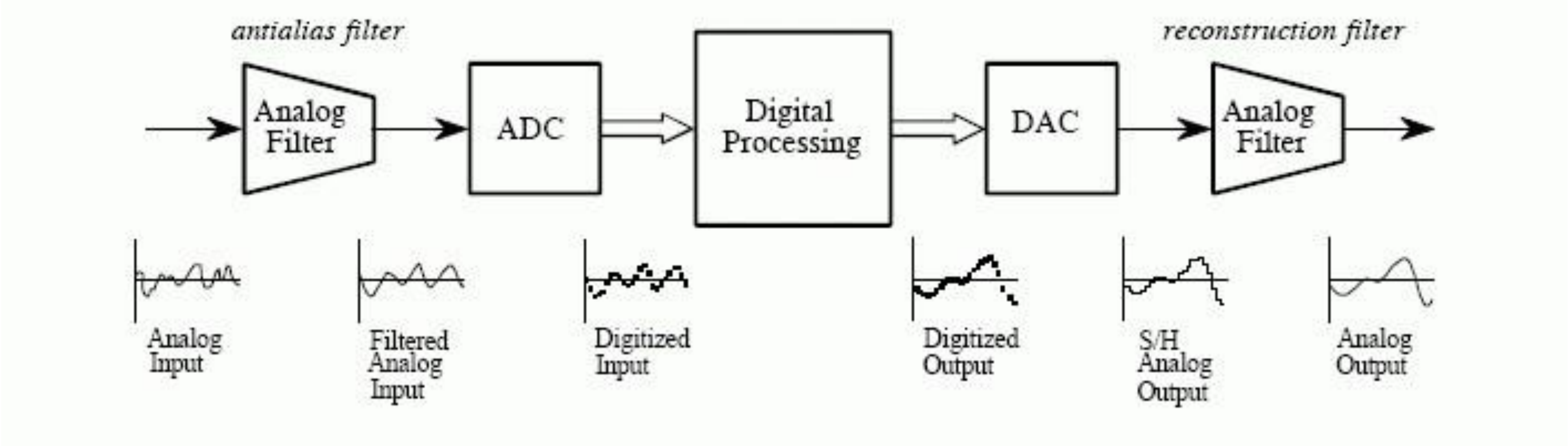
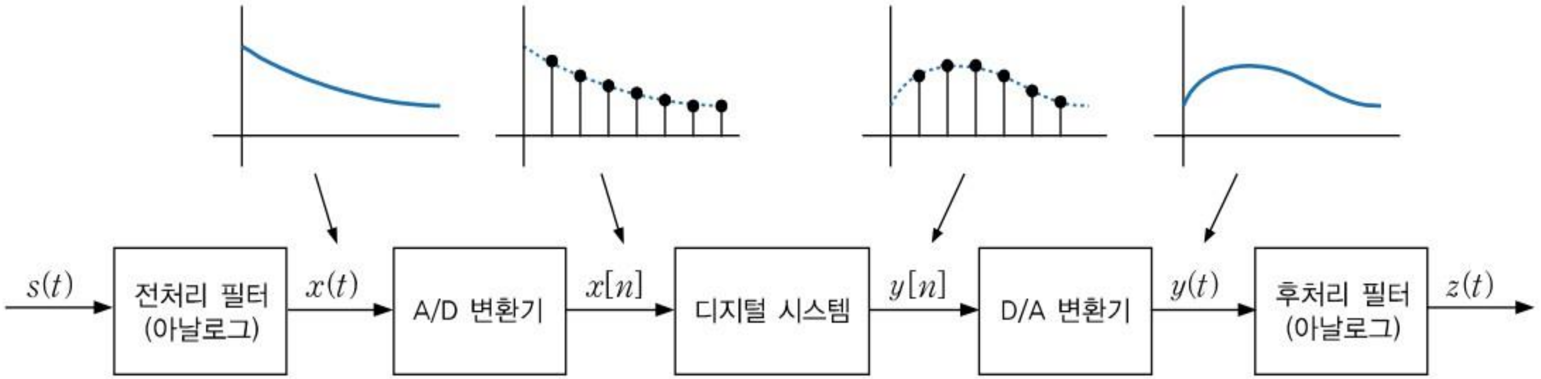
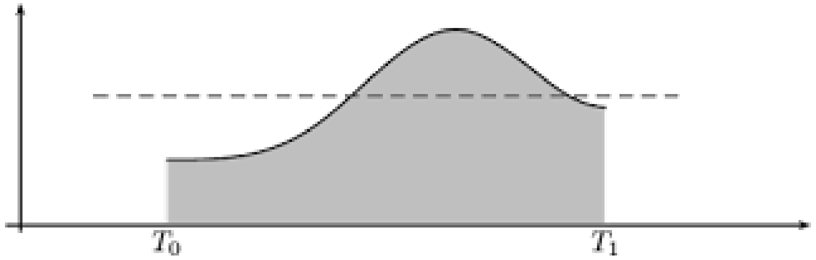
1

ADC(Analog to digital converter)
DAC(Digital to analog converter)

아날로그 신호와 디지털 신호 간 자유로운 변환

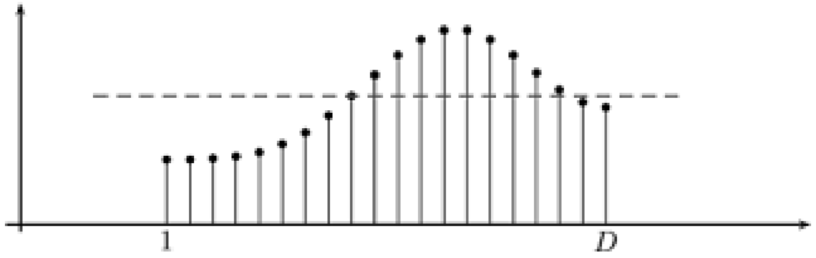






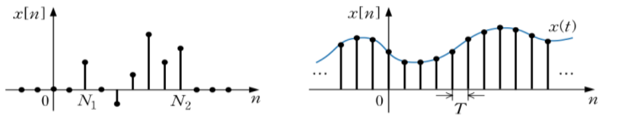
Continuous-time signal

- 연속된 시간
- 간격이 연속적인 값

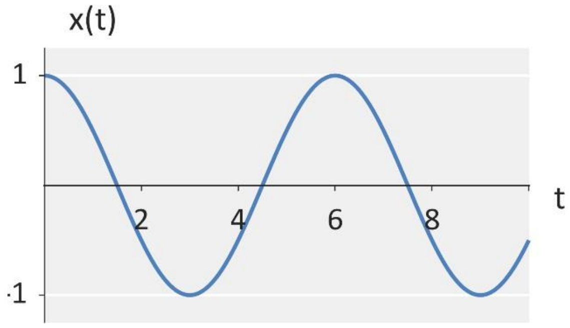


Discrete-time signal

- 규칙적인 시간 간격
- 간격이 일정한 값
- 정수로 이루어진 x값



(a) Finite duration signal (b) infinite duration signa



Continuous signal

- 실수(Real number)로 이루어진 x축
- 일반적으로 x축은 시간
- 무한 기간
  - $f(t) = \sin(t), \ t \in \Re$
- 유한 기간
  - $f(t) = \sin(t), \ t \in [-\pi, \pi]$
  - 다른 구간 $f(t) = 0$

Signal energy and power

- Energy : 해당 구간에서 주어진 신호의 에너지... 증폭
- Power : 해당 구간동안 주어진 신호의 시간당 평균 전력

유한한(finite interval) 구간에서... [t1, t2]

	Energy	Power
➢ *Continuous signal*	$\int_{t_1}^{t_2} \lvert x(t) \rvert^2 \, dt$	$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \lvert x(t) \rvert^2 \, dt$
➢ *Discrete signal*	$\sum_{n=n_1}^{n_2} \lvert x[n] \rvert^2$	$\frac{1}{n_2 - n_1 + 1} \sum_{n=n_1}^{n_2} \lvert x[n] \rvert^2$

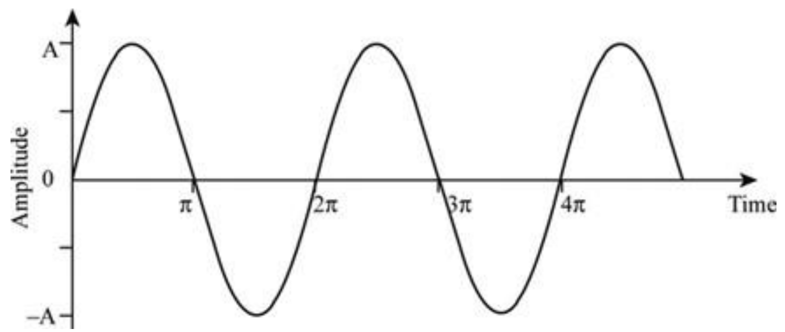
무한한(infinite interval) 구간에서... [-∞, ∞]

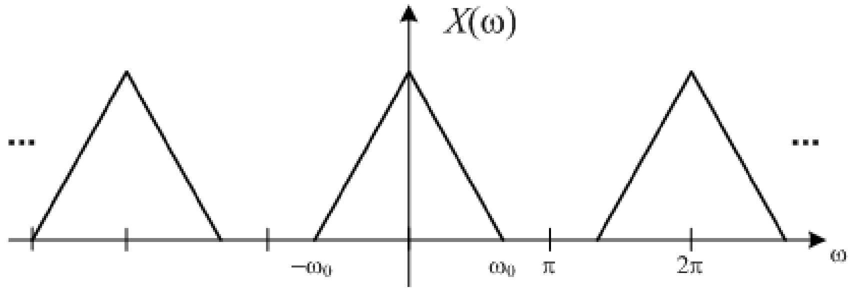
- Finite total energy $E_\infty < \infty \rightarrow P_\infty = \lim_{T \to \infty} \frac{E_\infty}{2T} = 0$
- Finite average power $0 < P_\infty < \infty \rightarrow E_\infty = \infty$

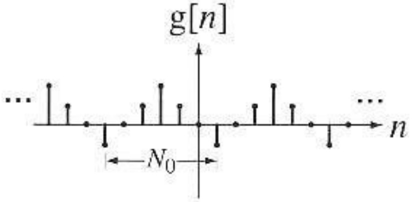
- $x(t) = x(t + T)$
- $x[n] = x[n + N]$

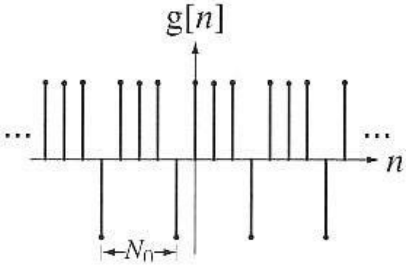
주기가 있는, 반복되는 시그널은 이렇게 표현 가능

T와 N은 특정 주기



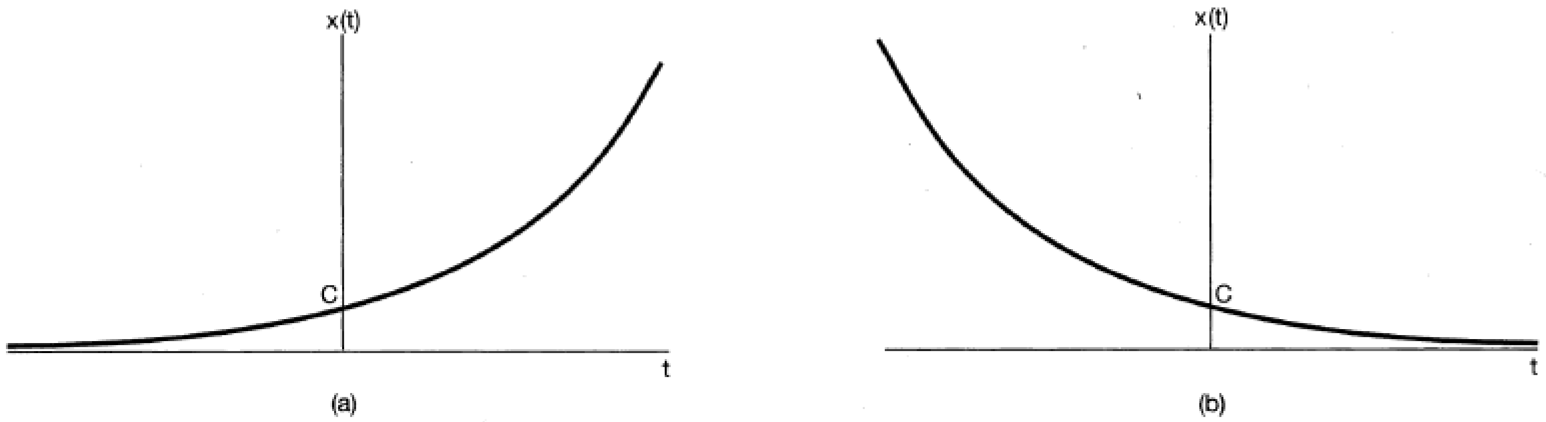






Real valued exponential signal…

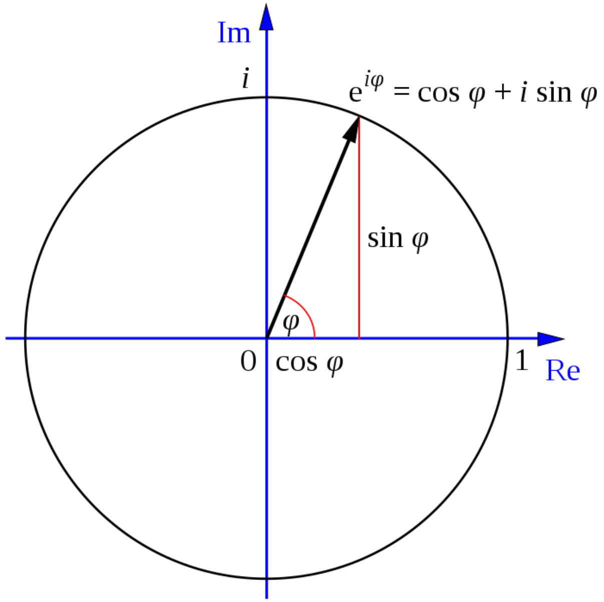
Ce^{at}, where (C, a) is real number



Then how about complex valued exponential signal? With imaginary numbers!

단순한 지수함수?

F(frequency) or W(오메가) : 주파수=1초 동안 진동수
T(time period) : 주기=1회 반복 걸리는 시간



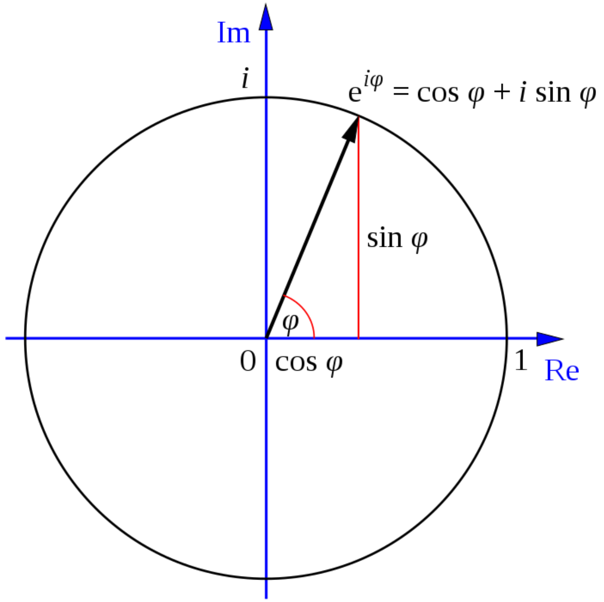
- F = 1/T
  - 대표적인 주기함수 : cos, sin
  - sin(ax), cos(ax)의 주기 = 2π/a

- 계산하기 편한, 간편한 주기함수 : e(exponential)… HOW???
  - 오일러 공식(Euler's formula)
  - $e^{ix} = \cos x + i \sin x$
  - 허수와 실수 면에서 표현 가능
  - 수의 범위는 복소수까지 확장
  - 복소수 z = x+iy… with Euler, z = re^{ix} (r = $\sqrt{x^2 + y^2}$, 반지름)
  - 회전하는 원에서, r = 반지름, x = 회전각
  - 주기함수로써 삼각함수가 아닌 e함수를 다룸으로, 단순한 계산과 같은 여러 이점(푸리에 변환 등)

- 삼각함수(w)의 진동수 = $2\pi f$(rad*(1/sec))에서, f = w/2π
  - 쉽게 말해, 2π/T = w… (원의 둘레/한바퀴 걸리는 시간) = 각속도
  - 각도(radians) = 각속도(각진동수)*회전시간 = (rad/sec)*(sec) = w*t = Ω(rad)
  - $x(t) = e^{iwt} = \cos(2\pi ft) + i\sin(2\pi ft)$

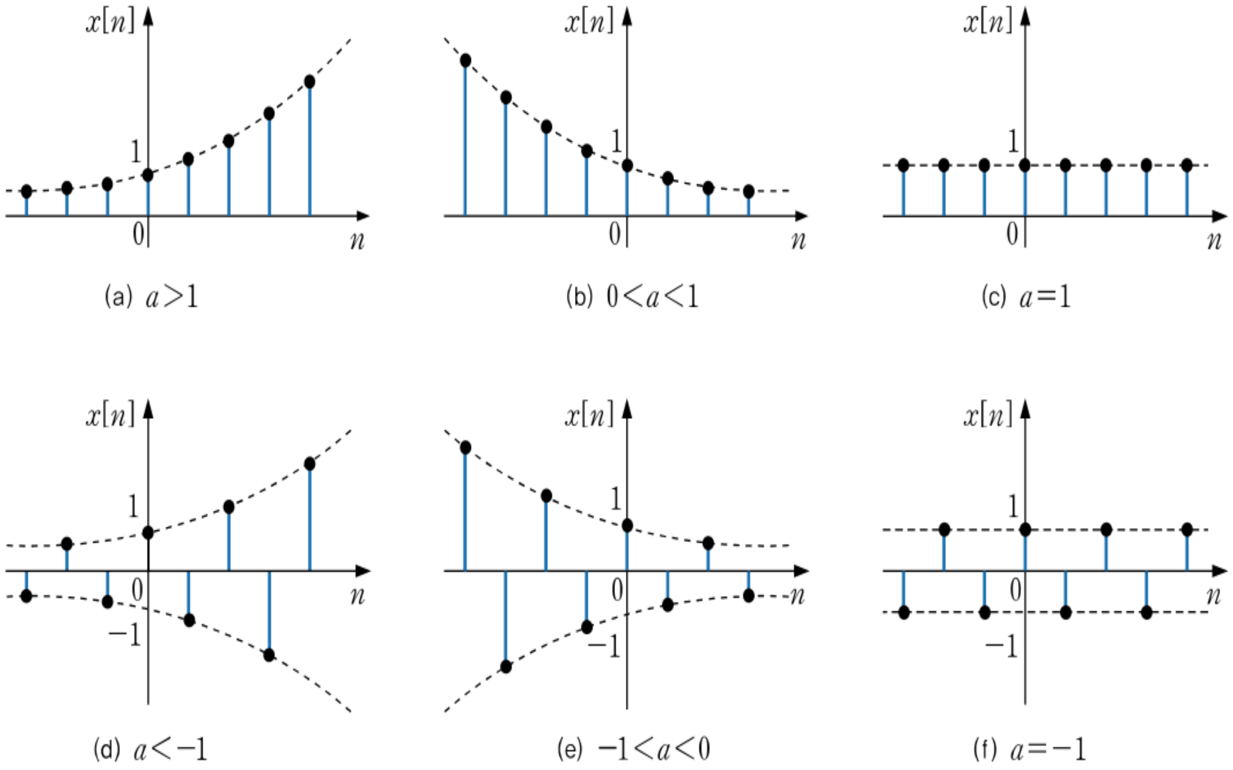
x(t) = $e^{jw_0 t}$

- 주기함수(periodical signal)
- 따라서 $e^{jw_0 t} = e^{jw_0(t+T)}$, period = T
- Angler frequency(각진동수 = 각속도)
- ω_0, $\omega_0 = 2\pi f_0$, f_0 = frequency
- 각진동수는 단위시간동안 나아가는 각도(rad/sec)
- 주기함수이기 위해서는, $e^{jw_0 t} = e^{jw_0(t+T)} = e^{jw_0 t}e^{jw_0 T}$
    - $e^{jw_0 T}$ = must 1... why???
    - T = $2\pi/\omega_0$, $e^{j2\pi}$, 오른쪽 그림에서 각도 2π인 경우 1
    - cos값 1, sin값 0



이제까지 실수와 허수의 continuous signal 탐색
오일러 공식을 이용하여 허수까지 확장 가능...

이제 Discrete-time exponential signal!



x[n] = Ca^n에서...

C, a가 real number인 경우... 변수가 지수함수의 밑!

|a| > 1, increase exponentially
|a| < 1, decrease exponentially

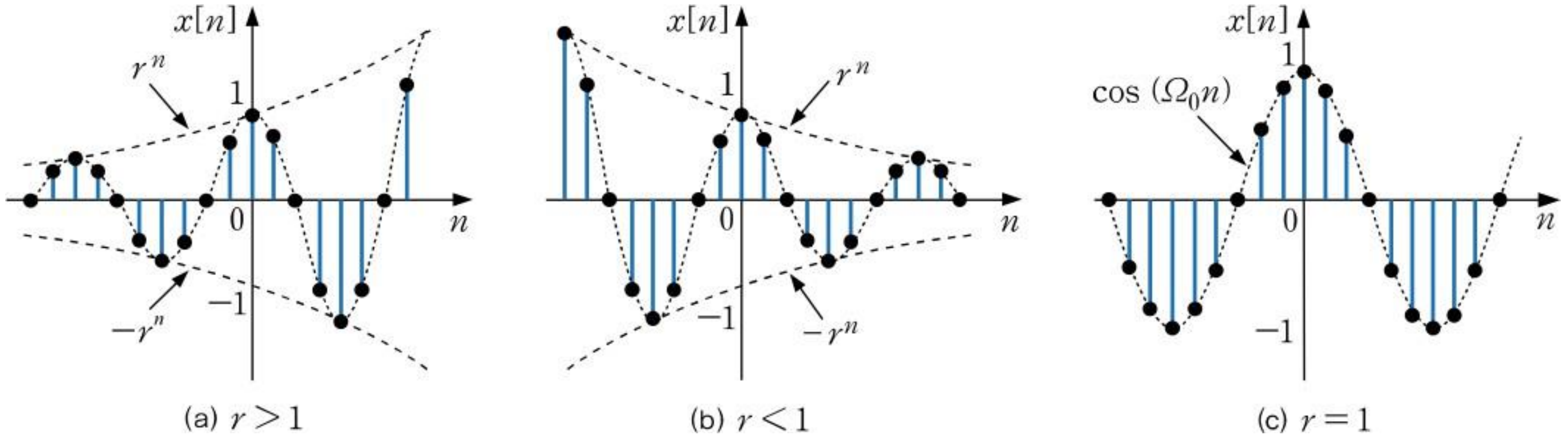
If a < 0 the sign of the value changes alternately...
(위, 아래)

Real number가 아닌, complex-valued exponential sequence(복소수 지수 신호)는?

a값에 오일러 변환한 $r\mathrm{e}^{jw_0t} = re^{j\Omega}$ 삽입

$x[n] = \left(re^{j\Omega}\right)^n = r^n e^{j\Omega n}$

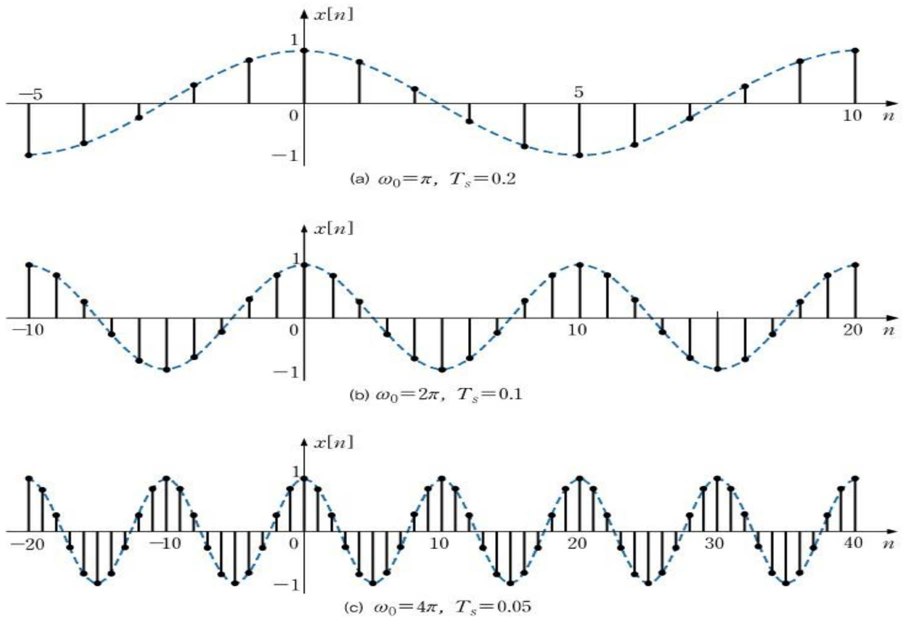
- r값에 따라 기하급수적으로 상승 혹은 하강
- r^n : envelope(봉투)
- $e^{j\Omega n}$: 코사인과 사인의 합으로 이루어진 주기함수



Sampling(샘플링) – 본질적으로 “거리=속력x시간” 이용

- 연속적인 신호의 특정 순간의 값을 캐치하여 일련의 수로 나타낸 것
- 일반적으로 일정한 시간 간격으로 수행
- T = $2\pi/\omega_0$ = $1/f_0$
- T_s = $2\pi/\omega_s$ = $1/f_s$ = Sampling period(sec)
  - 샘플의 시간간격... Sampling frequency(rate)과는 다른 의미(samples/sec)

- Sampling of sinusoidal signal(continuous to discrete)
  - x(t) = $A\cos(w_0 t)$, y(t) = $A\,sin(w_0 t)$ 연속함수
  - w*t = (rad/sec)*(sec) = rad... 각도(θ)... $A\cos(\theta)$
  - x[n] = x(nT_s) = $A\cos(w_0 \mathrm{n} T_s)$ = $A\cos(\Omega_0 n)$ 이산함수
  - t (time) = nT_s (샘플 수)*(sec) “t는 시간이자 주기 T, 보는 관점에서 주기는 시간”
  - n을 x축으로 하는 새로운 이산함수 탄생(단순한 방식으로 time scale 제거 완료)
    - Continuous angular frequency = w_0 (rad/sec)
    - Discrete angular frequency = Ω_0(=$w_0 T_s$) (rad) – 물리적인 타임 스케일 영향X
      - Like $2\pi = \omega_0 \mathrm{T}$ = 각속도*전체주기
      - $\Omega_0 = w_0 T_s$ = 각속도*샘플링주기 = 하나의 샘플링 시간 간격 당 이동한 거리 = $2\pi\frac{T_s}{T}$



아날로그 신호 샘플링

- $w_0 = 2\pi f_0$
- $w_0 = \pi$
- f_0 = ½
- T = 2
- 샘플링 주기 T_s = 0.2
- T/T_s = 10(한 주기당 샘플링한 횟수)
- 샘플링 주기와 아날로그 신호 주기를 구별할 것

- 같은 신호, 다른 샘플링 주기
- 하나의 샘플로부터 많은 연속함수 제작 가능
  - 시간 축을 우리가 정하기 나름이기 때문

공식 정리

- $w_0 = 2\pi f_0$ (연속신호 각속도(rad/sec) = 2π*연속신호 주파수)
- $\Omega_0 = w_0 T_s$ (이산신호 각속도(rad) = w_0*샘플링 주기)
- $\Omega_0 = 2\pi f_0 T_s = 2\pi \frac{T_s}{T}$
- f_0 = 1/T (연속신호 주파수 = 연속신호 주기의 역수)

핵심 개요

- x(θ) = e$^{j\theta}$ = $\cos(\theta)$ + … (각도 θ에 대한 식)
- x(t) = e$^{j\omega_0 t}$ = $\cos(\omega_0 t)$ + … (각도를 각속도*시간으로 표현)
- x[n] = x(nT_s) = e$^{j\omega_0 nT_s}$ = $\cos(\omega_0 nT_s)$ + … (시간 t를 이산 샘플링 간격*샘플링 개수로 표현)
- x[n] = x(nT_s) = e$^{j\Omega_0 n}$ = $\cos(\Omega_0 n)$ ($\Omega_0 = w_0 T_s = 2\pi\frac{T_s}{T}$(각진동수 = 각속도*샘플링 주기 시간) 이용)

- 각각의 변환식에서의 핵심
  - 거리 = 속력*시간
  - 각도 = 각속도*시간
  - 연속시간 = 샘플링 주기(간격)*샘플링 개수
  - 이산 각진동수 = 연속 각진동수*샘플링 주기

Discrete-time sinusoidal signal의 주기성 (x[n] = $e^{j\Omega n}$ = $\cos(\Omega_0 n)$)

$e^{j\Omega_0(n)}$ = $e^{j\Omega_0(n+N)}$ = $e^{j\Omega_0 n}e^{j\Omega_0 N}$...
즉, $e^{j\Omega_0 N}$ = 1(이산신호이기에 N=정수)

$e^{j(\omega_0+2\pi)n} = e^{j2\pi n}e^{j\omega_0 n} = e^{j\omega_0 n}$

- $e^{j\Omega_0 N}$ = 1
- Ω_0N(각도) = 2πK(2π마다 값 1)
- $\Omega_0/2\pi$ = K/N(int/int) = F_0(rational number_유리수_NOT파이)

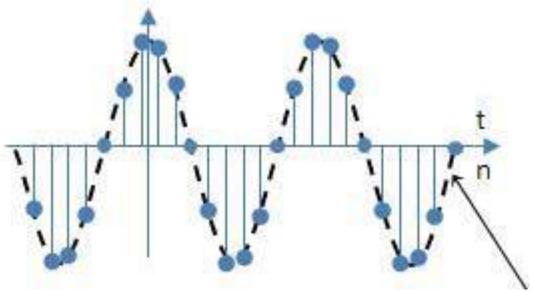
$e^{j(\omega_0+2\pi)n} = e^{j2\pi n}e^{j\omega_0 n} = 1*e^{j\omega_0 n}$

In continuous-time signal(연속신호)에서 모든 오메가(w)의 구별되는 값에 따라 함수값 역시 구별된다.

In discrete-time signal(이산신호)에서는 w+2π, w+4π, w+6π ... 등 신호들이 동일하다.

2PI를 항상 주기로 가진다.

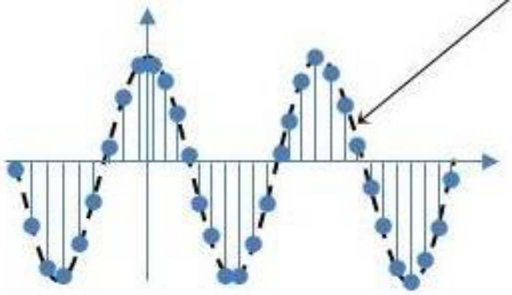
$\cos(0.2\pi n)$, $\Omega_0 = 0.2$파이



Rational number, periodicity!!!

$F_0 = k/N = 1/10 = 0.1$

$\cos(0.5n)$, $\Omega_0 = 0.5$



$F = \Omega_0/2\pi = 1/4\pi$

Not periodicity!!!

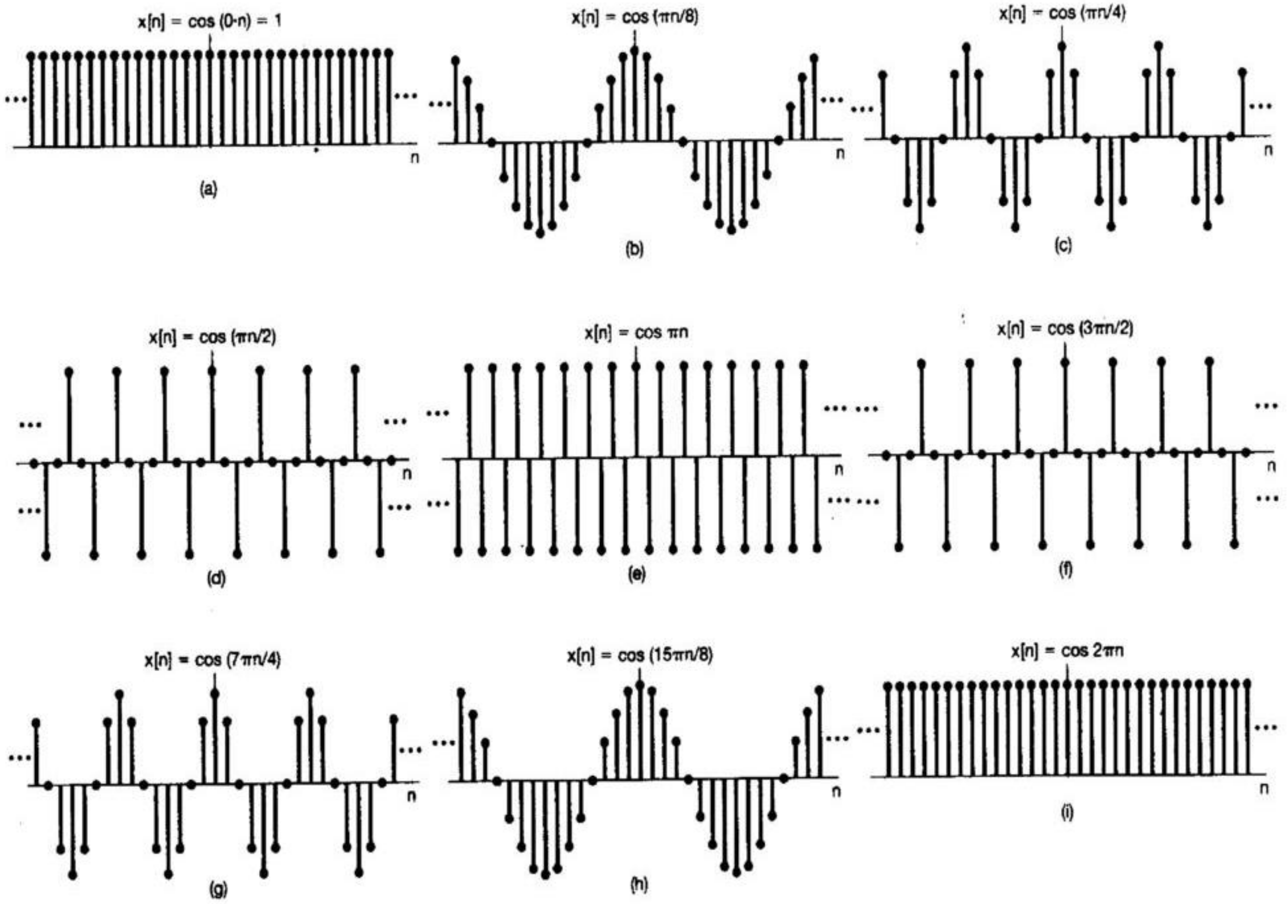
Frequency of discrete sinusoidal signals

ex) case (b)

- x[n] = x(nT_s) = $A\cos(w_0 nT_s)$
- Ω_0 = $w_0 T_s$ = $\pi/8$
- $2\pi fT_s$ = $\pi/8$
- fT_s = 1/16
- T_s/T = 1/16 (최대 약분 정수배에서 16)
- 샘플링 간격*16 = 아날로그 주기

ex) case (h)

- x[n] = x(nT_s) = $A\cos(w_0 nT_s)$
- Ω_0 = $w_0 T_s$ = $15\pi/8$
- $2\pi fT_s$ = $15\pi/8$
- fT_s = 15/16
- T_s /T = 15/16 (최대 약분 정수배에서 16)
- 샘플링 간격*16 = 아날로그 주기



즉 $\Omega_0/2\pi$ = K/N(int/int) 공식 적용 가능

$15\pi/16\pi$ = K/N

$N = \frac{16}{15}K$ (최소 정수배에 따라 K=15, N=16)

Frequency of discrete sinusoidal signal

- 이산 정형파는 다음과 같이 표현 가능(각 진동수 변화가 2PI를 넘어갈 수 없기에 진동수 변화에 한계가 있다.)
  - $\mathbf{0} \leq \boldsymbol{\Omega_0} \leq \boldsymbol{2\pi} = -\boldsymbol{\pi} \leq \boldsymbol{\Omega_0} \leq \boldsymbol{\pi}$ $(-\mathbf{0.5} \leq \mathrm{K/N} \leq \mathbf{0.5})$, $(-\mathbf{0.5} \leq \boldsymbol{f} = 1/\mathrm{N} \leq \mathbf{0.5})$

- 연속 정형파는 다음과 같이 표현 가능(각 진동수 변화에 한계가 없다.)
  - $-\infty < \boldsymbol{w}_0 < \infty$

- 이산 정형파의 기본 주기(N, N과 K는 서로소, number of period integer N)
  - $N = \mathrm{K}(\frac{2\pi}{\Omega_0})$
- 이산 정형파의 기본 주파수
  - $\frac{2\pi}{N} = \frac{\Omega_0}{K}$
- 디지털 주파수가 유리수일 때만 주기 신호
- 주파수가 2π의 정수배만큼 (차이나는)다른 신호는 서로 같은 신호

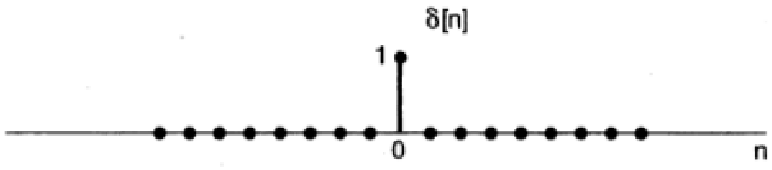
2

In discrete-time sequence…

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

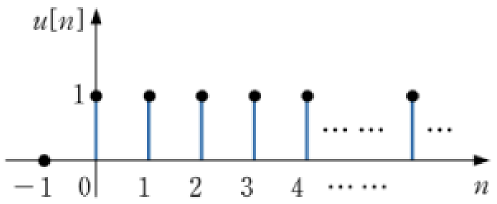
– ***Kronecker delta also can be used as a function***

➢ $\delta[n - n_0] = \begin{cases} 1, n = n_0 \\ 0, n \neq n_0 \end{cases}$



- ***Unit step sequence***
  – ***DC signal with constant (=1) value at*** $n \geq 0$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

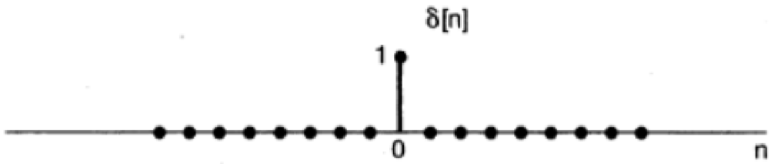


In continuous-time sequence…

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

– *Kronecker delta also can be used as a function*

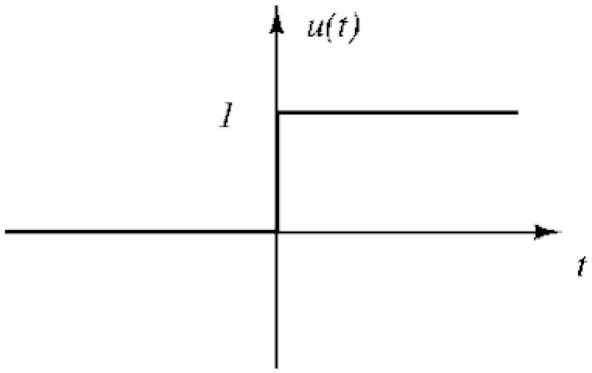
➢ $\delta[n-n_0] = \begin{cases} 1, n = n_0 \\ 0, n \neq n_0 \end{cases}$



- *Unit step signal*

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases}$$

– *Note that the unit step is discontinuous at t = 0.*



- $x[n]\delta[n] = x[0]\delta[n]$
  - $x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$
  - $x[n]=\sum_{k=-\infty}^{\infty} x\,[k]\delta[n-k]$
    - n=k인 곳만 값이 유지
    - x[k]에 대하여 전체 범위 적용

- $\delta[n] = u[n] - u[n-1]$
  - $u[n] = \sum_{m=-\infty}^{n} \delta[m] = \sum_{k=\infty}^{0} \delta[n-k]$

- $u(t) = \int_{-\infty}^{t} \delta(\tau)\, \mathrm{d}\tau$
  - dt로 미분하면 $\frac{\mathrm{d}u(t)}{\mathrm{d}t} = \delta(t)$
  - 즉 기울기 0에서의 한번 변화로 해석
  - 접선의 기울기 = $\frac{1}{\Delta}$
  - 넓이=1, u(t) 변화 0-1

- ***Relationship between $\boldsymbol{u(t)}$* and $\boldsymbol{\delta(t)}$**
  - ***u(t) is discontinuous at t = 0 and consequently is formally not differentiable***
  - ***Use approximation to obtain unit impulse from unit step.***

$$\delta(t) = \frac{du(t)}{dt} \longrightarrow \delta_\Delta(t) = \frac{du_\Delta(t)}{dt}$$

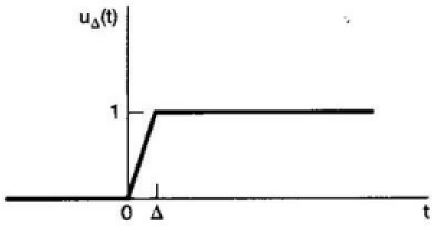


Figure 1.33 Continuous approximation to the unit step, $u_\Delta(t)$.

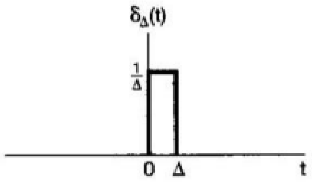


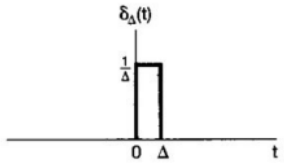
Figure 1.34 Derivative of $u_\Delta(t)$.

델타(Δ) 변화가 0에 가까워지면 기울기
= infinite

- $u(t) = \int_{\tau=-\infty}^{t} \delta(\tau)\, \mathrm{d}\tau$
  - $\int_{\sigma=\infty}^{0} \delta(t-\sigma) \cdot (-\,\mathrm{d}\sigma)$
    - $\sigma = t\text{-}\tau$
    - $\mathrm{d}\sigma = \text{-}1 \cdot \mathrm{d}\tau$
  - $\int_{\sigma=0}^{\infty} \delta(t-\sigma) \cdot \mathrm{d}\sigma$

Continuous	Discrete
d/dt	F[n]-F[n-1]
integral	sigma

- ***Unit impulse function***



– *$\delta_\Delta(t)$ is a short pulse, of duration Δ and with unit area for any value of Δ*

– *Now, we can obtain*

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

- ***Unit impulse function***

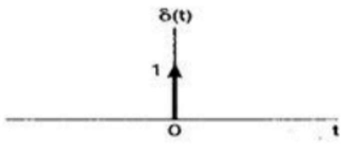


Figure 1.35 Continuous-time unit impulse.

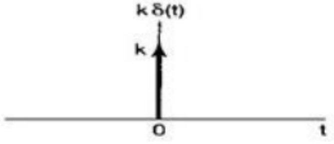
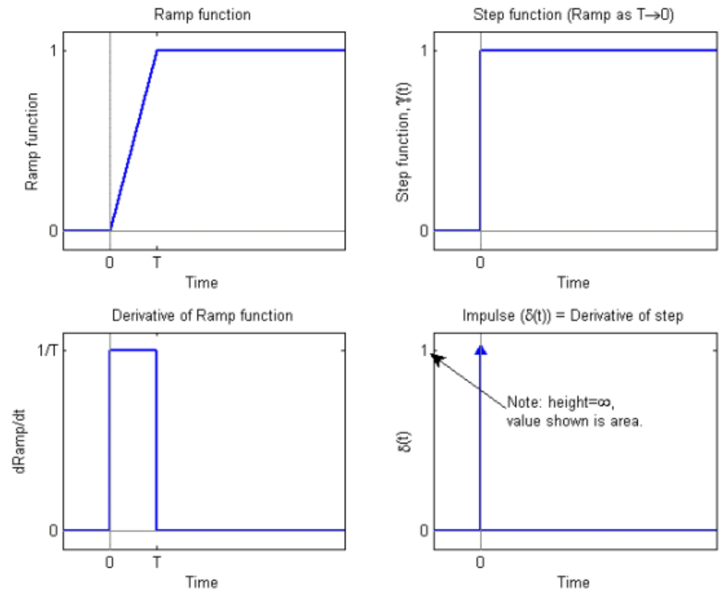
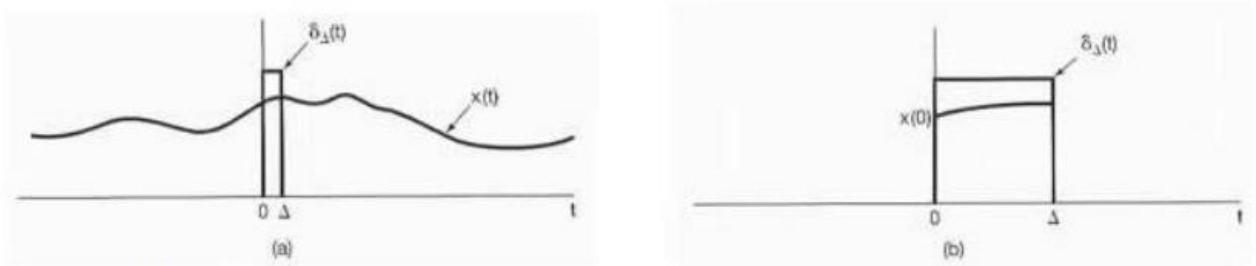


Figure 1.36 Scaled impulse.



넓이(적분) = 1

- Sampling property of unit impulse function



By sending $\Delta \to 0$ $\quad x(t)\delta_\Delta(t) \approx x(0)\delta_\Delta(t)$

$$\longrightarrow \quad x(t)\delta(t) = x(0)\delta(t)$$

In general

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

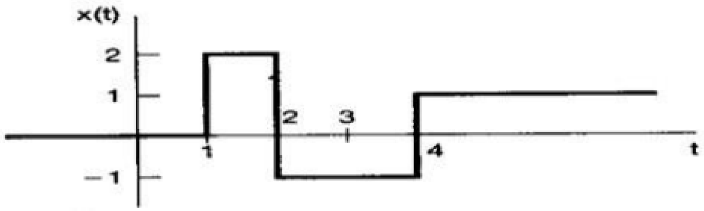
Discrete 경우와 유사한 식 도출

- $x[n]\delta[n] = x[0]\delta[n]$
  - $x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$

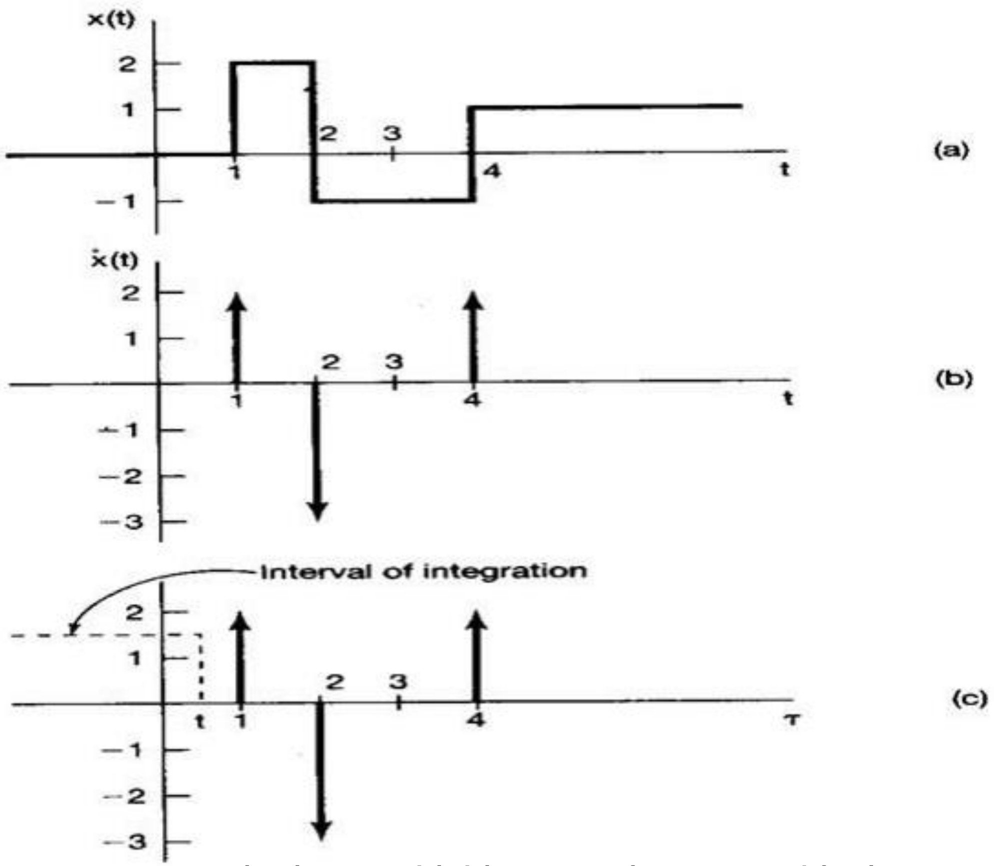
- $x(t)\delta(t) = x(0)\delta(t)$
  - $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

  - $x(t) = \int_{t_0=-\infty}^{\infty} x(t_0)\delta(t-t_0)dt$

Ex) Let`s calculate and graph the derivative of following signal



(a)



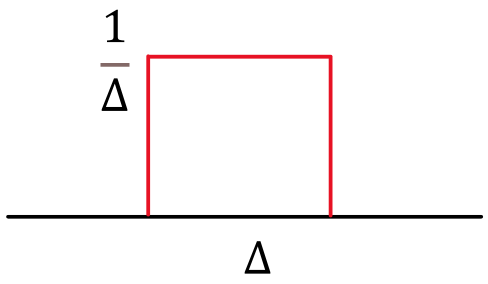
- 값이 무한한 크기 = 화살표
- 값의 집중된 면적 = 화살표 길이

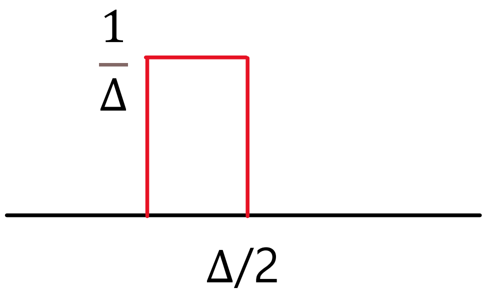
- ***Ex) Show that***

$$\delta(2t) = \frac{1}{2}\delta(t)$$

- $\delta(2t) = \frac{1}{2} * \delta(t)$

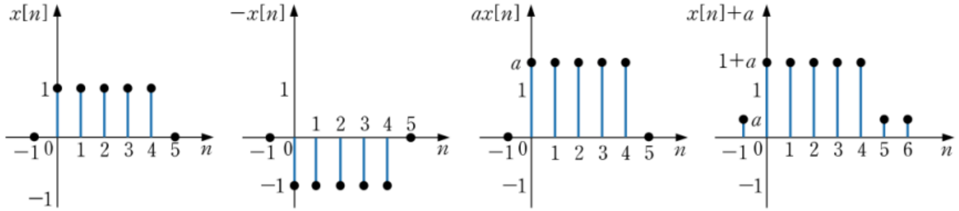
- 적분
- $\int_{t=-\infty}^{\infty} \delta(t)\,\mathrm{d}t = 1$
- $\int_{t=-\infty}^{\infty} \delta(2t)\,\mathrm{d}t = 1/2$





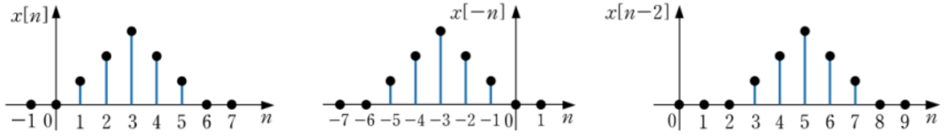
3

- ***Manipulating amplitude***



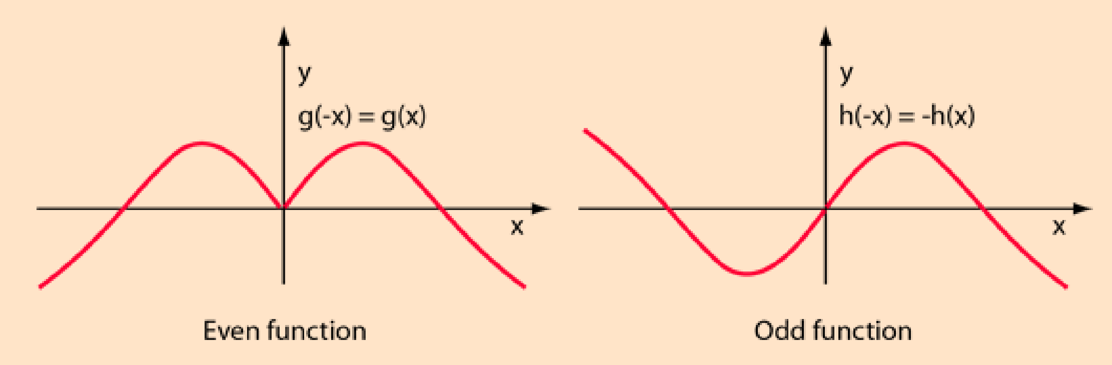
folding **amplifying** **Shifting**

- ***Manipulating time***

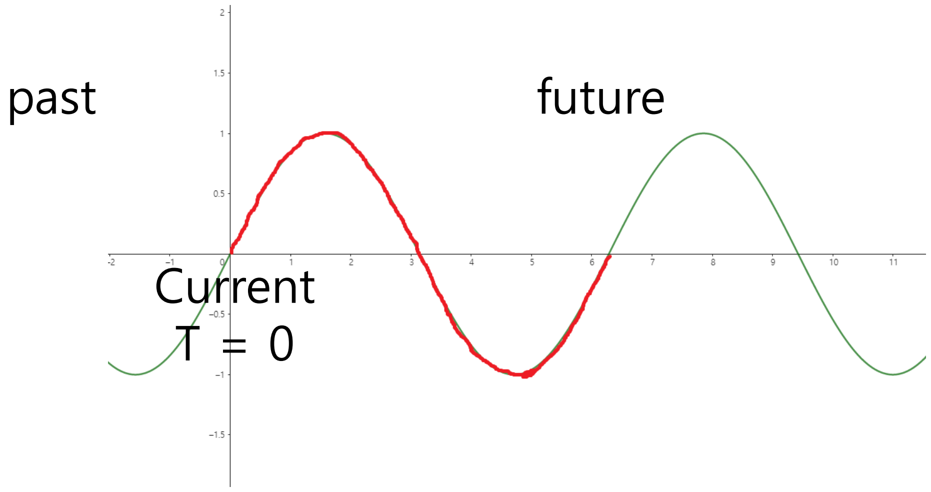


folding **Shifting**

- *Even and odd synthesis*
  - *A real-valued sequence* $x_e(n)$ *is called even (symmetric) if*
$$x_e(-n) = x_e(n)$$
  - *A real-valued sequence* $x_o(n)$ *is called odd (antisymmetric) if*
$$x_o(-n) = -x_o(n)$$
- *Then*
  - *Any arbitrary real-valued sequence* $x(n)$ *can be decomposed into its even and odd components*
$$x(n) = x_e(n) + x_o(n)$$
  - *Where the even and odd parts are given by*
$$x_e(n) = \frac{1}{2}[x(n) + x(-n)], \quad x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$



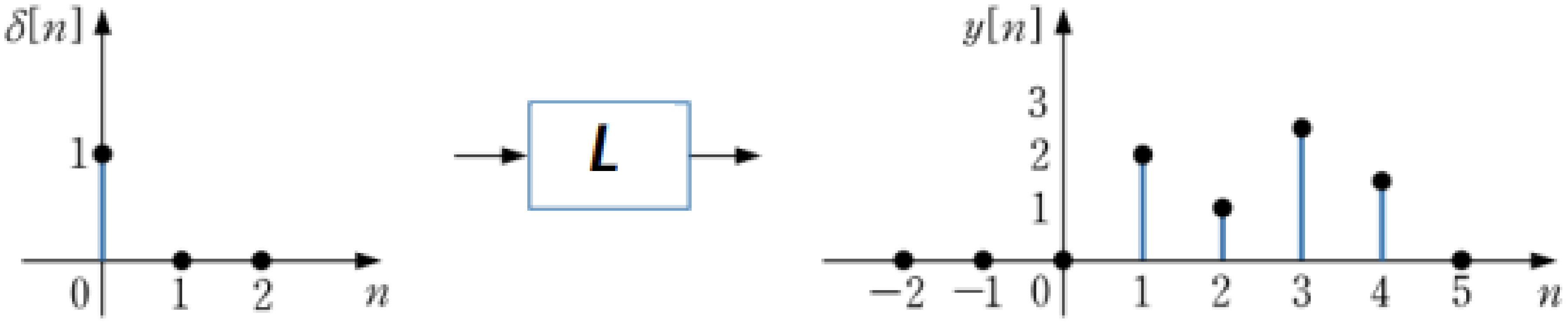
- Dynamic (memory) system - 동적
  - 과거에 쌓였던 정보가 미래에 영향 有
  - 한마디로 메모리 기능 탑재
  - Accumulator(누산기), Capacitor(반도체 = 메모리카드(0, 1)), Inductor
  - Sequential logic circuit(순차 회로) : flip-flop...
- Instantaneous (memory-less) system - 순간
  - Combination logic circuit(조합회로) : and, or...



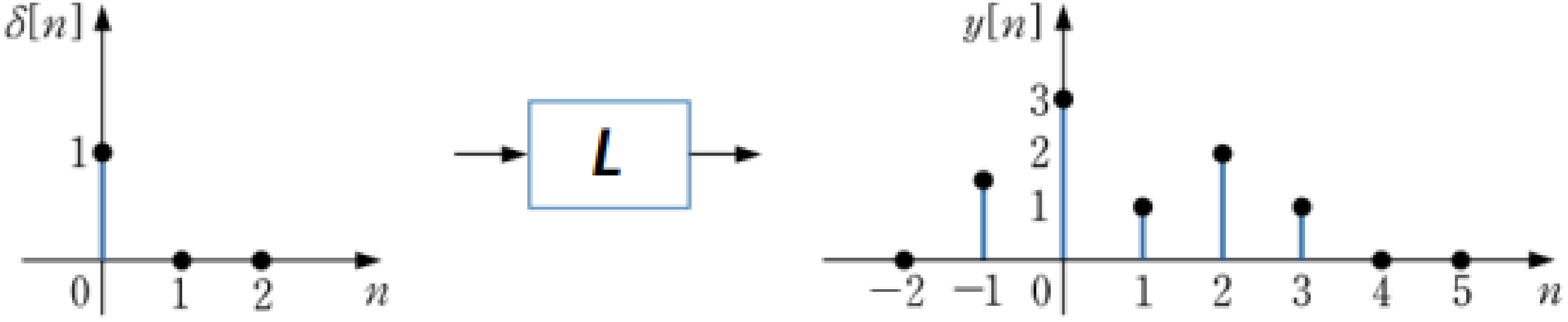
- Causal system (인과관계를 가진 시스템)
  - 미래의 정보가 현재의 아웃풋에 영향을 주지 않는다.
  - 현재 아무것도 넣지 않으면 미래에 아무것도 나오지 않는다.
  - 과거의 정보가 존재하지 않아야 한다.
- Noncausal system
  - 예측이 힘들다.
  - 이미지 픽셀처리를 할 때, 픽셀 이미지(연속)
  - 과거의 특정 정보가 현재일 때(shift)를 생각해보면, 미래의 정보가 존재하게 된다.

- Sin(0) = 0 = 현재
- Sin(1) = 0.3 = 미래
- 즉 미래의 시점이 output의 current, past에 존재하면 안된다.
- 현재 시간 축보다 앞서간 신호가 있으면 안된다. (simple!!!)

$$y[n]=0,\ n\le n_0,\ \text{ where } x[n]=0,\ n\le n_0$$



(a) Causal system



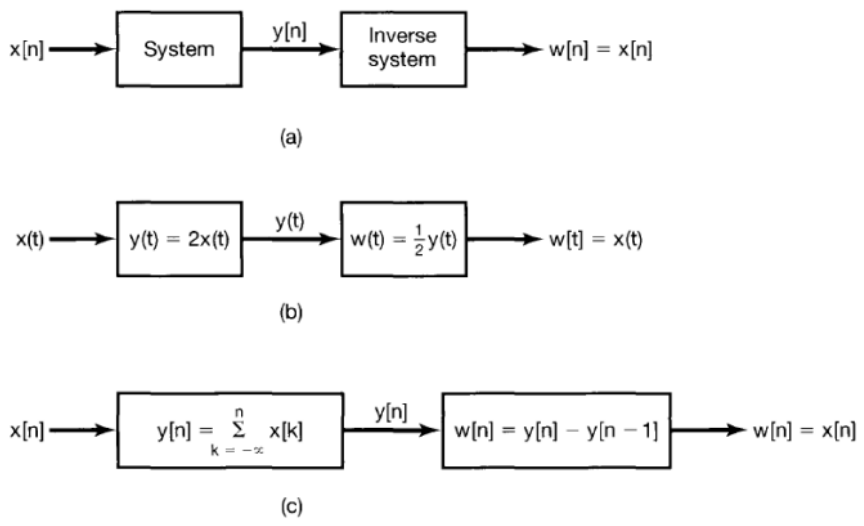
(b) Noncausal svstem

- Invertible(inverse - 가역성)
  - distinct inputs lead to distinct outputs(일대일함수)

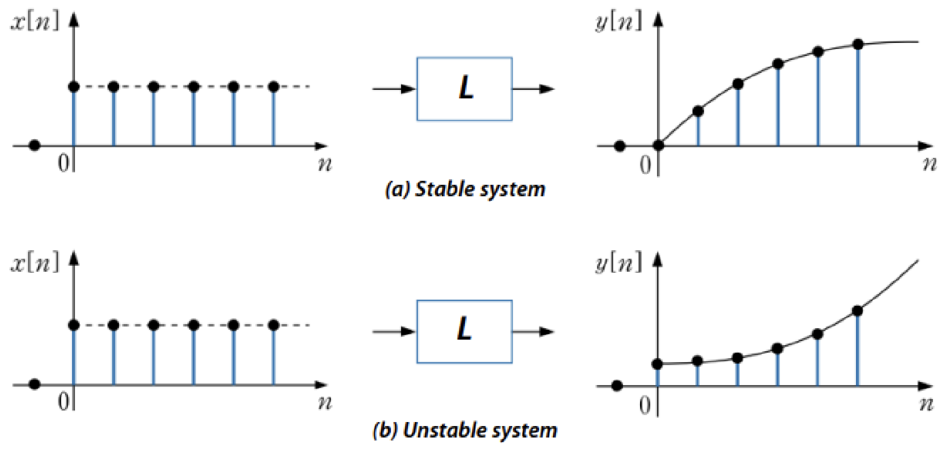
- BIBO stability(Bounded input Bounded output)
  - Input(x)가 not infinite하고 output(y)도 not infinite하다고 여겨지면 stable
  - 포화되면 stable : 안정성
  - 발산하면 non-stable : 과부하 → 시스템의 과도한 열 → 폭발

- Linear systems(Important!!!)
  - Principle of superposition을 만족
  - Additivity : L(x1) = y1, L(x2) = y2 → L(x1+x2) = L(x1) + L(x2) = y1+y2
  - Homogeneity : L(x) = y → L(ax) = a*L(x) = ay
  - Principle : L(ax+bx) = aL(x)+bL(x)

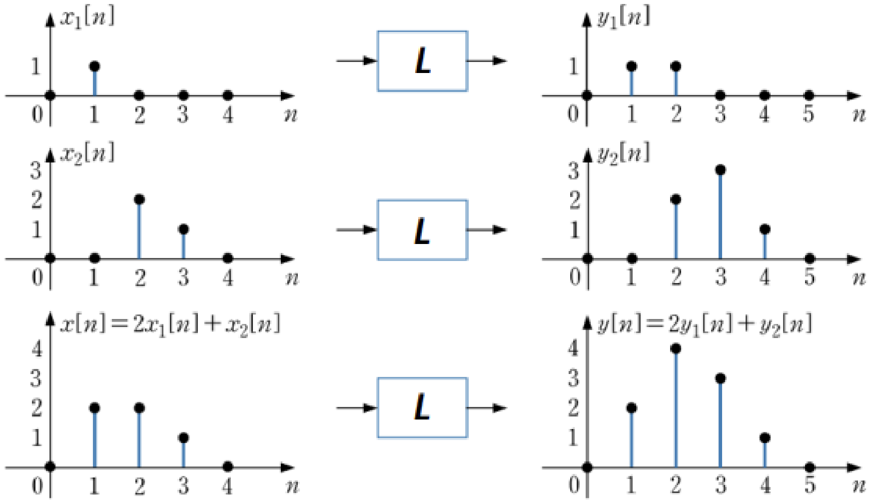
- LTI(Linear time invariant)
  - 시간 축(x축)을 기준으로 shifting 할 때 본래의 waveform을 유지



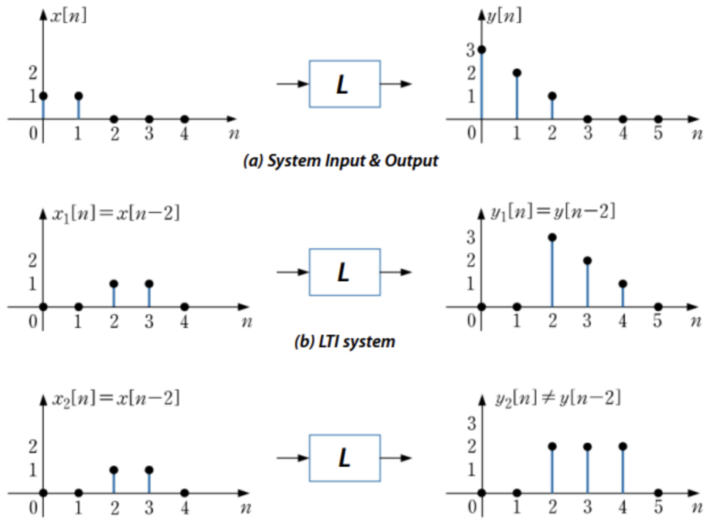
x[n]
System
y[n]
Inverse system
w[n] = x[n]
(a)
x(t)
y(t) = 2x(t)
y(t)
w(t) = ½y(t)
w[t] = x(t)
(b)
x[n]
y[n] = Σ x[k]
y[n]
w[n] = y[n] − y[n − 1]
w[n] = x[n]
(c)



x[n]
L
y[n]
(a) Stable system
x[n]
L
y[n]
(b) Unstable system



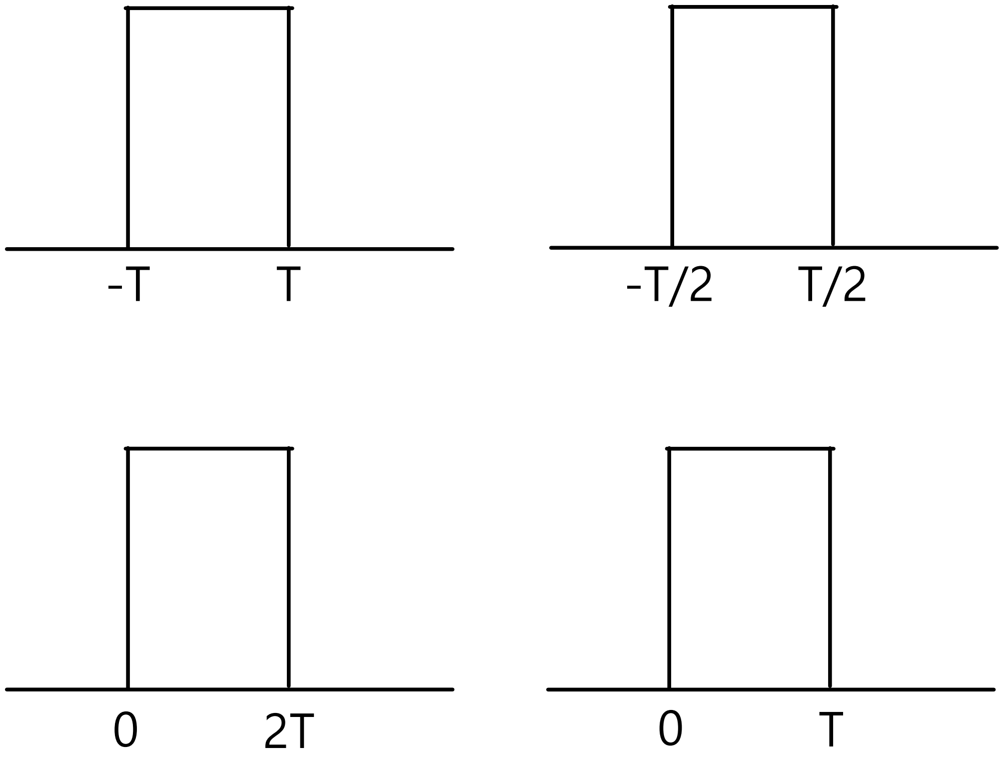
x₁[n]
L
y₁[n]
x₂[n]
L
y₂[n]
x[n] = 2x₁[n] + x₂[n]
L
y[n] = 2y₁[n] + y₂[n]



x[n]
L
y[n]
(a) System Input & Output
x₁[n] = x[n−2]
L
y₁[n] = y[n−2]
(b) LTI system
x₂[n] = x[n−2]
L
y₂[n] ≠ y[n−2]

- Question. x(t) → system → y(t) = x(2t), is it TI system?

- x(t-T) → y(t-T)=x(2t-T)
- not y(t-T), but y(t-T/2)
- Wave form changed, not TI
- TIP! 시간 축 t에 대해, shift가 아닌 복제 (multiplication)가 있으면 TI가 불만족



Not T/2 to 3T/2

- ***Determine whether linearity, time invariance, causality, stability, and memory property are satisfied for an accumulator that accumulates input values and shows the result as follows.***

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

- Linearity, L[x[n]] = y[n]
  - L[ax1[n]+bx2[n]] = ay1[n] + by2[n]
  - Sigma[ax1[n]+bx2[n]] = a*sigma + b*sigma(linear!)

- LTI, L[x[n-m]] = y[n-m]
  - $\sum_{k=-\infty}^{n} x[k-m] = y[n-m]$
  - $\sum_{s=-\infty}^{n-m} x[s] = y[n-m]$, (k-m=s)

- Causal! Future information doesn't affect now!

- Unstable! Y is increase infinitely

- Memory property satisfied
  - now is affect by past info
  - Dynamic (memory) system

4

$$x[n] = \cdots + x[-1]\delta[n+1] + x[0]\delta[n] + \cdots + x[k]\delta[n-k] + \cdots$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$y[n] = L\{x[n]\} = \cdots + L\{x[0]\delta[n]\} + \cdots + L\{x[k]\delta[n-k]\} + \cdots$$

$$= \sum_{k=-\infty}^{\infty} L\{x[k]\delta[n-k]\}$$

- ***Homogeneity***

$$y[n] = \cdots + x[0]L\{\delta[n]\} + \cdots + x[k]L\{\delta[n-k]\} + \cdots$$

$$= \sum_{k=-\infty}^{\infty} x[k]\,L\{\delta[n-k]\}$$

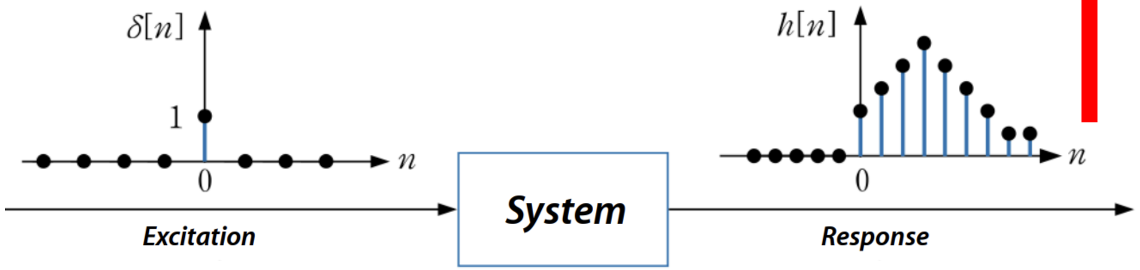
- ***Time-invariant***

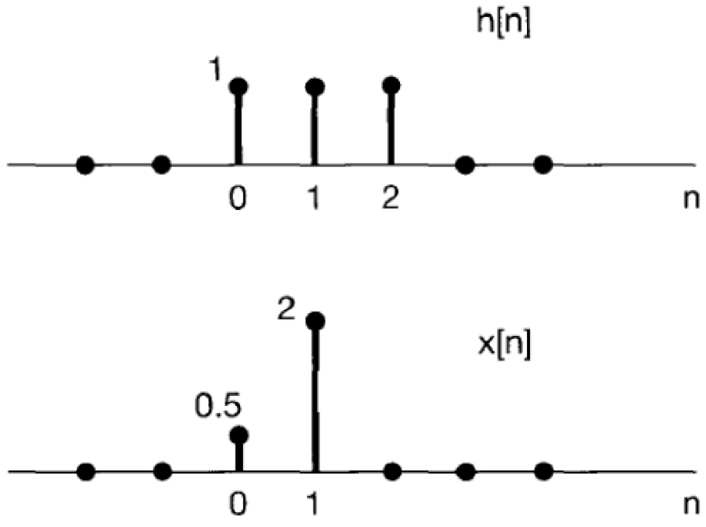
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

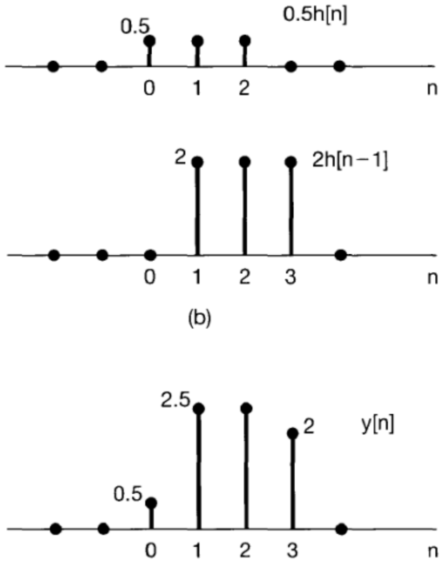
- ***Output of a discrete LTI system : becomes the convolutional sum of the input and the impulse response***

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = x[n] * h[n]$$

- x 이산 신호는 델타함수(n=k일 때만 값 1)로 표현
- Linearity
  - Input x에 대해, output y에 대한 표현
    - sigma L 밖으로 빼기 가능(additivity)
  - x[k]는 constant, 상수에 대한 곱 표현
    - output L 밖으로 빼기 가능(homogeneity)
- 델타 output은 h함수로 표현(impulse response)
- By LTI, h[n] = h[n-k], x[n] convolution h[n]







(b)

- X[0] = 0.5
- X[1] = 2

- Case of 0, x[0]*h[n-0]
- Case of 1, x[1]*h[n-1]

- 음수부터 양수까지의 h 함수 이동(shift)
- 그 동안에 x[k], 일종의 constant value와의 곱
- 이동된 특정 waveform과 상수와의 곱 = y[n]

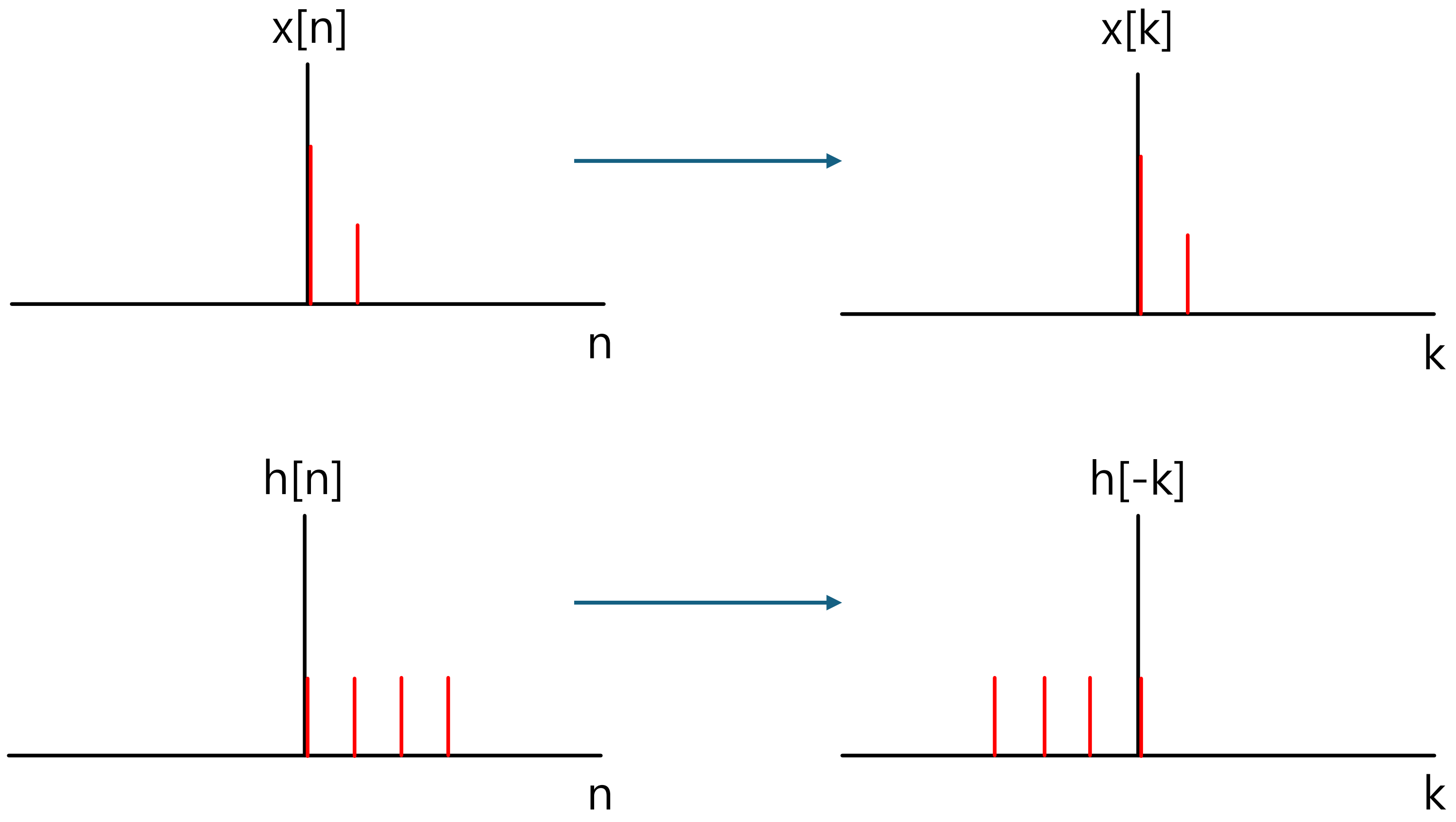
- 새로운 관점에서의 해석?
  - Domain을 n에서 k로 해석
  - N이 양수일 때 h는 양수
  - N이 음수일 때 h는 음수

- *Output of a discrete LTI system : becomes the convolutional sum of the input and the impulse response*

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = x[n] * h[n]$$

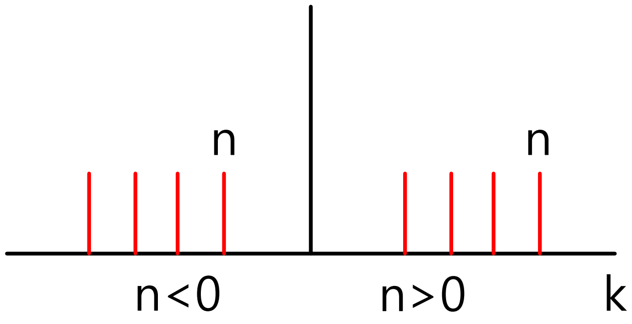
- ***Sliding method***

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} x[k]h[-(k-n)]$$

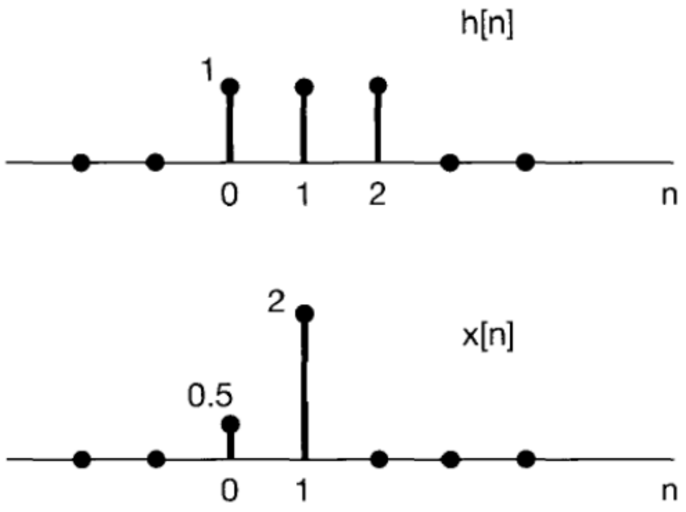


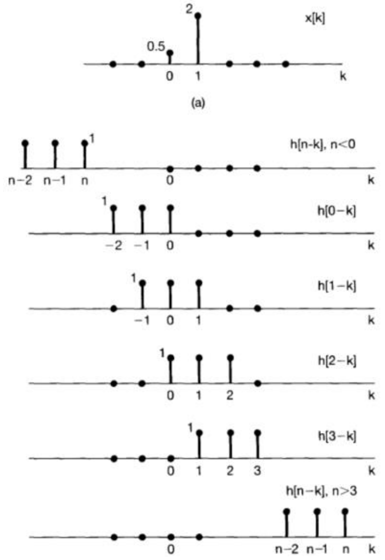
- N=0, x[k]*h[-k]
- N=1, x[k]*h[-k+1]
- …
- N값에 따라 즉각적인 결과

h[-k+n] = h[-(k-n)]
k축으로 n만큼 이동

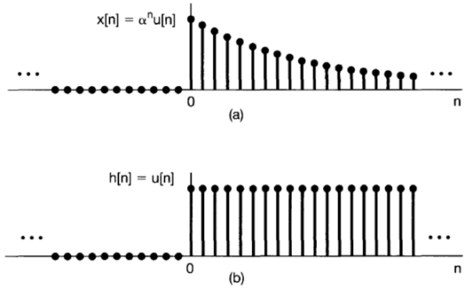


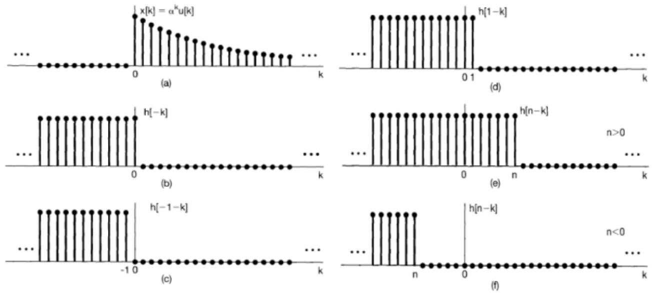
- N<0, all 0
- N=0, 0.5
- N=1, 2.5
- N=2, 2.5
- N=3, 2
- N>3, 0
- N값에 따라 바로 값을 얻을 수 있다는 장점



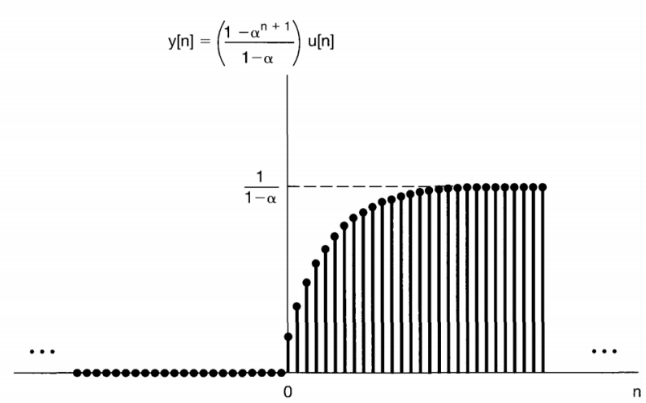


$$x[n] = \alpha^n u[n] \text{ with } 0 < \alpha < 1$$
$$h[n] = u[n]$$





- N<0, all 0
- N=0, u[n]^2=1
- N=1, u[n]^2+a*(u[n]^2)=1+a
- N=2, u[n]^2+…+(a^2)*(u[n]^3)=1+a+a^2
- …
- 등비수열
- 등비가 0~1, 최대값은 1/1-a



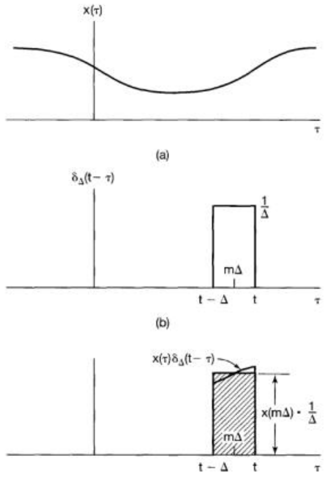
$$\frac{a(r^n - 1)}{r - 1} \quad (a : \text{초항} \ / \ r : \text{공비} \ / \ n : \text{더하는것의 개수})$$

- x(t) = x(t)*delta(triangle)*triangle
- Delta(triangle) = 1/triangle

- 우리의 시스템(h(t))이 delta와 비슷하다면
input = output
- h(t)인 순간부터 output은 y
- Time invariant system에서만 가능

- *Using approximated rectangular pulses*
  - *Any continuous-time signal can be represented as the sum of scaled and shifted unit impulse signals*

$$\delta_\Delta(t)=\begin{cases}\frac{1}{\Delta}, & 0\le t<\Delta\\ 0, & \text{otherwise}\end{cases}$$



$$x(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty}x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

$$\downarrow$$

$$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$$

- ***The convolution Integral***

$$x(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty}x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

$$\downarrow$$

$$y(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty}x(k\Delta)h_{k\Delta}(t)\Delta$$

$$\downarrow \text{ Time-Invariant system}$$

$$y(t)=\lim_{\Delta\to 0}\sum_{k=-\infty}^{\infty}x(k\Delta)h_\Delta(t-k\Delta)\Delta$$

$$x(t)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau)d\tau$$

$$\downarrow$$

$$\boxed{y(t)=\int_{-\infty}^{\infty}x(\tau)h(t-\tau)d\tau \qquad y(t)=x(t)*h(t)}$$

- ***Find the convolution integral***

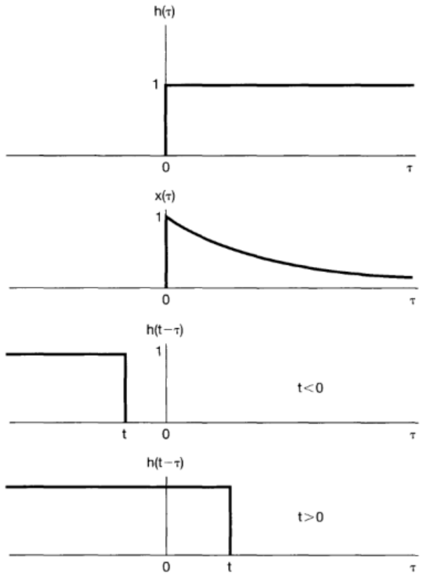
$$x(t) = e^{-at}u(t), \quad a > 0$$

$$h(t) = u(t)$$

$$x(\tau)h(t-\tau) = \begin{cases} e^{-a\tau}, & 0 < \tau < t \\ 0, & \text{otherwise} \end{cases}$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$= \int_0^t e^{-a\tau}d\tau$$



- ***The convolution Integral***

$$x(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

$$\downarrow$$

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)h_{k\Delta}(t)\Delta$$

$\downarrow$ ***Time-Invariant system***

$$y(t) = \lim_{\Delta \to 0} \sum_{k=-\infty}^{\infty} x(k\Delta)h_\Delta(t-k\Delta)\Delta$$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

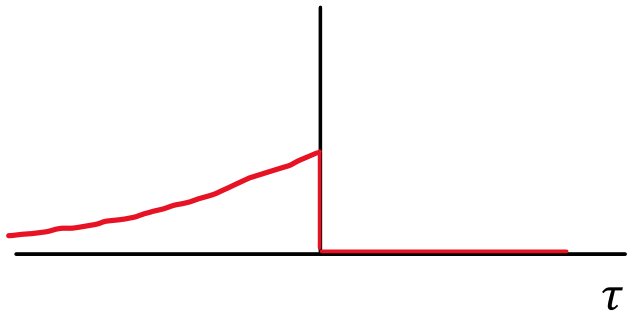
$$\downarrow$$

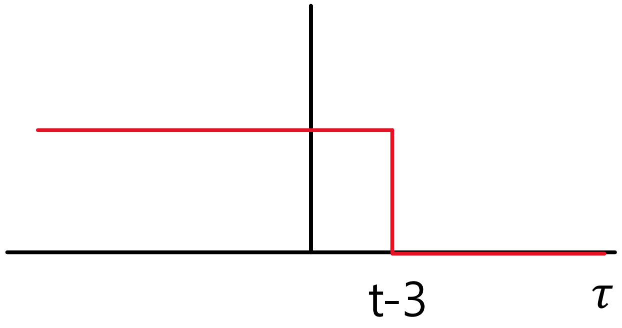
$$\boxed{\begin{gathered} y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau \\ y(t) = x(t) * h(t) \end{gathered}}$$

- ***Find the convolution integral***

$$x(t) = e^{2t}u(-t)$$

$$h(t) = u(t-3)$$





x(t)*h(t)

$\int_{-\infty}^{t-3} e^{2\tau}\, d\tau$, t<=3

$\int_{-\infty}^{0} e^{2\tau}\, d\tau$, t>3

5

x(t) → Delta → x(t)

- ***Invertibility of LTI systems***

$$x(t) \rightarrow \boxed{h(t)} \xrightarrow{y(t)} \boxed{h_1(t)} \rightarrow w(t) = x(t)$$

$$h(t) * h_1(t) = \delta(t)$$

$$h[n] * h_1[n] = \delta[n]$$

- ***Memory:***
  - *If* $h[n] = \alpha\delta[n]$, *Instantaneous system*
  - *If* $h[n] \neq \alpha\delta[n]$, *Dynamic (memory) system*

L(delta[n]) = delta[n]*a
= contains only one sample

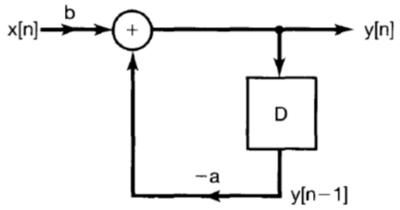
- 과거에 정보가 없는 것 → causal
- 인풋이 유한할 때, 아웃풋 또한 inf보다 작은 상수 → stable

- System analysis
    - Time-domain analysis
    - Frequency-domain analysis
        - Using Fourier transform
        - Laplace transform

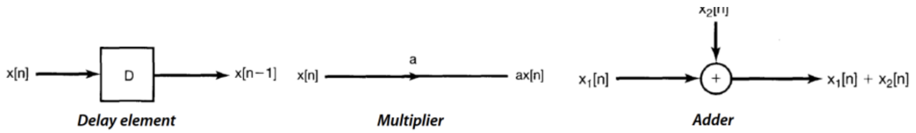
- 재귀적인 호출과 유사한 형태
    - Discrete time : y[n] – y[n-1]
    - Continuous time : y(t)/dt

- Basic concept
    - 시간 t 에 대해 x와 y에 대해…
    - 시그널 y[n]은 x(0 to)와 y(1 to) 로 표현 가능

- ***Generalized modeling***
    - ***Including the systems that have recursive structure***



$$y[n] = -ay[n-1] + bx[n]$$
$$y[n] + ay[n-1] = bx[n]$$



input

output

- ***Ex) Impulse response of a given LTI system is***
  - $h[n] = (0.5)^n u[n]$
  - ***Then, model the system with LCCDE expression***

$$y[n] = \sum_{k=0}^{\infty} h[k]x[n-k] = \sum_{k=0}^{\infty} (0.5)^k x[n-k]$$

$$= x[n] + 0.5x[n-1] + 0.5^2 x[n-2] + \cdots + 0.5^k x[n-k] + \cdots$$

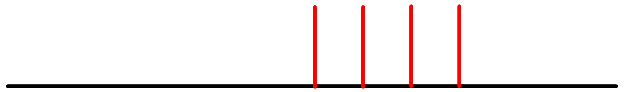
$$y[n-1] = \sum_{k=0}^{\infty} (0.5)^k x[n-1-k]$$

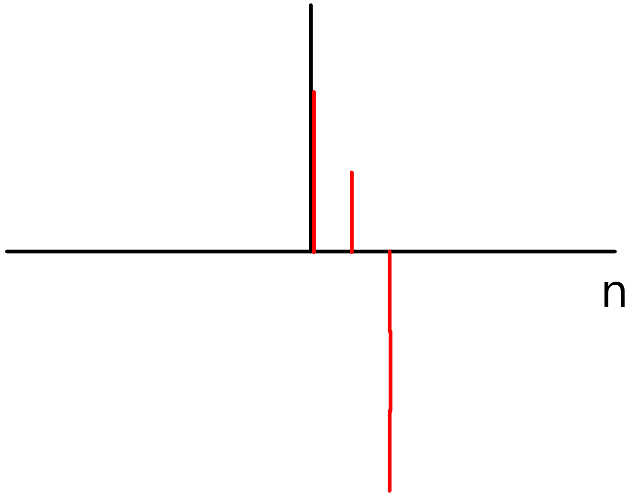
$$= x[n-1] + 0.5x[n-2] + \cdots + 0.5^{k-1} x[n-k] + \cdots$$

$$y[n] - 0.5y[n-1] = x[n]$$

FIR system(11P) – finite impulse response system

- 현재 아웃풋 = 현재(0)부터 과거까지의 인풋 – 현재 이전(1)부터 과거까지의 아웃풋(1)
- 공식 y[n] = h[n] convolution x[n]
- 만약 유한하다면 … $\sum_{k=-\infty}^{\infty} h[k]x\,[n-k]$
- $h[k]$가 다음이라면 … $\sum_{k=0}^{4} B_k \cdot x[n-k]$





- **Let** $y[n] = 2x[n] + x[n-1] - 3x[n-2]$
  - ***Then, what is the impulse response of this system?***

- $h[n] = 2\delta[n] + \delta[n-1] - 3[n-2]$
- $h[0] = 2$
- $h[1] = 1$
- $h[2] = -3$

- ***Let*** $y[n] = x[n] + ay[n-1]$ ***and assume a causal system.***
  - ***Then, what is the impulse response of this system?***

$h[0] = \delta[0] + ah[-1] = 1$ ***If the system is causal, h[n]=0 where n<0***

$h[1] = \delta[1] + ah[0] = a \times 1 = a$

$h[2] = \delta[2] + ah[1] = a \times a = a^2$

$\vdots$

$h[n] = a^n u[n]$

- ***Let*** $y[n] - \frac{1}{2} y[n-1] = x[n]$ ***and assume a causal system.***
- ***And Input is*** $x[n] = K\delta[n]$
- ***Then what is the response?***

Output

- IIR(13, 14P) – infinite impulse response system
  - Impulse response = δ를 시스템에 넣었을 때 나오는 signal
  - 핵심 : x와 y의 관계 = δ 와 h(L{δ}) 관계
  - FIR의 경우 y[n] = x[n] + x[n-1]… 꼴 이용
  - IIR의 경우 y[n] = x[n] + y[n-1] 꼴 이용
- $y[n] - \frac{1}{2} y[n-1] = x[n]$
- $y[n] - \frac{1}{2} y[n-1] = K\delta[n]$
  - $y[n] = K\mathrm{h}[n]$
  - 참고로 $y[n] = \mathrm{h}[n]$, where $x[n] = \delta[n]$
- $K\mathrm{h}[n] - \frac{1}{2} K\mathrm{h}[n-1] = K\delta[n]$
- n<0, all 0 (causal system)
- h[0]=1
- h[1]=1/2
- y[n] = $K \left(\frac{1}{2}\right)^n \cdot u[n]$

- ***System responses of a recursive system***

$$y[n] = \boxed{\sum_{m=0}^{M} b_m x[n-m]} - \boxed{\sum_{k=1}^{N} a_k y[n-k]}$$

a) Input signals injected into system

b) System response Influenced from the previous state of the system

- 수식이 복잡해지면 대입법으로 해결 복잡
  - 실제로는 본인만의 고유한 신호 노이즈가 존재
  - Causal조차 t<0에 값이 존재(노이즈)

- 실제 신호 $y[n] = y_{natural}[n] + y_{forced}[n]$
- 17P) 수학의 Homogeneous + Particular 특성 적용 가능(일치)

- 예를 들어 $a_2 y_2(t) + a_1 y_1(t) + ay(t) = bx(t)$ 경우…
- y_H(natural) <= $a_2 y_2(t)$ +… = 0
- y_P(Forced) <= $a_2 y_2(t)$ +… = bx(t)

- ***Continuous time system – Differential equation***

$$\frac{dy(t)}{dt} + 2y(t) = x(t), \qquad x(t) = Ke^{3t}u(t)$$

- ***General solution?***
  - ***Heterogeneous solution + Particular solution***
  - ***To find the exact value of coefficients we have to consider auxiliary conditions***
    - ***Initial rest, etc.***

- Homogeneous
  - $y_H = ae^{st}$
  - $ase^{st} + 2ae^{st} = 0$
  - $s = -2$
  - $y_H = ae^{-2t}$

- Particular
  - $y_P = Me^{3t}\ (t \geq 0)$
  - $3Me^{3t} + 2Me^{3t} = Ke^{3t}(t \geq 0)$
  - $M = K/5$

- $y = y_H + y_p$
  - $ae^{-2t} + K/5 \cdot e^{3t}$
  - If causal…
    - x(0) = 0 → y(0) = 0
    - y(0) = 0, a = -K/5

- ***Continuous time system – Differential equation***

$$\frac{d^2y(t)}{dt^2}+13\frac{dy(t)}{dt}+12y(t)=x(t)$$

- ***Find general solution with*** $x(t)=u(t)$
- ***Auxiliary conditions***
  - ***Initial rest***

    $y'(0)=y(0)=0$

- Homogeneous
  - $y_H = a\mathrm{e}^{st}$
  - $s^2ae^{st} + 13sae^{st} + 12ae^{st} = 0$
  - $s = -12, -1$
  - $y_H = a_1\mathrm{e}^{-12t} + a_2\mathrm{e}^{-t}$

- Particular
  - $y_P = \mathrm{K}\ (\mathrm{t} \geq 0)$
  - $12\mathrm{K} = 1\ (\mathrm{t} \geq 0)$
  - $\mathrm{K} = 1/12$

- $y = y_H + y_p$
  - $a_1\mathrm{e}^{-12t} + a_2\mathrm{e}^{-t} + 1/12$
  - $y'(0) = 0$
  - $-12a_1 - a_2 = 0$
  - $y(0) = 0$
  - $a_1 + a_2 + \frac{1}{12} = 0$
  - $a_1 = \frac{1}{132},\ a_2 = -\frac{1}{11}$

- ***Example 1***
  - ***Find general solution of the following problem***
  - ***Let y[-1]=-1 and input is*** $x[n]=u[n]$

$$y[n]+0.5y[n-1]=x[n]$$

- $y[0]+0.5y[-1]=x[0]$
- $y[0]=1.5$
- 이런 방식으로 우회

- Homogeneous
  - $y_H = C\lambda^n$
  - $y_H[n]+y_H[n-1]=0$
  - $C\lambda^n + 0.5C\lambda^{n-1} = 0$
  - $\lambda = -0.5$
  - $y_H = C(-0.5)^n$

- Particular
  - $y_P = \mathrm{K}\ (\mathrm{t} \geq 0)$
  - $\mathrm{K} + 0.5\mathrm{K} = 1\ (\mathrm{t} \geq 0)$
  - $\mathrm{K} = 2/3$

- $y = y_H + y_p$
  - $C(-0.5)^n + 2/3$
  - $y[-1] = -1$ but… $(\mathrm{t} \geq 0)$→ 직접 대입 불가
  - $C + \frac{2}{3} = 1.5$
  - $C = \frac{5}{6}$
  - $\mathrm{y} = \frac{5}{6}(-0.5)^n + \frac{2}{3}$

- ***Solve following problem***

$$y[n]-5y[n-1]+6y[n-2]=x[n-1]-5x[n-2], \quad x[n]=(3n+5)u[n]$$

  - ***Complete solution***
    - ***Initial condition should be provided***
    - ***Let*** $y[-1]=2, \quad y[-2]=1$

- $y[n+2]-5y[n+1]+6y[n]=x[n+1]-5x[n]$
- $a_1(n+2)+a_2-5a_1(n+1)-5a_2+6a_1n+6a_2=3(n+1)+5-5(3n+5)$ $(\mathrm{t}\geq 0)$
- $2a_1n-3a_1+2a_2=-12n-17$
- $a_1=-6$
- $a_2=-35/2$

- Homogeneous
  - $y_H=C\lambda^n$
  - $C\lambda^n-5C\lambda^{n-1}+6C\lambda^{n-2}=0$
  - $\lambda=2,3$
  - $y_H=C_1 2^n+C_2 3^n$

- Particular
  - $y_P=a_1n+a_2$ $(\mathrm{t}\geq 0)$
  - $y_P=-6n-\frac{35}{2}$ $(\mathrm{t}\geq 0)$

- $y=y_H+y_p$
  - $C_1 2^n+C_2 3^n-6n-\frac{35}{2}$
  - $(\mathrm{t}\geq 0)$→ 직접 대입 불가
  - $y[0]-5y[-1]+6y[-2]=0, y[0]$=4
  - $y[1]-5y[0]+6y[-1]=5, y[1]$=13
  - $y[2]-5y[1]+6y[0]=0, y[2]$=24

5

FOURIER SERIES

Continuous : $x(t) = e^{st}$, $(e^{s} = e^{jw})$
Discrete : $x[n] = z^{n}$, $(|z = a + jb| = 1)$

- $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$
- $= \int_{-\infty}^{\infty} h(\tau)e^{st} \cdot e^{-s\tau}\,d\tau$
- $= e^{st}\left[\int_{-\infty}^{\infty} h(\tau)e^{-s\tau}\,d\tau\right]$
- $= x(t) \cdot H(s)$

τ 에 대해 상수 취급

- *Let* $x(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_3 t}$
  - *Then* $y(t) = a_1 H(s_1) e^{s_1 t} + a_2 H(s_2) e^{s_2 t} + a_3 H(s_3) e^{s_3 t}$
- *This is possible because it is Linear system*

- *If* $x(t) = \sum_k a_k e^{s_k t}$, *then* $y(t) = \sum_k a_k H(s_k) e^{s_k t}$

- *If* $x[n] = \sum_k a_k z_k^n$, *then* $y[n] = \sum_k a_k H(z_k) z_k^n$

- 주기신호가 주어져 있을 때, 시스템에 넣었더니 y(t)는 convolution으로 표현 가능
- e^{st}(상수) 밖으로 빼기 가능, 주파수 성분으로만 구성된(시간이 존재하지 않는) $H(s)$변환 가능
- 푸리에 변환

- $y(t) = x(t) \cdot H(s)$
- $L\{x(t)\} = x(t) \cdot H(s)$
- $x(t)$ = eigen function
- $H(s)$ = eigen value

- 무슨 의미…
- $A \cdot v = \lambda \cdot v$
- Matrix A
- Vector v
- 행렬과 벡터의 선형 결합
- Eigen value와 Eigen vector의 곱으로 표현

- **Let** $y(t) = x(t-3)$ **and** $x(t) = e^{j2t}$
  - *Then what is the* $H(s)$

- $y(t) = x(t-3) = H(s)x(t)$
- $\mathrm{e}^{j2(t-3)} = H(s)\mathrm{e}^{j2t}$
- $H(s) = \mathrm{e}^{-6j}$, 시간 성분 삭제

- $y(t) = x(t) * h(t) = x(t-3)$일 때… $h(t)$는?
  - $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)\,\mathrm{d}\tau$
  - $x(t) = \int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)\,\mathrm{d}\tau$

  - $? = \int_{-\infty}^{\infty} \delta(\tau-3)x(t-\tau)\,\mathrm{d}\tau$, if $\mathrm{k} = (\tau - 3)$
  - $? = \int_{-\infty}^{\infty} \delta(k)x(t-k-3)\,\mathrm{d}k$
  - $? = \int_{-\infty}^{\infty} \delta(k)x((t-3)-k)\,\mathrm{d}\tau$
  - $? = x(t-3)$
  - $h(t) = \delta(t-3)$

- 유사한 방식으로…
  - $\mathrm{H(s)} = \int_{-\infty}^{\infty} h(\tau)\mathrm{e}^{-s\tau}\,\mathrm{d}\tau$
  - $\mathrm{H(s)} = \int_{-\infty}^{\infty} \delta(\tau-3)\mathrm{e}^{-s\tau}\,\mathrm{d}\tau$
  - $\mathrm{H(s)} = \mathrm{e}^{-3s}$, ($\tau = 3$값만 생존)
  - $x(t) = \mathrm{e}^{st}$에서 $\mathrm{s} = j2$

- 즉 $y(t) = x(t) * h(t)$일 때…
  - $y(t) = x(t-3)$
  - $h(t) = \delta(t-3)$

- **Let** $y(t) = \cos(4(t-3)) + \cos(7(t-3))$
  - **Where** $x(t) = \cos(4t) + \cos(7t)$
- **Then what is the eigenvalues?**

- $x(t) = \frac{1}{2}e^{j4t} + \frac{1}{2}e^{-j4t} + \frac{1}{2}e^{j7t} + \frac{1}{2}e^{-j7t}$
- $s = j4,\ s = -j4,\ s = j7,\ s = -j7$

- $y(t) = x(t-3)$
- $h(t) = \delta(t-3)$
- $H(s) = e^{-3s}$

- $y(t) = \frac{1}{2}H(4j)e^{j4t} + \frac{1}{2}H(-4j)e^{-j4t} + \frac{1}{2}H(7j)e^{j7t} + \frac{1}{2}H(-7j)e^{-j7t}$
- $H(4j) = e^{-12j}$
- $H(-4j) = e^{12j}$
- $H(7j) = e^{-21j}$
- $H(-7j) = e^{21j}$

- Sinusoid signal 로 input을 구현할 때, output도 그래도 따라온다.
- 세상에 있는 모든 신호를 수학적으로 표현이 가능해진다.