# Spring 2023 - MATH 3501 Linear Algebra

## HW2 – 3pt

### Deadline is April 02, 2023, 23:59

### Name of file: "Name_Surname_Spring 23_LA_HW2_CRN_(xxx).pdf"

## Subspaces

**Task 1.** Determine whether or not W is a subspace of $\mathbf{R}^3$ where W consists of all vectors (a, b, c) in $\mathbf{R}^3$ such that:

(a) $a=3b$ (b) $a\leq b\leq c$ (c) $ab=0$ (d) $a+b+c=0$ (e) $b=a^2$(f) $a=2b=3c$

**Task 2.** Let V be the vector space of n-square matrices over a field **R**. Show that W is a subspace of V if W consists of all matrices $A[a_{ij}]$ that are

(a) symmetric ($A^T=A$ or $a_{ij}=a_{ji}$); (b) (upper) triangular; (c) diagonal; (d) scalar

**Task 3.** Let AX=B be a nonhomogeneous system of linear equations in ***n*** unknowns; that is, $B\neq\mathbf{0}$. Show that the solution set is not a subspace of $\mathbf{R}^n$.

**Task 4.** Suppose U and W are subspaces of V for which $U\cup W$ is a subspace. Show that $U\subseteq W$ or $W\subseteq U$.

**Task 5.** Let V be the vector space of all functions from **R** into **R**. Show that W is a subspace of V where W consists of all: (a) bounded functions, (b) even functions. Recall that $f:R\to R$ is *bounded* if $\exists M\in R$ such that $\forall x\in R$. we have $|f(x)|\leq M$; and f(x) is *even* if $f(-x)=f(x)$, $\forall x\in R$.

## Linear Combinations, Linear Spans

**Task 6.** Write the polynomial $f(t)=at^2+bt+c$ as a linear combination of the polynomials

$p_1(t)=(t-1)^2$, $p_2(t)=(t-1)$ $p_3=1$

[Thus, $p_1$, $p_2$, $p_3$ span the space $P_2(t)$ of polynomials of degree $\leq 2$.]

**Task 7.** Show that (a) If $S\subseteq T$, then $\mathrm{Span}(S)\subseteq\mathrm{Span}(T)$. (b) $\mathrm{Span}(\mathrm{span}(S))=\mathrm{Span}(S)$.

## Basis and Dimension

**Task 8.** Find a subset of $u_1$, $u_2$, $u_3$, $u_4$ that gives a basis for $W=\mathrm{Span}\{u_i\}$ of $\mathbf{R}^5$, where

(a) $u_1=(1,0,1,0,1)$, $u_2=(1,1,2,1,0)$, $u_3=(2,1,3,1,1)$, $u_4=(1,2,1,1,1)$

(b) $u_1=(1,0,1,1,1)$, $u_2=(2,1,2,0,1)$, $u_3=(1,1,2,3,4)$, $u_4=(4,2,5,4,6)$

**Task 9.** Consider the subspaces

$U=\{(a,b,c,d):b-2c+d=0\}$ and $W=\{(a,b,c,d):a=d,b=2c\}$ of $\mathbf{R}^4$.

Find a basis and the dimension of

(a) U, (b) W, (c) $U\cap W$).

**Task 10.** Determine the dimension of the vector space W of the following n-square matrices:

(a) symmetric matrices, (b) antisymmetric matrices,

(d) diagonal matrices, (c) scalar matrices.

## Rank of a Matrix, Row and Column Spaces

**Task 11.** For k=1, 2, …, 5,

1. Find the number $n_k$ of linearly independent subsets consisting of k columns for each of the following matrices:

$$\text{(a)}\quad A = \begin{bmatrix} 1 & 1 & 0 & 2 & 3 \\ 1 & 2 & 0 & 2 & 5 \\ 1 & 3 & 0 & 2 & 7 \end{bmatrix}, \qquad \text{(b)}\quad B = \begin{bmatrix} 1 & 2 & 1 & 0 & 2 \\ 1 & 2 & 3 & 0 & 4 \\ 1 & 1 & 5 & 0 & 6 \end{bmatrix}$$

2. For each k point out explicitly all linear independent columns

**Task 12.** For the matrix below, where $C^1, C^2, \ldots, C^6$ denote its columns

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 6 \\ 2 & 4 & 3 & 8 & 3 & 15 \\ 1 & 2 & 2 & 5 & 3 & 11 \\ 4 & 8 & 6 & 16 & 7 & 32 \end{bmatrix}$$

(i) Find a basis of the RS(A).

(ii) Find the columns that are linear combinations of preceding columns.

(iii) Find columns (excluding $C^6$) that form a basis for the CS(A).

(iv) Express $C^6$ as a linear combination of the basis vectors obtained in (iii).

**Task 13.** Determine which of the following matrices have the same row space:

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & -4 & 5 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & -1 & 3 \\ 2 & -1 & 10 \\ 3 & -5 & 1 \end{bmatrix}$$

**Task 14.** Determine which of the following subspaces of $\mathbf{R}^3$ are identical:

$$U_1 = \operatorname{span}[(1,1,-1),\ (2,3,-1),\ (3,1,-5)], \qquad U_2 = \operatorname{span}[(1,-1,-3),\ (3,-2,-8),\ (2,1,-3)]$$

$$U_3 = \operatorname{span}[(1,1,1),\ (1,-1,3),\ (3,-1,7)]$$

**Task 15.** Find a basis for (i) the row space and (ii) the column space of the matrix M:

$$M = \begin{bmatrix} 0 & 0 & 3 & 1 & 4 \\ 1 & 3 & 1 & 2 & 1 \\ 3 & 9 & 4 & 5 & 2 \\ 4 & 12 & 8 & 8 & 7 \end{bmatrix}$$

Good luck!

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