$$T(n) = k \cdot T\left(\frac{n}{k}\right) + c \cdot n \; ; \; T(1) = 0$$

$$n = k^m$$

$$m = \log_k n$$

vyjadrite $T(n)$ pomocou $c; n; k$ a Big-O

a) $T(n) \Rightarrow T(k^m) = k \cdot T(k^{m-1}) + c \cdot k^m$

$$\frac{T(k^m)}{k^m} = \frac{T(k^{m-1})}{k^{m-1}} + c$$

$$\frac{T(k^{m-1})}{k^{m-1}} = \frac{T(k^{m-2})}{k^{m-2}} + c$$

$$\vdots \quad = \frac{T(k^1)}{k} + c$$

$$\frac{T(k^1)}{k} = \frac{T(1)}{1} + c \Rightarrow \frac{T(k)}{k} = c$$

$$\frac{T(k^m)}{k^m} = c + c + \dots + c + c = c \cdot m$$

$T(k^m) = c \cdot m \cdot k^m$

$T(n) = c \cdot \log(n) \cdot n \Rightarrow O(n \log(n))$

a, Telescoping

b, $T(1) = 0$

$T(k) = k \cdot \overbrace{T(1)}^{0} + ck$

$\approx ck$

$T(k^2) = k \cdot T(k) + ck^2$

$= k \cdot ck + ck^2 = 2ck^2$

$T(k^3) = k \cdot T(k^2) + ck^3$

$= k \cdot 2ck^2 + ck^3$

$= 3ck^3$

$T(k^4) = 4ck^4$

$T(k^m) = m \cdot c \cdot k^m$

indukcia

$T(k^0) = 0 \cdot c \cdot k^0 = 0$

$T(1) = 0$

$T(k^l) = l \cdot c \cdot k^l \; ; \; l = 0, 1, 2, \ldots, m-1$

dokáž pre $l = m$

$$T(k^m) = k \cdot T(k^{m-1}) + c k^m$$

$$T(k^m) = k \cdot (m-1) \cdot c \cdot k^{m-1} + c k^m$$

$$= c k^m (m-1) + c k^m$$

$$= c k^m (m - 1 + 1)$$

$$= c k^m \cdot m$$

$$\Rightarrow c \cdot n \log(n) = O(n \cdot \log(n))$$

PR

```
1.  for ( i=1; i < n; i *= 2 ) {
2.    for ( j = n; j > 0; j /= 2 ) {
3.      for ( k = j; k < n; k += 2 ) {
          sum += (i + j * k );
        }
      }
    }
```

1. for => $\log(n)$

2. for => $\log(n)$

3. for => n

$\log(n) \cdot \log(n) \cdot n = n \log^2 n = \Theta$