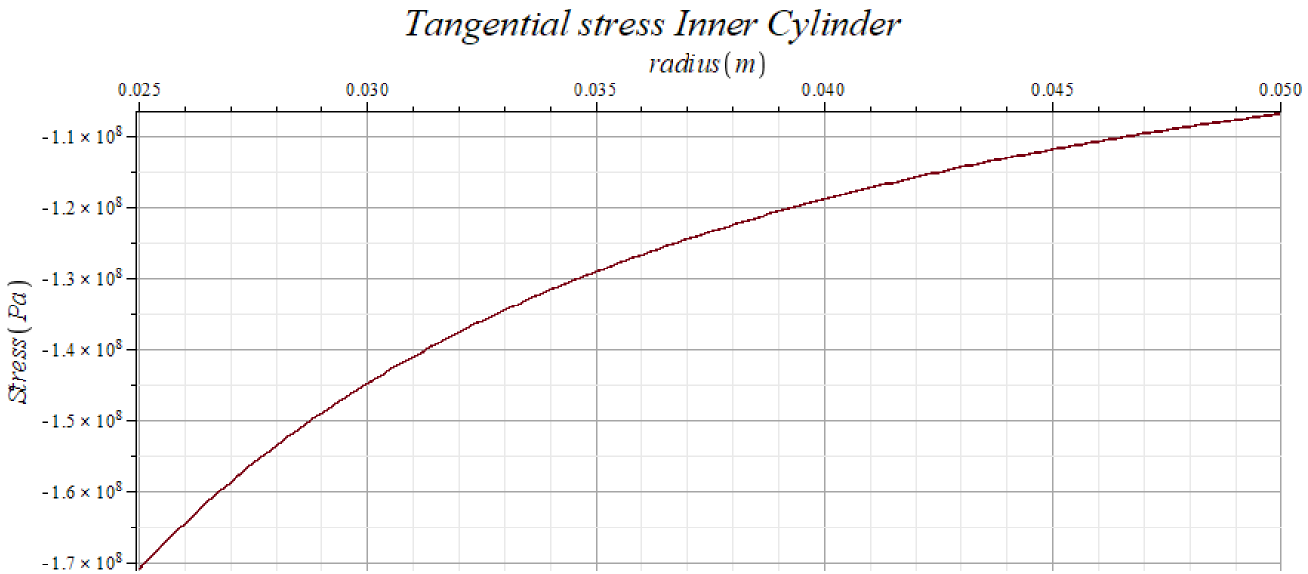
Hooke's law for plane stress

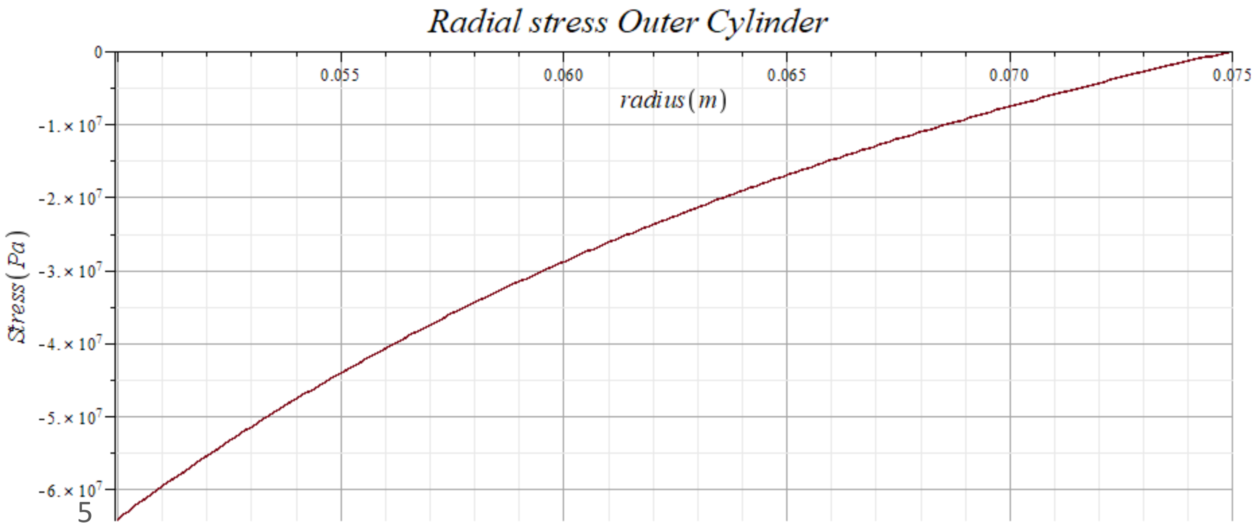
$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

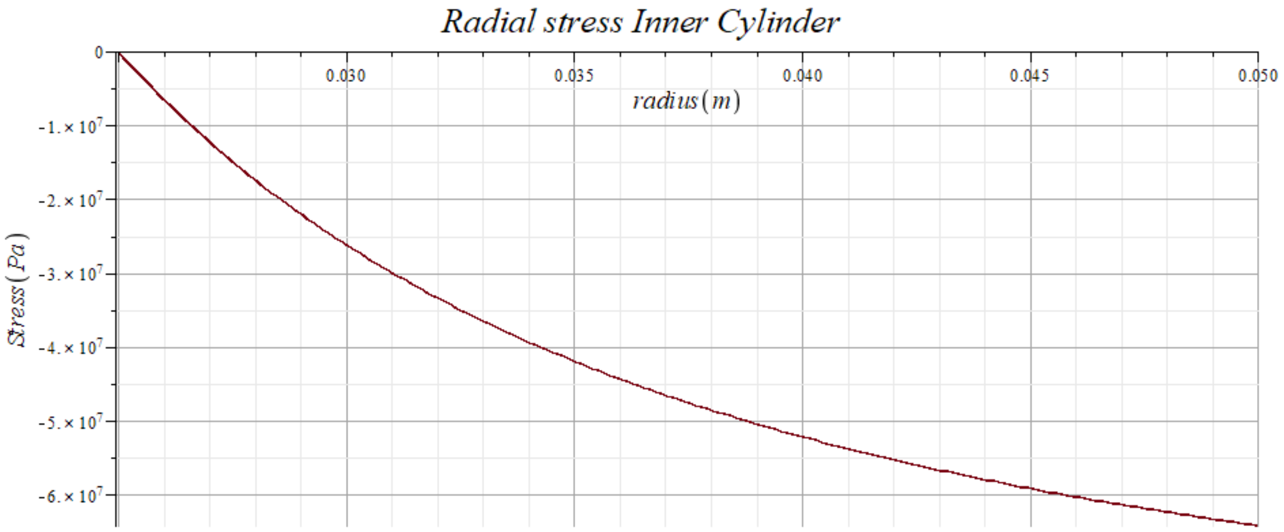
$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

ANALYTICAL STRESSES DISTRIBUTIONS

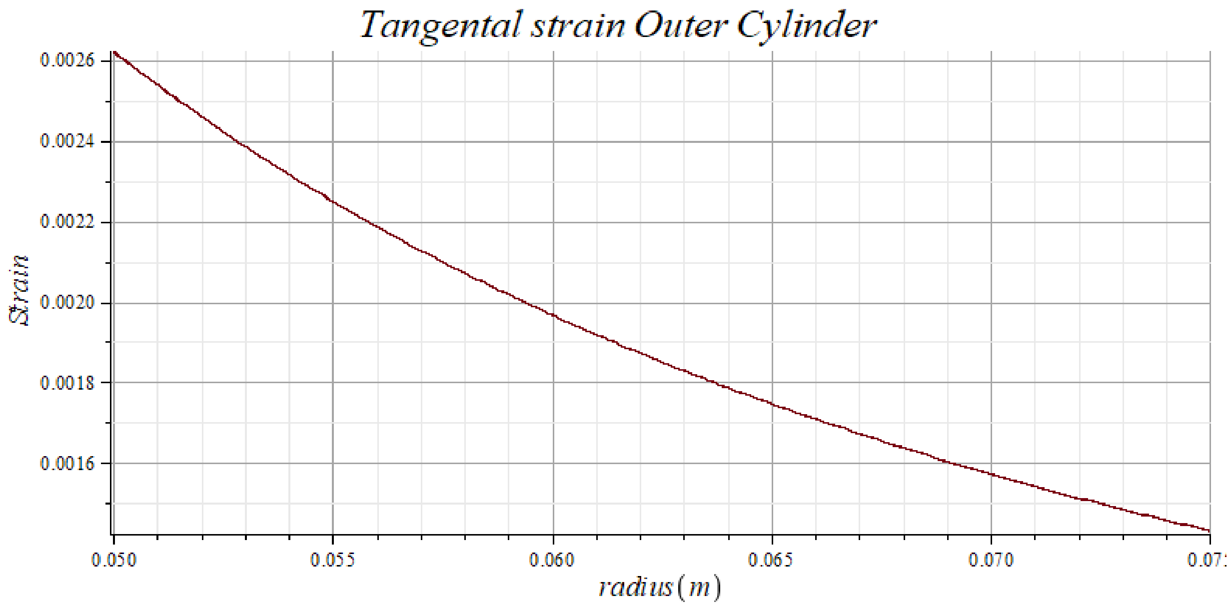


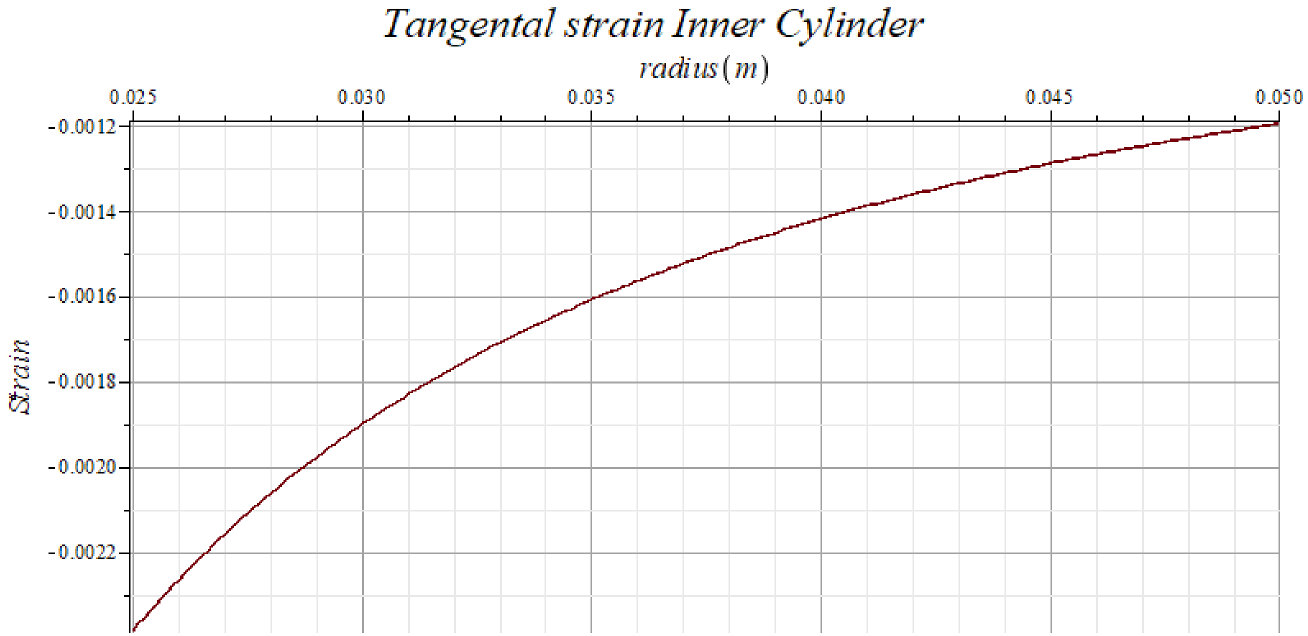


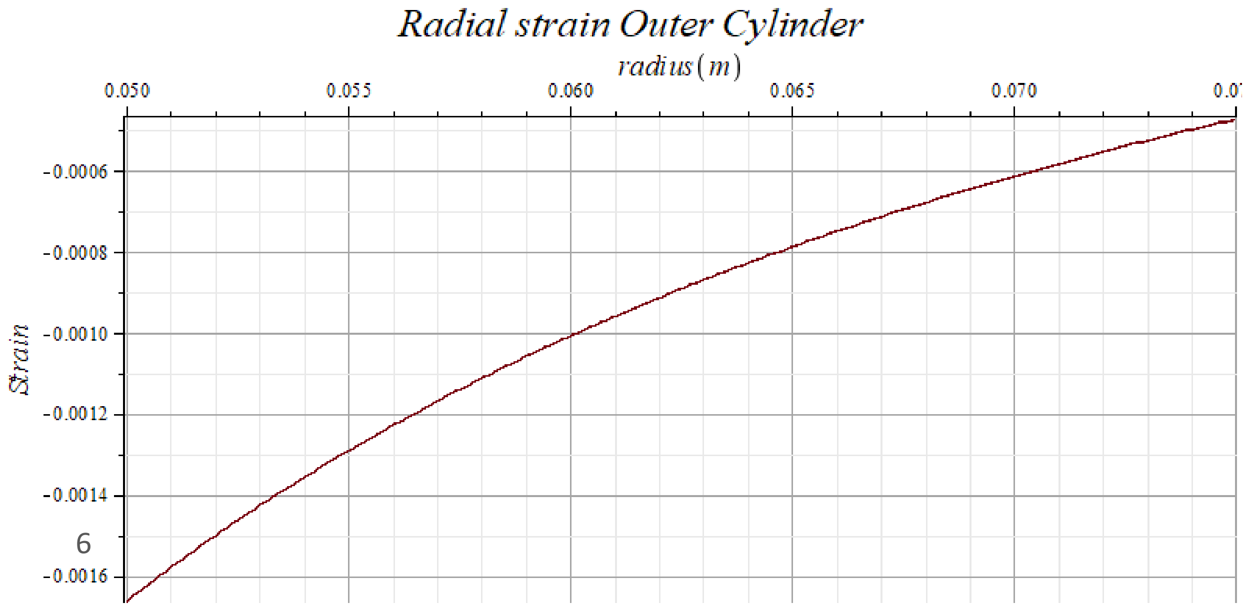


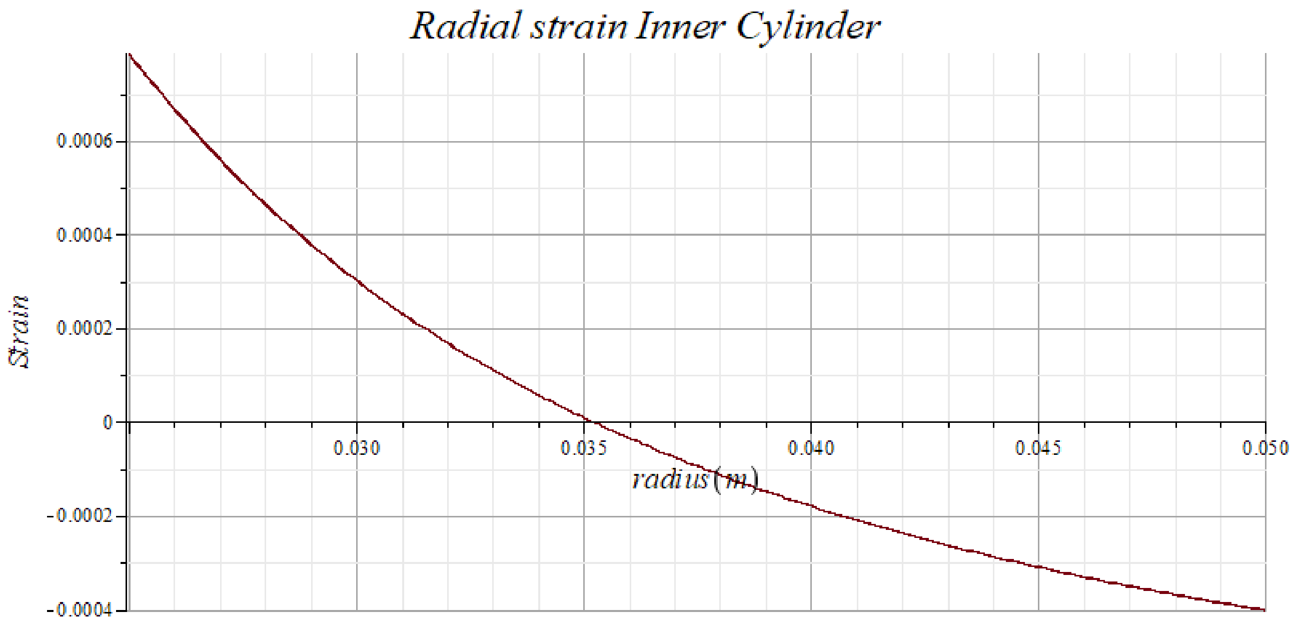


ANALYTICAL STRAIN DISTRIBUTIONS







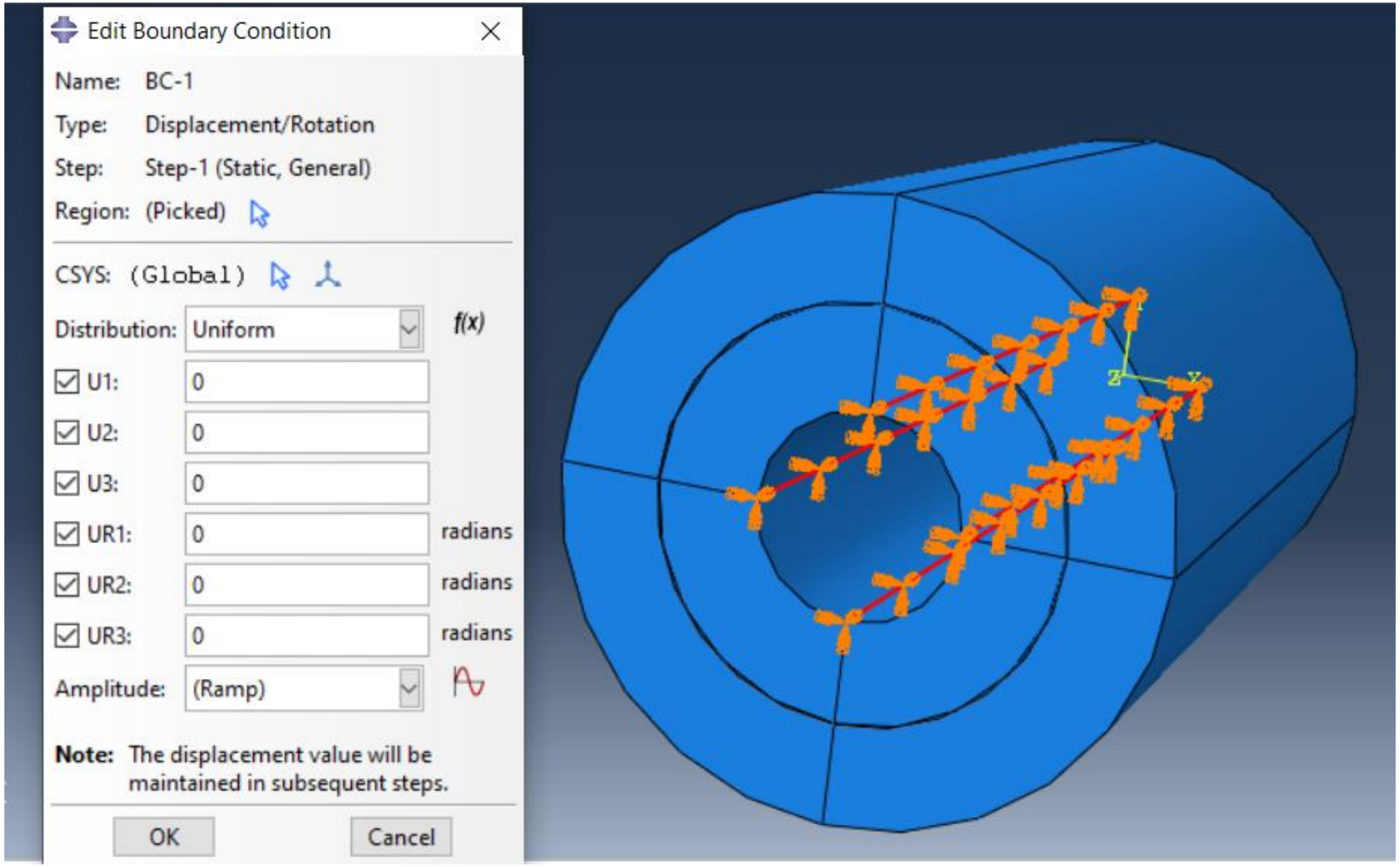


ABAQUS MODEL

Radial interference = 0.1909 mm

Mesh global size = 0.004 m

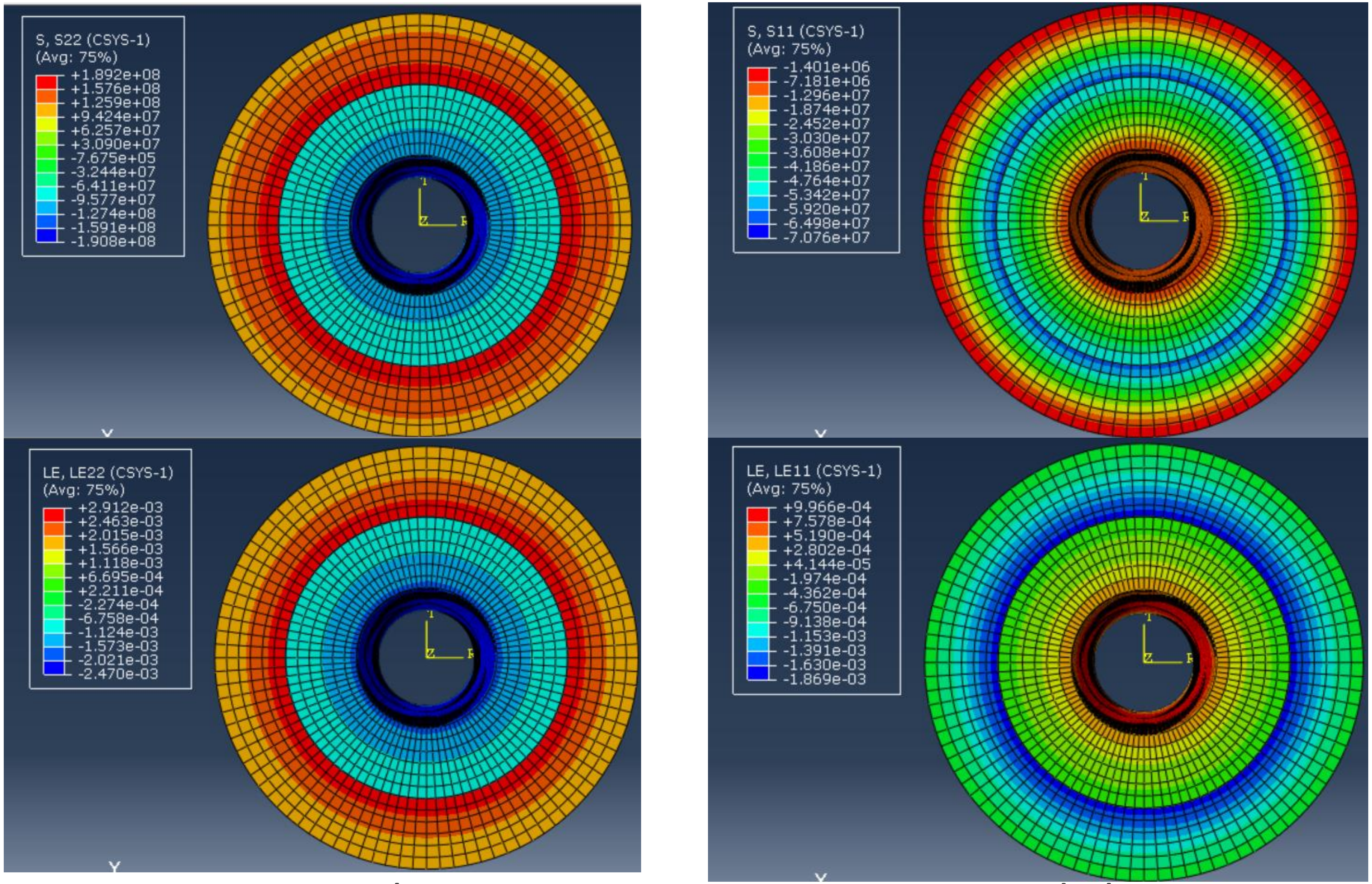
Hard contact with friction 0.2



ABAQUS RESULTS

Stresses

Strains

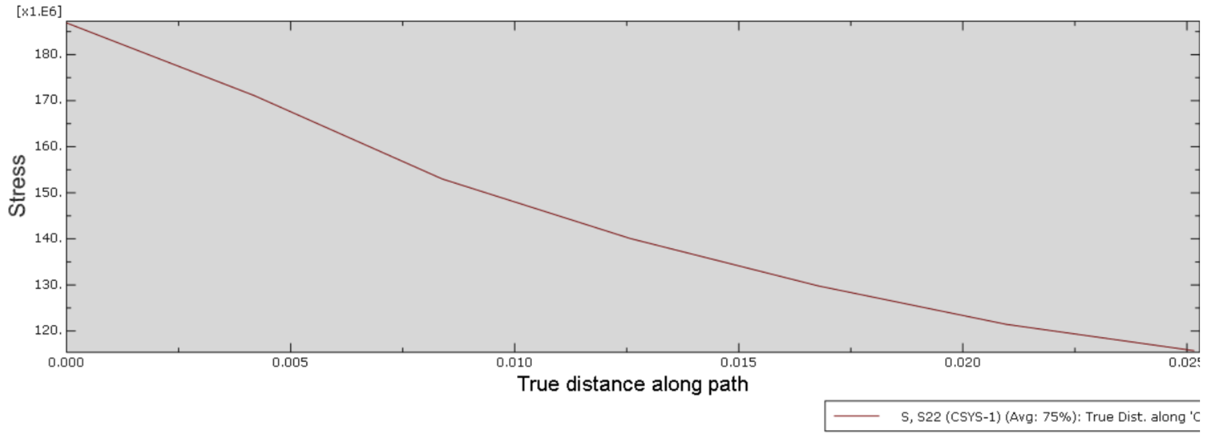


Tangential

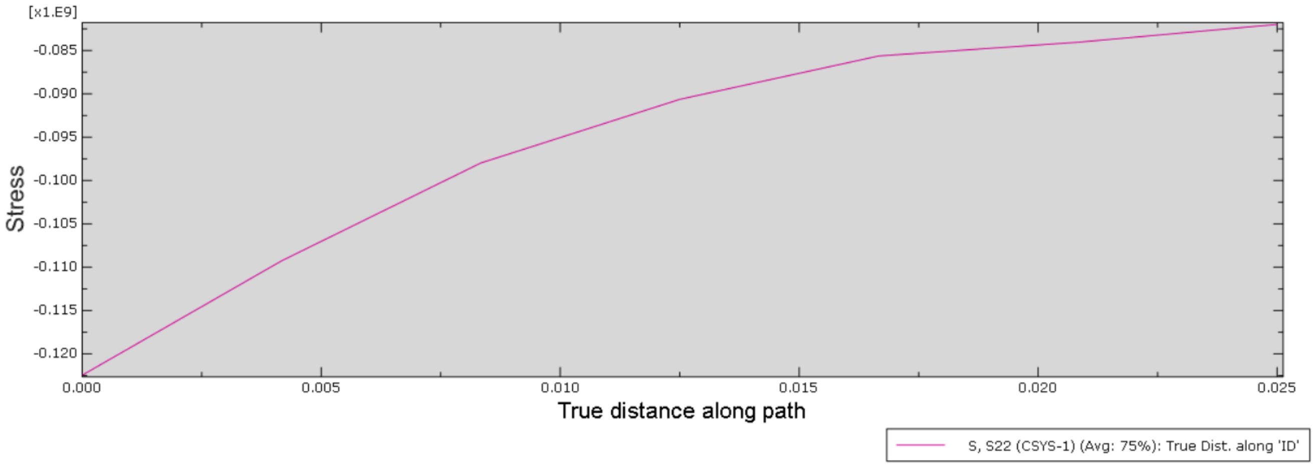
Radial

STRESSES IN ABAQUS

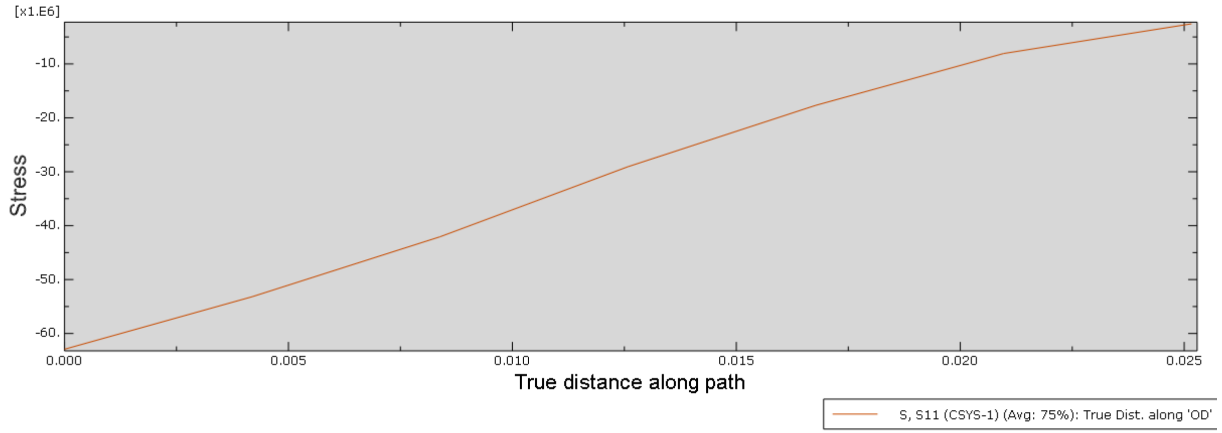
Tangential stress Outer Cylinder



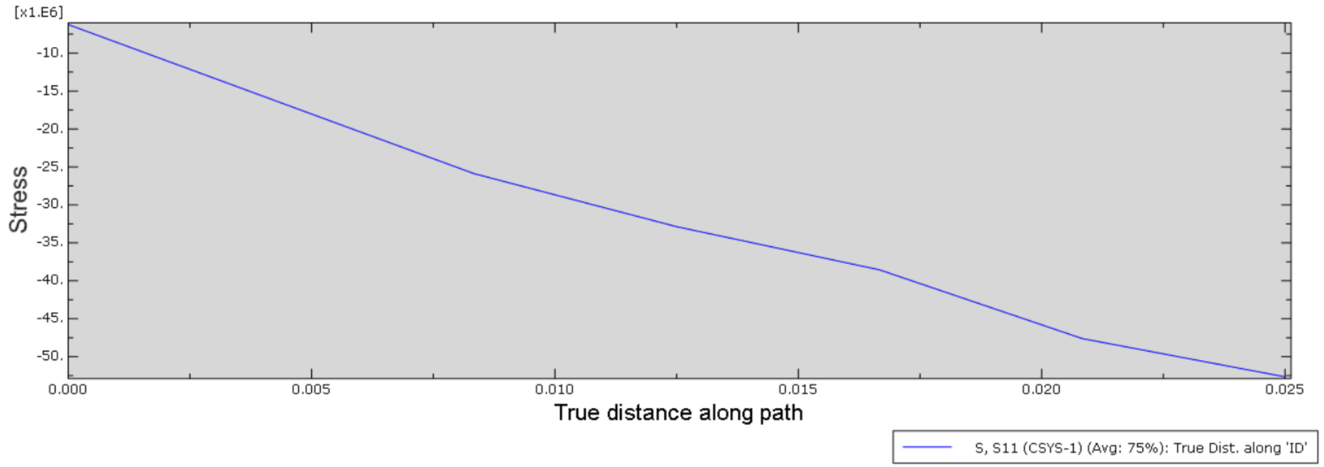
Tangential stress Inner Cylinder



Radial stress Outer Cylinder

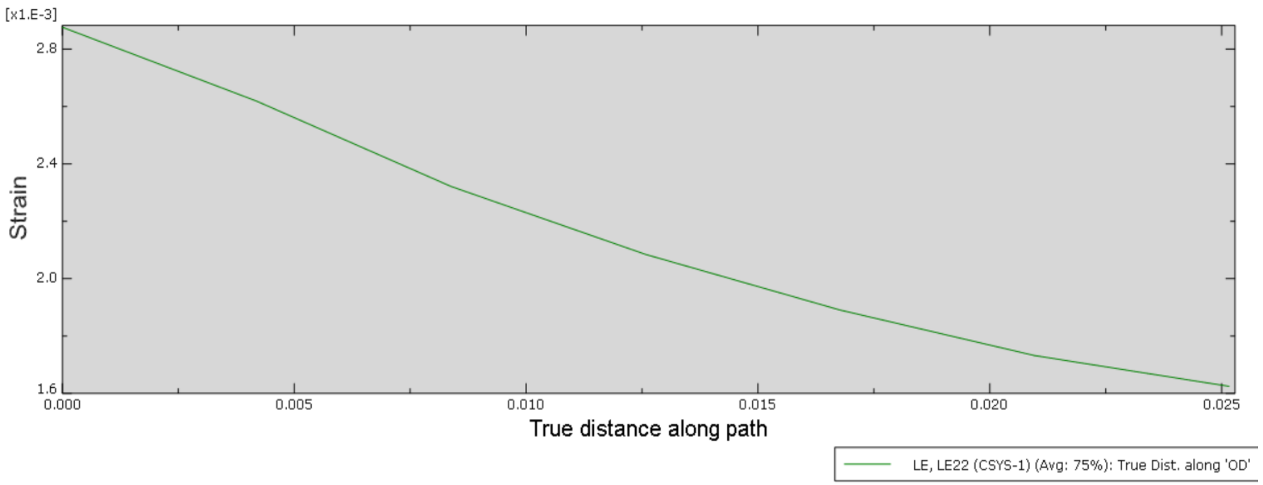


Radial stress Inner Cylinder

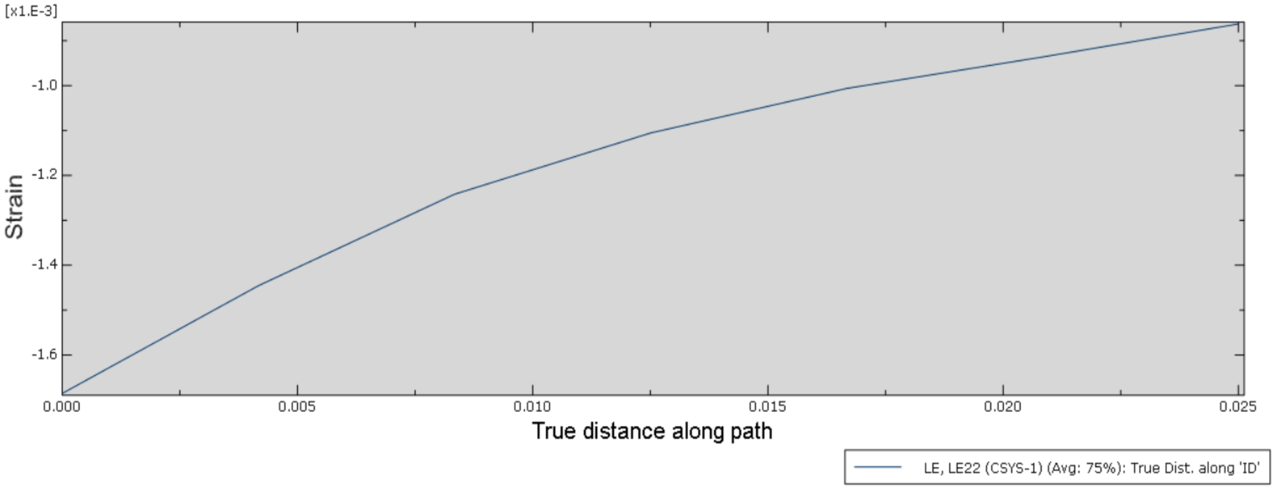


STRAINS IN ABAQUS

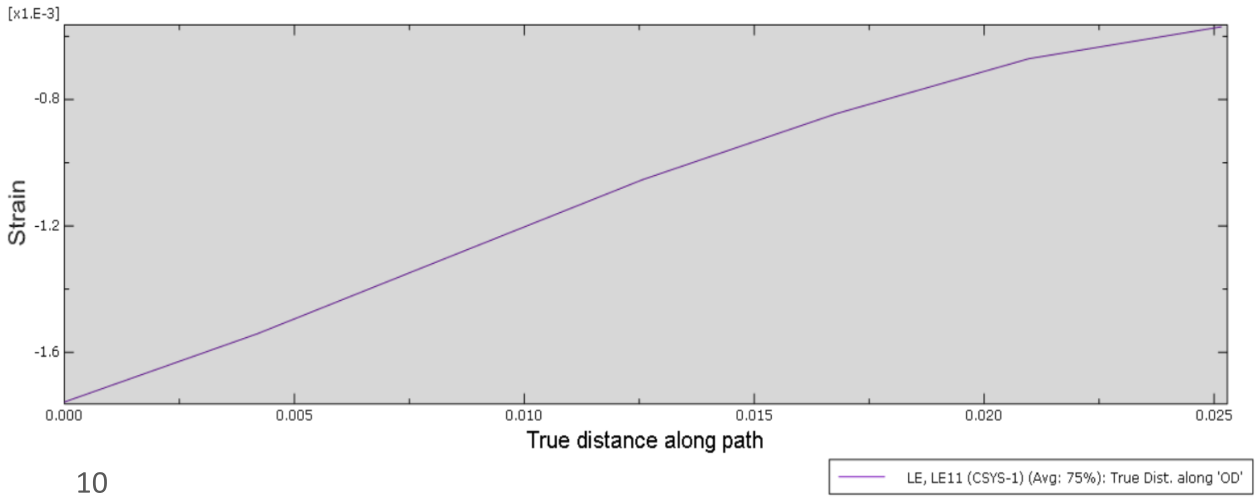
Tangential stain Outer Cylinder



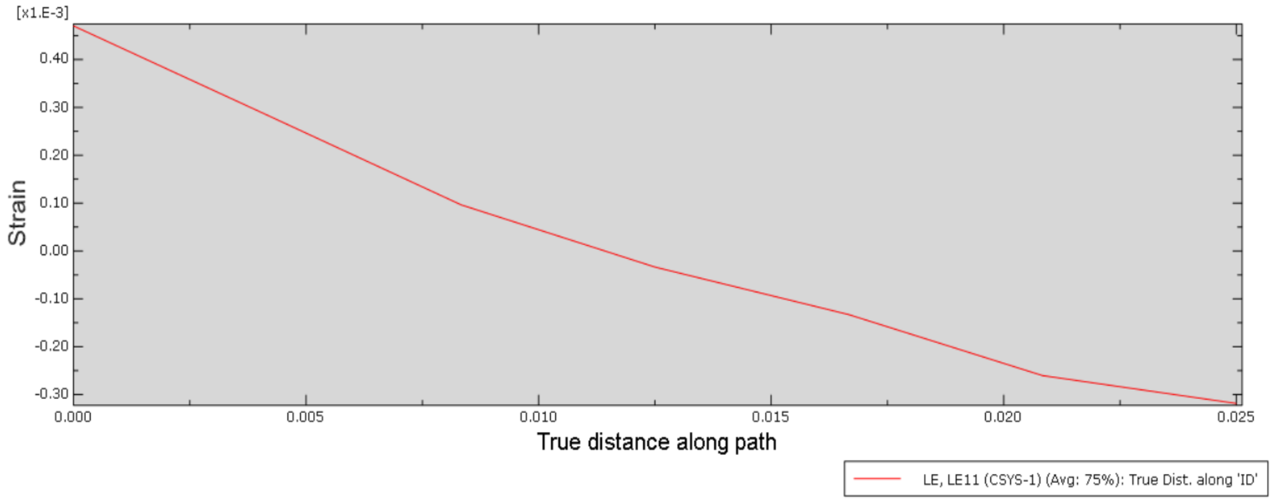
Tangential strain Inner Cylinder



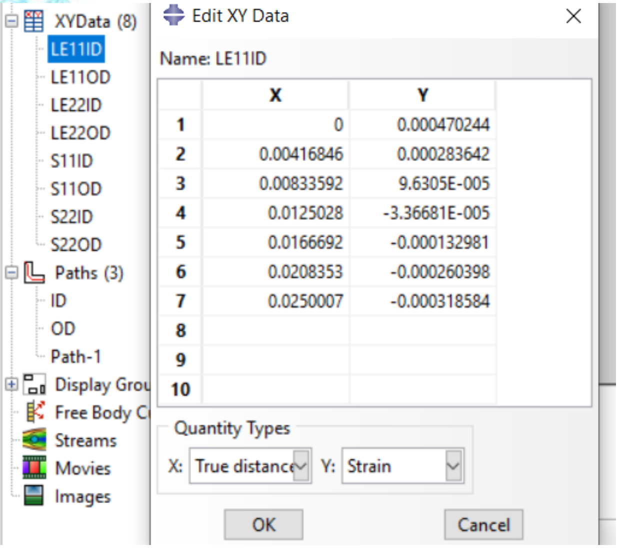
Radial strain Outer Cylinder

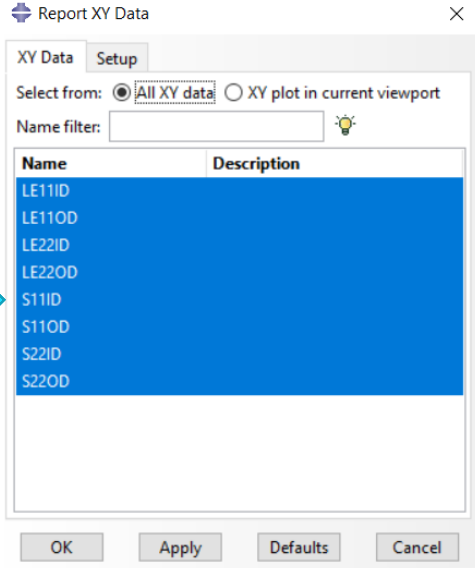


Radial strain Inner Cylinder



DATA ANALYSIS





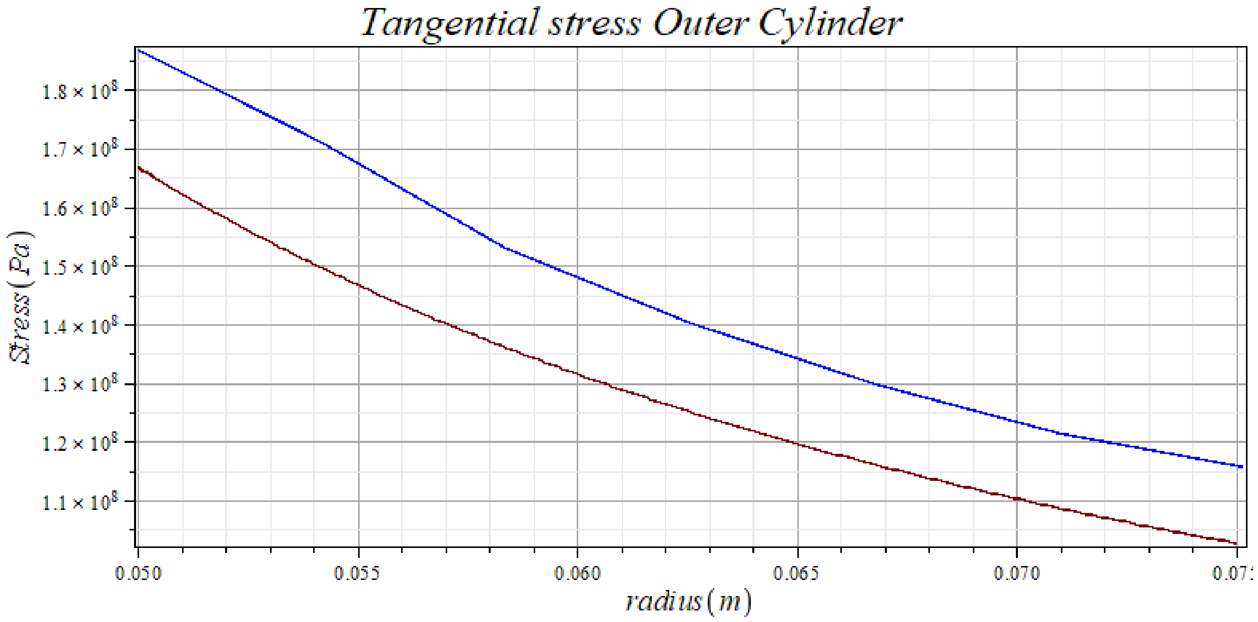
X	LE11ID
0.	470.244E-06
4.16846E-03	283.642E-06
8.33592E-03	96.3051E-06
12.5028E-03	-33.6681E-06
16.6692E-03	-132.981E-06
20.8353E-03	-260.398E-06
25.0007E-03	-318.584E-06

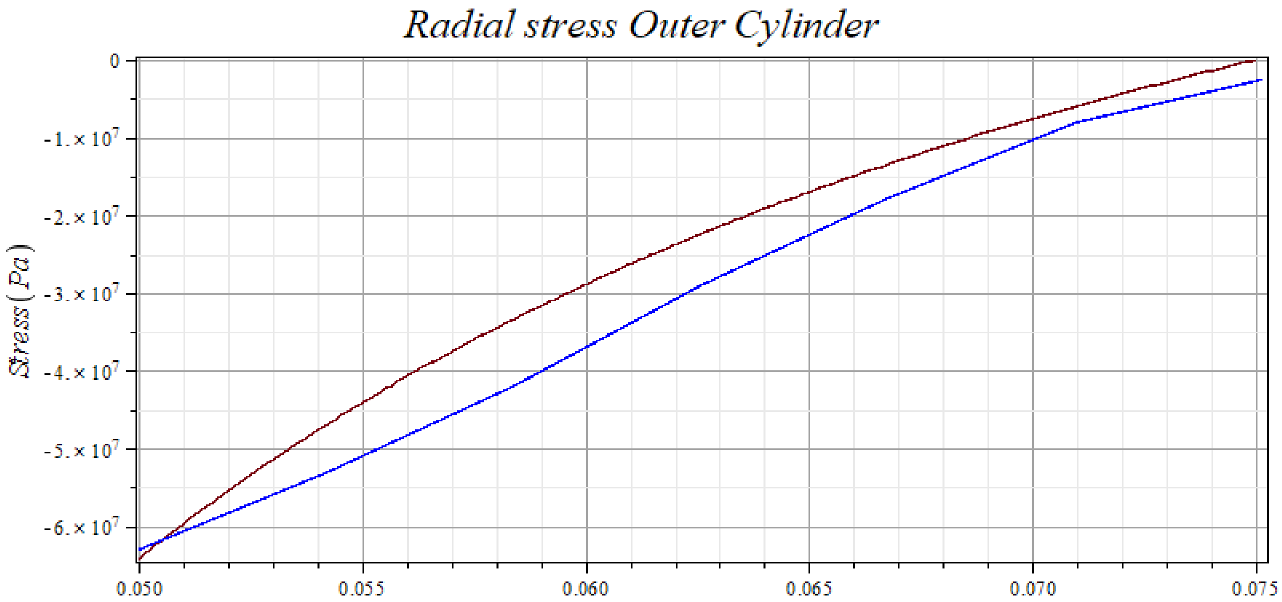
$S22OD := [[0, 186.878E+06], [4.19141E\text{-}03, 171.089E+06], [8.38383E\text{-}03, 153.002E+06], [12.5774E\text{-}03, 140.082E+06], [16.7719E\text{-}03, 129.796E+06], [20.9672E\text{-}03, 121.452E+06], [25.1632E\text{-}03, 115.724E+06]]:$

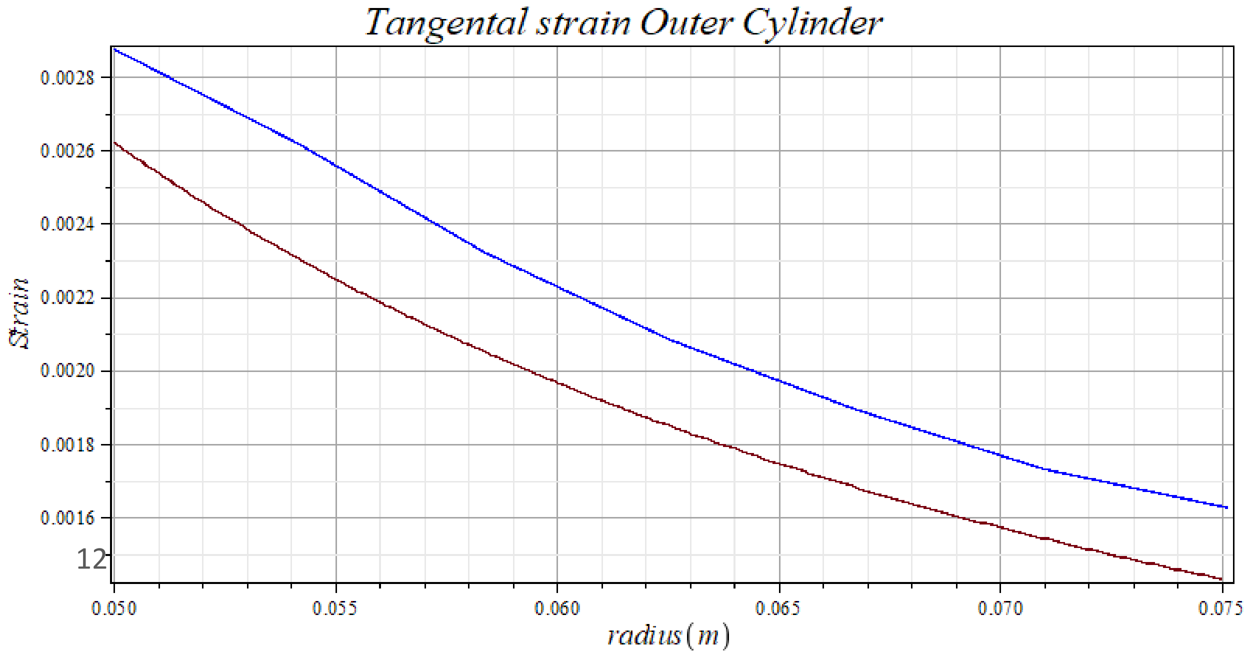
for i **from** 1 **to** 7 **do**
$L11ID[i][1] := L11ID[i][1] + 0.025 :$
$L22ID[i][1] := L22ID[i][1] + 0.025 :$
$S11ID[i][1] := S11ID[i][1] + 0.025 :$
$S22ID[i][1] := S22ID[i][1] + 0.025 :$
end do:
for i **from** 1 **to** 7 **do**
$L11OD[i][1] := L11OD[i][1] + 0.05 :$
$L22OD[i][1] := L22OD[i][1] + 0.05 :$
$S11OD[i][1] := S11OD[i][1] + 0.05 :$
$S22OD[i][1] := S22OD[i][1] + 0.05 :$
end do:

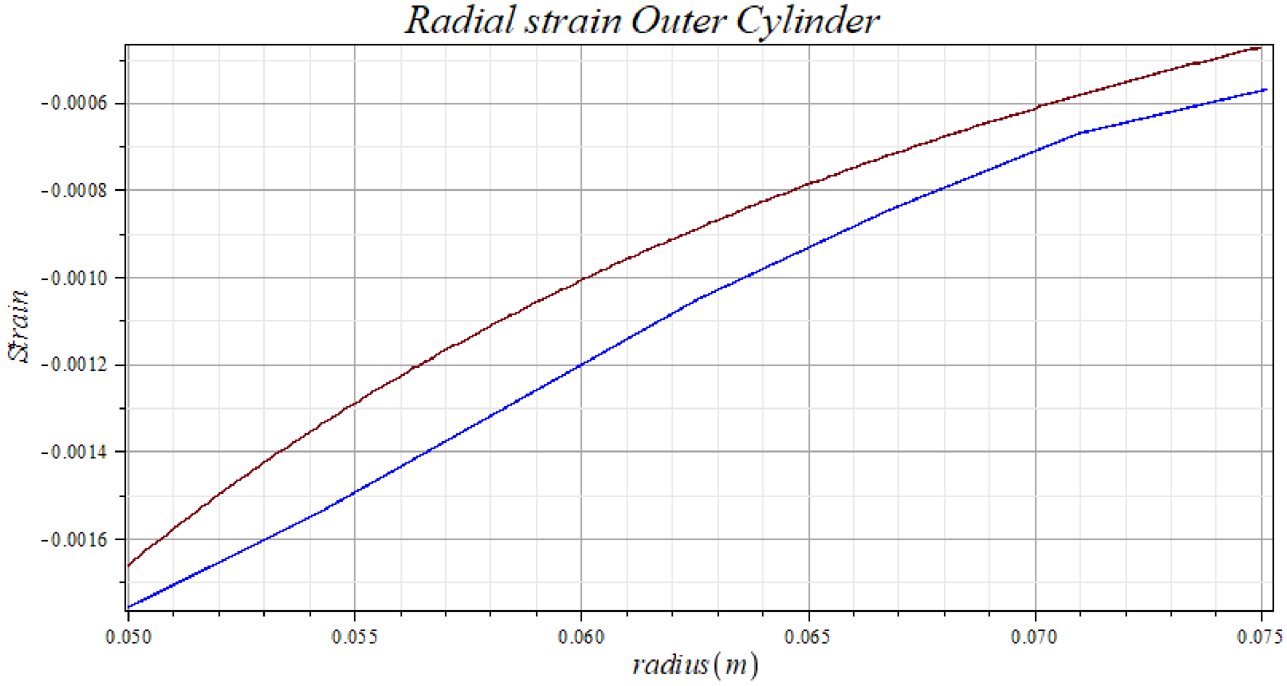
$S11ODP := pointplot(S11OD, color = [blue], axes = boxed, connect = true) :$
$S22ODP := pointplot(S22OD, color = [blue], axes = boxed, connect = true) :$

OUTER CYLINDER: THEORY VS ABAQUS

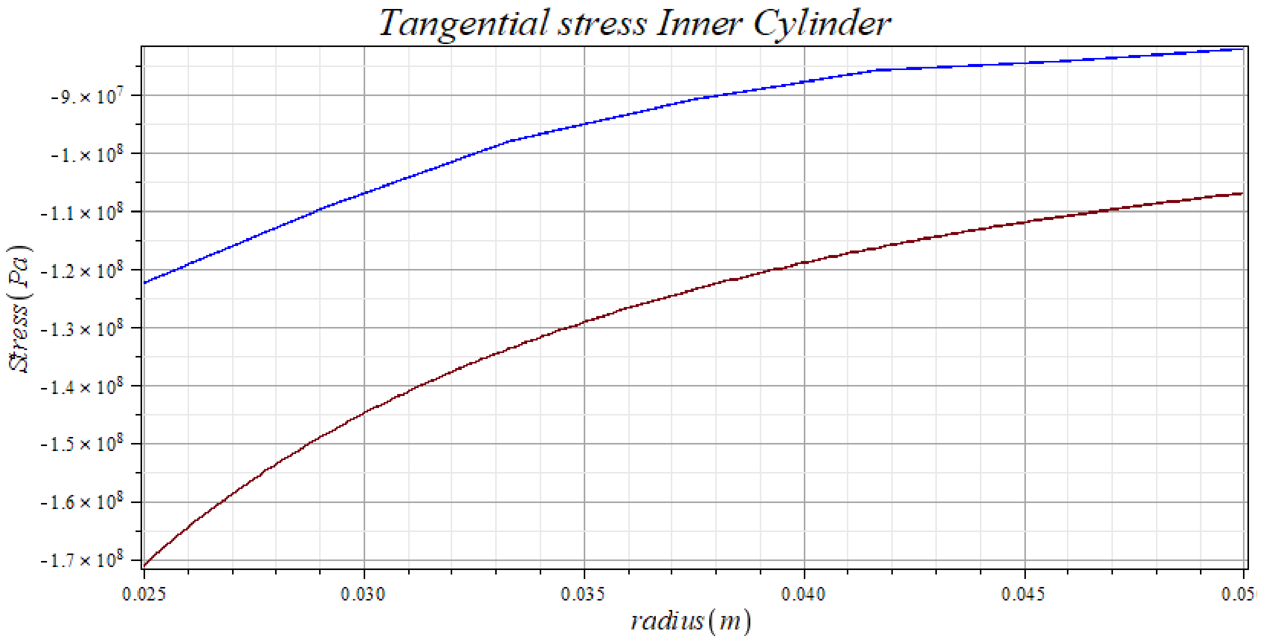


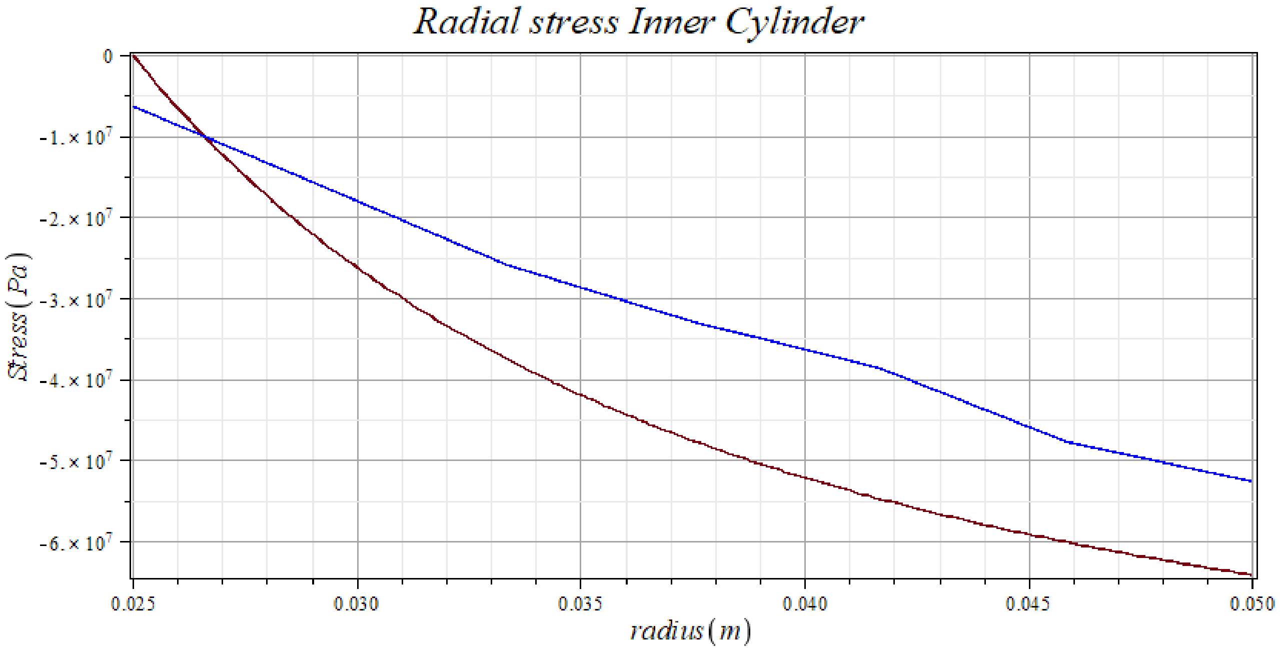


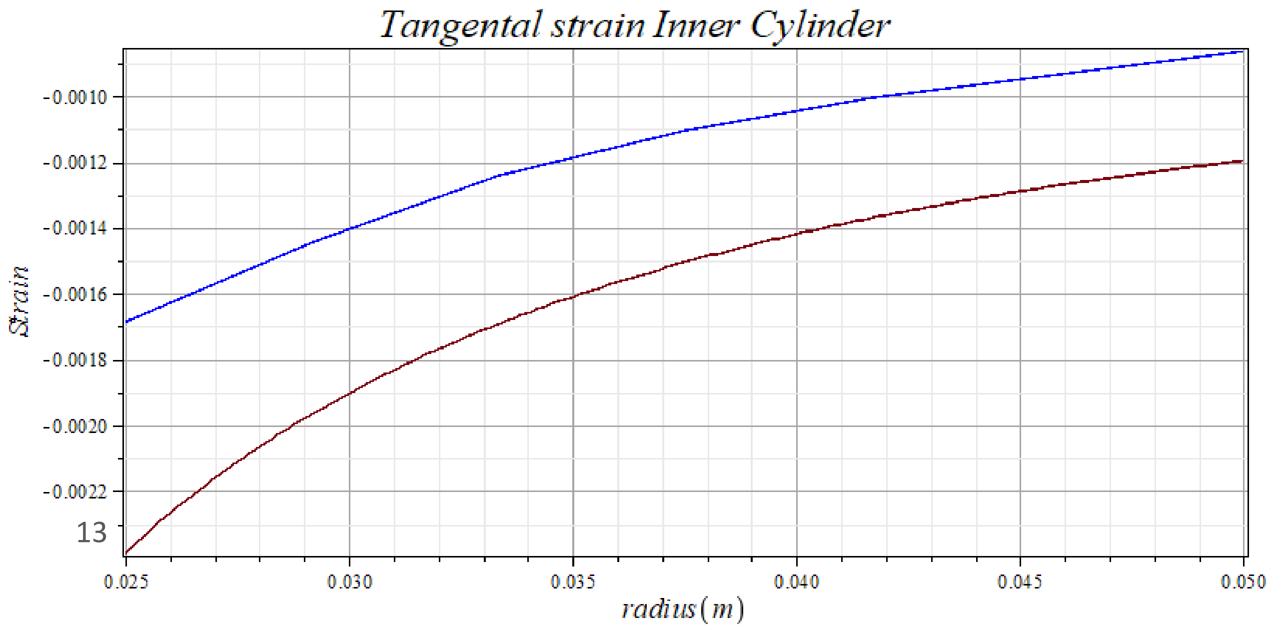


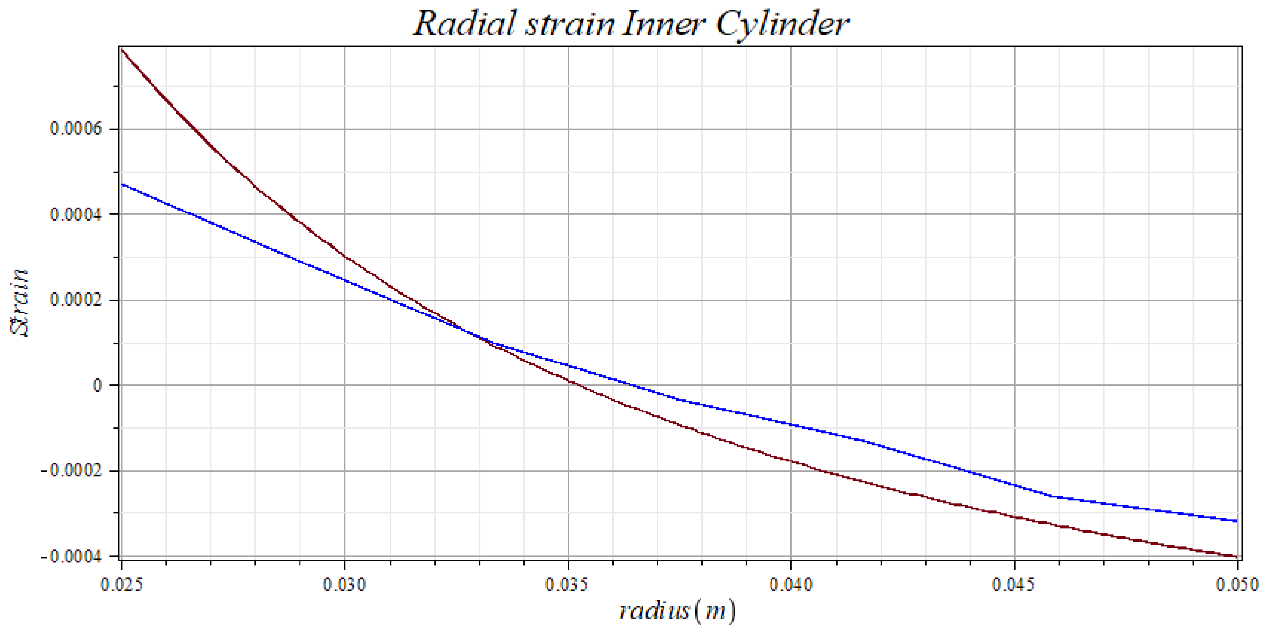


OUTER CYLINDER: THEORY VS ABAQUS

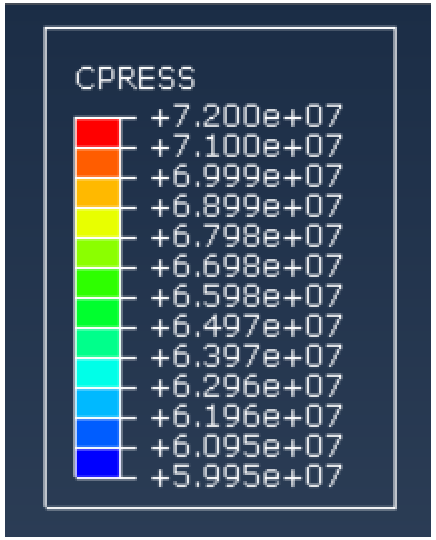








INTERFACE PRESSURE: THEORY VS ABAQUS



Average value from Abaqus:

65.98 MPa

Theoretical interface pressure:

64.17 MPa

CONCLUSIONS

- Radial stresses and strain values are close
- Tangential stresses and strains Abaqus values are greater
- Interface pressure – close agreement
- BC might affect the values but without BC the model is flying.