```
restart
with(plots):
interface(showassumed=0):
assume(k>0, kp>0, d>0)
eq1 := C·sin(kp·d) = D·sin(k·d + delta)
```

$$C \sin(kp\,d) = \mathrm{D} \sin(d\,k + \delta) \tag{1}$$

```
eq2 := C·kp·cos(kp·d) = D·k·cos(k·d + delta)
```

$$C\,kp \cos(kp\,d) = \mathrm{D}\,k \cos(d\,k + \delta) \tag{2}$$

```
solMaple := allvalues(solve({eq1, eq2}, {C, delta}))
```

$$\left\{ C = \sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ k\,\mathrm{D},\ \delta = -d\,k + \arctan\left(k \sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ \sin(kp\,d),\ \sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ kp \cos(kp\,d) \right) \right\}, \left\{ C = -\sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ k\,\mathrm{D},\ \delta = -d\,k + \arctan\left(-k \sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ \sin(kp\,d),\ -\sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ kp \cos(kp\,d) \right) \right\} \tag{3}$$

```
solMaple1 := -d·k + arctan(sqrt(1/(cos(kp·d)^2·kp^2 - cos(kp·d)^2·k^2 + k^2))·kp·cos(kp·d))
```

$$-d\,k + \arctan\left(\sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ kp \cos(kp\,d) \right)$$

$$-d\,k + \arctan\left(\sqrt{\frac{1}{\cos(kp\,d)^2\,kp^2 - \cos(kp\,d)^2\,k^2 + k^2}}\ kp \cos(kp\,d) \right) \tag{4}$$

```
C := solve(eq1, C)
```

$$\frac{\mathrm{D} \sin(d\,k + \delta)}{\sin(kp\,d)} \tag{5}$$

```
eq2 := C·kp·cos(kp·d) = D·k·cos(k·d + delta)
```

$$\frac{\mathrm{D} \sin(d\,k + \delta)\ kp \cos(kp\,d)}{\sin(kp\,d)} = \mathrm{D}\,k \cos(d\,k + \delta) \tag{6}$$

```
sol := solve(eq2, delta)
```

$$-d\,k + \arctan\left(\frac{k \tan(kp\,d)}{kp} \right) \tag{7}$$

```
series(sol, k=0)
```

$$\left(-d + \frac{\tan(kp\,d)}{kp} \right) k - \frac{1}{3}\ \frac{\tan(kp\,d)^3}{kp^3}\ k^3 + \frac{1}{5}\ \frac{\tan(kp\,d)^5}{kp^5}\ k^5 + \mathrm{O}(k^7) \tag{8}$$

```
series(solMaple1, k=0)
```

$$\arctan\left(\sqrt{\frac{1}{\cos(kp\,d)^2}}\,\cos(kp\,d)\right) - d\,k + \frac{1}{4}\,\frac{\sqrt{\frac{1}{\cos(kp\,d)^2}}\,\left(\cos(kp\,d)^2-1\right)}{kp^2\cos(kp\,d)}\,k^2 + \frac{1}{8}\,\frac{\sqrt{\frac{1}{\cos(kp\,d)^2}}\,\left(\cos(kp\,d)^2-1\right)^2}{kp^4\cos(kp\,d)^3}\,k^4 + O(k^6) \tag{9}$$

$k := E \to \mathrm{sqrt}\left(\frac{2\cdot m\cdot E}{h^2}\right)$:

$kp := E \to \mathrm{sqrt}\left(\frac{2\cdot m\cdot (E+V0)}{h^2}\right)$:

$\mathrm{delta} := E \to \left(-d + \frac{\tan(kp(E)\cdot d)}{kp(E)}\right) k(E)$

$$E \to \left(-d + \frac{\tan(kp(E)\,d)}{kp(E)}\right) k(E) \tag{10}$$

this one is fromexpanding the solution that I did woth maple with systems of equations. Wierd how it does not cancel out.

$delta2 := E \to -\arctan\left(\sqrt{\frac{1}{\cos(kp(E)\cdot d)^2}}\,\cos(kp(E)\,d)\right) + d\,k(E)$

$$E \to -\arctan\left(\sqrt{\frac{1}{\cos(kp(E)\,d)^2}}\,\cos(kp(E)\,d)\right) + d\,k(E)$$

$\mathrm{sigma} := E \to \frac{4\cdot \mathrm{Pi}}{k(E)^2}\cdot(2\cdot 0+1)\cdot\sin^2(\mathrm{delta}(E))$

$$E \to \frac{4\,\pi\sin(\delta(E))^2}{k(E)^2} \tag{12}$$

$sigma1 := E \to \frac{4\cdot \mathrm{Pi}}{k(E)^2}\cdot(2\cdot 0+1)\cdot\sin^2(delta2(E))$

$$E \to \frac{4\,\pi\sin(\delta 2(E))^2}{k(E)^2} \tag{13}$$

$gg := n \to solve\left(kp(E)\cdot d = \frac{\left(n+\frac{1}{2}\right)\cdot \mathrm{Pi}}{2}, E\right)$:

$d := 10\cdot 10^{-10}$:

$m := 9.1\cdot 10^{-31}$:

$h := 1.054\cdot 10^{-34}$:

$V0 := 5\cdot 1.6\cdot 10^{-19}$:

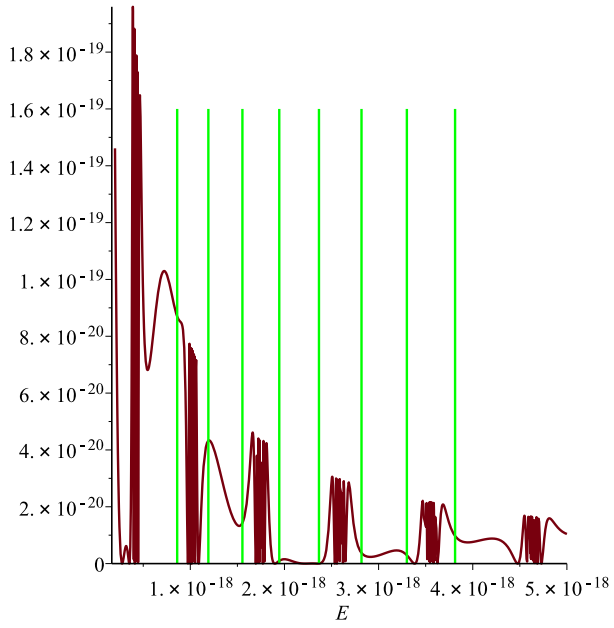
$ss5eVRange := seq(gg(i), i = 10..18)$

$$8.604591127\,10^{-19},\ 1.191797893\,10^{-18},\ 1.553258380\,10^{-18},\ 1.944840574\,10^{-18},\ 2.366544476\,10^{-18},\ 2.818370085\,10^{-18},\ 3.300317401\,10^{-18},\ 3.812386425\,10^{-18},\ 4.354577156\,10^{-18} \tag{14}$$

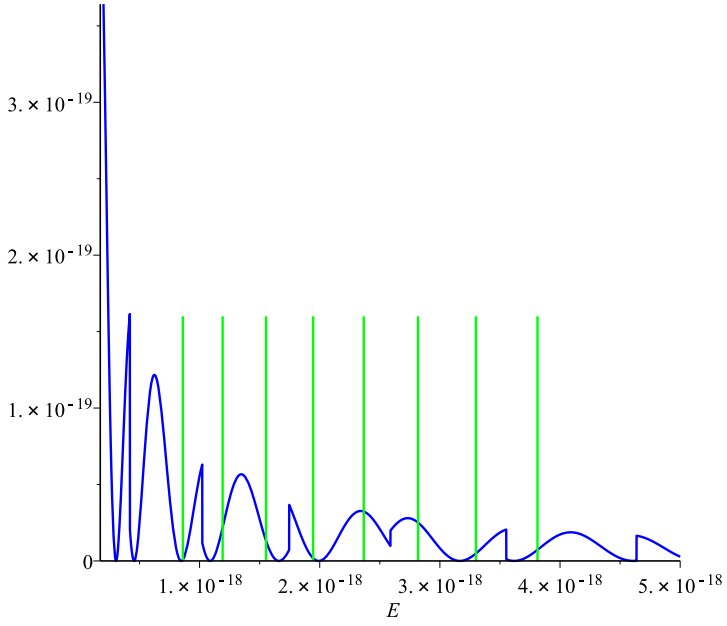
$ssMapleSmall := seq(gg(i), i = 1000000..1000008)$

$$1.506086869\,10^{-8},\ 1.506089881\,10^{-8},\ 1.506092894\,10^{-8},\ 1.506095906\,10^{-8},\ 1.506098918\,10^{-8},\ 1.506101930\,10^{-8},\ 1.506104942\,10^{-8},\ 1.506107955\,10^{-8},\ 1.506110967\,10^{-8} \tag{15}$$

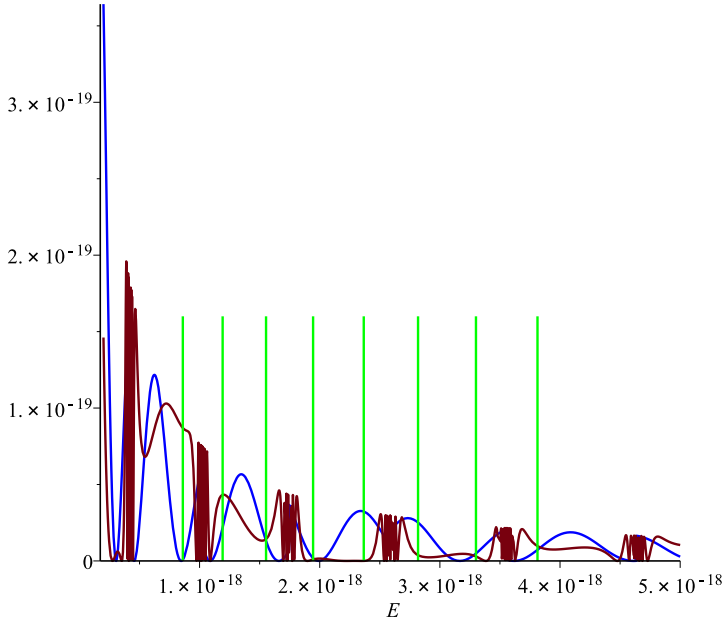
```
sp1eVR := plot( [ss5eVRange[1], y, y = 0 ..1.6·10^-19], color = green) :
sp2eVR := plot( [ss5eVRange[2], y, y = 0 ..1.6·10^-19], color = green) :
sp3eVR := plot( [ss5eVRange[3], y, y = 0 ..1.6·10^-19], color = green) :
sp4eVR := plot( [ss5eVRange[4], y, y = 0 ..1.6·10^-19], color = green) :
sp5eVR := plot( [ss5eVRange[5], y, y = 0 ..1.6·10^-19], color = green) :
sp6eVR := plot( [ss5eVRange[6], y, y = 0 ..1.6·10^-19], color = green) :
sp7eVR := plot( [ss5eVRange[7], y, y = 0 ..1.6·10^-19], color = green) :
sp8eVR := plot( [ss5eVRange[8], y, y = 0 ..1.6·10^-19], color = green) :

sp1MS := plot( [ssMapleSmall[1], y, y = 0 ..6·10^-30], color = green) :
sp2MS := plot( [ssMapleSmall[2], y, y = 0 ..6·10^-30], color = green) :
sp3MS := plot( [ssMapleSmall[3], y, y = 0 ..6·10^-30], color = green) :
sp4MS := plot( [ssMapleSmall[4], y, y = 0 ..6·10^-30], color = green) :
sp5MS := plot( [ssMapleSmall[5], y, y = 0 ..6·10^-30], color = green) :
sp6MS := plot( [ssMapleSmall[6], y, y = 0 ..6·10^-30], color = green) :
sp7MS := plot( [ssMapleSmall[7], y, y = 0 ..6·10^-30], color = green) :
sp8MS := plot( [ssMapleSmall[8], y, y = 0 ..6·10^-30], color = green) :
pp := plot(sigma(E), E = 2·10^-19 ..50·10^-19) :
display( pp, sp1eVR, sp2eVR, sp3eVR, sp4eVR, sp5eVR, sp6eVR, sp7eVR, sp8eVR)
```




```
hs := plot(sigma1(E), E = 2·10^-19 ..50·10^-19, color = blue) :
display( hs, sp1eVR, sp2eVR, sp3eVR, sp4eVR, sp5eVR, sp6eVR, sp7eVR, sp8eVR)
```

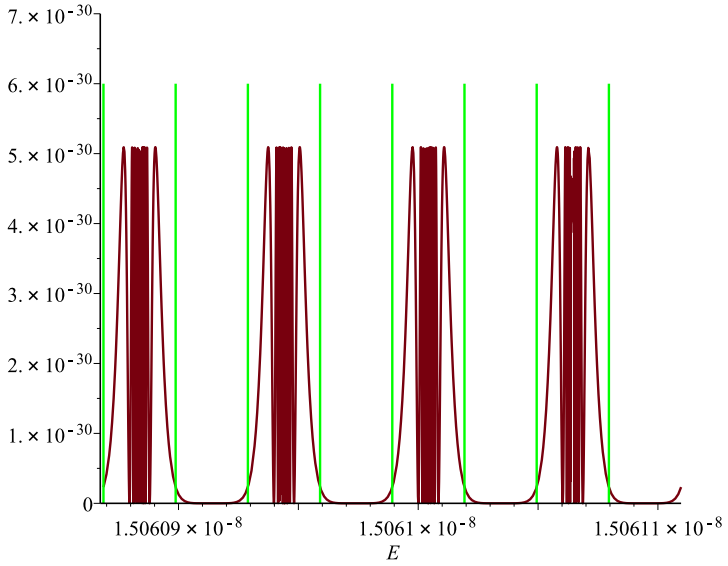


$display(hs, pp, sp1eVR, sp2eVR, sp3eVR, sp4eVR, sp5eVR, sp6eVR, sp7eVR, sp8eVR)$



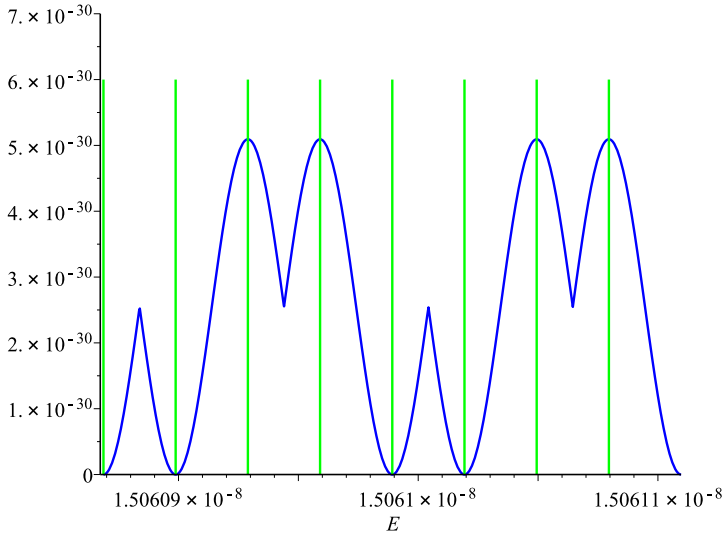
$pp1 := plot(\text{sigma}(E), E = 1.506086869 \cdot 10^{-8} .. 1.506110967\ 10^{-8}, view = [1.506086869 \cdot 10^{-8} .. 1.506110967\ 10^{-8}, 0 .. 7 \cdot 10^{-30}])$:

$display(pp1, sp1MS, sp2MS, sp3MS, sp4MS, sp5MS, sp6MS, sp7MS, sp8MS)$

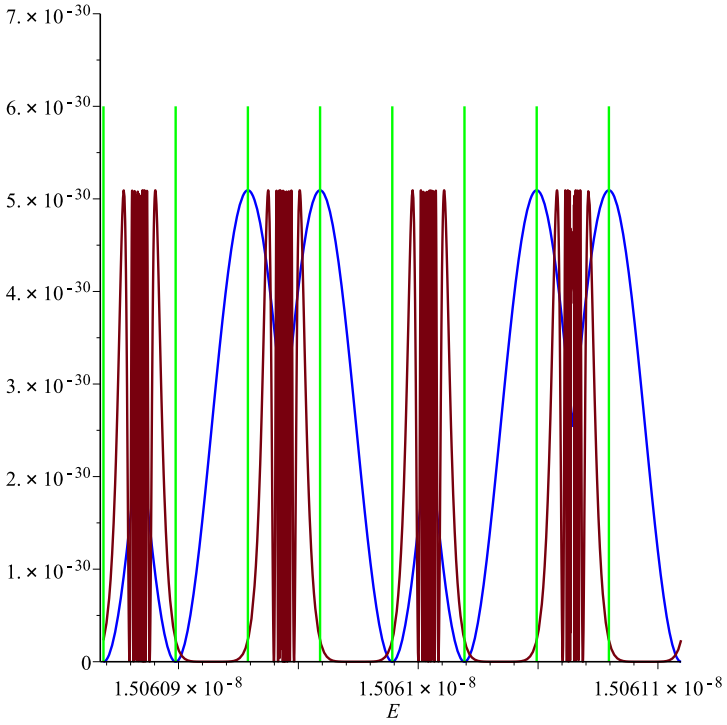


$hs1 := plot(sigma1(E), E = 1.506086869\ 10^{-8} ..1.506110967\ 10^{-8}, view = [1.506086869\ 10^{-8} ..1.506110967\ 10^{-8}, 0 ..7 \cdot 10^{-30}], color = blue)$:

$display(hs1, sp1MS, sp2MS, sp3MS, sp4MS, sp5MS, sp6MS, sp7MS, sp8MS)$



$display(hs1, pp1, sp1MS, sp2MS, sp3MS, sp4MS, sp5MS, sp6MS, sp7MS, sp8MS)$


7. × 10^-30
6. × 10^-30
5. × 10^-30
4. × 10^-30
3. × 10^-30
2. × 10^-30
1. × 10^-30
0
1.50609 × 10^-8
1.5061 × 10^-8
1.50611 × 10^-8
E