1

Filter

요점정리

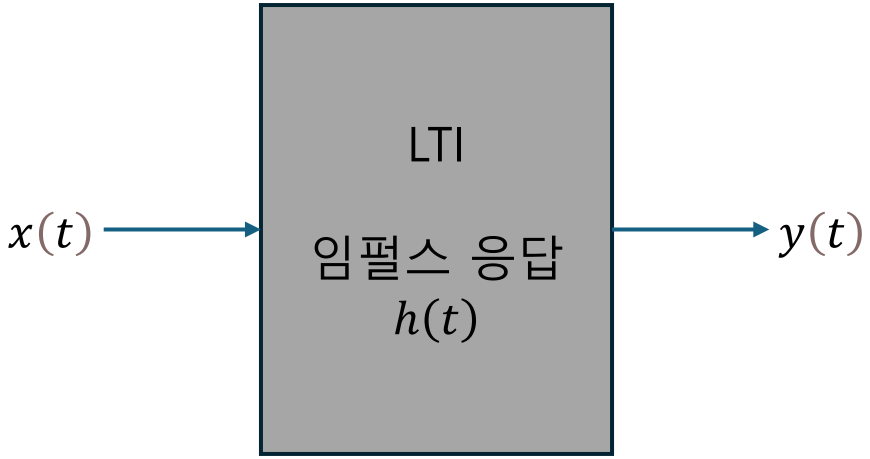
주기 신호 → 푸리에 시리즈
비 주기 신호 → 푸리에 변환

- 주기, 연속
- $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}$
- $a_k = \frac{1}{T}\int_T x(t) e^{-jkw_0 t} dt$
- 푸리에 변환 → $X(jw) = \sum_{k=-\infty}^{\infty} a_k * 2\pi * \delta(w - kw_0)$

- 주기, 불연속
- $x[n] = \sum_{k=0}^{N-1} a_k e^{jkw_0 n}$
- $a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] \cdot e^{-jkw_0 n}$

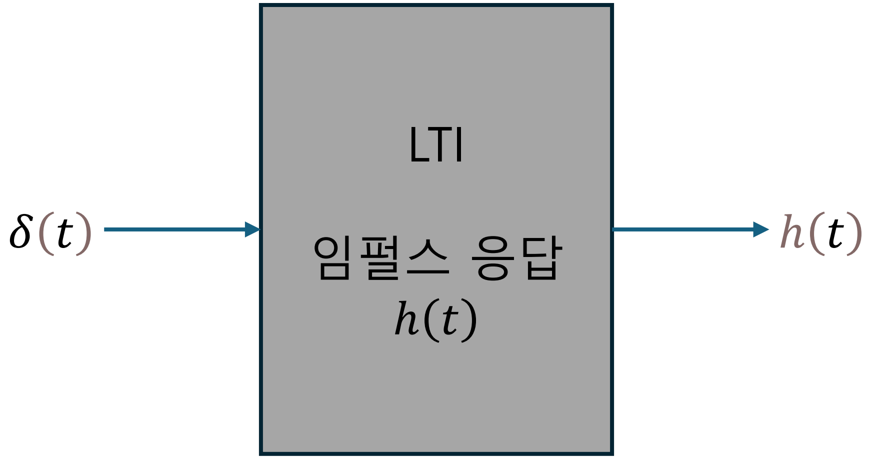
- 비 주기, 연속
- $X(jw) = \int_{-\infty}^{\infty} x(t) \cdot e^{-jwt} dt$
- $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(jw) e^{jwt} dw$
- Convolution 응용 → $x(t) * h(t) = X(jw)H(jw)$

- 비 주기, 불연속
- $X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$
- $x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{jw}) e^{jwn} dw$



$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Linear Time-Invariant System



$$\boldsymbol{y(t) = x(t) * h(t)}$$
$$\boldsymbol{Y(jw) = X(jw) \cdot H(jw)}$$

1. 시간 도메인에서는 출력 $y(t)$가 입력 $x(t)$와 임펄스 응답 $h(t)$의 컨볼루션으로 표현됩니다:

$$y(t) = x(t) * h(t)$$

2. 주파수 도메인에서는 출력 $Y(j\omega)$가 입력 $X(j\omega)$와 주파수 응답 $H(j\omega)$의 곱으로 표현됩니다:

$$Y(j\omega) = X(j\omega) \cdot H(j\omega)$$

Phase란?

주기함수의 시작 포인트(형태가 보존되는가?)

$$Y(jw) = |Y(jw)|e^{j\measuredangle Y(jw)}$$
$$X(jw) = |X(jw)|e^{j\measuredangle X(jw)}$$
$$H(jw) = |H(jw)|e^{j\measuredangle H(jw)}$$

- ***FT of Convolution***

$$Y(j\omega) = \underbrace{H(j\omega)}_{\text{Frequency Response}}X(j\omega)$$

- ***Magnitude representation***

$$|Y(j\omega)| = \underbrace{|H(j\omega)|}_{\text{Gain}}|X(j\omega)|$$

- ***Phase representation***

$$\measuredangle Y(j\omega) = \underbrace{\measuredangle H(j\omega)}_{\text{Phase shift}} + \measuredangle X(j\omega)$$

Linear Phase **→ 원본 유지**
Non-Linear Phase **→ 원본 훼손**

- ***Shifting***

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$$

$- x(t - t_0)$

$$x(t-t_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega(t-t_0)}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{-j\omega t_0}e^{j\omega t}\,d\omega$$

$$\boxed{x(t-t_0) \Leftrightarrow X(\omega)e^{-j\omega t_0}}$$

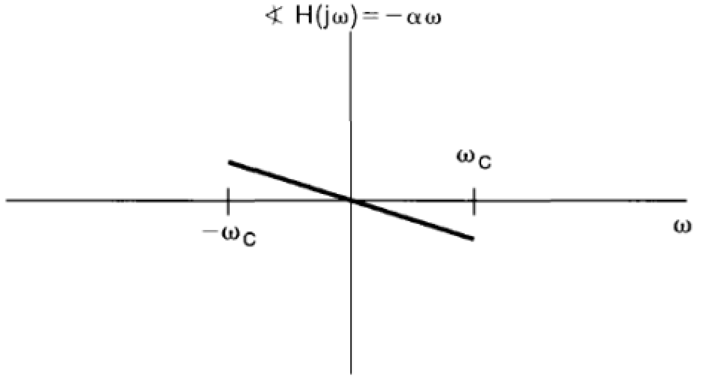
$$H(j\omega) = e^{-j\omega t_0}$$

$$\downarrow$$

$$|H(j\omega)| = 1 \qquad \boxed{\measuredangle H(j\omega) = -\omega t_0}$$

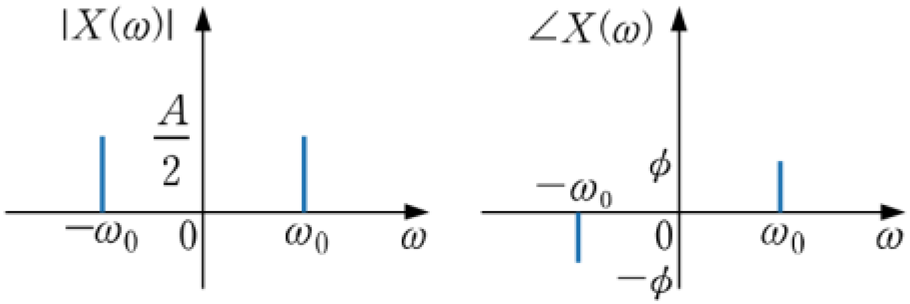
$$\downarrow$$

$$y(t) = x(t-t_0)$$



Phase 그래프가 직선 형태면 완벽한 Linear Phase

기울기 0 **→ No delay**
기울기 Not 0 **→ Delay**



$$x(t) = A\cos(2\pi f_0 t + \phi) = A\cos(\omega_0 t + \phi)$$

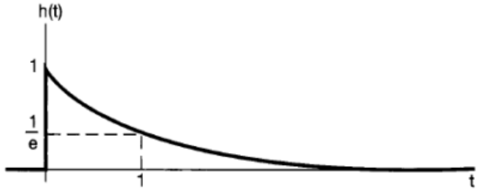
$$x(t) = \frac{A}{2} e^{j\phi} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j\phi} e^{-j2\pi f_0 t}$$

Magnitude spectrum(크기)
Phase spectrum(위상)

두 스펙트럼을 구해보자!

- ***Impulse response***

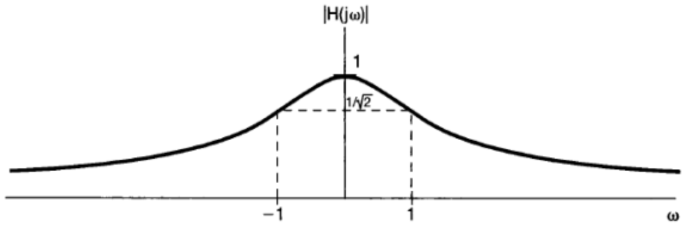
$$h(t)=e^{-t}u(t)$$



(a)

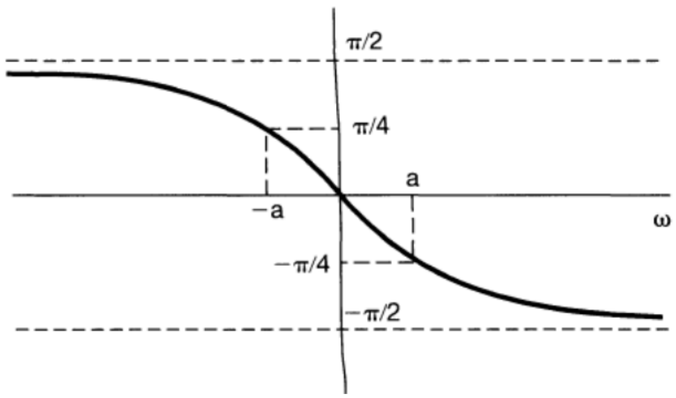
- ***Frequency response & its magnitude representation***

$$H(j\omega)=\frac{1}{j\omega+1}$$



(b)

$$\sphericalangle H(j\omega)=\arctan(-\omega)$$



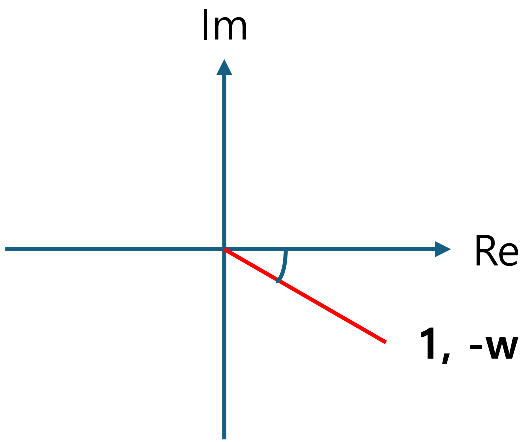
$$h(t)=e^{-t}u(t)$$

$$H(jw)=\int_{-\infty}^{\infty} e^{-t}u(t)e^{-jwt}\,dt$$

$$=\int_{0}^{\infty} e^{-t}e^{-jwt}\,dt$$

$$=\int_{0}^{\infty} e^{-(1+jw)t}\,dt$$

$$=\frac{1}{1+jw}=\frac{\mathbf{1}}{\mathbf{1}+\boldsymbol{w}^{\mathbf{2}}}[\mathbf{1}-\boldsymbol{jw}]$$

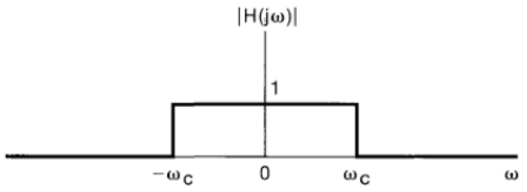


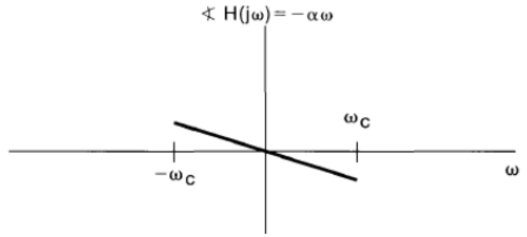
- ***Augmented with linear phase shifting***

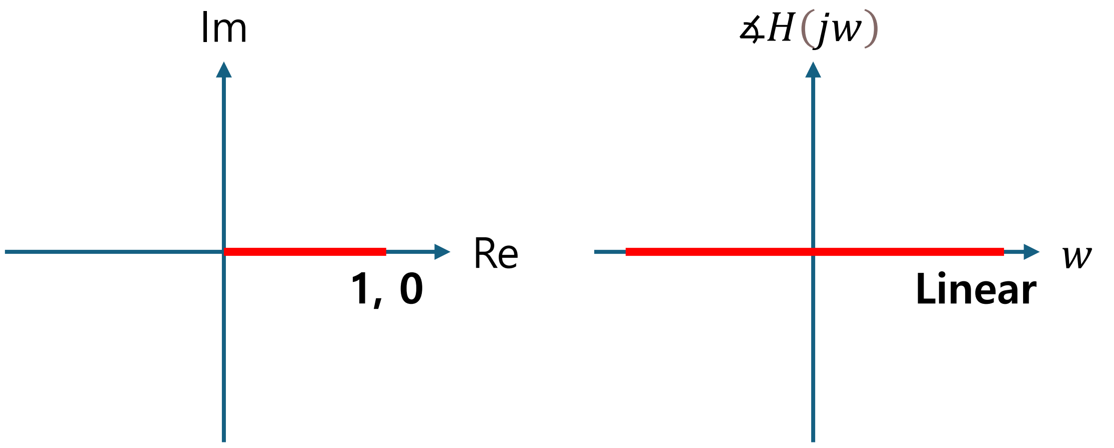
$$h(t) = \frac{\sin \omega_c t}{\pi t}$$

$$H(j\omega) = \begin{cases} 1, & |\omega| \le \omega_c, \\ 0, & |\omega| > \omega_c. \end{cases}$$

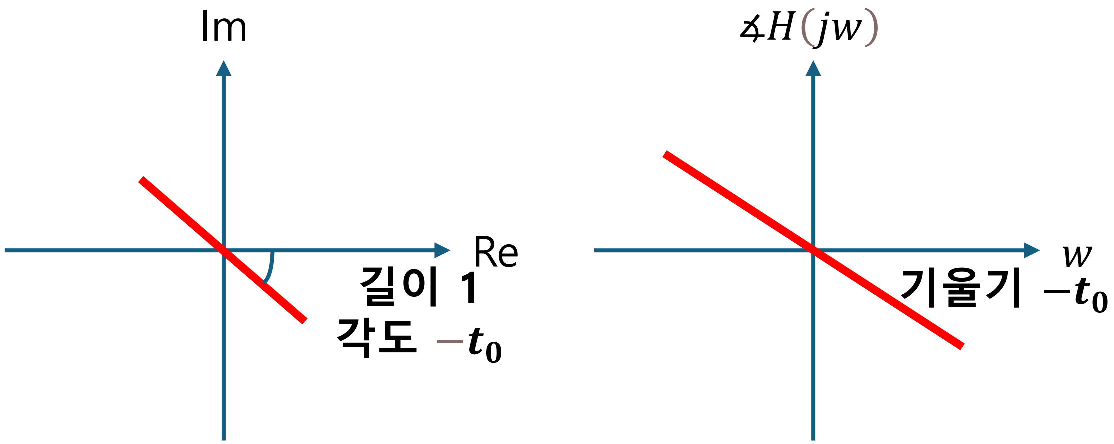
- ***Then, how to augment the linear phase?***







$$H(jw) = 1 \ (|w| \le w_0)$$



$$e^{-jwt_0} \cdot H(jw) = e^{-jwt_0} \ (|w| \le w_0)$$

- *Cutoff Frequency (a.k.a Break Frequency)*
  - *The frequency at which a filter begins to attenuate the input signal significantly*
  - *Typically defined as the point where the signal's amplitude is reduced to* $1/\sqrt{2}$ *(approximately 70.7%) which corresponds to a -3dB*

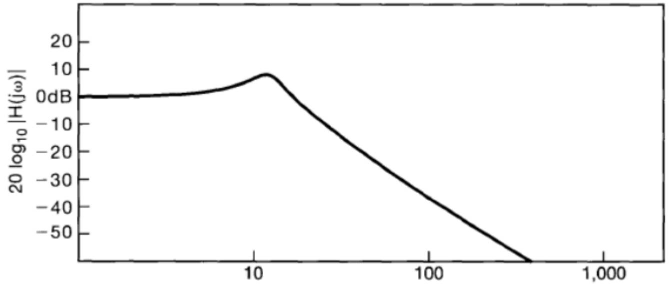
$$dB = 10\log_{10}\frac{P_1}{P_2} \quad \Big| \quad 3dB(dB=3) \longrightarrow dB = 3 = 10\log_{10}2$$

$$-3\,\mathrm{dB} = -10\log_{10}2$$
$$= 10\log_{10}2^{-1} = 10\log_{10}\left(\frac{1}{2}\right)$$

어떤 신호의 최고점(피크)를 a라 하고,
cutoff 지점을 b라 할 때, b는 a의 70%

$$\mathrm{P} \approx V^2$$

$$v_1 : v_2 = 1 : \sqrt{2}$$



Bode plot

Cutoff Frequency

필터가 입력신호를 상당히 감쇠하기 시작하는 주파수...
일반적으로 전압 진폭이 70% 감소 시점, -3dB 지점

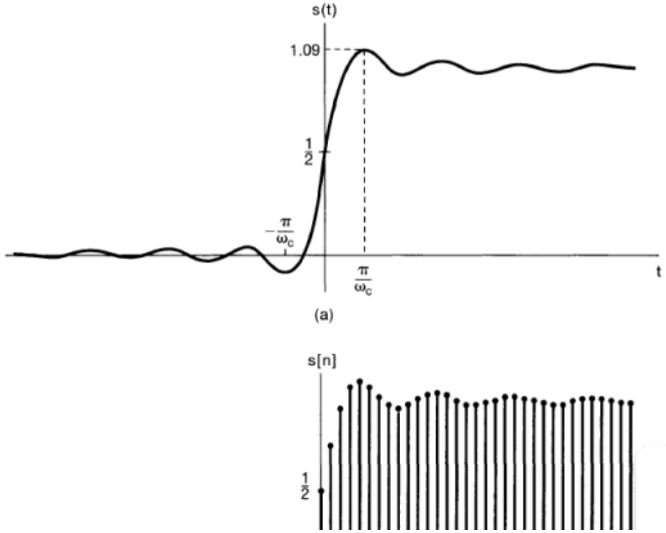
X축은 데시벨, Y축은 로그
→ 한눈에 패턴을 볼 수 있음

Step response

- *System Response to the unit step signal*
  - *System Stability Assessment*
  - *Transient Response Analysis*
  - *Response Speed Measureme*

$$s(t) = u(t) * h(t)$$

$$\boxed{\begin{aligned} s(t) &= \int_{-\infty}^{t} h(\tau)d\tau \\ s[n] &= \sum_{m=-\infty}^{n} h[m] \end{aligned}}$$



단위 계단 함수, Impulse Response

$$s(t) = u(t) * h(t)$$

$$s(t) = \int_{-\infty}^{\infty} h(\tau) \cdot u(t-\tau)\, \mathrm{d}\tau$$

$$s(t) = \int_{-\infty}^{\mathrm{t}} h(\tau)\, \mathrm{d}\tau$$

어떤 시스템에 대해서, 급격한 변화를
가진 인풋(u함수)에 대해 발산, 포화,
fluctuation 등이 없다면 좋은 성능

Low pass filter

- ***LTI system described by linear constant-coefficient differential equations.***

$$\tau \frac{dy(t)}{dt} + y(t) = x(t),$$

– ***Frequency response***

$$H(j\omega) = \frac{1}{j\omega\tau + 1}$$

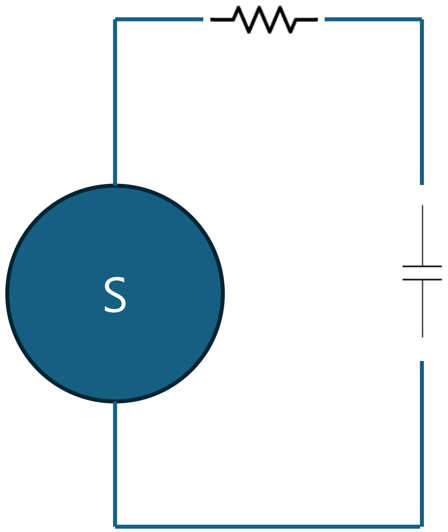
– ***Impulse response***

$$h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$

– ***Step response***

$$s(t) = h(t) * u(t) = [1 - e^{-t/\tau}]u(t)$$

X = Voltage of Source
Y = Voltage of Capacitor



$$v_s(t) = v_R(t) + v_c(t)$$
$$v_R(t) = I(t) \cdot R$$
$$I(t) = C \cdot \frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$$

$$\mathrm{x}(c) = \tau \cdot \frac{\mathrm{d}y(t)}{\mathrm{d}t} + y(t),$$
푸리에 변환 하면...?

$$\boldsymbol{y(t)} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{Y(jw)} e^{jwt}\, dw$$
$$\frac{\mathbf{dy(t)}}{\mathbf{dt}} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \boldsymbol{Y(jw)jw}\mathrm{e}^{jwt}\, \mathrm{d}w$$

$$X(jw) = \tau Y(jw) \cdot jw + Y(jw)$$
$$X(jw) = (jw\tau + 1)Y(jw)$$

$$H(jw) = \frac{1}{jw\tau + 1}$$

$$y(t) = x(t) * h(t)$$
$$Y(jw) = X(jw) \cdot H(jw)$$

Frequency Response

Impulse Response

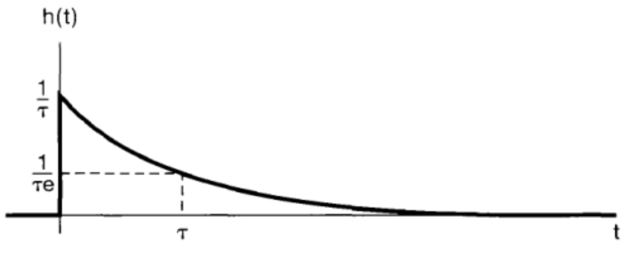
$$H(jw) = \int_{-\infty}^{\infty} h(t) \cdot e^{-jwt}\, dt$$

$$\boldsymbol{h(t) = \frac{1}{\tau} e^{-t/\tau} \cdot u(t)}$$

$\tau = R \cdot C$ (저항 * Capacitor)

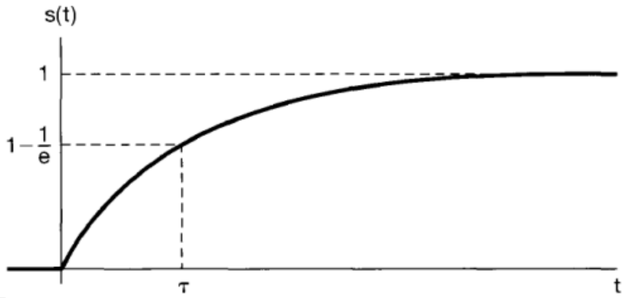
- ***Impulse response.***

$$h(t) = \frac{1}{\tau} e^{-t/\tau} u(t)$$



(a)

- ***Step response***



(b)

$$s(t) = h(t) * u(t) = [1 - e^{-t/\tau}]u(t)$$

Step Response

$$s(t) = u(t) * h(t)$$

$$s(t) = \int_{-\infty}^{t} h(k)\, dk$$

$$s(t) = \int_{-\infty}^{t} \frac{1}{\tau} e^{-k/\tau} \cdot u(k)\, dk$$

$$s(t) = \frac{1}{\tau} \int_{0}^{t} e^{-k/\tau}\, dk$$

$$s(t) = \frac{1}{\tau} \int_{0}^{t} e^{-k/\tau}\, dk$$

$$s(t) = \frac{1}{\tau} \cdot (-\tau) \cdot \left[e^{-k/\tau}\right]_0^t$$

$$\boldsymbol{S(t) = \left(1 - e^{-t/\tau}\right) u(t)}$$

τ값에 따라 그래프 변화

Bode plot for a continuous-time first-order system

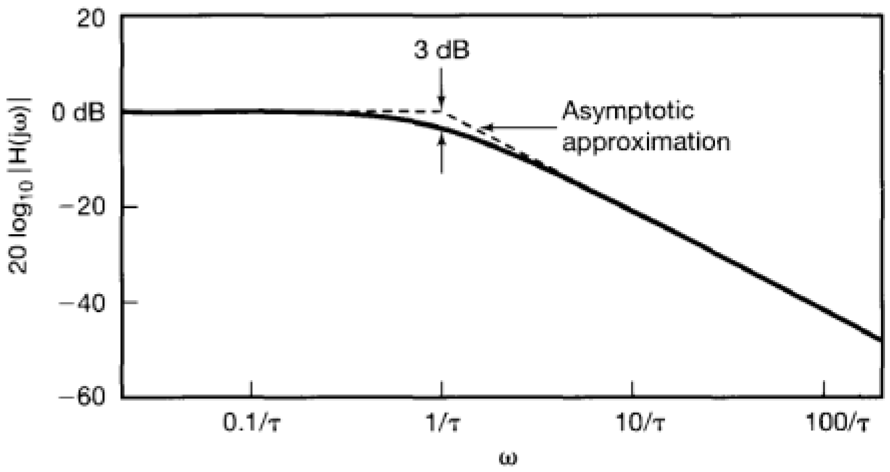
$$20\log_{10}|H(j\omega)| = -10\log_{10}[(\omega\tau)^2+1]$$

$$20\log_{10}|H(j\omega)| \simeq 0 \qquad \text{for} \quad \omega \ll 1/\tau,$$

$$20\log_{10}|H(j\omega)| \simeq -20\log_{10}(\omega\tau)$$
$$= -20\log_{10}(\omega) - 20\log_{10}(\tau) \quad \text{for} \quad \omega \gg 1/\tau.$$

Break frequency $\omega = \dfrac{1}{\tau}$

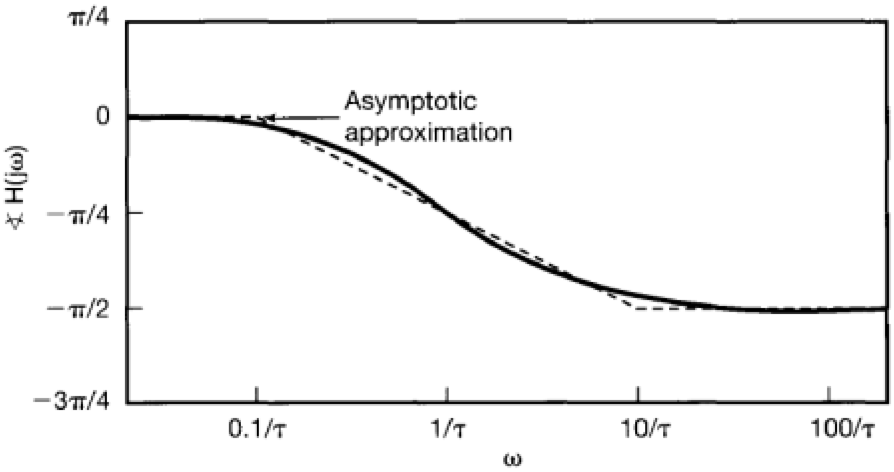
$$20\log_{10}\left|H\left(j\frac{1}{\tau}\right)\right| = -10\log_{10}(2) \simeq -3 \text{ dB}.$$



$$\sphericalangle H(j\omega) = -\tan^{-1}(\omega\tau)$$

$$\begin{array}{l} \text{No delay} \\ \text{Delay} \\ \text{No delay} \end{array} \simeq \begin{cases} 0, & \omega \le 0.1/\tau \\ -(\pi/4)[\log_{10}(\omega\tau)+1], & 0.1/\tau \le \omega \le 10/\tau. \\ -\pi/2, & \omega \ge 10/\tau \end{cases}$$

Linear phase ***over*** $\dfrac{0.1}{\tau} \le \omega \le \dfrac{10}{\tau}$, $\quad \sphericalangle H\left(j\dfrac{1}{\tau}\right) = -\dfrac{\pi}{4}$.



5P에서... Phase가 Linear하다면 형태 유지한다고 했지요?

그래프를 3개의 직선으로 근사해서 쪼갤 수 있고, 만약 기울기가 0이라면 delay도 없습니다.

→ 즉... 사이에 있을 때는 어쩔 수 없이 distortion(왜곡)이 발생합니다.

9P의 Cutoff Frequency를 바탕으로 Bode plot을 그려볼까요?

$$p \propto v^2$$

- $\mathrm{d}B = 10\log_{10}\frac{p_1}{p_2}$
- $\mathrm{d}B = 20\log_{10}\frac{v_1}{v_2}$

Magnitude spectrum 기준으로…

$$|H(jw)| = \sqrt{\frac{1}{1+jw\tau}\cdot\frac{1}{1-jw\tau}} = \sqrt{\frac{1}{1+w^2\tau^2}}$$

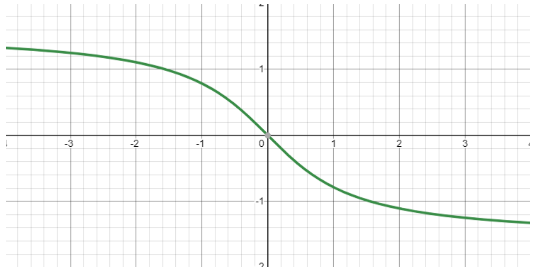
$\mathbf{0}\sim\frac{\mathbf{1}}{\boldsymbol{\tau}}$ → passband = 감쇠X = input과 output 비율 보존

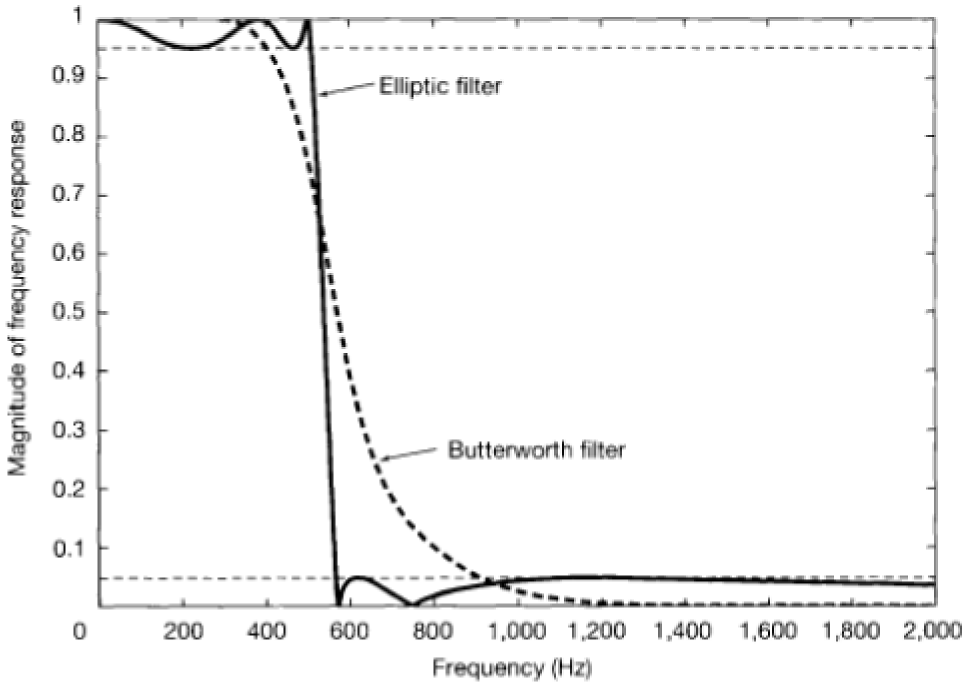
Cutoff Frequency → $\frac{\mathbf{1}}{\boldsymbol{\tau}}$

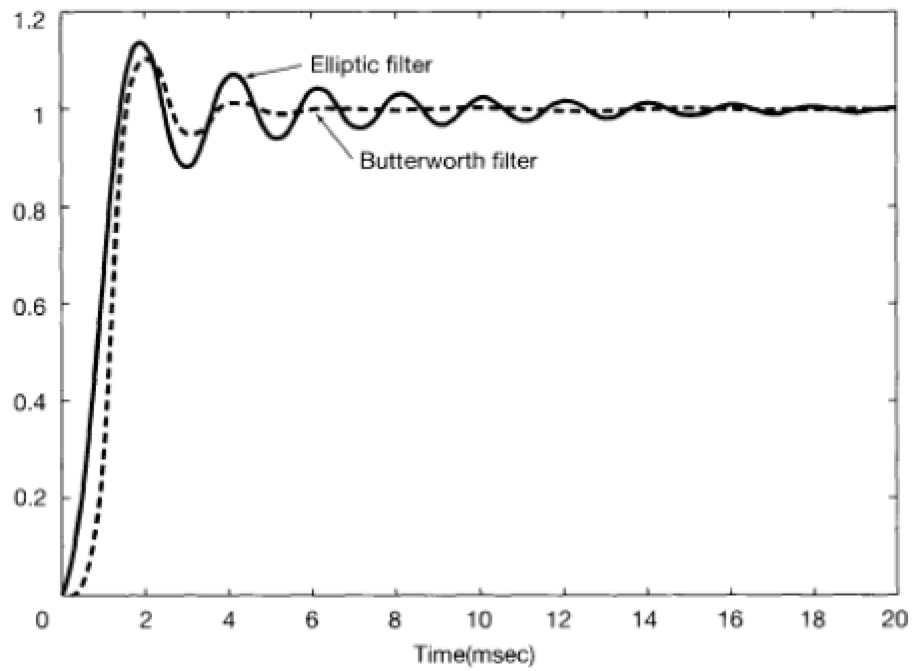
Phase spectrum 기준으로…

$$H(jw) = \frac{1}{1+jw\tau}\cdot\frac{1-jw\tau}{1-jw\tau} \rightarrow (1,\ -w\tau)$$

$$\tan\theta = -w\tau$$
$$\theta = \arctan(-w\tau)$$







다음과 같은 여러 필터가 존재…

Butterworth filter →
- 이 필터는 주파수 응답에서 매우 평탄
- 컷오프 주파수에서 안정성을 빠르게 찾아가는 특징
- 주로 신호의 왜곡을 최소화하면서 원활한 통과 특성을 제공

Elliptic filter →
- 이 필터는 주파수 응답의 패스밴드와 스톱밴드에서 매우 가파른 기울기
- 통과 대역과 차단 대역의 저주파수에서의 왜곡 범위를 최소화하는 특성
- 일반적으로 더 높은 차단 비율과 더 짧은 전이 대역을 제공

- **Butterworth Filter**: "정확한 응답 특성을 가지며, 안정성을 빠르게 찾아가는 필터"
- **Elliptic Filter**: "가파른 기울기를 통해 왜곡 범위를 최소화하는 필터"

High pass filter

$$RC\frac{dv_r(t)}{dt} + v_r(t) = RC\frac{dv_s(t)}{dt}.$$

- ***Freq. response***

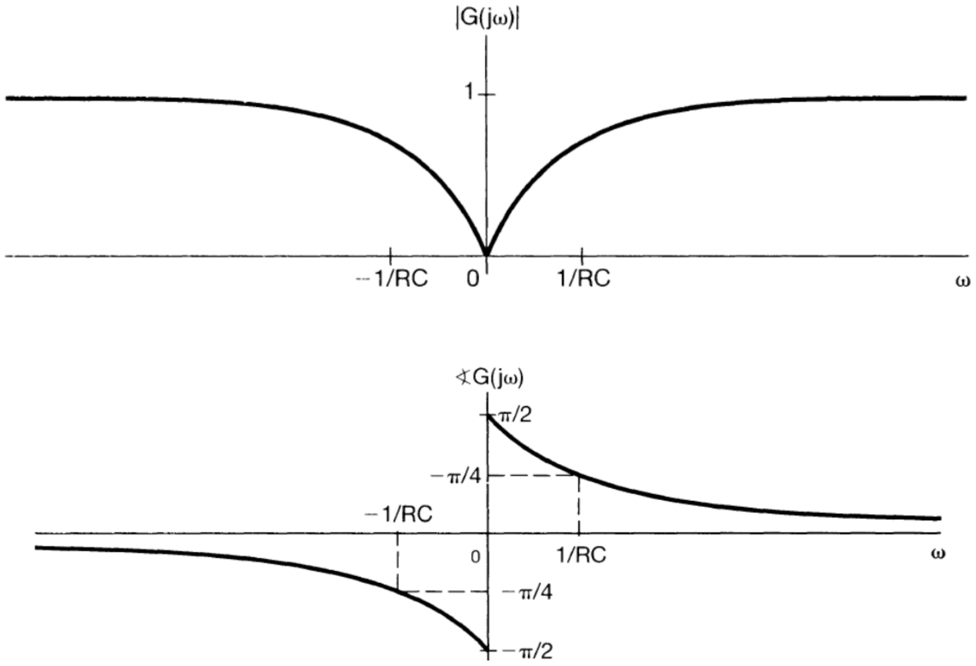
$$G(j\omega) = \frac{j\omega RC}{1 + j\omega RC}.$$

- ***Pass band***

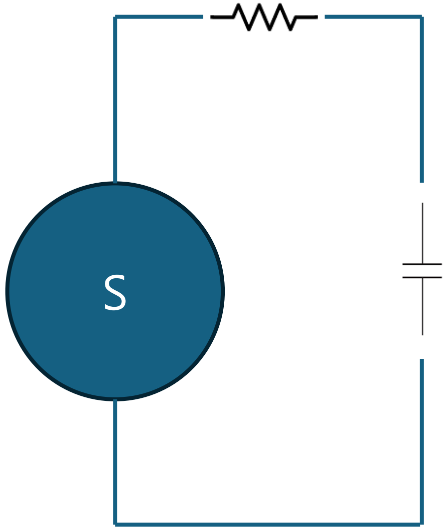
$$|\omega| \gg 1/RC$$

- ***Step response***

$$v_r(t) = e^{-t/RC}u(t),$$



X = Voltage of Source
Y = Voltage of Resister



$$v_S(t) = v_R(t) + v_C(t)$$
$$v_R(t) = I(t) \cdot R$$
$$I(t) = C \cdot \frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$$

$$v_R(t) = RC \cdot \frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$$

$$RC \cdot \frac{\mathrm{d}v_S(t)}{\mathrm{d}t} = RC \cdot \frac{\mathrm{d}v_R(t)}{\mathrm{d}t} + RC \cdot \frac{\mathrm{d}v_C(t)}{\mathrm{d}t}$$
$$RC \cdot \frac{\mathrm{d}v_S(t)}{\mathrm{d}t} = RC \cdot \frac{\mathrm{d}v_R(t)}{\mathrm{d}t} + v_R(t)$$

$$RC \cdot jw \cdot X(jw) = RC \cdot jw \cdot Y(jw) + Y(jw)$$
$$RC \cdot jw \cdot X(jw) = (jw\tau + 1)Y(jw)$$

$$H(jw) = \frac{RC \cdot jw}{jw\tau + 1} = \frac{jw\tau}{jw\tau + 1}$$

Frequency Response

Low pass filter에서...

$$\boldsymbol{H(jw) = \frac{1}{jw\tau + 1}}$$
$$\boldsymbol{h(t) = \frac{1}{\tau} e^{-t/\tau} \cdot u(t)}$$

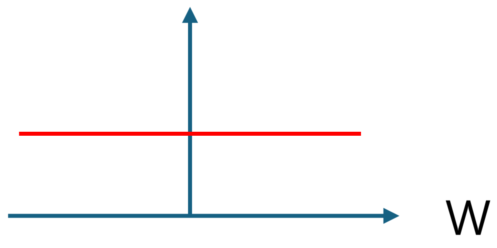
$$\boldsymbol{H(jw) = 1 - \frac{1}{1 + jw\tau}}$$
$$\boldsymbol{h(t) = \delta(t) - \frac{1}{\tau} e^{-t/\tau} \cdot u(t)}$$

참고!!!

- *Find the frequency spectrum of the following signal*

$$x(t) = \delta(t)$$

- $X(jw) = \int_{-\infty}^{\infty} x(t) \cdot \mathrm{e}^{-jwt}\,\mathrm{d}t$
- $X(jw) = \int_{0}^{\Delta} 1/\Delta \cdot \mathrm{e}^{-jwt}\,\mathrm{d}t$
- $X(jw) = \displaystyle\int_{0}^{\Delta} \frac{1}{\Delta} \cdot \mathrm{e}^{-jw(0)}\,\mathrm{d}t$
- $X(jw) = \int_{-\infty}^{\infty} \delta(t) \cdot \mathrm{e}^{-jwt}\,\mathrm{d}t = x(0)$
- $X(jw) = 1$
- 모든 성분의 계수(coefficient)가 1
- 모든 주파수 성분 존재



- ***Example of DTFT***

$$x[n]=a^n u[n], \qquad |a|<1$$

$$X(e^{jw})=\sum_{n=-\infty}^{\infty} x[n]\cdot e^{-jwn}$$

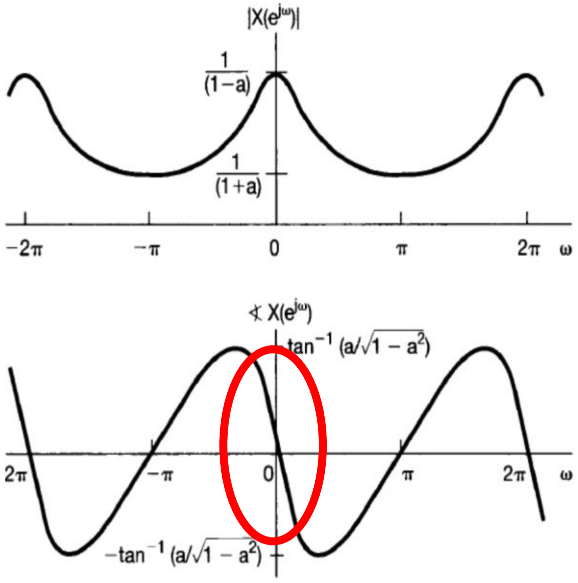
$$X(e^{jw})=\sum_{n=0}^{\infty} a^n e^{-jwn}$$

$$X(e^{jw})=\sum_{n=0}^{\infty} (ae^{-jw})^n$$

$$X(e^{jw})=\frac{1}{1-ae^{-jw}}$$

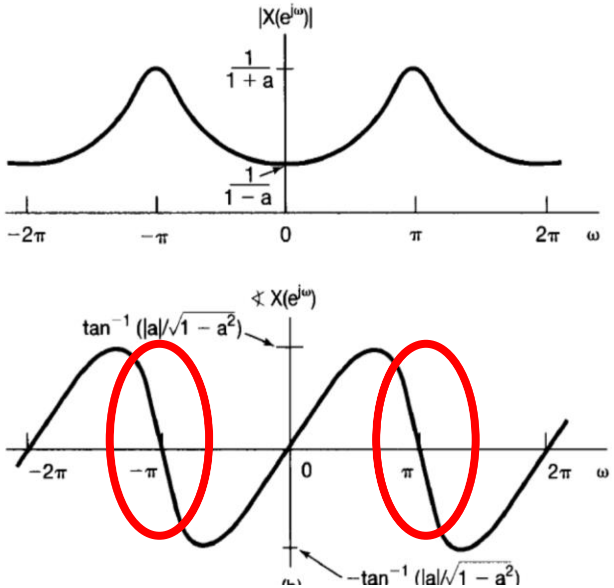
Low pass filter

a>0



High pass filter

a<0



$$|X(e^{jw})|=\frac{1}{1-ae^{-jw}}\cdot\frac{1}{1-ae^{jw}}$$

$$1+a^2-ae^{-jw}-ae^{jw}$$

$$1+a^2-2a\cos w$$

If w = 0…

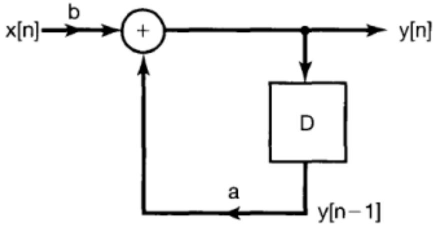
$$|X(e^{jw})|=\sqrt{\frac{1}{(a-1)^2}}=\frac{1}{1-a}$$

Pass band → 신호가 훼손 없이 그대로 통과 → Phase spectrum Linear

First-order recursive discrete-time filters (IIR)

- **Recursive system**

$$y[n]-ay[n-1]=x[n]$$



- **Frequency response**

$$H(e^{j\omega})=\frac{1}{1-ae^{-j\omega}}$$

- **Impulse response**

$$h(n)=a^n u[n]$$

- **Step response**

$$s[n]=\frac{1-a^{n+1}}{1-a}u[n]$$

$$s[n]=u[n]*h[n]$$

$$s[n]=\sum_{m=-\infty}^{n} h[m]$$

$$Y(e^{jw})=\sum_{n=-\infty}^{\infty} y[n]\cdot e^{-jwn}$$

$$Y(e^{jw})\cdot e^{-jw}=\sum_{n=-\infty}^{\infty} y[n-1]\cdot e^{-jw(n-1)}\cdot e^{-jw}$$

$$y[n-1]=y[n]*\delta[n-1]$$

$$\delta(e^{jw})=\sum_{n=-\infty}^{\infty} \delta[n-1]\cdot e^{-jwn}$$

Frequency response에서 Impulse response 구하기

$$y[n]-ay[n-1]=x[n]$$
$$Y(e^{jw})-a\cdot e^{-jw}Y(e^{jw})=X(e^{jw})$$

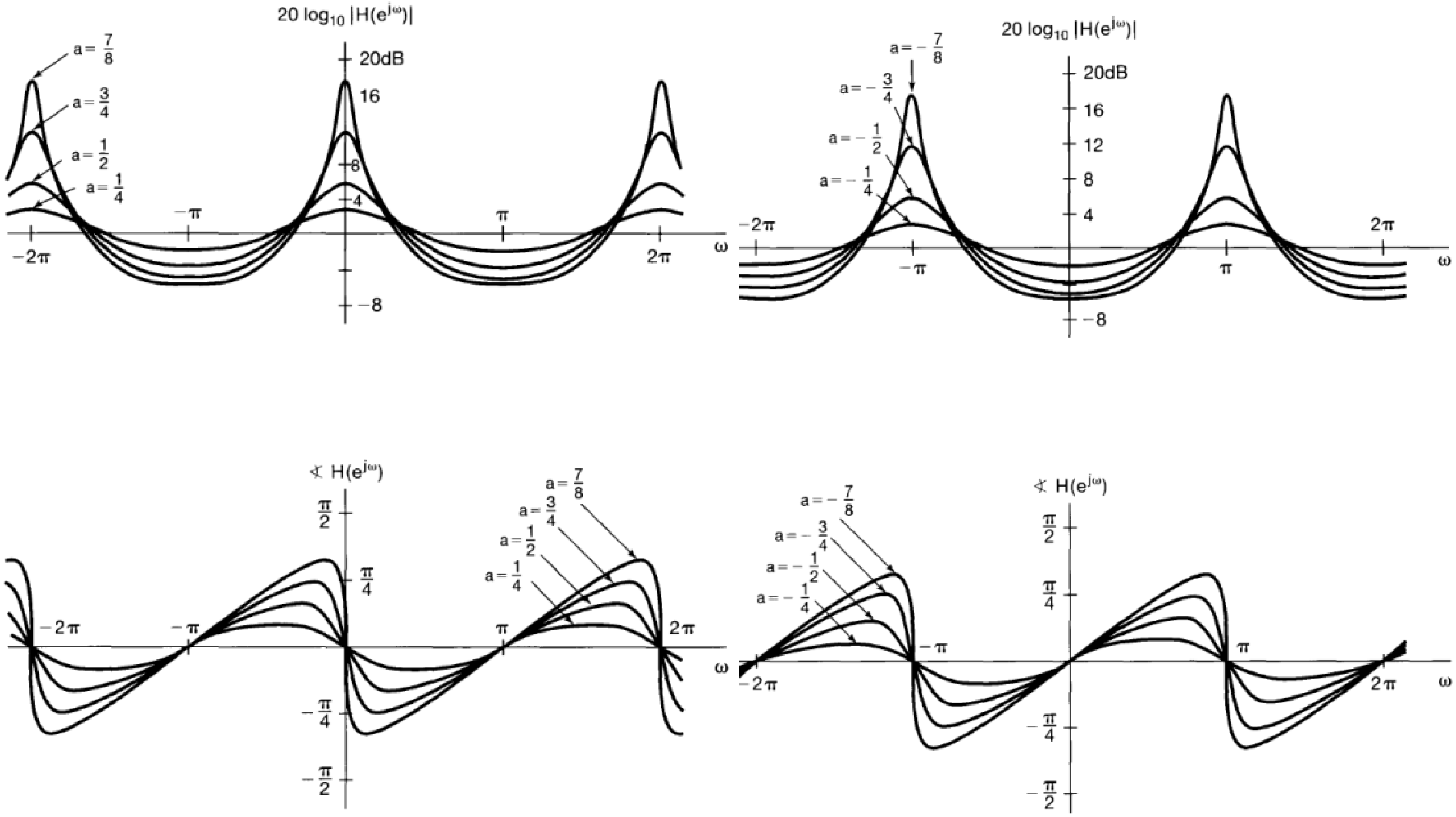
$$H(e^{jw})=\frac{1}{1-ae^{-jw}}$$

Impulse response에서 Frequency response 구하기

$$h[n]-ah[n-1]=\delta[n]$$
$$h[0]=1,\ h[1]=a,\ h[2]=a^2 \dots\ h[n]=a^{\mathrm{n}}$$

$$H(e^{jw})=\sum_{n=0}^{\infty}(ae^{-jw})^n$$

$$H(e^{jw})=\frac{1}{1-ae^{-jw}}$$



a값에 따라 band의 길이가 달라짐
→ 커지면 더 좁아져서 selective해짐

[Low pass]Non-recursive Discrete-Time Filters (FIR)

- *Weighted average*

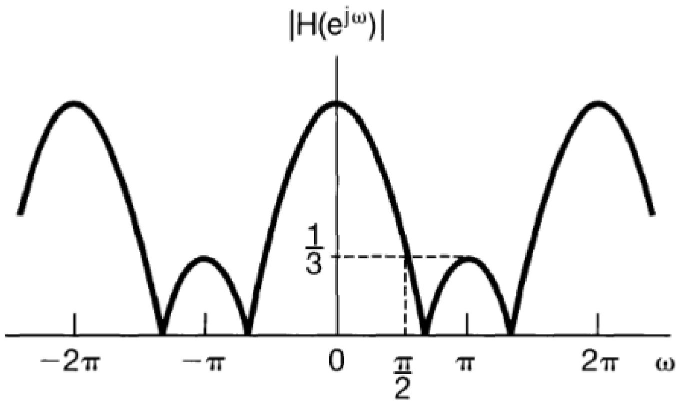
$$y[n]=\sum_{k=-N}^{M} b_k x[n-k]$$

- *Ex) Moving-average filter (3 point moving average)*

$$y[n]=\frac{1}{3}(x[n-1]+x[n]+x[n+1])$$

  - *Frequency response*

$$H(e^{j\omega})=\frac{1}{3}[e^{j\omega}+1+e^{-j\omega}]=\frac{1}{3}(1+2\cos\omega)$$



- *General equation for the moving average filter*

$$y[n]=\frac{1}{N+M+1}\sum_{k=-N}^{M} x[n-k]$$

$$H(e^{j\omega})=\frac{1}{N+M+1}\sum_{k=-N}^{M} e^{-j\omega k}$$

$$H(e^{j\omega})=\frac{1}{N+M+1}e^{j\omega[(N-M)/2]}\frac{\sin[\omega(M+N+1)/2]}{\sin(\omega/2)}$$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$$

$$H(e^{jw}) = \frac{1}{N+M+1} \sum_{k=-N}^{M} e^{-jwk}$$

$$H(e^{jw}) = \frac{1}{N+M+1} \cdot \frac{e^{jwN}\left(1 - e^{-jw[N+M+1]}\right)}{1 - e^{-jw}}$$

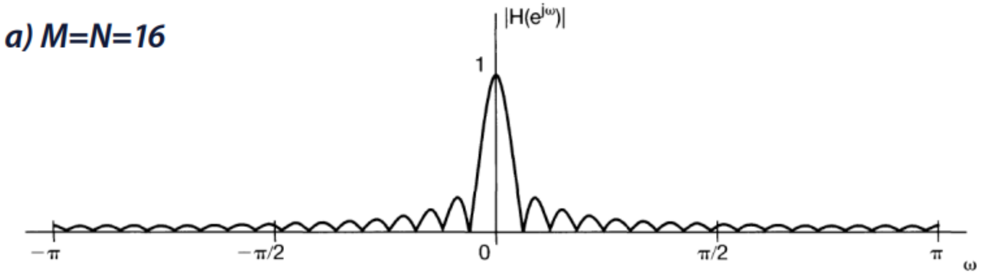
$$H(e^{jw}) = \frac{1}{N+M+1} \cdot \frac{e^{-jw[N+M+1]/2}(\sim)}{e^{-jw/2}(\sim)}$$

$$H(e^{j\omega}) = \frac{1}{N+M+1} \boxed{e^{j\omega[(N-M)/2]}} \frac{\sin[\omega(M+N+1)/2]}{\sin(\omega/2)}$$

나머지는 다 constant... 각 성분은 결국

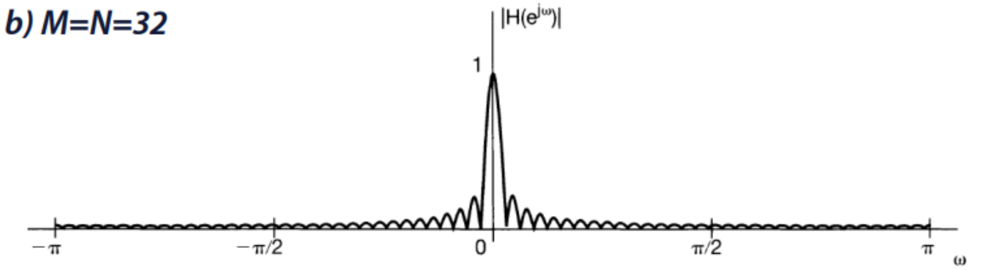
$$\measuredangle H(e^{jw}) = {(N-M)}/{2}$$

- ***Magnitude of the frequency response***
  - ***a) M=N=16***



(a)

  - ***b) M=N=32***



(b)

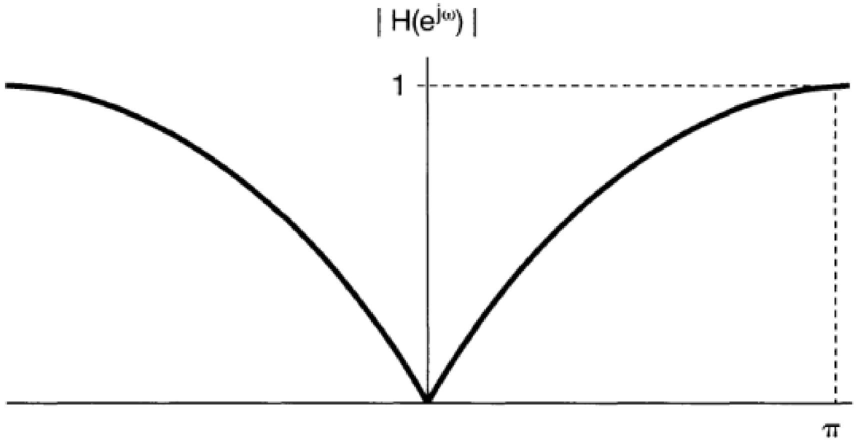
N-M 값을 키우면 → passband narrow...

- ***Ex) High pass filter of FIR***

$$y[n] = \frac{x[n] - x[n-1]}{2}$$

$$h[n] = \frac{1}{2}\{\delta[n] - \delta[n-1]\}$$

$$H(e^{j\omega}) = \frac{1}{2}\left[1 - e^{-j\omega}\right] = je^{-j\omega/2}\sin(\omega/2)$$



Continuous High pass → 미분
Discrete High pass → 차분

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$$

$$\delta(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n-1] \cdot e^{-jwn}$$

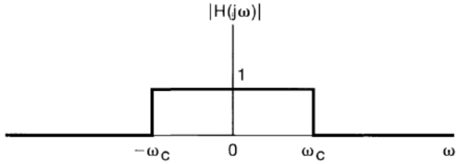
$$\delta(e^{jw}) = e^{-jw}$$

$$|H(e^{jw})| = \sqrt{(j \cdot e^{-\frac{jw}{2}} \cdot \sin(^{w}/_{2})) \cdot (-j \cdot e^{\frac{jw}{2}} \cdot \sin(^{w}/_{2}))} = |\sin(^{w}/_{2})|$$

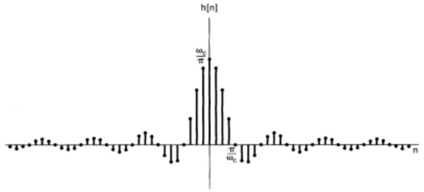
$$\measuredangle H(e^{jw}) = \frac{\pi}{2} - \frac{w}{2} + 0$$

4P → Phase representation 참고
Real-Imagine 그래프에서 각 그리기

- ***Prepare ideal Low-pass filter*** $h[n] = \frac{\sin \omega_c n}{\pi n}$



- ***Take mask and truncate impulse response***

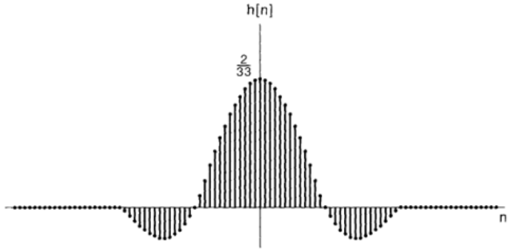


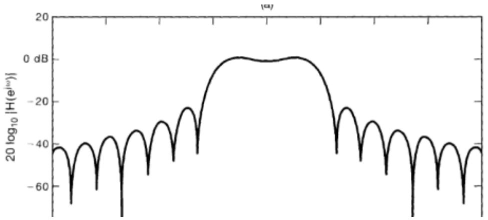
$$y[n] = \sum_{k=-N}^{M} b_k x[n-k]$$

$$b_k = \begin{cases} \frac{\sin(2\pi k/33)}{\pi k}, & |k| \le 32 \\ 0, & |k| > 32 \end{cases}$$

- ***FIR Filter***

$$h[n] = \begin{cases} \frac{\sin(2\pi n/33)}{\pi n}, & |n| \le 32 \\ 0, & |n| > 32 \end{cases}$$





이상적인 경우는 그렇지만... 한번 FIR 필터를 만들어 보자...

$h[n]$ = Real, Even인 경우 → $h[n] = h[N-1-n]$

$$\sum_{n=0}^{N-1} h[n] \cdot e^{-jwn}$$

$$\sum_{n=0}^{(N-1)/2} h[n] \cdot (e^{-jwn} + e^{-jw[N-1-n]})$$

$$\sum_{n=0}^{(N-1)/2} h[n] \cdot e^{-\frac{jw[N-1]}{2}}(\sim)$$

$\measuredangle H(e^{jw}) = -w[N-1]/2$ → Linear!!!

2

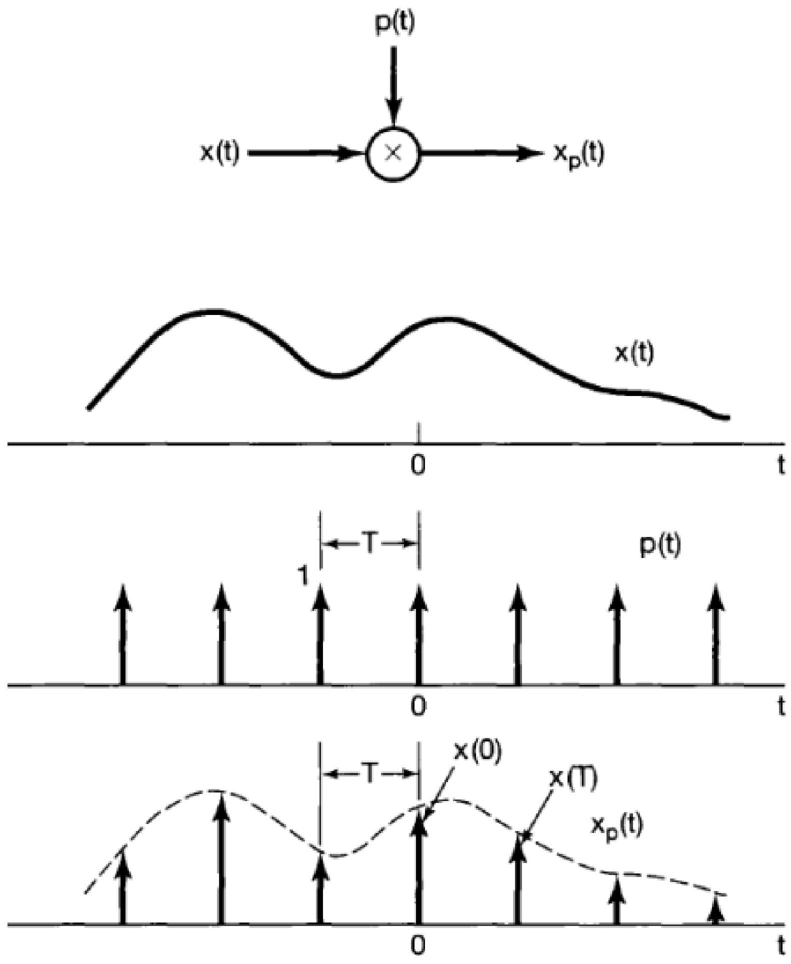
Sampling

- ***Impulse-Train Sampling***
  - *Impulse-train = Array of impulses*
  - *Sampling period =* T
  - *Sampling function =* $p(t)$
  - *Sampling frequency* $\omega_s = \dfrac{2\pi}{T}$
  - *Sampled signal*

$$x_p(t) = x(t)\,p(t)$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT)$$



공식 유도해보자...

매우 중요하다...

$$x_p(t) = x(t)p(t) \xleftrightarrow{FT} X_p(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)P(j(\omega-\theta))d\theta$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT) \xleftrightarrow{FT} P(j\omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{\infty} \delta(\omega - k\omega_s)$$

$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

- $X_p(j\omega)$ **is replicas of** $X(j\omega)$

$$x(t)p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x_p(jw)\mathrm{e}^{jwt}\,dw$$

$$x(t)p(t) = \frac{1}{2\pi}\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(j\theta)P\big(j(w-\theta)\big)\,d\theta\,\mathrm{e}^{jwt}\,dw$$

$w - \theta = k$

$dw = dk$

$$x(t)p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\boxed{[\frac{1}{2\pi}\int_{-\infty}^{\infty} P(jk)\mathrm{e}^{jkt}\,dk}]\mathrm{e}^{j\theta t}\,d\theta$$

$$x(t)p(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\boxed{[p(t)]}\mathrm{e}^{j\theta t}\,d\theta$$

$$x(t)p(t) = x(t)p(t)$$

$$x_p(jw) = \frac{1}{2\pi}X(jw) * P(jw)$$

이제 $p(t)$만 구하면 $x_p(t)$ 표현 가능!

$p(t) =$ 주기함수

$$\sum_{\mathrm{k}=-\infty}^{\infty} a_k\mathrm{e}^{jkw_0t} \Leftrightarrow \sum_{\mathrm{k}=-\infty}^{\infty} a_k * 2\pi * \delta(w - kw_0)$$

$$a_k = \frac{1}{T}\int_T p(t)\mathrm{e}^{-jkw_0t}\,\mathrm{dt} = \frac{1}{T} * \text{구간 델타 값 } 1 = \frac{1}{T}$$

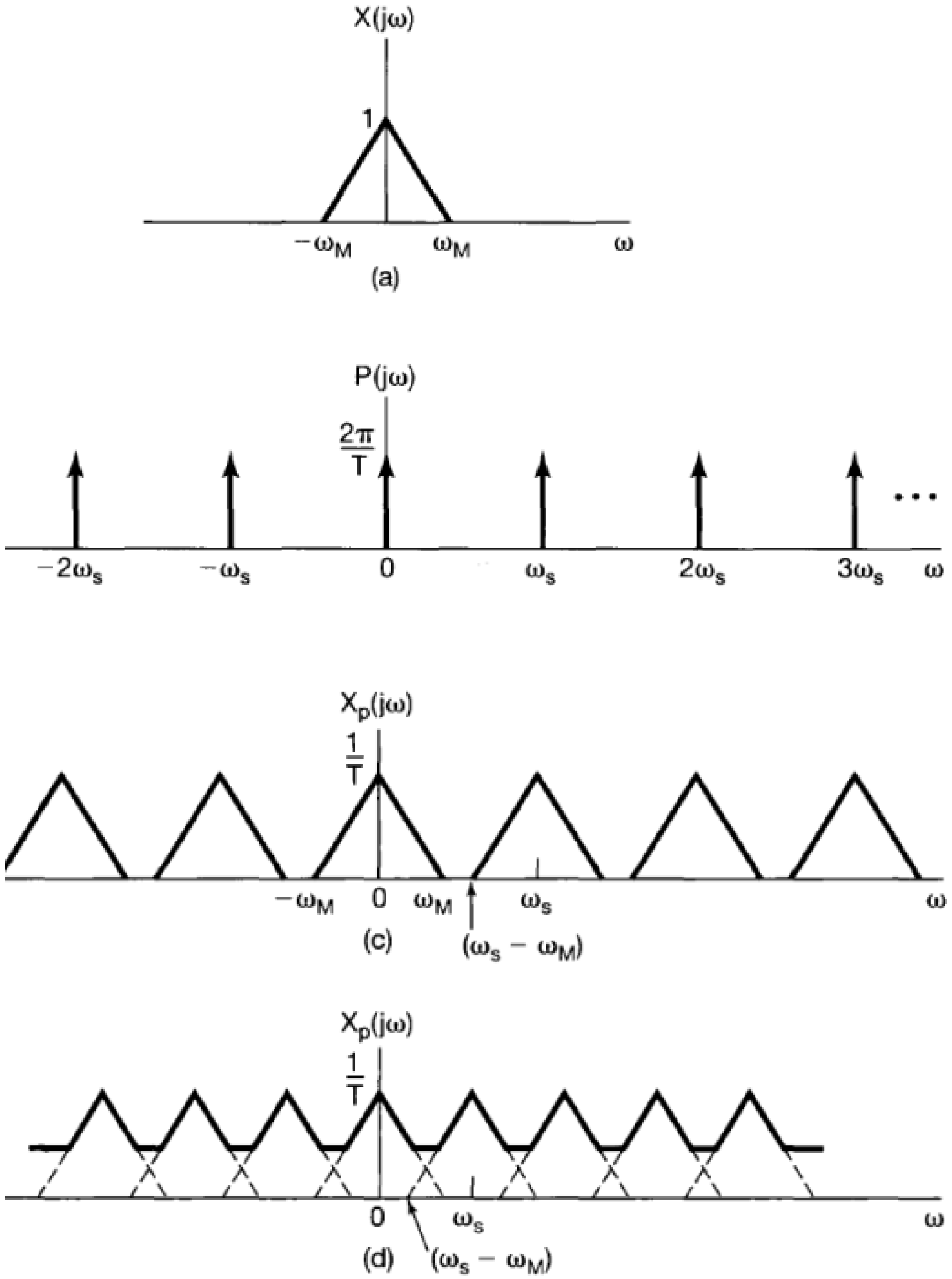
이제 마무리~

$$x_p(jw) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)P\big(j(w-\theta)\big)\,d\theta$$

$$x_p(jw) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\theta)\sum_{\mathrm{k}=-\infty}^{\infty} 2\pi/T * \delta(w-\theta-kw_0)\,d\theta$$

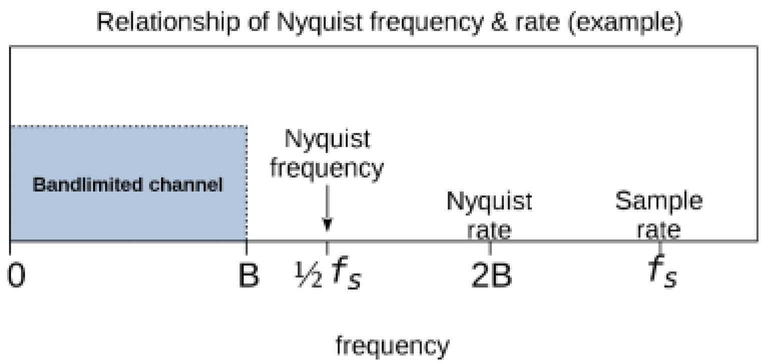
$$x_p(jw) = \frac{1}{\mathrm{T}}\sum_{-\infty}^{\infty}\int_{-\infty}^{\infty} X(j\theta)\cdot\delta(w-\theta-kw_s)\,\mathrm{d}\theta$$

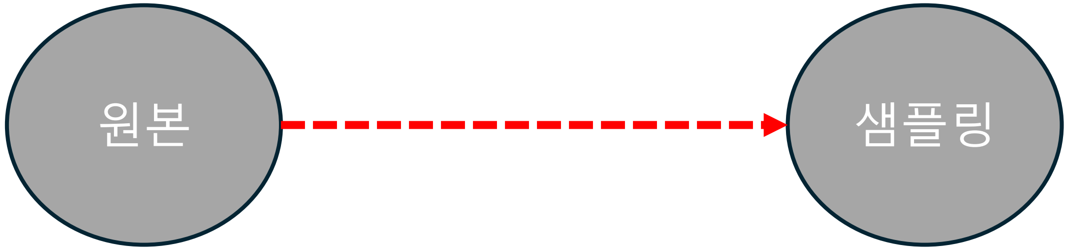
$$x_p(jw) = \frac{1}{\mathrm{T}}\sum_{-\infty}^{\infty} X(j(w-kw_s))$$



샘플링 주파수 w↓ = 샘플링 주기 T↑

즉 원본이 훼손되어 복구 불가...
→ Passband를 통한 복구 불가
→ $w_s = 2w_M$이 최적의 상황





원본이 B → 2B만큼의 Nyquist rate 감당 가능한가요?

샘플링이 f_s → $1/2f_s$ 만큼의 Nyquist frequency감당 가능한가요?

- ***Recovery process***