$\mathcal{O}(n)$ - najhorší prípad

$\Theta(n)$ - priemerné

$\Omega(n)$ - najlepší prípad

a) $\frac{(n+1)n}{2} = \frac{n^2+n}{2} = \frac{n^2}{2} + \frac{n}{2} \approx n^2+n$

$\Rightarrow \Theta(n^2)$

$\in \mathcal{O}(n^3)$ T

$\in \mathcal{O}(n^2)$ T

$\in \Theta(n^3)$ F

$\in \Omega(n)$ T

PR $\Theta(g(n))$

a) $(n^2+1)^{10} \approx (n^2)^{10} = n^{20} \Rightarrow \Theta(n^{20})$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim_{n\to\infty} \frac{(n^2+1)^{10}}{n^{20}} = \lim_{n\to\infty} \left(\frac{n^2+1}{n^2}\right)^{10} =$

$\lim_{n\to\infty} \left(1+\frac{1}{n^2}\right)^{10} = 1$

$\Rightarrow \lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$

b, $\sqrt{10n^2+7n+3} \simeq \sqrt{10n^2} = \sqrt{10} \cdot \sqrt{n^2} = \sqrt{10} \cdot n$

$\Theta(n)$

$$\lim_{n \to \infty} \frac{\sqrt{10n^2+7n+3}}{n} = \sqrt{10}$$

c, $2n \lg (n+2)^2 + (n+2)^2 \lg \frac{n}{2} \approx$

$4n \lg(n+2) + n^2 \lg \frac{n}{2}$

$n \lg(n+2) + n^2 (\lg n - \overbrace{\lg 2}^{1})$

$n \lg(n) + n^2 \lg(n)$

$\Theta(n^2 \log(n))$

d, $2^{n+1} + 3^{n-1} = 2^n \cdot 2 + 3^n \cdot 3^{-1} \approx 2^n + 3^n = \Theta(3^n)$

e $\lfloor \log_2 n \rfloor \approx \Theta(\log n)$

$x - 1 < \lfloor x \rfloor \leq x$

$\lfloor \log_2 n \rfloor > \log_2 n - 1$

$\log_2 n - \frac{1}{2} \log_2 n \, ; n \geq 4$

$\frac{1}{2} \log_2 n \Rightarrow \Theta(\log_2 n)$

④ $\log n < n < n\log_2 n < n^2 < n^3 < 2^n < n!$

$$\lim_{n\to\infty} \frac{\log n}{n} = \lim_{n\to\infty} \frac{(\log n)'}{n'} = \lim_{n\to\infty} \frac{\log n}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} \begin{cases} 0 & o \to f(n) \text{ pomalší rast ako } g(n) \\ c & \Theta \to f(n) = g(n) \\ \infty & \Omega \to f(n) \text{ rýchlejšie ako } g(n) \end{cases}$$

$$\lim_{n\to\infty} \frac{n}{n\log n} = \lim_{n\to\infty} \frac{1}{\log n} = 0$$

DÚ; Skontrolovať a precvičiť limity!

⑤ a, $(n-2)! = \Theta(n!) = \Theta((n-2)!)$

b, $5\lg(n+100)^{10} = 50\lg(n+100) = \lg(n)$

$\Theta \lg(n)$

⑥ $p(n) = a_k n^k + a_{k-1} n^{k-1} + \dots a_0$

Dokážte, že $\Theta(n^k)$

$$\lim_{n\to\infty} \frac{a_k n^k + a_{k-1} n^{k-1} + \dots a_0}{n^k} =$$

Θ

$$\lim_{n\to\infty}\left(a_k + \frac{a_{k-1}}{n} + \dots \frac{a_0}{n^k}\right) = a^k \Leftrightarrow \in \Theta(n^k)$$

$a^n \quad a > 0$

$$\lim_{n\to\infty} \frac{a_1^n}{a_2^n} = \lim_{n\to\infty}\left(\frac{a_1}{a_2}\right)^n = \begin{cases} 0 & a_1 < a_2 \\ 1 & a_1 = a_2 \\ \infty & a_1 > a_2 \end{cases}$$

$a_1^n \in O(a_2^n)$

$a_1^n \in \Theta(a_2^n)$

$a_2^n \in O(a_1^n)$

$a_1^n \in \Omega(a_2^n)$

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a, $1+3+5+7+\dots+999$

$$\sum_{i=1}^{500}(2i-1) = \sum_{i=1}^{500} 2i - \sum_{i=1}^{500} 1 = 2\sum_{i=1}^{500} i - \sum_{i=1}^{500} 1 = 2\cdot\frac{500\cdot 501}{2} - 500 =$$

$250\,000$

b, $2+4+8+16+\dots+1024$

$$\sum_{i=1}^{10} 2^i = \sum_{i=0}^{10}\left(2^i\right) - 1 = 2^{n+1} - 1 - 1 = 2^n - 2 = 2046$$

c, $\sum_{i=3}^{n+1} 1 = n+1-3+1 = n-1$

d, $\sum_{i=3}^{n+1} i$

e, $\sum_{i=0}^{n-1} i(i+1) = \sum_{i=0}^{n-1}(i^2+i) = \sum_{i=0}^{n-1} i^2 + \sum_{i=0}^{n-1} i$

$$\sum_{i=0}^{n-1} \frac{(n-1)n(2n-1)}{6}$$

$$\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$$

$$\sum_{i=0}^{n} 2^{i} - 1$$

g) $\sum_{i=1}^{n} \sum_{j=1}^{n} ij \Rightarrow \sum_{i=1}^{n} i \cdot \sum_{j=1}^{n} j = \frac{n\cdot(n+1)}{2} \cdot \frac{n\cdot(n+1)}{2}$

$$= \frac{n^2\cdot(n+1)^2}{4}$$

h) $\sum_{i=1}^{n-1} \frac{1}{i\cdot(i+1)} = \sum_{i=1}^{n-1} \frac{1}{i} - \frac{1}{i+1} = \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right)$

$$\dots + \left(\frac{1}{n-1} - \frac{1}{(n-1)+1}\right)$$

$$= 1 - \frac{1}{n}$$

PR $\Theta(g(n))$

a) $\sum_{i=0}^{n+1} (i^2+1)^2 = \sum_{i=0}^{n+1} i^4 + 2i^2 + 1 = \sum_{i=0}^{n+1} i^4 + \sum_{i=0}^{n+1} 2i^2 +$

$$\sum_{i=0}^{n+1} 1 \Rightarrow \Theta(n^5) + \Theta(n^3) + \Theta(n)$$

$$\sum_{i=0}^{n} i^k \Rightarrow \Theta(n^{k+1})$$

i) $\sum_{i=2}^{n-1} \lg i^2 = 2\sum_{i=2}^{n-1} \lg i = 2\sum_{i=1}^{n} \lg i - \lg n$

$$= 2 \cdot \sum_{i=1}^{n} \lg i - \lg n \Rightarrow \Theta(n \lg n)$$

$$\sum_{i=1}^{n} \log n \Rightarrow \Theta(n \log n)$$