First I give variables for alphas with different subscripts that I will use in the matrix

$a1 := (k-4)^2$

$$(k-4)^2 \quad (1)$$

$a2 := (k-2)^2$

$$(k-2)^2 \quad (2)$$

$a3 := (k)^2$

$$k^2 \quad (3)$$

$a4 := (k+2)^2$

$$(k+2)^2 \quad (4)$$

$a5 := (k+4)^2$

$$(k+4)^2 \quad (5)$$

Then set U equals to 1 and make the matrix.

$U := 1$

$$1 \quad (6)$$

$M := Matrix(5, [[a1-e, U, 0, 0, 0], [U, a2-e, U, 0, 0], [0, U, a3-e, U, 0], [0, 0, U, a4-e, U], [0, 0, 0, U, a5-e]])$

$$\begin{bmatrix} (k-4)^2-e & 1 & 0 & 0 & 0 \\ 1 & (k-2)^2-e & 1 & 0 & 0 \\ 0 & 1 & k^2-e & 1 & 0 \\ 0 & 0 & 1 & (k+2)^2-e & 1 \\ 0 & 0 & 0 & 1 & (k+4)^2-e \end{bmatrix} \quad (7)$$

Calculate the Determinant; set it equals to 0 and numerically solve for e for different k - which produces the sequence where for each k there are 5 answers for e.

$A := Determinant(M)$

$$k^{10} - 5\,e\,k^8 + 10\,e^2\,k^6 - 40\,k^8 - 10\,e^3\,k^4 + 80\,e\,k^6 + 5\,e^4\,k^2 + 524\,k^6 - e^5 - 80\,e^3\,k^2 - 548\,e\,k^4 + 40\,e^4 + 548\,e^2\,k^2 - 2608\,k^4 - 524\,e^3 + 160\,e\,k^2 + 2448\,e^2 + 3715\,k^2 - 3203\,e - 2016 \quad (8)$$

$t := seq([k, fsolve(A=0, e)], k=-2..2, 0.1):$

So I need to plot all these points. To do that I made 5 sequances where I just have 2 numbers: 1st - k, 2nd - e. And I like produce 5 separate pointplots. And when I plot them together I get a nice loking plot of all the points.

$nops(\{t\})$

$$41 \quad (9)$$

$g1 := seq([(\{t\}[i,1]), (\{t\}[i,2])], i=1..41):$
$g2 := seq([(\{t\}[i,1]), (\{t\}[i,3])], i=1..41):$
$g3 := seq([(\{t\}[i,1]), (\{t\}[i,4])], i=1..41):$
$g4 := seq([(\{t\}[i,1]), (\{t\}[i,5])], i=1..41):$
$g5 := seq([(\{t\}[i,1]), (\{t\}[i,6])], i=1..41):$

Loading plots

a := *pointplot*({*g1*})

PLOT(...)

b := *pointplot*({*g2*})

PLOT(...) **(11)**

c := *pointplot*({*g3*})

PLOT(...) **(12)**

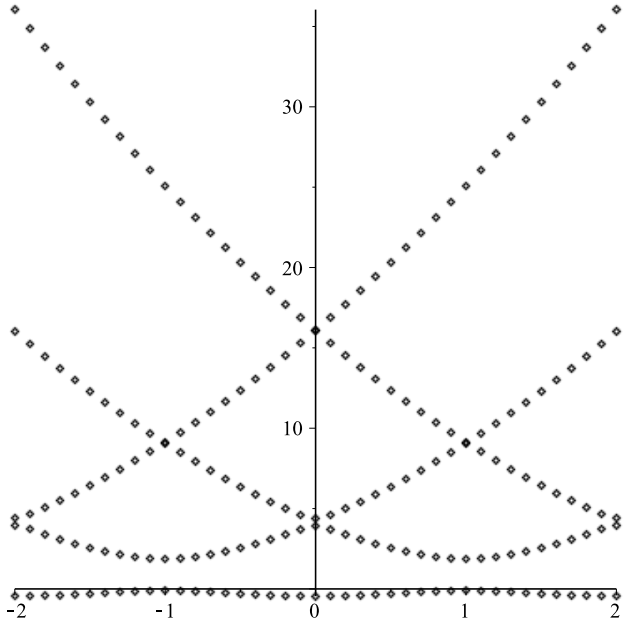
d := *pointplot*({*g4*})

PLOT(...) **(13)**

r := *pointplot*({*g5*})

PLOT(...) **(14)**

display(*a*, *b*, *c*, *d*, *r*)



With U=2, I repeat all of the above just for U equals 2:

U2 := 2

2 **(15)**

M2 := *Matrix*(5, [[*a1* − *e*, *U2*, 0, 0, 0], [*U2*, *a2* − *e*, *U2*, 0, 0], [0, *U2*, *a3* − *e*, *U2*, 0], [0, 0, *U2*, *a4*

$-e, U2], [0, 0, 0, U2, a5 - e]])$

$$\begin{bmatrix} (k-4)^2 - e & 2 & 0 & 0 & 0 \\ 2 & (k-2)^2 - e & 2 & 0 & 0 \\ 0 & 2 & k^2 - e & 2 & 0 \\ 0 & 0 & 2 & (k+2)^2 - e & 2 \\ 0 & 0 & 0 & 2 & (k+4)^2 - e \end{bmatrix} \quad (16)$$

$A2 := Determinant(M2)$

$$k^{10} - 5\,e\,k^8 + 10\,e^2\,k^6 - 40\,k^8 - 10\,e^3\,k^4 + 80\,e\,k^6 + 5\,e^4\,k^2 + 512\,k^6 - e^5 - 80\,e^3\,k^2 - 512\,e\,k^4 + 40\,e^4 + 512\,e^2\,k^2 - 2752\,k^4 - 512\,e^3 + 640\,e\,k^2 + 2112\,e^2 + 2608\,k^2 - 560\,e - 7680 \quad (17)$$

$t2 := seq([k, fsolve(A2 = 0, e)], k = -2..2, 0.1):$
$nops(\{t2\})$

$$41 \quad (18)$$

$f1 := seq([(\{t2\}[i, 1]), (\{t2\}[i, 2])], i = 1..41):$
$f2 := seq([(\{t2\}[i, 1]), (\{t2\}[i, 3])], i = 1..41):$
$f3 := seq([(\{t2\}[i, 1]), (\{t2\}[i, 4])], i = 1..41):$
$f4 := seq([(\{t2\}[i, 1]), (\{t2\}[i, 5])], i = 1..41):$
$f5 := seq([(\{t2\}[i, 1]), (\{t2\}[i, 6])], i = 1..41):$
$a1 := pointplot(\{f1\})$

$$PLOT(\ldots) \quad (19)$$

$b1 := pointplot(\{f2\})$

$$PLOT(\ldots) \quad (20)$$

$c1 := pointplot(\{f3\})$

$$PLOT(\ldots) \quad (21)$$

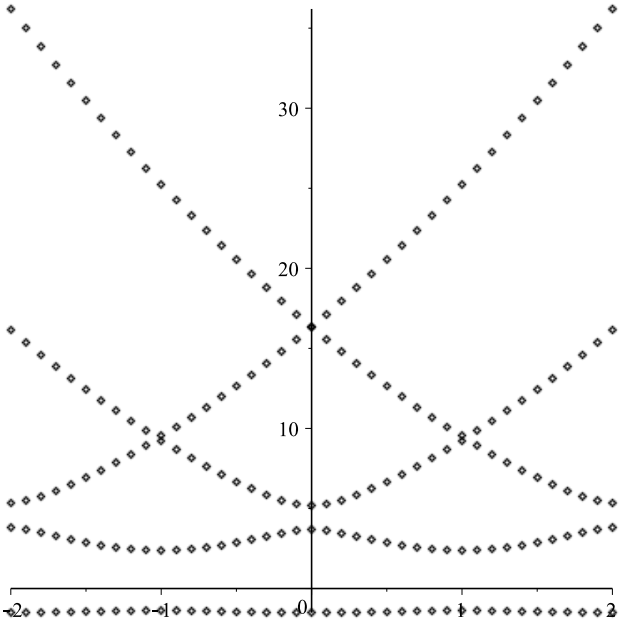
$d1 := pointplot(\{f4\})$

$$PLOT(\ldots) \quad (22)$$

$r1 := pointplot(\{f5\})$

$$PLOT(\ldots) \quad (23)$$

$display(a1, b1, c1, d1, r1)$


30
20
10
0
-2
-1
1
2