Link to the Mathematica .nb file

The problem

Taylor expanding $J_{u,v}$ about $b = 0$

Let's approximate

$$J_{u,v} = \int_{\pi-\phi}^{2\pi+\phi} c^u s^v \left(1 - r^2 - b^2 - 2brs\right)^{\frac{3}{2}} d\psi$$

as

$$J_{u,v} = \sum_{n=0} d_n b^n$$

We compute the coefficients **d** below:

In[186]:=
```
FullSimplify[SeriesCoefficient[(1 - r^2 - b^2 - 2 b r s)^(3/2), {b, 0, 0}]]
```

Out[186]= $\left(1 - r^2\right)^{3/2}$

$$d_0 = \left(1 - r^2\right)^{3/2} I_{u,v}$$

In[175]:=
```
FullSimplify[SeriesCoefficient[(1 - r^2 - b^2 - 2 b r s)^(3/2), {b, 0, 1}]]
```

Out[175]= $-3 r \sqrt{1 - r^2}\, s$

$$d_1 = -3 r \sqrt{1 - r^2}\, I_{u,v+1}$$

In[187]:=
```
Collect[SeriesCoefficient[(1 - r^2 - b^2 - 2 b r s)^(3/2), {b, 0, 2}], {c, s}, FullSimplify]
```

Out[187]= $-\frac{3}{2}\sqrt{1 - r^2} + \frac{3 r^2 s^2}{2\sqrt{1 - r^2}}$

$$d_2 = -\frac{3}{2}\sqrt{1 - r^2}\, I_{u,v} + \frac{3 r^2}{2\sqrt{1 - r^2}} I_{u,v+2}$$

In[179]:=
```
Collect[SeriesCoefficient[(1 - r^2 - b^2 - 2 b r s)^(3/2), {b, 0, 3}], {c, s}]
```

Out[179]= $\frac{3 r s}{2\sqrt{1 - r^2}} + \frac{r^3 s^3}{2\left(1 - r^2\right)^{3/2}}$

$$d_3 = \frac{3 r}{2\sqrt{1 - r^2}} I_{u,v+1} + \frac{r^3}{2\left(1 - r^2\right)^{3/2}} I_{u,v+3}$$

In[182]:= `Collect[SeriesCoefficient[(1 - r^2 - b^2 - 2 b r s)^(3/2), {b, 0, 4}], {c, s}, FullSimplify]`

Out[182]=
$$\frac{3}{8\sqrt{1-r^2}} + \frac{3 r^2 s^2}{4\left(1-r^2\right)^{3/2}} + \frac{3 r^4 s^4}{8\left(1-r^2\right)^{5/2}}$$

$$d_4 = \frac{3}{8\sqrt{1-r^2}} I_{u,v} + \frac{3 r^2}{4\left(1-r^2\right)^{3/2}} I_{u,v+2} + \frac{3 r^4}{8\left(1-r^2\right)^{5/2}} I_{u,v+4}$$

In[184]:= `Collect[SeriesCoefficient[(1 - r^2 - b^2 - 2 b r s)^(3/2), {b, 0, 5}], {c, s}, FullSimplify]`

Out[184]=
$$\frac{3 r s}{8\left(1-r^2\right)^{3/2}} + \frac{3 r^3 s^3}{4\left(1-r^2\right)^{5/2}} + \frac{3 r^5 s^5}{8\left(1-r^2\right)^{7/2}}$$

$$d_5 = \frac{3 r}{8\left(1-r^2\right)^{3/2}} I_{u,v+1} + \frac{3 r^3}{4\left(1-r^2\right)^{5/2}} I_{u,v+3} + \frac{3 r^5}{8\left(1-r^2\right)^{7/2}} I_{u,v+5}$$