ChatGPT
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Today
MPC Controller Cost Functio

Yesterday
Minimize Loss with Adam
Minimize Loss with Adam
Minimize Loss with Adam

Previous 7 Days
Control Protocols Overview
Industry Sensors Overview

Previous 30 Days
Network MST Algorithms Compa
Calculate Venturi Meter Diameter
MPC Algorithm for Overleaf

June
Entity Save Error Troubleshoot
Load Active Org Items

Upgrade plan
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4. Cost Function in Quadratic Form

The cost function can be rewritten in a quadratic form involving $\mathbf{U}_{\text{pred}}$:

$$J = (\mathbf{X}_{\text{pred}} - \mathbf{X}_{\text{ref}})^T Q (\mathbf{X}_{\text{pred}} - \mathbf{X}_{\text{ref}}) + \mathbf{U}_{\text{pred}}^T R \mathbf{U}_{\text{pred}}$$

Substitute $\mathbf{X}_{\text{pred}} = \Phi \mathbf{x}(0) + \Gamma \mathbf{U}_{\text{pred}}$:

$$J = (\Phi \mathbf{x}(0) + \Gamma \mathbf{U}_{\text{pred}} - \mathbf{X}_{\text{ref}})^T Q (\Phi \mathbf{x}(0) + \Gamma \mathbf{U}_{\text{pred}} - \mathbf{X}_{\text{ref}}) + \mathbf{U}_{\text{pred}}^T R \mathbf{U}_{\text{pred}}$$

Let $\mathbf{e} = \mathbf{X}_{\text{ref}} - \Phi \mathbf{x}(0)$, the cost function simplifies to:

$$J = (\Gamma \mathbf{U}_{\text{pred}} - \mathbf{e})^T Q (\Gamma \mathbf{U}_{\text{pred}} - \mathbf{e}) + \mathbf{U}_{\text{pred}}^T R \mathbf{U}_{\text{pred}}$$

Expand and collect terms involving $\mathbf{U}_{\text{pred}}$:

$$J = \mathbf{U}_{\text{pred}}^T (\Gamma^T Q \Gamma + R) \mathbf{U}_{\text{pred}} - 2\mathbf{e}^T Q \Gamma \mathbf{U}_{\text{pred}} + \mathbf{e}^T Q \mathbf{e}$$

Define:

$$H = \Gamma^T Q \Gamma + R, \quad \mathbf{f} = -\Gamma^T Q \mathbf{e}$$

Then the cost function is:

$$J = \mathbf{U}_{\text{pred}}^T H \mathbf{U}_{\text{pred}} + 2\mathbf{f}^T \mathbf{U}_{\text{pred}} + \text{constant terms}$$

The constant terms can be omitted in the optimization since they do not affect the decision variables:

$$J = \mathbf{U}_{\text{pred}}^T H \mathbf{U}_{\text{pred}} + 2\mathbf{f}^T \mathbf{U}_{\text{pred}}$$

5. Constraints

State and input constraints can be formulated as: