2. Use the informal definitions of O, Θ, and Ω to determine whether the following assertions are true or false.

   a. $n(n+1)/2 \in O(n^3)$ b. $n(n+1)/2 \in O(n^2)$

   c. $n(n+1)/2 \in \Theta(n^3)$ d. $n(n+1)/2 \in \Omega(n)$

3. For each of the following functions, indicate the class $\Theta(g(n))$ the function belongs to. (Use the simplest $g(n)$ possible in your answers.) Prove your assertions.

   a. $(n^2+1)^{10}$ b. $\sqrt{10n^2+7n+3}$

   c. $2n\lg(n+2)^2 + (n+2)^2\lg\frac{n}{2}$ d. $2^{n+1}+3^{n-1}$

   e. $\lfloor \log_2 n \rfloor$

4. a. Table 2.1 contains values of several functions that often arise in analysis of algorithms. These values certainly suggest that the functions

$$\log n, \quad n, \quad n\log n, \quad n^2, \quad n^3, \quad 2^n, \quad n!$$

   are listed in increasing order of their order of growth. Do these values prove this fact with mathematical certainty?

   b. Prove that the functions are indeed listed in increasing order of their order of growth.

5. Order the following functions according to their order of growth (from the lowest to the highest):

$$(n-2)!, \quad 5\lg(n+100)^{10}, \quad 2^{2n}, \quad 0.001n^4+3n^3+1, \quad \ln^2 n, \quad \sqrt[3]{n}, \quad 3^n.$$

6. a. Prove that every polynomial of degree k, $p(n) = a_k n^k + a_{k-1}n^{k-1} + \cdots + a_0$ with $a_k > 0$, belongs to $\Theta(n^k)$.

   b. Prove that exponential functions a^n have different orders of growth for different values of base $a > 0$.

TABLE 2.1 Values (some approximate) of several functions important for analysis of algorithms

n	$\log_2 n$	n	$n \log_2 n$	n^2	n^3	2^n	$n!$
10	3.3	10^1	$3.3 \cdot 10^1$	10^2	10^3	10^3	$3.6 \cdot 10^6$
10^2	6.6	10^2	$6.6 \cdot 10^2$	10^4	10^6	$1.3 \cdot 10^{30}$	$9.3 \cdot 10^{157}$
10^3	10	10^3	$1.0 \cdot 10^4$	10^6	10^9		
10^4	13	10^4	$1.3 \cdot 10^5$	10^8	10^{12}		
10^5	17	10^5	$1.7 \cdot 10^6$	10^{10}	10^{15}		
10^6	20	10^6	$2.0 \cdot 10^7$	10^{12}	10^{18}		

1. Compute the following sums.

   a. $1 + 3 + 5 + 7 + \cdots + 999$

   b. $2 + 4 + 8 + 16 + \cdots + 1024$

   c. $\sum_{i=3}^{n+1} 1$ d. $\sum_{i=3}^{n+1} i$ e. $\sum_{i=0}^{n-1} i(i+1)$

   f. $\sum_{j=1}^{n} 3^{j+1}$ g. $\sum_{i=1}^{n} \sum_{j=1}^{n} ij$ h. $\sum_{i=0}^{n-1} 1/i(i+1)$

2. Find the order of growth of the following sums.

   a. $\sum_{i=0}^{n-1} (i^2+1)^2$ b. $\sum_{i=2}^{n-1} \lg i^2$

   c. $\sum_{i=1}^{n} (i+1)2^{i-1}$ d. $\sum_{i=0}^{n-1} \sum_{j=0}^{i-1} (i+j)$

   Use the $\Theta(g(n))$ notation with the simplest function $g(n)$ possible.