Link to Mathematica .nb file

Definitions

Define A1 (see A1.nb)

```
A1[lmax_] := Module[{A, B, Yxyz, Y, by, bp, p, x, y, z},
   A[l_, m_] := Sqrt[((2 - KroneckerDelta[m, 0]) (2 l + 1) (l - m)!)/(4 π (l + m)!)];
   B[l_, m_, j_, k_] := (2^l m! ((l+m+k-1)/2)!)/(j! k! (m - j)! (l - m - k)! ((-l+m+k-1)/2)!);
   Yxyz[l_, m_, x_, y_, z_] := {
      Sum[Sum[(-1)^(j/2) A[l, m] B[l, m, j, k] x^(m-j) y^j z^k, {k, 0, l - m}
      Sum[Sum[(-1)^((j-1)/2) A[l, Abs[m]] B[l, Abs[m], j, k] x^(Abs[m]-j) y^j
        {j, 1, Abs[m], 2}]
   Y[l_, m_, x_, y_, z_] :=
    Expand[FullSimplify[Yxyz[l, m, x, y, Sqrt[1 - x^2 - y^2]]]] /. Sqrt[1 - x^2 - y^2] → z;
   by[n_, x_, y_, z_] := Module[{l, m},
     l = Floor[Sqrt[n]];
     m = n - l^2 - l;
     Y[l, m, x, y, z]];
   bp[n_, x_, y_, z_] := Module[{l, m, μ, ν},
     l = Floor[Sqrt[n]];
     m = n - l^2 - l;
     μ = l - m;
     ν = l + m;
     If[EvenQ[ν], x^(μ/2) y^(ν/2), x^((μ-1)/2) y^((ν-1)/2) z]];
   p[l_, m_] := Module[{Ylm},
     Ylm = Y[l, m, x, y, z];
     Table[If[n == 0, Ylm, Coefficient[Ylm, bp[n, x, y, z]]] /. {x → 0, y → 0, z → 0},
      {n, 0, (lmax + 1)^2 - 1}]];
   Transpose[Flatten[Table[p[l, m], {l, 0, lmax}, {m, -l, l}], 1]]];
```

Define A2 (see A2.nb)

```
A2[lmax_] := Module[{bg, p, x, y, z},
  bg[n_, x_, y_, z_] := Module[{l, m, μ, ν},
    l = Floor[√n];
    m = n - l^2 - l;
    μ = l - m;
    ν = l + m;
    Which[
     EvenQ[ν], (μ + 2)/2 x^(μ/2) y^(ν/2),
     ν == 1 && μ == 1, z,
     μ > 1, z ((μ - 3)/2 x^((μ-5)/2) y^((ν-1)/2) - (μ - 3)/2 x^((μ-5)/2) y^((ν+3)/2) - (μ + 3)/2 x^((μ-1)/2) y^((ν-1)/2)),
     OddQ[l], z (-x^(l-3) + x^(l-1) + 4 x^(l-3) y^2),
     True, 3 x^(l-2) y z
    ]];
  p[n_] := Module[{g},
    g = bg[n, x, y, z];
    Join[{Evaluate[g /. {z → 0, x → 0, y → 0}]},
     Table[Coefficient[g, bp[j, x, y, z]] /. {z → 0, x → 0, y → 0},
      {j, 1, (lmax + 1)^2 - 1}]]];
  Inverse[Transpose[Flatten[Table[p[l^2 + l + m], {l, 0, lmax}, {m, -l, l}], 1]]]];
```

A is just their dot product

```
A[lmax_] := Dot[A2[lmax], A1[lmax]];
```

Examples

Print A for lmax=2

```
A[2] // MatrixForm
```

$$
\begin{pmatrix}
\frac{1}{2\sqrt{\pi}} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{\frac{5}{\pi}}}{2} & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{\frac{3}{\pi}}}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{\frac{3}{\pi}}}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\sqrt{\frac{3}{\pi}}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{\frac{5}{\pi}}}{4} & 0 & \frac{1}{4}\sqrt{\frac{5}{3\pi}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{2}\sqrt{\frac{5}{3\pi}} & 0 \\
0 & 0 & 0 & 0 & \frac{\sqrt{\frac{15}{\pi}}}{4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{2}\sqrt{\frac{5}{3\pi}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{3\sqrt{\frac{5}{\pi}}}{4} & 0 & -\frac{\sqrt{\frac{15}{\pi}}}{4}
\end{pmatrix}
$$

LaTeXify

Make ***A*** LATEX-friendly

```
ATeX[lmax_] := TeXForm[1/(2 √π)] TeXForm[FullSimplify[2 √π A[lmax]]];
```

Print $\boldsymbol{A}$ for $l_{\max} = 2$

```
ATeX[2]
```

```
\frac{1}{2 \sqrt{\pi }} \left(
\begin{array}{ccccccccc}
 1 & 0 & 0 & 0 & 0 & 0 & \sqrt{5} & 0 & 0 \\
 0 & 0 & 0 & \frac{\sqrt{3}}{2} & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & \sqrt{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 &
   -\frac{\sqrt{5}}{2} & 0 & \frac{\sqrt{\frac{5}{3}}}{2} \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sqrt{\frac{5}{3}} & 0 \\
 0 & 0 & 0 & 0 & \frac{\sqrt{15}}{2} & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{5}{3}} & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & -\frac{3 \sqrt{5}}{2} & 0 & -\frac{\sqrt{15}}{2} \\
\end{array}
\right)
```