Carrier Synchronization: DFT-based Approach

Siddhartan Govindasamy

I. Overview of carrier synchronization

Consider a quadrature communications system as shown in Figure 1. If the cosine signals at the transmitter and receiver are not synchronized (i.e. if there is a phase offset between the signals), the received signal cannot be decoded correctly unless the phase offset is corrected for. Here, we will see one approach for correcting timing offsets digitally, when the transmitted signal is a Binary-Phase-Shift-Keyeing (BPSK) signal. Note that a very similar approach is possible when the signals are 4-QAM, or QPSK. We start by developing an appropriate model for the phase offsets when the signals are sampled digitally, and then present an approach to correcting for the phase offset.

II. DT Flat fading with carrier offset

We have thus far seen that we can represent a quadrature modulated system (i.e. one with in-phase and quadrature phase components) using a baseband equivalent model as follows

$$y_b(t) = h_b * x_b(t)e^{-j\phi(t)} + n(t) \tag{1}$$

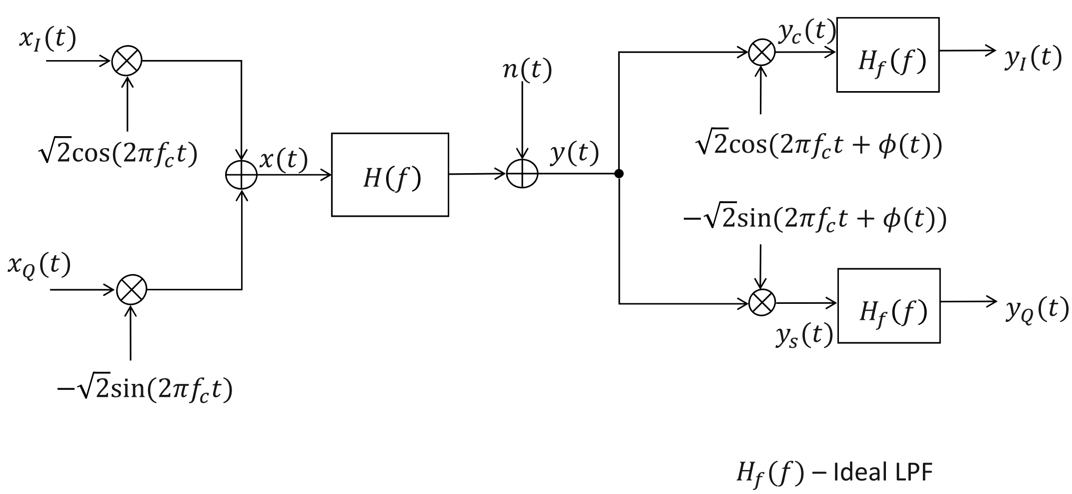

Fig. 1. Transmitter and Receiver of Quadrature System.

where $x_b(t) = x_I(t)+jx_Q(t)$, $y_b(t) = y_I(t)+jy_Q(t)$, $h_b(t)$ is a complex impulse response related to the true impulse response of the channel, and $\phi(t)$ is a phase offset between the sinusoids at the transmitter and the receiver. Under certain conditions, namely that the bandwidth of the transmitted signal is much lower than the *coherence bandwidth* of the channel, where coherence bandwidth refers to a range of frequencies over which a channel frequency response can be assumed to be constant, we can model the channel as a single multiplication This leads to the following simplification of the above equation

$$y_b(t) = hx_b(t)e^{-j\phi(t)} + n(t) \tag{2}$$

where h is now a complex scale factor. This model is called a flat fading model, since multiplying by a single scale factor in the time domain corresponds to a flat frequency response.

Suppose that $x_b(t)$ originates from a complex DT signal and $y_b(t)$ is sampled to produce a complex DT signal. We can then model our system as

$$y[k] = hx[k]e^{-j\phi[k]} + n[k] \tag{3}$$

where $x[k]$ and $y[k]$ are complex DT signals, h is a complex number, $\phi[k]$ is a DT signal related to $\phi(t)$ and $n[k]$ is a sampled version of the noise in the system.

This model, while simple, is very powerful and is used in many wireless communications systems. Additionally, it is worth noting that for a system whose implementation is all done in DT/digitally, the underlying properties of the system are not particularly important as the system designer only observes a system in (3). In other words, you can (for now at least) assume that your system is given by (3), without worrying too much about the underlying CT system.

III. Carrier Synchronization

Here we discuss an approach to carrier synchronization using the Discrete-Fourier Transform. This approach works well if the frequency and phase of the carriers do not drift significantly over the duration of the communication. First lets assume that the channel follows the model in (3), and let us express the channel h, which is complex number, in exponential form, i.e.

$$h = |h|e^{j\angle h} \tag{4}$$

where $\angle h$ is the phase of the complex number h, and $|h|$ is its magnitude. Let us then use the following substitution

$$\psi[k] = \phi[k] - \angle h \tag{5}$$

$\psi[k]$ represents the combination of the phase offset due the oscillators at the transmitter and receiver not being matched, and the phase offset introduced by the channel. Note that the minus sign on $\angle h$ is just to make notation slightly easier moving forward. It doesn't really make a difference since $\angle h$ is a random quantity depending on the channel and could be negative or positive.

This leads to (3) being expressed as

$$y[k] = \left(|h|e^{j\angle h}\right) x[k]e^{-j\phi[k]} + n[k] \tag{6}$$

$$= |h|x[k]e^{-j(\phi[k]-\angle h)} + n[k] \tag{7}$$

$$= |h|x[k]e^{-j\psi[k]} + n[k] \tag{8}$$

Now, if you do the processing on a computer, you can collect a block of samples of $y[k]$ and estimate what $|h|$ is and divide it out from your signal. Define the normalized signal as $\bar{y}[k]$, which can be expressed as

$$\bar{y}[k] = x[k]e^{-j\psi[k]} + \frac{1}{|h|}n[k] \tag{9}$$

Now, we are in a position to correct for the phase offset. Suppose that we generate an estimate for $\psi[k]$ where the estimate at time k is $\hat{\psi}[k]$. Then we could multiply $\bar{y}[k]$ with this estimate to produce a phase compensated estimate of $x[k]$ which we will call $\hat{x}[k]$, as follows

$$\hat{x}[k] = \bar{y}[k]e^{j\hat{\psi}[k]} = x[k]e^{-j\psi[k]} \cdot e^{j\hat{\psi}[k]} + \frac{e^{j\hat{\psi}[k]}}{|h|}n[k] \tag{10}$$

Let's simplify our notation and write $\tilde{n}[k] = \frac{e^{j\hat{\psi}[k]}}{|h|}n[k]$ which is a scaled version of the original noise, and is hence still noise. The previous expression becomes

$$\hat{x}[k] = \bar{y}[k]e^{j\hat{\psi}[k]} = x[k]e^{-j\psi[k]} \cdot e^{j\hat{\psi}[k]} + \tilde{n}[k] = x[k]e^{-j(\psi[k]-\hat{\psi}[k])} + \tilde{n}[k] \tag{11}$$

If $\hat{\psi}[k]$ is an accurate estimate of $\psi[k]$, we will have

$$\hat{x}[k] = x[k] + \tilde{n}[k] \tag{12}$$

which effectively cancels out the effects of the phase offset due to the oscillators not being matched, and phase offsets introduced by the channel.

In the next section, we will look at one way to generate $\hat{\psi}[k]$, using the FFT

IV. Generating the phase estimate

The FFT based approach uses the fact that the N-point FFT of a complex exponential given by $e^{j\frac{2\pi}{N}\ell k}$ is $N\delta[m-\ell]$. Note that $\delta[m-\ell]$ means that the ℓ-th entry of the output of the FFT equals 1, and all other entries equal zero.

Recall the following expression from the previous section

$$\bar{y}[k] = x[k]e^{-j\psi[k]} + \frac{1}{|h|}n[k] \tag{13}$$

Let's again assume the noise is negligible and write

$$\bar{y}[k] = x[k]e^{-j\psi[k]} \tag{14}$$

Suppose that the phase offset follows the following model

$$\psi[k] = f_{\Delta}k + \theta\,. \tag{15}$$

This model incorporates a frequency offset f_{Δ} and a constant phase offset θ and is sufficient for many systems.

Define a new signal

$$s[k] = \bar{y}[k] = (x[k]e^{-j\psi[k]})^2 = (x[k])^2 e^{-j2(f_{\Delta}k+\theta)} \tag{16}$$

$$\tag{17}$$

Since $x[k] = \pm 1$,

$$s[k] = e^{-j2(f_{\Delta}k+\theta)} = e^{-j2\theta}e^{-j2f_{\Delta}k} \tag{18}$$

Suppose that we have N samples of $s[k]$. We can further write this term in a form where we can apply the FFT property described above.

$$s[k] = e^{-j2\theta}e^{-j\frac{2\pi}{N}\frac{Nf_{\Delta}}{\pi}k} \tag{19}$$

If we take the N point FFT of the above signal, we should get:

$$FFT\{s[k]\}_N = Ne^{-j2\theta}\delta\left[m + \frac{Nf_\Delta}{\pi}\right] \tag{20}$$

By looking for the peak of the FFT of $s[k]$, we can find what f_Δ is. By looking at the value of the peak, you can find what θ is, which will enable you to correct for the frequency and phase offsets.

Note that we have assumed that $\frac{Nf_\Delta}{\pi}$ is an integer here, but if N is long, you will get a close enough approximation to f_Δ using this approach.