Expression for $\mathcal{P}(\mathbf{G}_n)$

CASE 1: ν even

$$
\begin{aligned}
\mathcal{P}(\mathbf{G}_n) &= \int_{\pi-\phi}^{2\pi+\phi} (rc_\varphi)^{\frac{\mu+2}{2}}(b+rs_\varphi)^{\frac{\nu}{2}} rc_\varphi \,\mathrm{d}\varphi \\
&= r^{\frac{\mu+\nu}{2}+2} \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{\frac{\mu+4}{2}} \left(\frac{b}{r}+s_\varphi\right)^{\frac{\nu}{2}} \,\mathrm{d}\varphi \\
&= r^{l+2} \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{\frac{\mu+4}{2}} \sum_{i=0}^{\frac{\nu}{2}} \binom{\frac{\nu}{2}}{i} \left(\frac{b}{r}\right)^{\frac{\nu}{2}-i} (s_\varphi)^i \,\mathrm{d}\varphi \\
&= r^{l+2} \sum_{i=0}^{\frac{\nu}{2}} \binom{\frac{\nu}{2}}{i} \left(\frac{b}{r}\right)^{\frac{\nu}{2}-i} \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{\frac{\mu+4}{2}} (s_\varphi)^i \,\mathrm{d}\varphi \\
&= r^{l+2} \sum_{i=0}^{\frac{\nu}{2}} \binom{\frac{\nu}{2}}{i} \left(\frac{b}{r}\right)^{\frac{\nu}{2}-i} \mathcal{I}_{\frac{\mu+4}{2},i} \\
&= r^{l+2} \mathcal{K}_{\frac{\mu+4}{2},\frac{\nu}{2}} \,.
\end{aligned}
$$

CASE 2: ν odd, $\mu = 1$, l even

$$
\begin{aligned}
\mathcal{P}(\mathbf{G}_n) &= -\int_{\pi-\phi}^{2\pi+\phi} (rc_\varphi)^{l-2}(1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} rs_\varphi \,\mathrm{d}\varphi \\
&= -r^{l-1} \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{l-2} s_\varphi (1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi \\
&= -r^{l-1} \mathcal{J}_{l-2,1}
\end{aligned}
$$

CASE 3: ν odd, $\mu = 1$, l odd

$$
\begin{aligned}
\mathcal{P}(\mathbf{G}_n) &= -\int_{\pi-\phi}^{2\pi+\phi} (rc_\varphi)^{l-3}(b+rs_\varphi)(1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} rs_\varphi \,\mathrm{d}\varphi \\
&= -r^{l-2} \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{l-3}(b+rs_\varphi)(s_\varphi)(1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi \\
&= -r^{l-2} \left(b \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{l-3}(s_\varphi)(1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi + r \int_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{l-3}(s_\varphi)^2(1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi \right) \\
&= -r^{l-2} \left(b\mathcal{J}_{l-3,1} + r\mathcal{J}_{l-3,2} \right)
\end{aligned}
$$

CASE 4: otherwise

$$
\begin{aligned}
\mathcal{P}(\mathbf{G}_n) &= \int\limits_{\pi-\phi}^{2\pi+\phi} (rc_\varphi)^{\frac{\mu-3}{2}} (b+rs_\varphi)^{\frac{\nu-1}{2}} (1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} rc_\varphi \,\mathrm{d}\varphi \\
&= r^{\frac{\mu+\nu}{2}-1} \int\limits_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{\frac{\mu-1}{2}} \left(\frac{b}{r}+s_\varphi\right)^{\frac{\nu-1}{2}} (1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi \\
&= r^{l-1} \int\limits_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{\frac{\mu-1}{2}} \sum_{i=0}^{\frac{\nu-1}{2}} \binom{\frac{\nu-1}{2}}{i} \left(\frac{b}{r}\right)^{\frac{\nu-1}{2}-i} (s_\varphi)^i (1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi \\
&= r^{l-1} \sum_{i=0}^{\frac{\nu-1}{2}} \binom{\frac{\nu-1}{2}}{i} \left(\frac{b}{r}\right)^{\frac{\nu-1}{2}-i} \int\limits_{\pi-\phi}^{2\pi+\phi} (c_\varphi)^{\frac{\mu-1}{2}} (s_\varphi)^i (1-r^2-b^2-2brs_\varphi)^{\frac{3}{2}} \,\mathrm{d}\varphi \\
&= r^{l-1} \sum_{i=0}^{\frac{\nu-1}{2}} \binom{\frac{\nu-1}{2}}{i} \left(\frac{b}{r}\right)^{\frac{\nu-1}{2}-i} \mathcal{J}_{\frac{\mu-1}{2},i} \\
&= r^{l-1} \mathcal{L}_{\frac{\mu-1}{2},\frac{\nu-1}{2}} \,.
\end{aligned}
$$