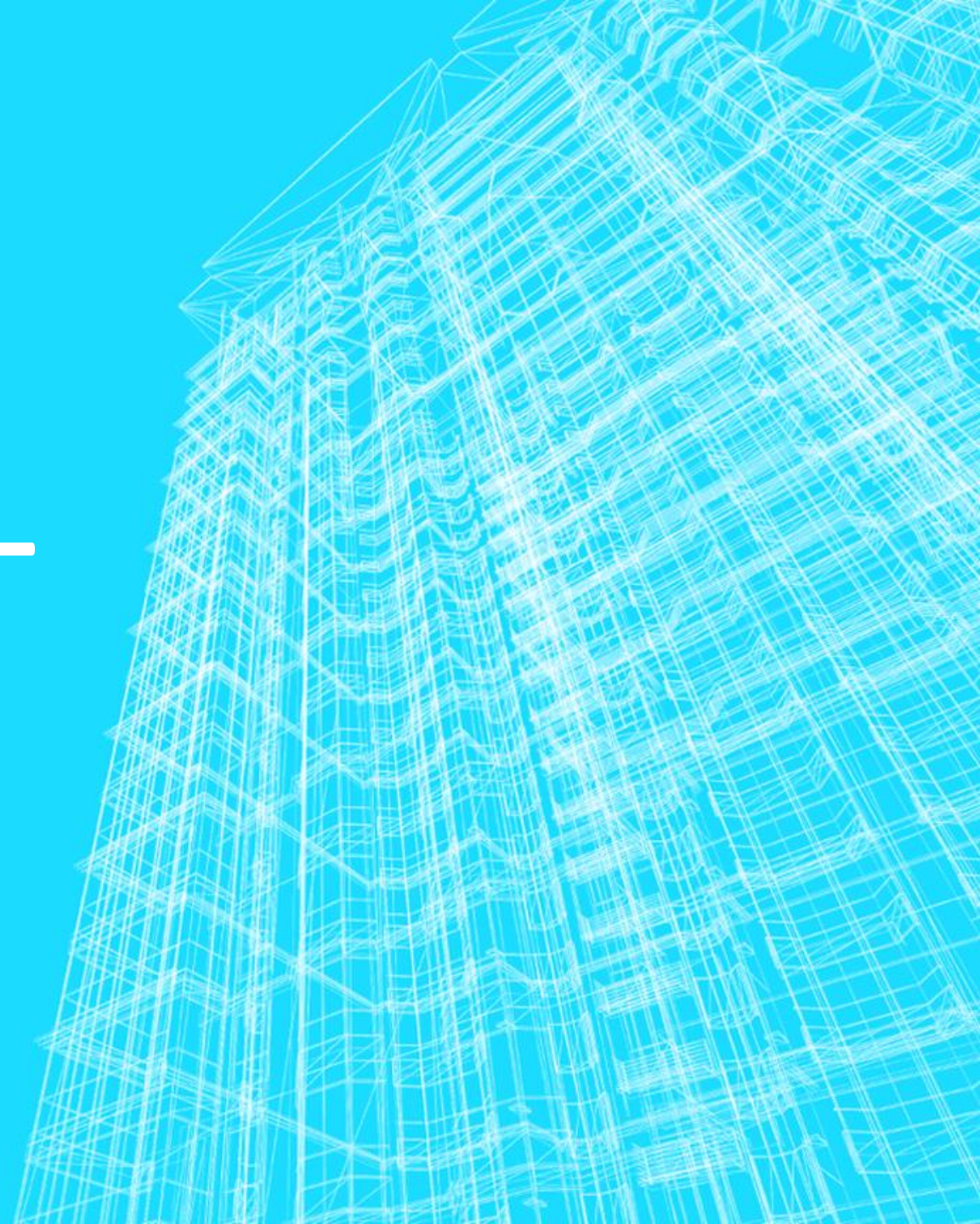


FINAL PROJECT

Anastasija Cumika

Structural analysis and design 2020





CYLINDER SHRUNK IN ANOTHER CYLINDER

Dimensions:

1. Inner cylinder: $R=25..50$ m
2. Outer cylinder: $R=50..75$ m
3. Length = 250 mm

Material – Al 7075-T6:

1. Young's Modulus: 71.7 GPa
2. Poisson's ratio : 0.33
3. Yield stress (for 25.43-50.8mm thickness): ≥ 462 MPa

Factor of Safety = 2

<http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA7075T6>

<http://www.matweb.com/search/DataSheet.aspx?MatGUID=4f19a42be94546b686bbf43f79c51b7d&ckck=1>

Find stresses and strains distribution for max allowable radial interference

MAXIMUM ALLOWABLE RADIAL INTERFERENCE

$$p_i = \frac{E\delta}{b} \left[\frac{(c^2 - b^2)(b^2 - a^2)}{2b^2(c^2 - a^2)} \right] \quad (7.37)$$

Where:

E = Young's Modulus

δ = radial interference between the two cylinders

a = inner radius of the inner cylinder

b = outer radius of inner cylinder and inner radius of outer cylinder

c = outer radius of outer cylinder

Using tresca critria:

$$tmax := \frac{sy}{2 \cdot SF}$$

$$eq := tmax = \frac{(s1 - s2)}{2}$$

Lame's equations

Case 1: Internal pressure only ($P_o=0$):

At inside surface, $r = r_i$:

$$\sigma_\theta = P_i \left[\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \right] \quad \sigma_r = -P_i \quad \sigma_z = \frac{P_i r_i^2}{r_o^2 - r_i^2} \quad (7.32)$$

At outside surface, $r = r_o$:

$$\sigma_\theta = \left[\frac{2P_i r_i^2}{r_o^2 - r_i^2} \right] \quad \sigma_r = 0 \quad \sigma_z = \frac{P_i r_i^2}{r_o^2 - r_i^2} \quad (7.33)$$

Case 2: External Pressure only ($P_i=0$):

At inside surface, $r = r_i$:

$$\sigma_\theta = \left[\frac{-2P_o r_o^2}{r_o^2 - r_i^2} \right] \quad \sigma_r = 0 \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2 - r_i^2} \quad (7.35)$$

At outside surface, $r = r_o$:

$$\sigma_\theta = -P_o \left[\frac{r_o^2 + r_i^2}{r_o^2 - r_i^2} \right] \quad \sigma_r = -P_o \quad \sigma_z = \frac{-P_o r_o^2}{r_o^2 - r_i^2} \quad (7.36)$$

Max allowable radial interference:

0.1909mm

Interface pressure:

64.17 MPa

ANALYTICAL RESULT OF STRESSES AND STRAINS DISTRIBUTION

Lame's equations

Case 1: Internal pressure only ($P_o=0$):

$$\sigma_{\theta} = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left[1 + \frac{r_o^2}{r^2} \right] \quad \sigma_r = \frac{P_i r_i^2}{r_o^2 - r_i^2} \left[1 - \frac{r_o^2}{r^2} \right]$$

Case 2: External Pressure only ($P_i=0$):

$$\sigma_{\theta} = \frac{-P_o r_o^2}{r_o^2 - r_i^2} \left[1 + \frac{r_i^2}{r^2} \right] \quad \sigma_r = \frac{-P_o r_o^2}{r_o^2 - r_i^2} \left[1 - \frac{r_i^2}{r^2} \right]$$

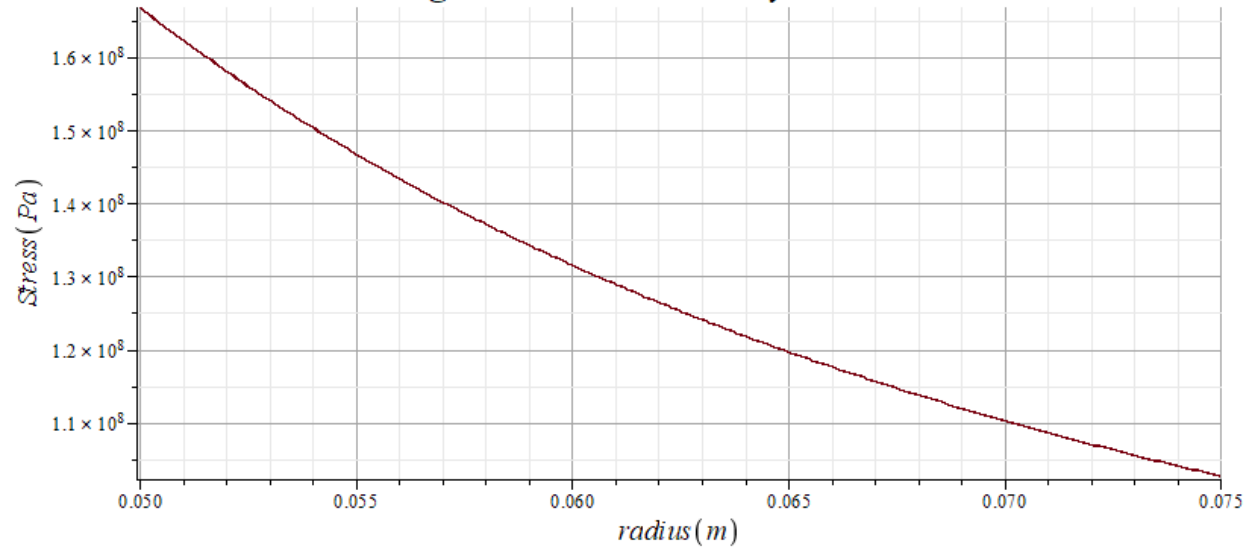
Hooke's law for plane stress

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y)$$

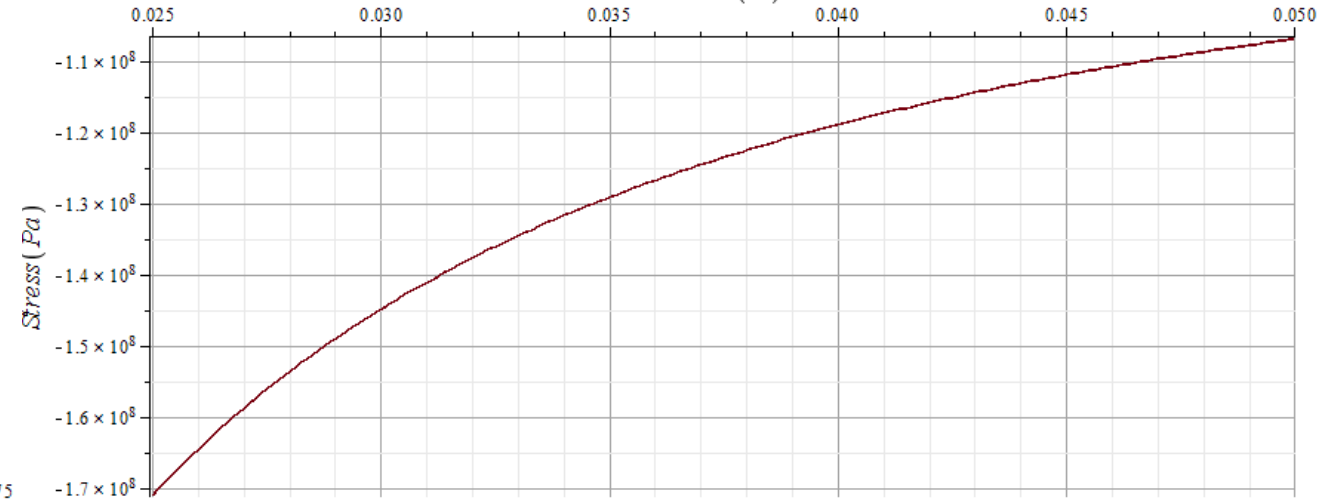
$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x)$$

ANALYTICAL STRESSES DISTRIBUTIONS

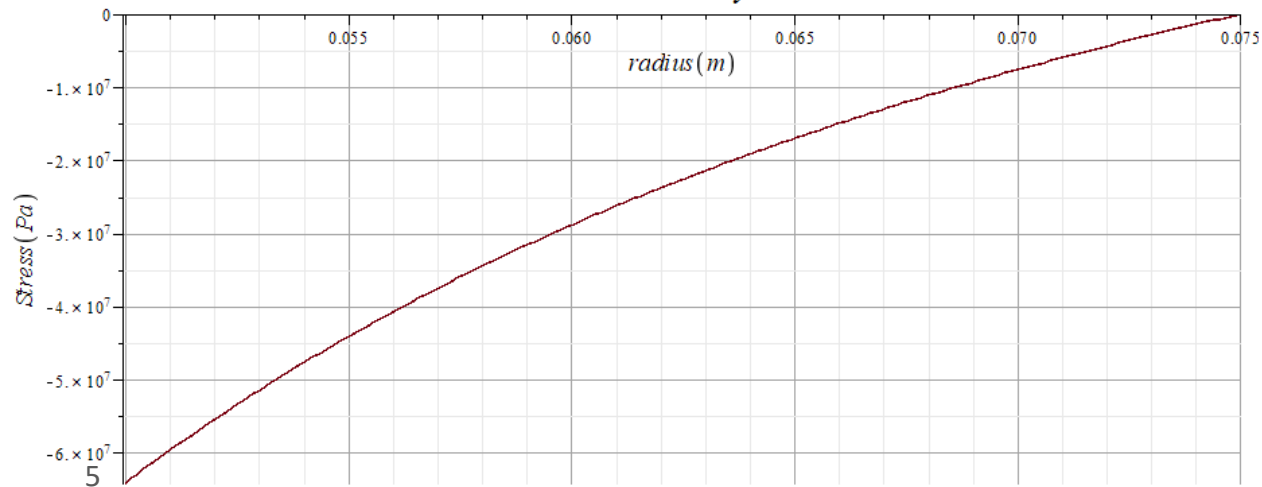
Tangential stress Outer Cylinder



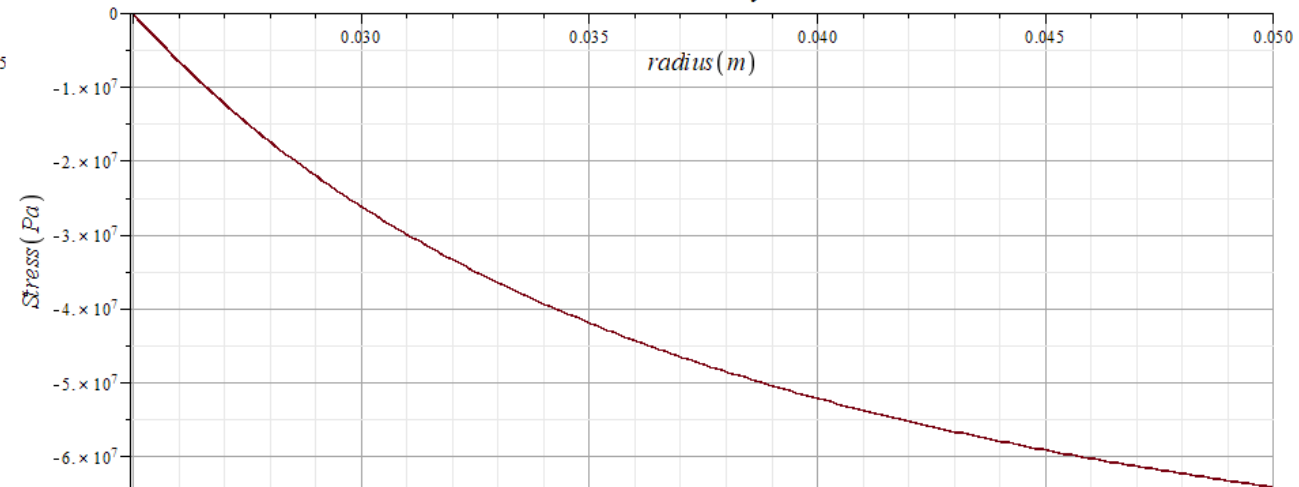
Tangential stress Inner Cylinder



Radial stress Outer Cylinder

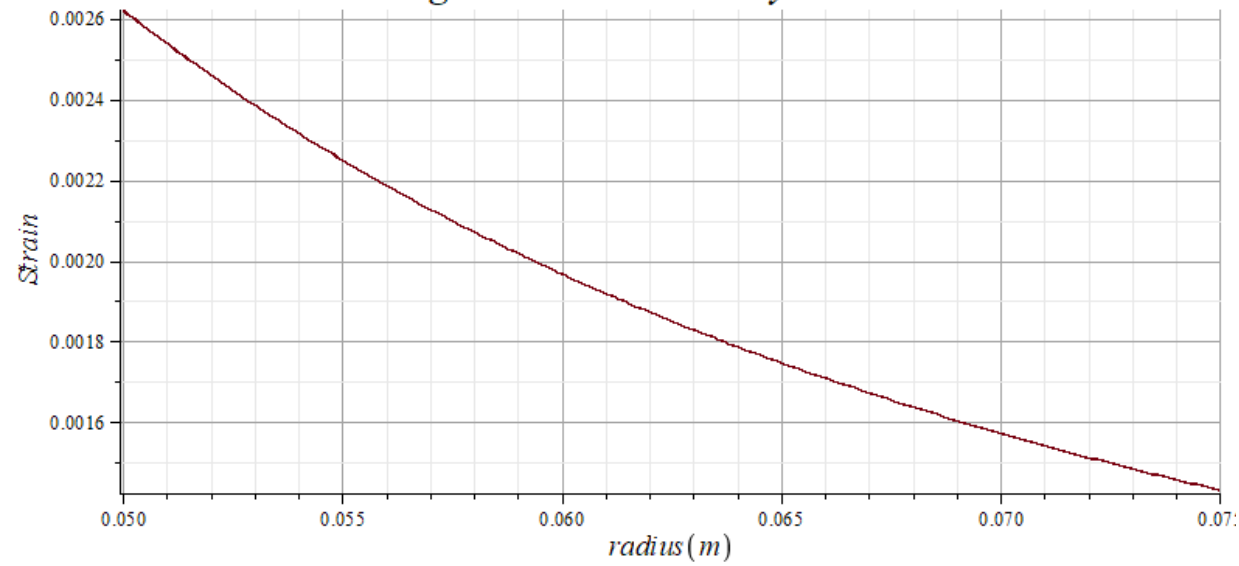


Radial stress Inner Cylinder

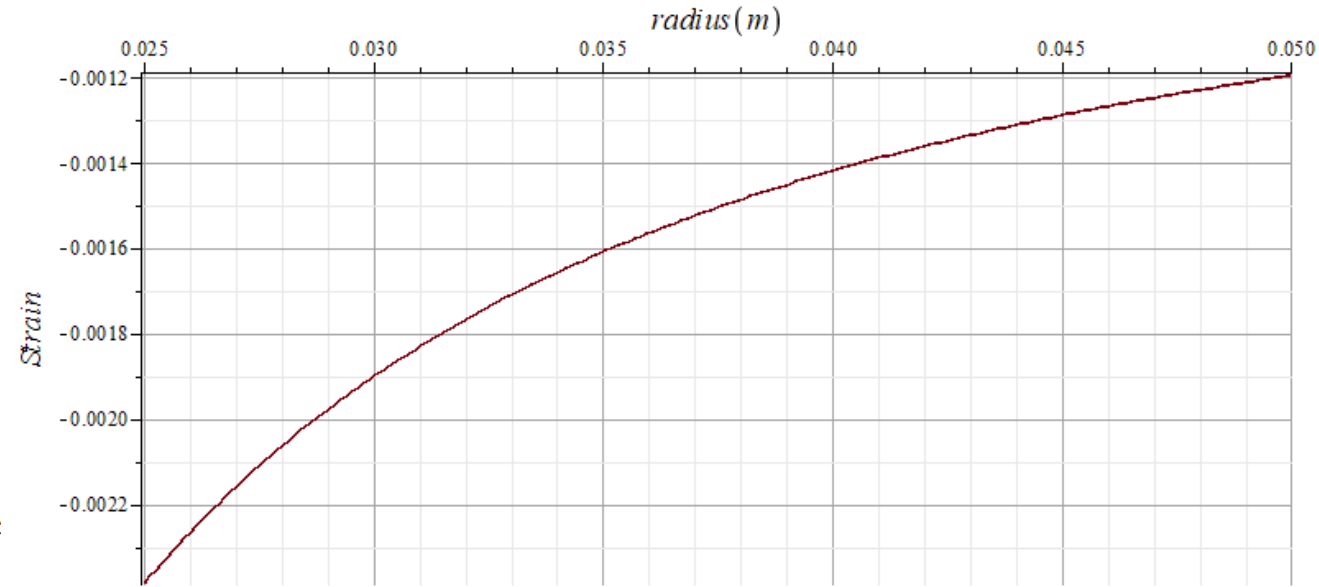


ANALYTICAL STRAIN DISTRIBUTIONS

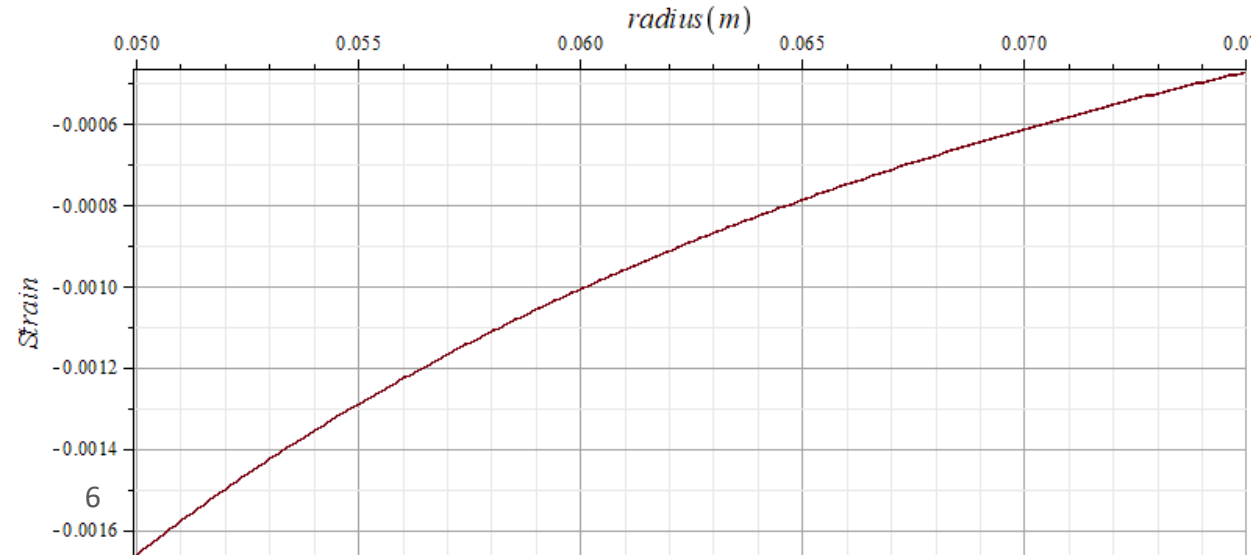
Tangential strain Outer Cylinder



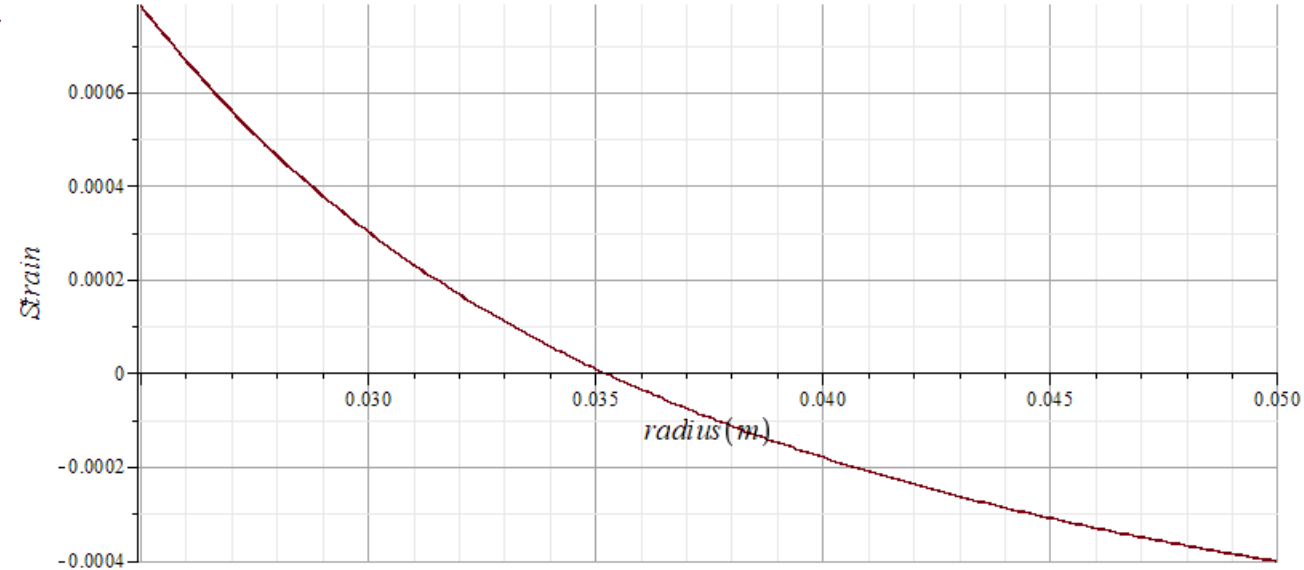
Tangential strain Inner Cylinder



Radial strain Outer Cylinder



Radial strain Inner Cylinder

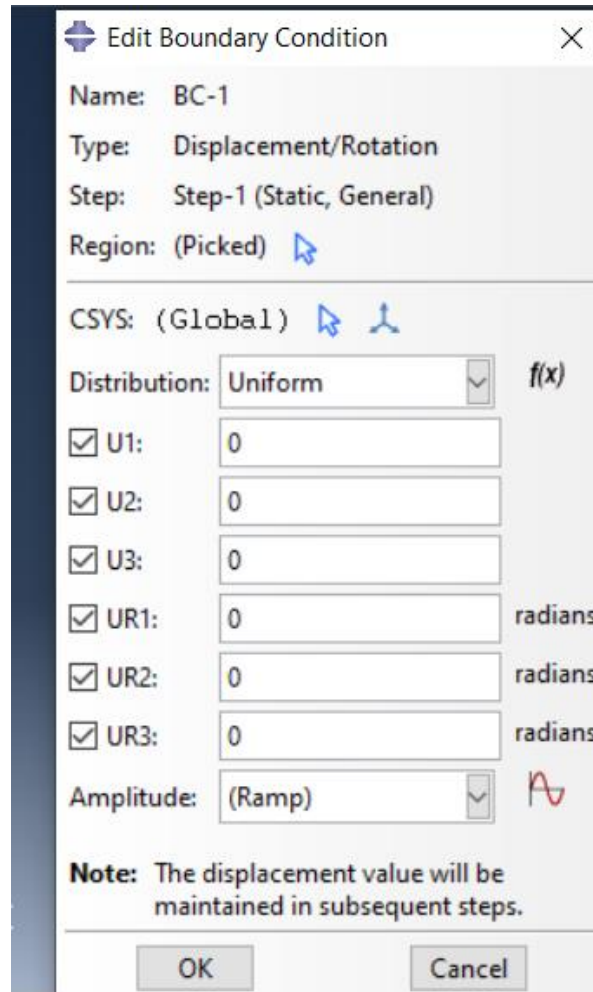


ABAQUS MODEL

Radial interference = 0.1909 mm

Hard contact with
friction 0.2

Mesh global size = 0.004 m

The image shows the 'Edit Boundary Condition' dialog box in ABAQUS. It is titled 'Edit Boundary Condition' with a close button (X) in the top right corner. The 'Name' field is 'BC-1'. The 'Type' is 'Displacement/Rotation'. The 'Step' is 'Step-1 (Static, General)'. The 'Region' is '(Picked)'. The 'CSYS' is '(Global)'. The 'Distribution' is 'Uniform'. The 'Amplitude' is '(Ramp)'. There are checkboxes for U1, U2, U3, UR1, UR2, and UR3, all of which are checked. The values for U1, U2, U3, UR1, UR2, and UR3 are all '0'. The units for UR1, UR2, and UR3 are 'radians'. A note at the bottom states: 'Note: The displacement value will be maintained in subsequent steps.' There are 'OK' and 'Cancel' buttons at the bottom.

Name: BC-1

Type: Displacement/Rotation

Step: Step-1 (Static, General)

Region: (Picked)

CSYS: (Global)

Distribution: Uniform $f(x)$

☒ U1: 0


☒ U2: 0

☒ U3: 0

☒ UR1: 0 radians

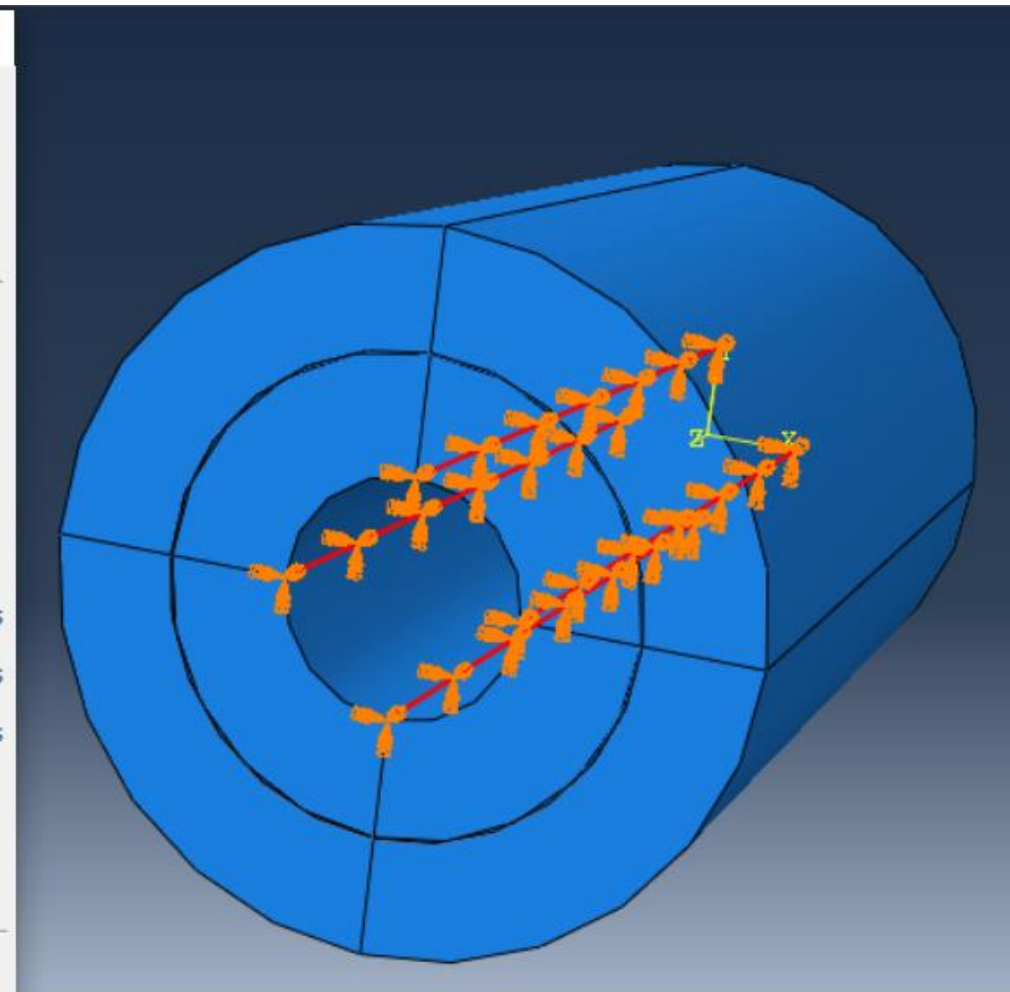
☒ UR2: 0 radians

☒ UR3: 0 radians

Amplitude: (Ramp) 

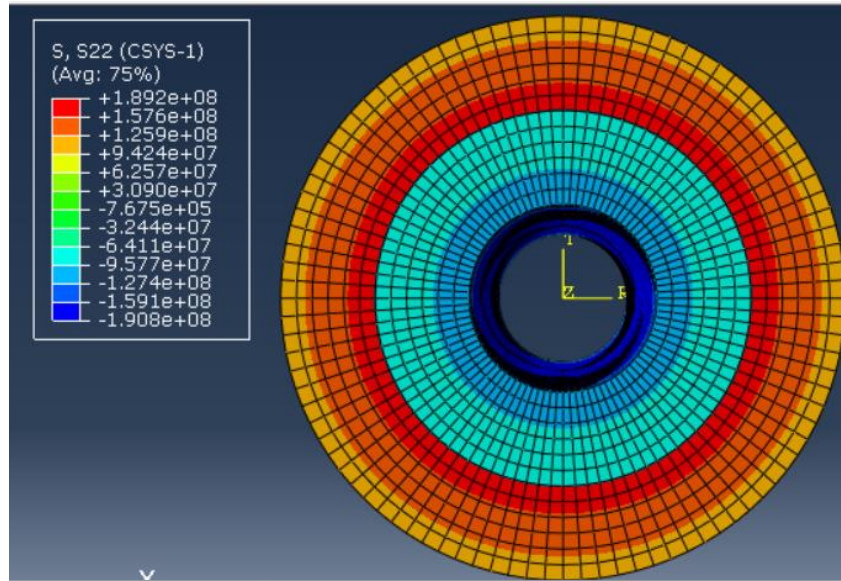
Note: The displacement value will be maintained in subsequent steps.

OK Cancel

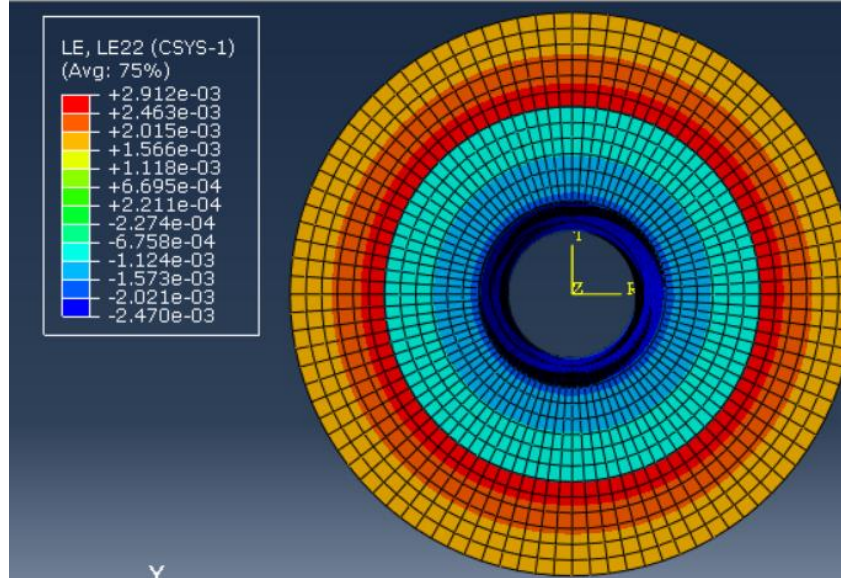


ABAQUS RESULTS

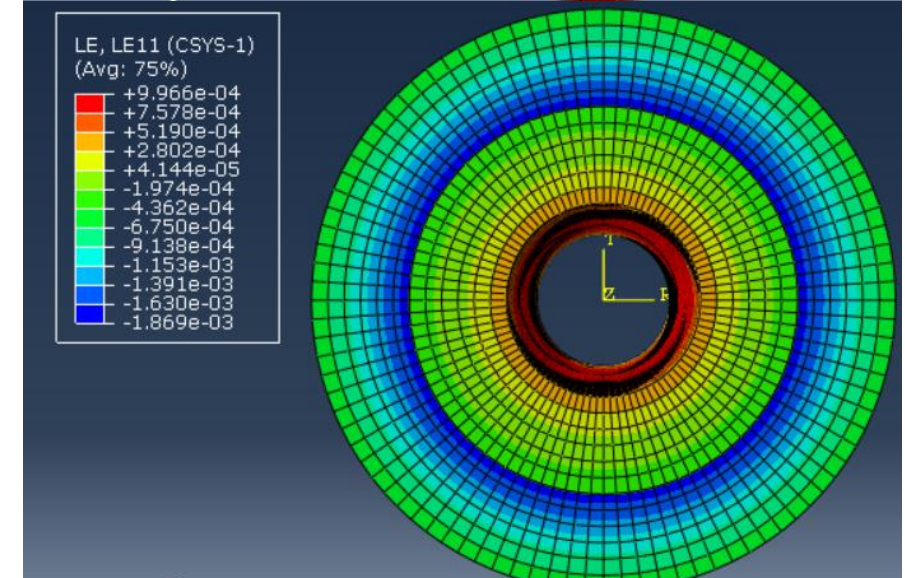
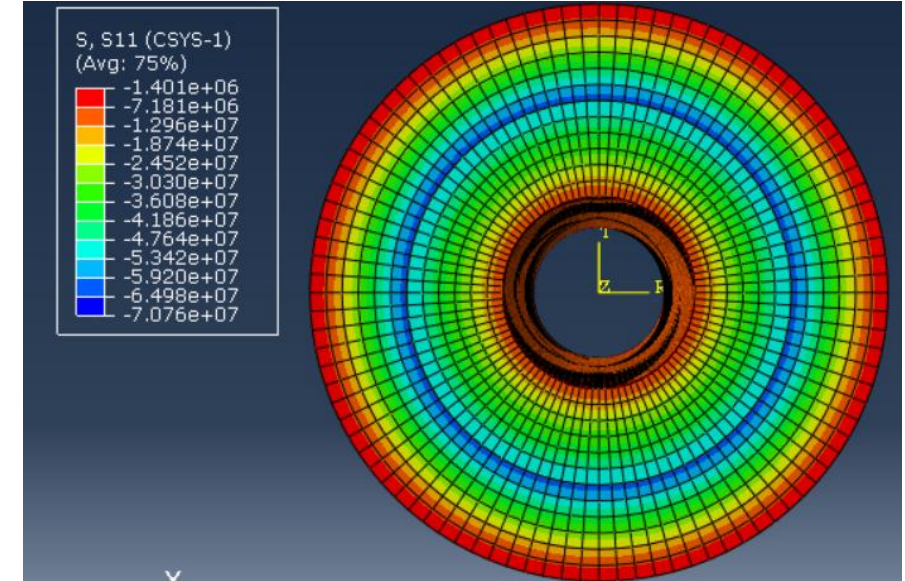
Stresses



Strains



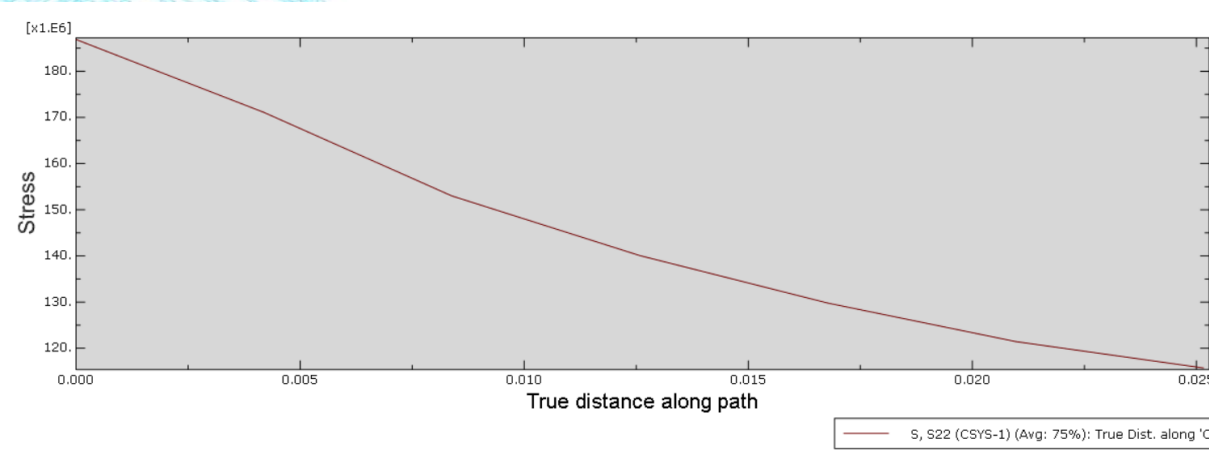
Tangential



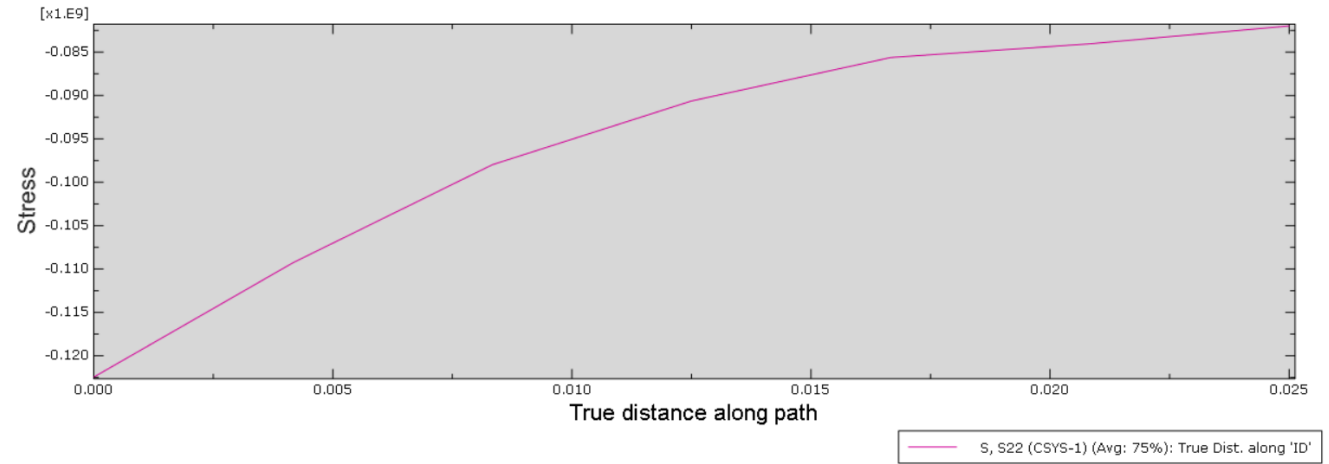
Radial

STRESSES IN ABAQUS

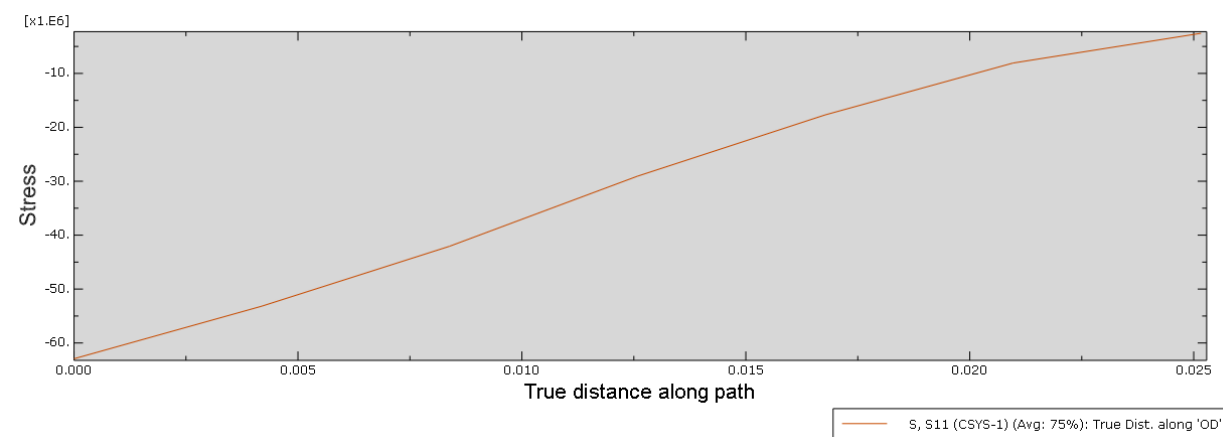
Tangential stress Outer Cylinder



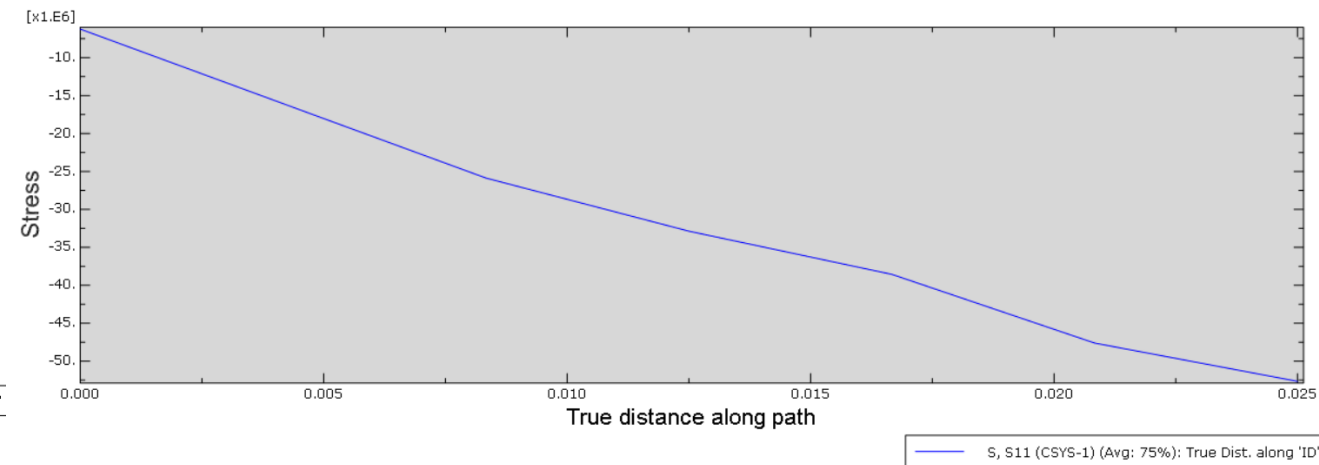
Tangential stress Inner Cylinder



Radial stress Outer Cylinder

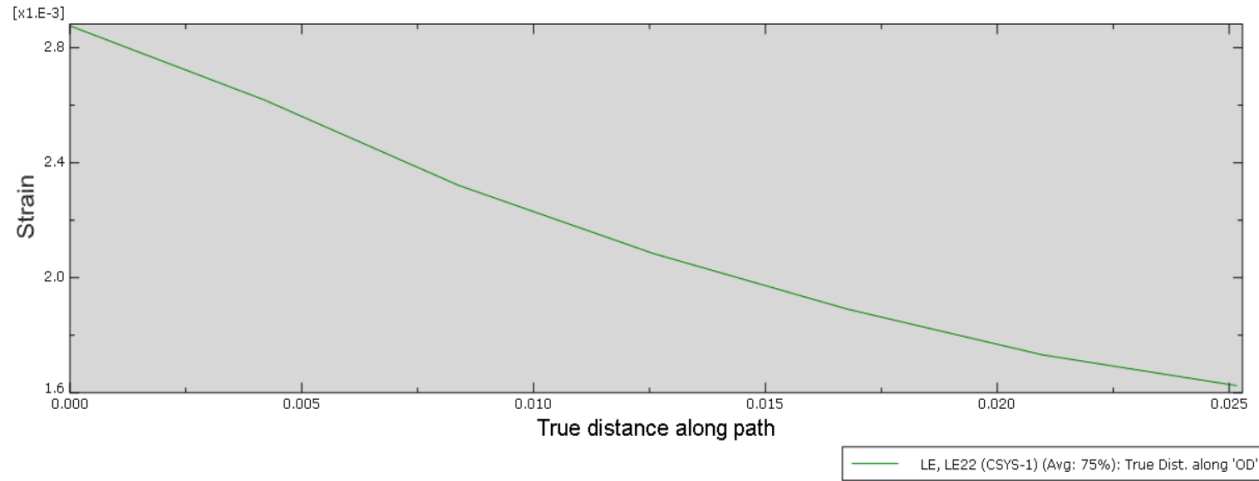


Radial stress Inner Cylinder

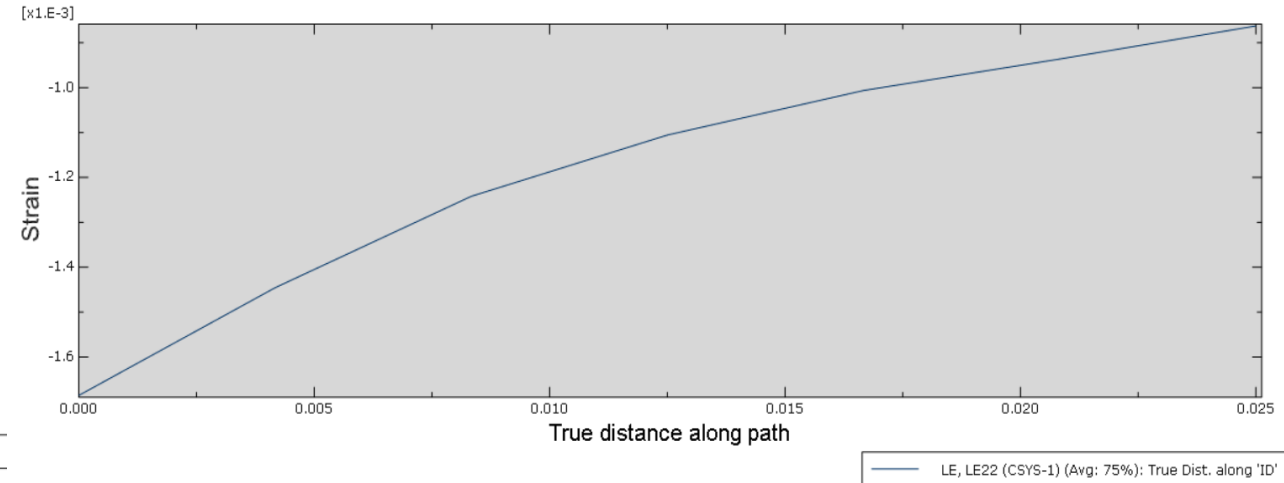


STRAINS IN ABAQUS

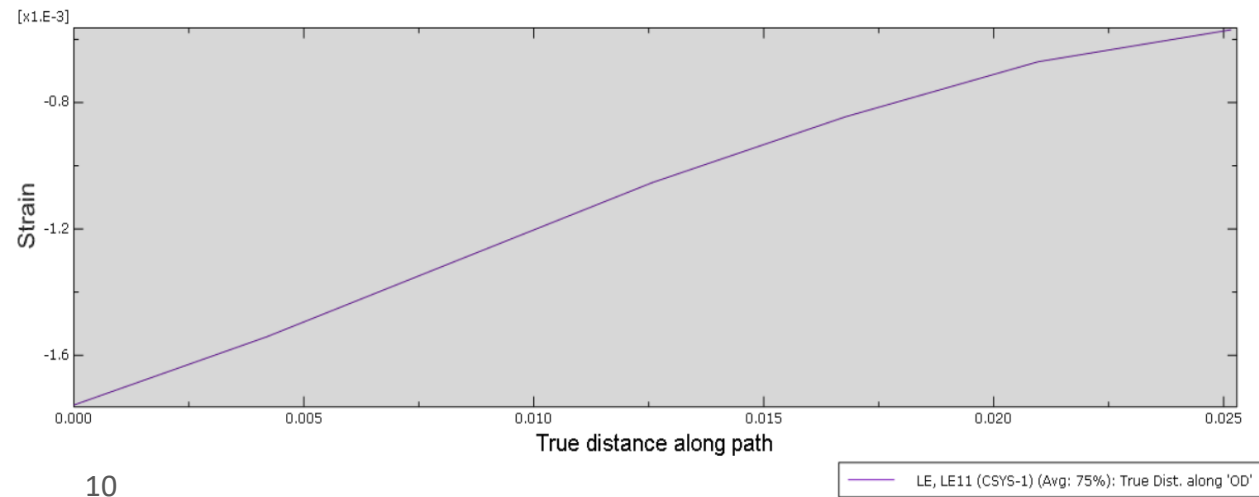
Tangential stain Outer Cylinder



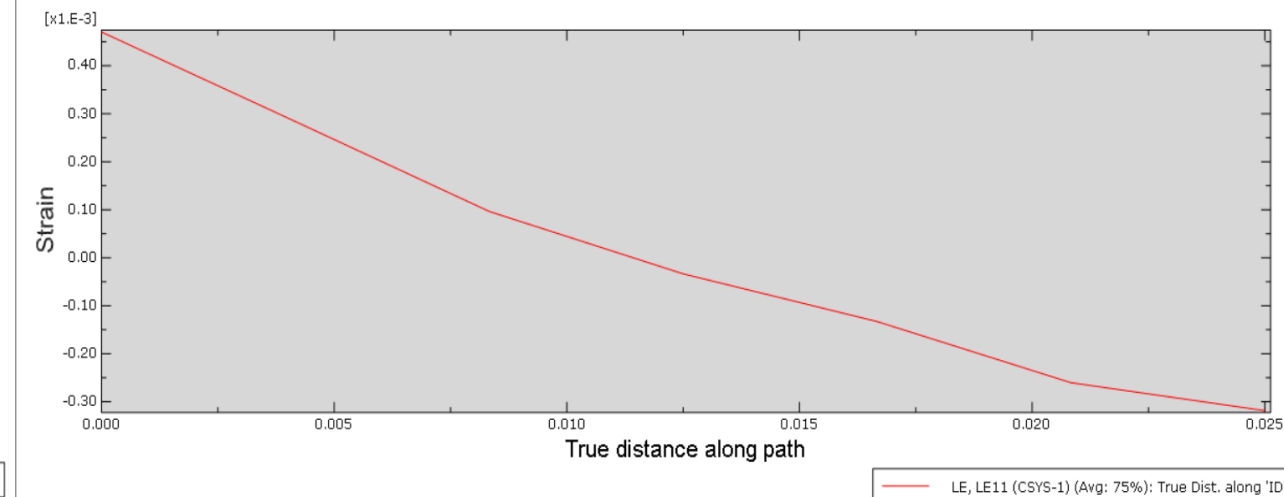
Tangential strain Inner Cylinder



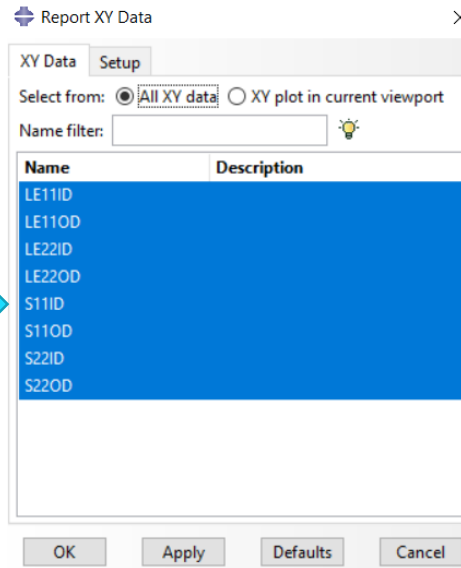
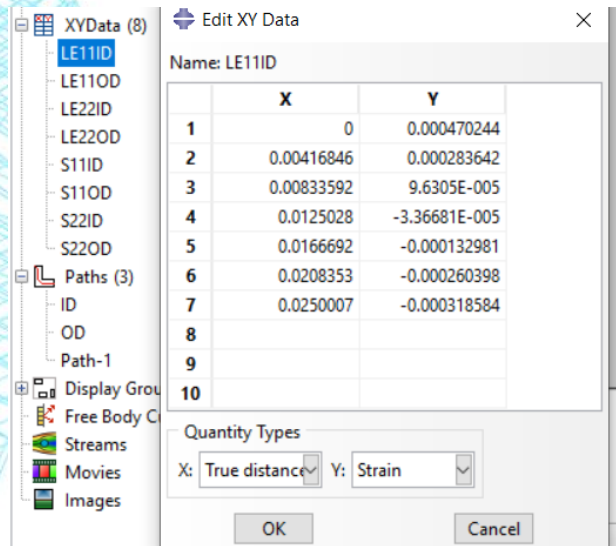
Radial strain Outer Cylinder



Radial strain Inner Cylinder



DATA ANALYSIS



X	LE11ID
0.	470.244E-06
4.16846E-03	283.642E-06
8.33592E-03	96.3051E-06
12.5028E-03	-33.6681E-06
16.6692E-03	-132.981E-06
20.8353E-03	-260.398E-06
25.0007E-03	-318.584E-06

$S22OD := [[0, 186.878E+06], [4.19141E-03, 171.089E+06], [8.38383E-03, 153.002E+06], [12.5774E-03, 140.082E+06], [16.7719E-03, 129.796E+06], [20.9672E-03, 121.452E+06], [25.1632E-03, 115.724E+06]] :$

for i **from** 1 **to** 7 **do**
 $L11ID[i][1] := L11ID[i][1] + 0.025 :$
 $L22ID[i][1] := L22ID[i][1] + 0.025 :$
 $S11ID[i][1] := S11ID[i][1] + 0.025 :$
 $S22ID[i][1] := S22ID[i][1] + 0.025 :$

end do:
for i **from** 1 **to** 7 **do**
 $L11OD[i][1] := L11OD[i][1] + 0.05 :$
 $L22OD[i][1] := L22OD[i][1] + 0.05 :$
 $S11OD[i][1] := S11OD[i][1] + 0.05 :$
 $S22OD[i][1] := S22OD[i][1] + 0.05 :$

end do:

$S11ODP := \text{pointplot}(S11OD, \text{color} = [\text{blue}], \text{axes} = \text{boxed}, \text{connect} = \text{true}) :$
 $S22ODP := \text{pointplot}(S22OD, \text{color} = [\text{blue}], \text{axes} = \text{boxed}, \text{connect} = \text{true}) :$

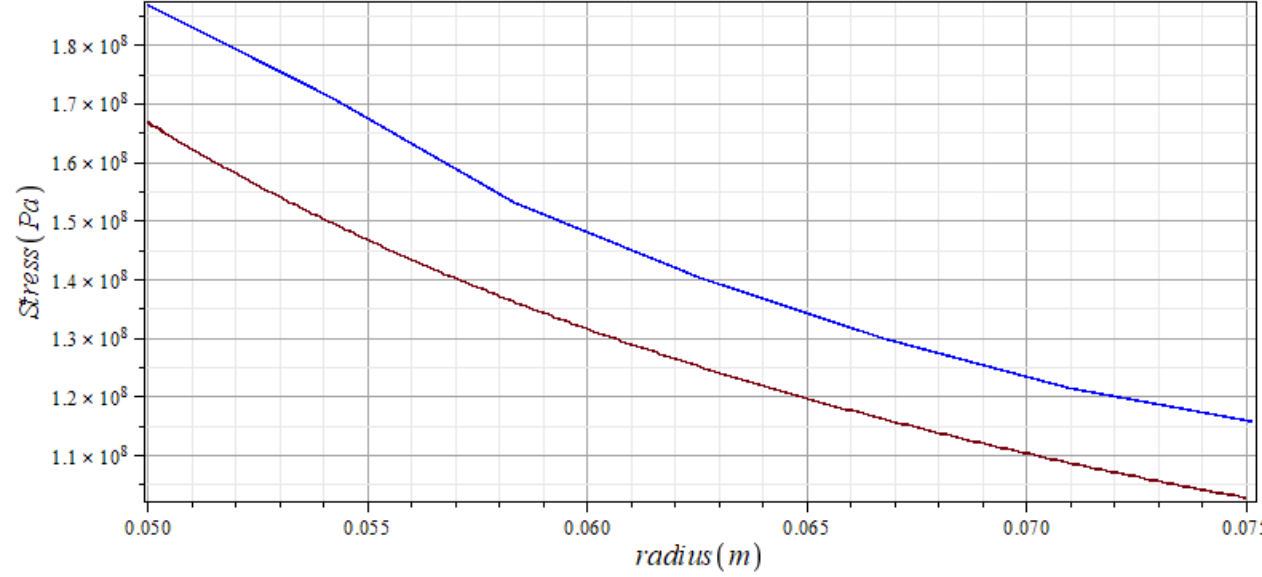


Activate Windows

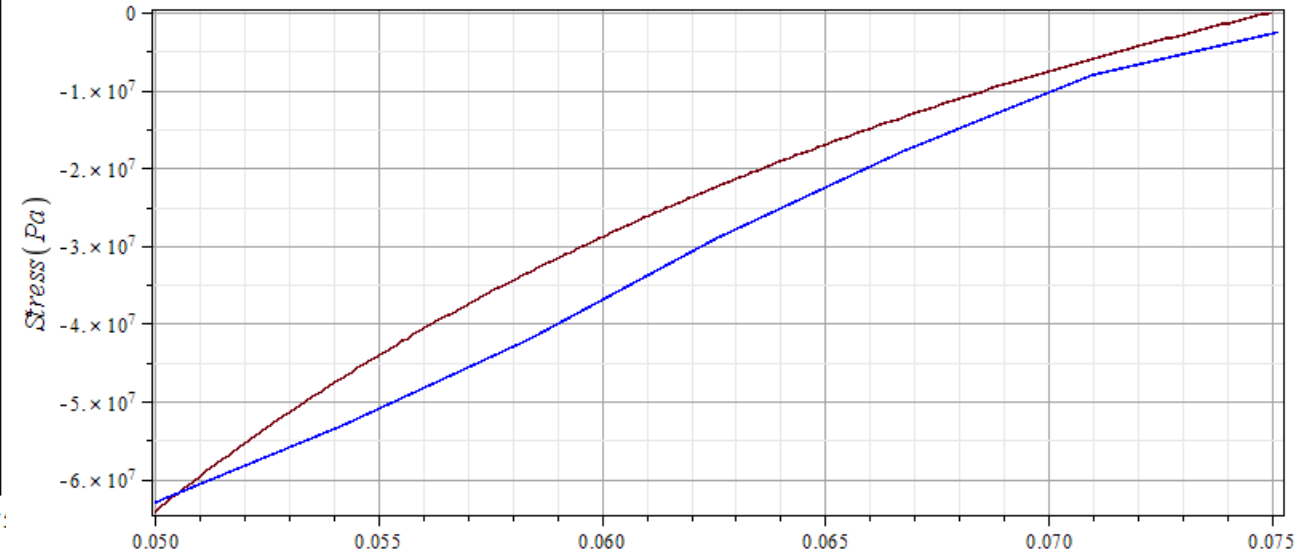
Go to Settings to activate Windows.

OUTER CYLINDER: THEORY VS ABAQUS

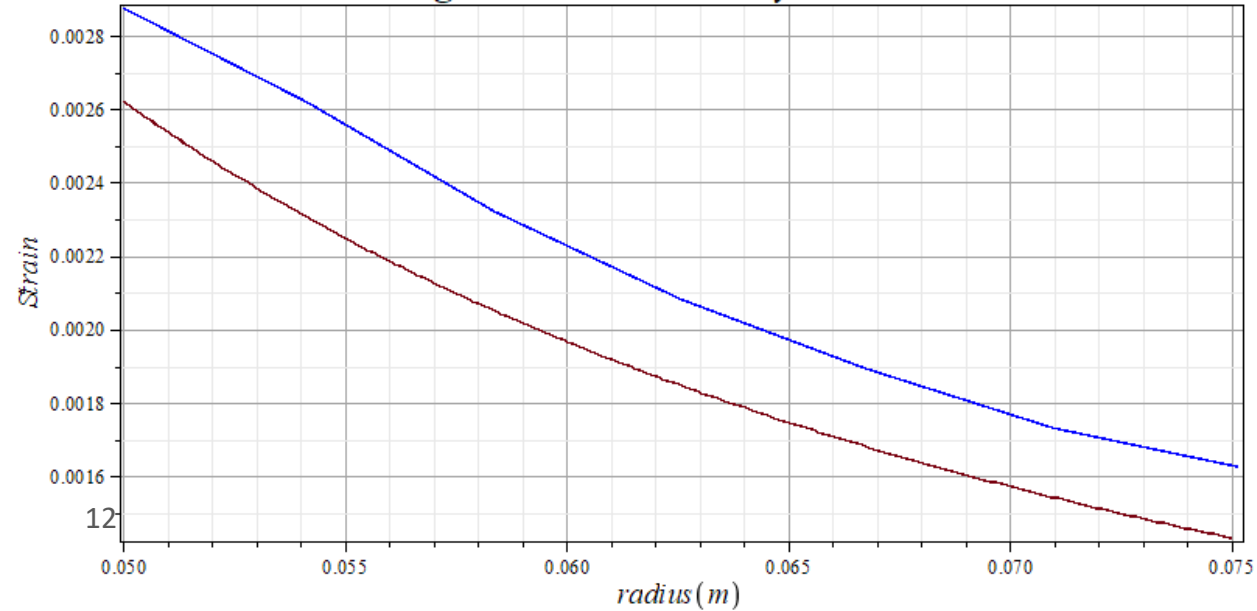
Tangential stress Outer Cylinder



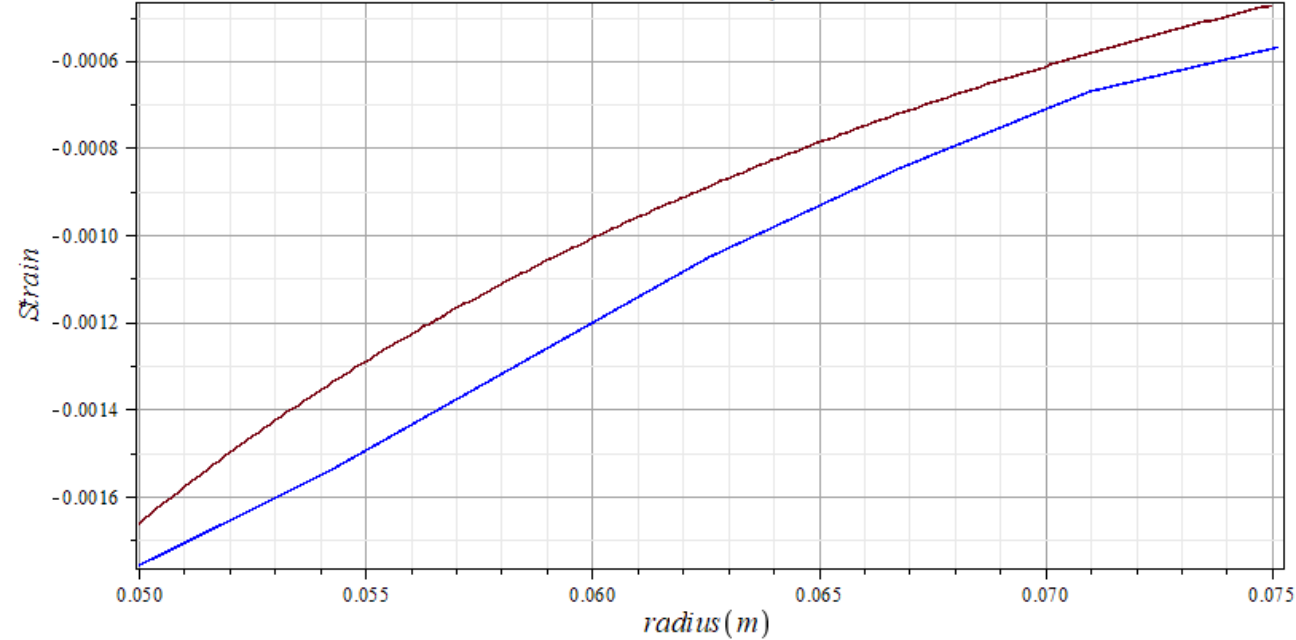
Radial stress Outer Cylinder



Tangential strain Outer Cylinder

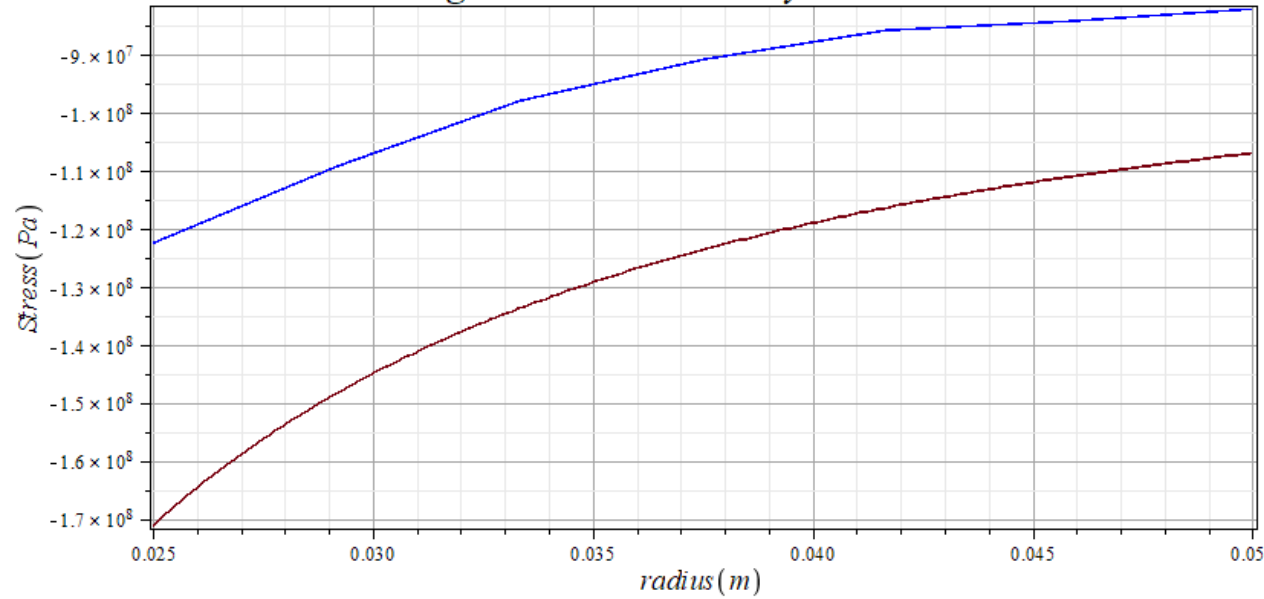


Radial strain Outer Cylinder

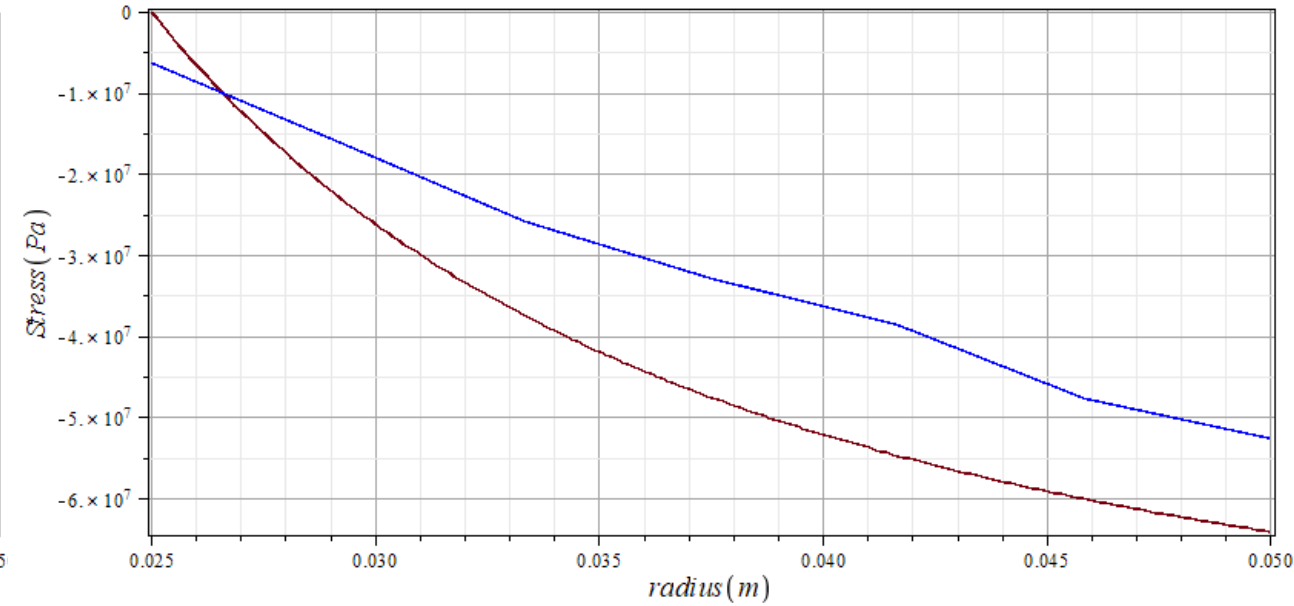


OUTER CYLINDER: THEORY VS ABAQUS

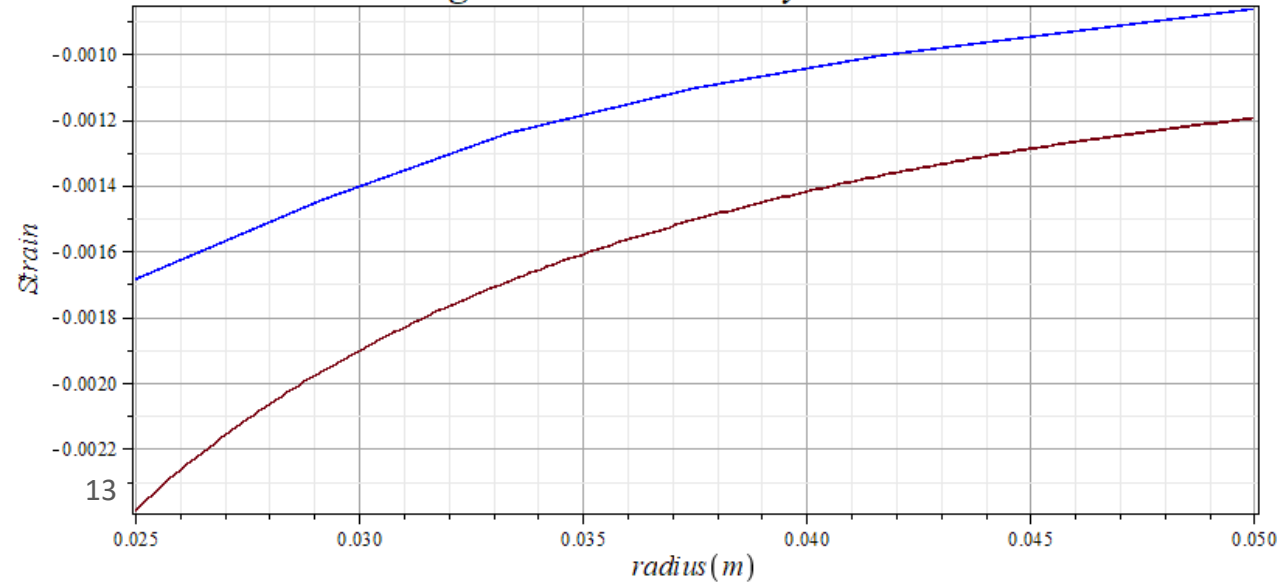
Tangential stress Inner Cylinder



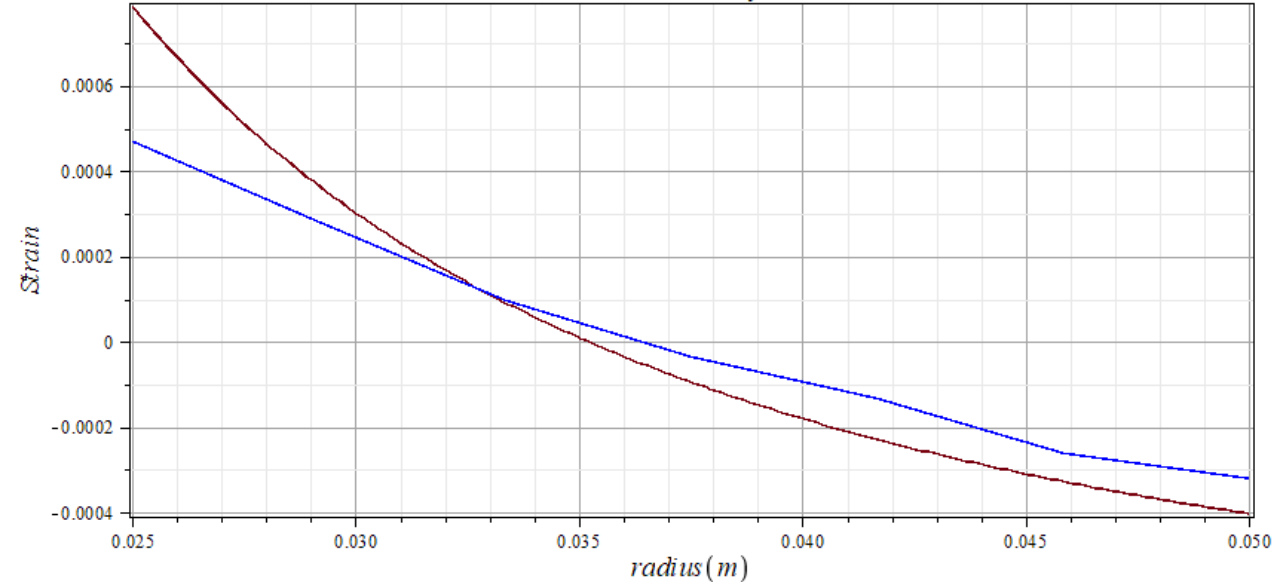
Radial stress Inner Cylinder



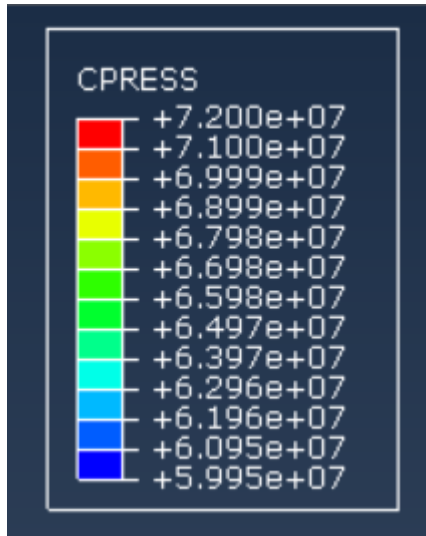
Tangential strain Inner Cylinder



Radial strain Inner Cylinder



INTERFACE PRESSURE: THEORY VS ABAQUS



**Average value from
Abaqus:**
65.98 MPa

**Theoretical interface
pressure:**
64.17 MPa



CONCLUSIONS

- Radial stresses and strain values are close
- Tangential stresses and strains Abaqus values are greater
- Interface pressure – close agreement
- BC might affect the values but without BC the model is flying.