Definitions

Exterior Derivative

In[1]:= `ExtDeriv[F_] := FullSimplify[D[F[[2]], x] - D[F[[1]], y]] /. ` $\sqrt{1 - x^2 - y^2}$ ` → z;`

v even

In[2]:= `ExtDeriv[`$\{0, x^{\frac{\mu+2}{2}} y^{\frac{v}{2}}\}$`]`

Out[2]= $\frac{1}{2} x^{\mu/2} y^{v/2} (2 + \mu)$

As expected, this is the first term in $\tilde{g}$.

$v = \mu = 1$

In[3]:= `ExtDeriv[`$\dfrac{1 - \left(\sqrt{1 - x^2 - y^2}\right)^3}{3 \left(1 - \left(\sqrt{1 - x^2 - y^2}\right)^2\right)}$ $\{-y, x\}$`]`

Out[3]= `z`

As expected, this is the second term in $\tilde{g}$.

v odd, $\mu = 1$, l even

In[4]:= `ExtDeriv[`$\{x^{l-2} \sqrt{1 - x^2 - y^2}^{3}, 0\}$`]`

Out[4]= $3 x^{-2+l} y z$

As expected, this is the third term in $\tilde{g}$.

v odd, $\mu = 1$, l odd

In[5]:= `ExtDeriv[`$\{x^{l-3} y \sqrt{1 - x^2 - y^2}^{3}, 0\}$`];`
`Collect[Expand[%], z]`

Out[6]= $\left(-x^{-3+l} + x^{-1+l} + 4 x^{-3+l} y^2\right) z$

As expected, this is the fourth term in $\tilde{g}$.

v odd, $\mu = 1$, I odd

In[7]:= $\text{ExtDeriv}\left[\left\{0, x^{\frac{\mu-3}{2}} y^{\frac{v-1}{2}} \sqrt{1 - x^2 - y^2}^{\,3}\right\}\right]$

$\text{FullSimplify}\left[\% == z \left(\frac{\mu - 3}{2} x^{\frac{\mu-5}{2}} y^{\frac{v-1}{2}} - \frac{\mu - 3}{2} x^{\frac{\mu-5}{2}} y^{\frac{v+3}{2}} - \frac{\mu + 3}{2} x^{\frac{\mu-1}{2}} y^{\frac{v-1}{2}}\right)\right]$

Out[7]= $-\frac{1}{2} x^{\frac{1}{2}(-5+\mu)} y^{\frac{1}{2}(-1+v)} z \left(\left(-1 + y^2\right)\left(-3 + \mu\right) + x^2 \left(3 + \mu\right)\right)$

Out[8]= True

As expected, this is the fifth term in $\tilde{g}$.