Link to Mathematica .nb file

Definitions

Define A1 and A2

```
In[1]:= A[l_, m_] := Sqrt[((2 - KroneckerDelta[m, 0]) (2 l + 1) (l - m)!)/(4 π (l + m)!)];

B[l_, m_, j_, k_] := (2^l m! ((l+m+k-1)/2)!)/(j! k! (m - j)! (l - m - k)! ((-l+m+k-1)/2)!);

Yxyz[l_, m_, x_, y_, z_] := { Sum[Sum[(-1)^(j/2) A[l, m] B[l, m, j, k] x^(m-j) y^j z^k, {k, 0, l - m}], {
                              Sum[Sum[(-1)^((j-1)/2) A[l, Abs[m]] B[l, Abs[m], j, k] x^(Abs[m]-j) y^j z^k,
                                {j, 1, Abs[m], 2}]

Y[l_, m_, x_, y_] :=
  Expand[FullSimplify[Yxyz[l, m, x, y, Sqrt[1 - x^2 - y^2]]]] /. Sqrt[1 - x^2 - y^2] → z;
bp[n_, x_, y_] := Module[{l, m, μ, ν},
   l = Floor[Sqrt[n]];
   m = n - l^2 - l;
   μ = l - m;
   ν = l + m;
   If[EvenQ[ν], x^(μ/2) y^(ν/2), x^((μ-1)/2) y^((ν-1)/2) Sqrt[1 - x^2 - y^2]]];
p[l_, m_, lmax_] := Module[{Ylm},
   Ylm = Y[l, m, x, y] /. z → Sqrt[1 - x^2 - y^2];
   Join[{Evaluate[Ylm /. {Sqrt[1 - x^2 - y^2] → 0, x → 0, y → 0}]},
    Table[Coefficient[Ylm, bp[n, x, y]] /. {Sqrt[1 - x^2 - y^2] → 0, x → 0, y → 0},
     {n, 1, (lmax + 1)^2 - 1}]]];
A1[lmax_] := Transpose[Flatten[Table[p[l, m, lmax], {l, 0, lmax}, {m, -l, l}], 1]];
bg[n_, x_, y_] := Module[{l, m, μ, ν},
   l = Floor[Sqrt[n]];
   m = n - l^2 - l;
   μ = l - m;
   ν = l + m;
   Which[
    EvenQ[ν], (μ + 2)/2 x^(μ/2) y^(ν/2),
    ν == 1 && μ == 1, Sqrt[1 - x^2 - y^2],
    μ > 1, Sqrt[1 - x^2 - y^2] ((μ - 3)/2 x^((μ-5)/2) y^((ν-1)/2) - (μ - 3)/2 x^((μ-5)/2) y^((ν+3)/2) - (μ + 3)/2 x^((μ-1)/2) y^((ν-1)/2)),
```

```
    OddQ[l], Sqrt[1 - x^2 - y^2] (-x^(l-3) + x^(l-1) + 4 x^(l-3) y^2),
    True, 3 x^(l-2) y Sqrt[1 - x^2 - y^2]
   ]];
p2[n_, lmax_] := Module[{g},
   g = bg[n, x, y];
   Join[{Evaluate[g /. {Sqrt[1 - x^2 - y^2] → 0, x → 0, y → 0}]},
    Table[Coefficient[g, bp[j, x, y]] /. {Sqrt[1 - x^2 - y^2] → 0, x → 0, y → 0},
     {j, 1, (lmax + 1)^2 - 1}]]];
A2Inv[lmax_] := Transpose[Flatten[Table[p2[l^2 + l + m, lmax],
     {l, 0, lmax}, {m, -l, l}], 1]];
A2[lmax_] := Inverse[A2Inv[lmax]];
```

A is just their dot product

```
In[12]:= A[lmax_] := Dot[A1[lmax], A2[lmax]];
```

Examples

Print A for lmax=3

In[13]:= `A[3] // MatrixForm`

Out[13]//MatrixForm=

$$
\begin{pmatrix}
\frac{1}{2\sqrt{\pi}} & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{\frac{5}{\pi}}}{4} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\sqrt{\frac{3}{\pi}}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{\frac{3}{\pi}}}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{\frac{3}{\pi}}}{10} & 0 \\
0 & \frac{\sqrt{\frac{3}{\pi}}}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\frac{7}{6\pi}} \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{3\sqrt{\frac{5}{\pi}}}{8} & 0 & \frac{\sqrt{\frac{15}{\pi}}}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2}\sqrt{\frac{5}{3\pi}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{2}\sqrt{\frac{5}{3\pi}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -\frac{1}{2}\sqrt{\frac{5}{3\pi}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -\frac{3\sqrt{\frac{5}{\pi}}}{8} & 0 & -\frac{\sqrt{\frac{15}{\pi}}}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{4}\sqrt{\frac{7}{15\pi}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{3}{16}\sqrt{\frac{35}{2\pi}} & 0 & -\frac{5}{4}\sqrt{\frac{7}{6\pi}} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\sqrt{\frac{7}{15\pi}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4}\sqrt{\frac{7}{15\pi}} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{16}\sqrt{\frac{35}{2\pi}} & 0 & -\frac{5}{4}\sqrt{\frac{7}{6\pi}}
\end{pmatrix}
$$