Link to Mathematica .nb file

Definitions

Let's just define H. The I function is identical, except we replace λ by ϕ.

In[18]:=
$$H[u\_, v\_, \lambda\_] := \int_{\pi-\lambda}^{2\pi+\lambda} \text{Cos}[\psi]^u \, \text{Sin}[\psi]^v \, d\psi;$$

Let's define our recurrence relation

In[20]:=
$$\text{HRec}[u\_, v\_, \lambda\_] := \text{Which}\Big[$$
$$\text{OddQ}[u], 0,$$
$$(u == v) \;\&\&\; (u == 0), 2\lambda + \pi,$$
$$(u == 0) \;\&\&\; (v == 1), -2\,\text{Cos}[\lambda],$$
$$u \geq 2, \frac{2}{u+v}\,\text{Cos}[\lambda]^{u-1}\,\text{Sin}[\lambda]^{v+1} + \frac{u-1}{u+v}\,\text{HRec}[u-2, v, \lambda],$$
$$v \geq 2, -\frac{2}{u+v}\,\text{Cos}[\lambda]^{u+1}\,\text{Sin}[\lambda]^{v-1} + \frac{v-1}{u+v}\,\text{HRec}[u, v-2, \lambda]$$
$$\Big];$$

Check that they are equivalent up to u = 10, v = 10:

In[23]:=
```
Table[FullSimplify[HRec[u, v, λ] == H[u, v, λ]], {u, 0, 10}, {v, 0, 10}]
```

Out[23]=
```
{{True, True, True, True, True, True, True, True, True, True, True},
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 {True, True, True, True, True, True, True, True, True, True, True}}
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