Link to Mathematica .nb file

Definitions

Define the change of basis matrix A1

In[1]:=

$$A[l\_, m\_] := \sqrt{\frac{(2 - \text{KroneckerDelta}[m, 0])\ (2\,l + 1)\ (l - m)\,!}{4\,\pi\ (l + m)\,!}};$$

$$B[l\_, m\_, j\_, k\_] := \frac{2^{l}\ m\,!\ \left(\frac{l+m+k-1}{2}\right)!}{j\,!\ k\,!\ (m - j)\,!\ (l - m - k)\,!\ \left(\frac{-l+m+k-1}{2}\right)!};$$

$$\text{Yxyz}[l\_, m\_, x\_, y\_, z\_] := \begin{cases} \text{Sum}\left[\text{Sum}\left[(-1)^{\frac{j}{2}}\, A[l, m]\, B[l, m, j, k]\, x^{m-j}\, y^{j}\, z^{k}, \{k, 0, l - m\}\right], \{ \right. \\ \text{Sum}\left[\text{Sum}\left[(-1)^{\frac{j-1}{2}}\, A[l, \text{Abs}[m]]\, B[l, \text{Abs}[m], j, k]\, x^{\text{Abs}[m]-j}\, y^{j}\, z^{k}, \right.\right. \\ \quad \left.\{j, 1, \text{Abs}[m], 2\}\right] \end{cases}$$

$$Y[l\_, m\_, x\_, y\_] := \text{Expand}\left[\text{FullSimplify}\left[\text{Yxyz}\left[l, m, x, y, \sqrt{1 - x^2 - y^2}\right]\right]\right] \mathbin{/.} \sqrt{1 - x^2 - y^2} \to z;$$

$$\begin{aligned} &\text{bp}[n\_, x\_, y\_] := \text{Module}\Big[\{l, m, \mu, \nu\}, \\ &\quad l = \text{Floor}\left[\sqrt{n}\right]; \\ &\quad m = n - l^2 - l; \\ &\quad \mu = l - m; \\ &\quad \nu = l + m; \\ &\quad \text{If}\left[\text{EvenQ}[\nu], x^{\frac{\mu}{2}}\, y^{\frac{\nu}{2}}, x^{\frac{\mu-1}{2}}\, y^{\frac{\nu-1}{2}} \sqrt{1 - x^2 - y^2}\right]\Big]; \end{aligned}$$

$$\begin{aligned} &p[l\_, m\_, \text{lmax}\_] := \text{Module}\Big[\{\text{Ylm}\}, \\ &\quad \text{Ylm} = Y[l, m, x, y] \mathbin{/.} z \to \sqrt{1 - x^2 - y^2}; \\ &\quad \text{Join}\Big[\Big\{\text{Evaluate}\Big[\text{Ylm} \mathbin{/.} \Big\{\sqrt{1 - x^2 - y^2} \to 0, x \to 0, y \to 0\Big\}\Big]\Big\}, \\ &\quad\ \text{Table}\Big[\text{Coefficient}[\text{Ylm}, \text{bp}[n, x, y]] \mathbin{/.} \Big\{\sqrt{1 - x^2 - y^2} \to 0, x \to 0, y \to 0\Big\}, \\ &\quad\quad \{n, 1, (\text{lmax} + 1)^2 - 1\}\Big]\Big]\Big]; \end{aligned}$$

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A1[lmax_] := Transpose[Flatten[Table[p[l, m, lmax], {l, 0, lmax}, {m, -l, l}], 1]];
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Convert our polynomial vector **p** to a spherical harmonic vector **y**

In[8]:=

$$\begin{aligned} &\text{pvec} = \{1 - \text{u1} - 2\,\text{u2}, 0, \text{u1} + 2\,\text{u2}, 0, \text{u2}, 0, 0, 0, \text{u2}\}; \\ &\text{yvec} = \text{Dot}[\text{Inverse}[\text{A1}[2]], \text{pvec}]; \\ &\text{FullSimplify}[\text{Dot}[\text{yvec}, \{Y_{0,0}, Y_{1,-1}, Y_{1,0}, Y_{1,1}, Y_{2,-2}, Y_{2,-1}, Y_{2,0}, Y_{2,1}, Y_{2,2}\}]] \end{aligned}$$

Out[10]=

$$\frac{2}{15}\sqrt{\pi}\ \left(-5\ (-3 + 3\,\text{u1} + 4\,\text{u2})\ Y_{0,0} + 5\sqrt{3}\ (\text{u1} + 2\,\text{u2})\ Y_{1,0} - 2\sqrt{5}\ \text{u2}\ Y_{2,0}\right)$$

In[11]:=

$$\text{Collect}[\%, \{Y_{0,0}, Y_{1,-1}, Y_{1,0}, Y_{1,1}, Y_{2,-2}, Y_{2,-1}, Y_{2,0}, Y_{2,1}, Y_{2,2}\}, \text{Simplify}]$$

Out[11]=

$$-\frac{2}{3}\sqrt{\pi}\ (-3 + 3\,\text{u1} + 4\,\text{u2})\ Y_{0,0} + 2\sqrt{\frac{\pi}{3}}\ (\text{u1} + 2\,\text{u2})\ Y_{1,0} - \frac{4}{3}\sqrt{\frac{\pi}{5}}\ \text{u2}\ Y_{2,0}$$