Polarimetric scattering and emission properties of targets with reflection symmetry

Simon H. Yueh, Ronald Kwok, and Son V. Nghiem Jet Propulsion Laboratory, California Institute of Technology, Pasadena

This paper investigates the symmetry of polarimetric scattering and emission coefficients of media with reflection symmetry. A reflection operator is defined and is used to create the images of electromagnetic fields and sources. The image fields satisfy Maxwell's equations, meaning that Maxwell's equations are invariant under the described reflection operations. By applying the reflection operations to media with reflection symmetry, the symmetry properties of the Stokes parameters, characterizing the polarization state of thermal emissions, are shown to agree with existing experimental data. The first two Stokes parameters are symmetric with respect to the reflection plane, while the third and fourth Stokes parameters have odd symmetry. In active remote sensing, the symmetry properties of the polarimetric scattering matrix elements of deterministic targets and the polarimetric covariance matrix elements of random media or distributed targets are examined. For deterministic targets, the cross-polarized responses are odd functions with respect to the symmetry direction, whereas the copolarized responses are even functions. For distributed targets or random media, it is found that the correlations of copolarized and cross-polarized responses are antisymmetric with respect to the reflection plane, while the other covariance matrix elements are symmetric. Consequently, in the cases of backscatter, the copolarized and cross-polarized components are completely uncorrelated when the incidence direction is on the symmetry plane. The derived symmetry properties of polarimetric backscattering coefficients agree with the predictions of a two-scale surface scattering model and existing sea surface HH and VV backscatter data. Finally, the conditions for a general type of media, i.e., bianisotropic media, to be reflection symmetric are examined.

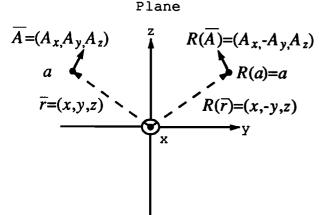
1. Introduction

This paper discusses the symmetry properties of the polarization components of active scattered fields and passive thermal radiations from media with reflection symmetry in light of recent significant interests in polarimetric active and passive remote sensing of geophysical media, particularly, wind-roughened ocean surfaces, which are symmetric with respect to the wind direction. In active remote sensing, the HH or VV backscatter from wind-induced sea surfaces has been known to be symmetric with respect to the wind direction [Wentz et al., 1984]. However, symmetries of the other polarimetric backscattering coefficients, characterizing the mutual correlation between the electric fields collected with two arbitrary antenna po-

Copyright 1994 by the American Geophysical Union.

Paper number 94RS02228. 0048-6604/94/94RS-02228\$08.00

larizations, have not yet been discussed. In passive remote sensing the polarization states of thermal radiations are described by a Stokes vector with four parameters. The first two Stokes parameters of sea surface brightness temperatures were found to be symmetric with respect to the ocean wind direction [Etkin et al., 1991; Wentz, 1992; Yueh et al., 1994c], while the third Stokes parameter was an odd function for emissions from corrugated soil surfaces [Nghiem et al., 1991], from periodic water surfaces [Johnson et al., 1993; Yueh et al., 1994a], and from sea surfaces at normal incidence [Dzura et al., 1992] and at incidence angles of 30°-50° [Yueh et al., 1994cl. Additionally, those observed symmetry properties of Stokes parameters are consistent with the results generated by a theoretical emission model for random rough surfaces [Yueh et al., 1993]. Although the experimental evidence described above had suggested the symmetries of polarimetric active scattering and passive emission coefficients, there was no rigorous explanation



Reflection

Figure 1. Relations between the original field and the image field defined by the reflection operator.

based on the Maxwell equations. Here this paper shows that the symmetries of electromagnetic fields scattered or emitted from reflection symmetric media can be derived from the Maxwell equations.

Section 2 demonstrates that Maxwell's equations are invariant under a described reflection operator. Section 3 describes how the polarization vector components of electric fields are transformed under the reflection operation. Section 4 presents the symmetry properties of thermal radiations with respect to the reflection plane, while section 5 presents the symmetry properties of polarimetric scattering coefficients. The conditions for media to be reflection symmetric are discussed in section 6.

2. Reflection Operations and the Invariance of Maxwell's Equations

This section introduces a reflection operator and its commutation relations with the divergence and curl operators, which are two of the most frequently used operators in many areas of physics, including quantum mechanics, fluid dynamics, and electromagnetics. In this paper we restrict the applications of the reflection operator to electromagnetics and use it to investigate the symmetry of electromagnetic fields. Specifically, using the reflection operator, we introduce a complementary problem with the fields corresponding to the images of a set of electromagnetic fields, and we show that the image fields satisfy Maxwell's equations to demonstrate the invariance of Maxwell's equations under the reflection operations.

Without loss of generality, throughout this paper the x-z plane, unless otherwise mentioned, is chosen as the reflection plane, with respect to which the reflection operation is applied. The reflection operator \Re is defined as follows: When \Re is applied to a scalar field a(x, y, z), it creates a reflection image a' by the following relation:

$$a'(x, y, z) = \Re(a(x, y, z)) = a(x, -y, z)$$
 (1)

When applied to a vector field A, it creates another vector field A',

$$\mathbf{A}'(x, y, z) = \Re(\mathbf{A}(x, y, z)) = \hat{x}A_x(x, -y, z)$$
$$-\hat{y}A_y(x, -y, z) + \hat{z}A_z(x, -y, z)$$
(2)

Given the above definition as illustrated in Figure 1, it is straightforward to show that for an arbitrary vector field A the following commutation relations hold true when R operates together with the divergence and curl operators:

$$\nabla \cdot \Re(\mathbf{A}) = \Re(\nabla \cdot \mathbf{A})$$

$$\nabla \times \Re(\mathbf{A}) = -\Re(\nabla \times \mathbf{A})$$
(3)

Hence \Re is commutable with the divergence operator and does not change the divergence of the vector field. In contrast, an additional minus sign is observed when the reflection and curl operations are reversed. This is because the reflection operation causes a sign change to the vector component perpendicular to the reflection plane, thus changing the handedness of the vector field.

Using \Re , we can define a complementary problem with the fields and sources related to the images of those in the original problem. Specifically, the fields in the complementary problem, indicated by a prime, are defined as follows:

$$\mathbf{E}'(x, y, z) = \Re(\mathbf{E}(x, y, z))$$

$$\mathbf{D}'(x, y, z) = \Re(\mathbf{D}(x, y, z))$$

$$\mathbf{J}'(x, y, z) = \Re(\mathbf{J}(x, y, z))$$

$$\rho'(x, y, z) = \Re(\rho(x, y, z))$$
(4)

and

$$\mathbf{H}'(x, y, z) = -\Re(\mathbf{H}(x, y, z))$$

$$\mathbf{B}'(x, y, z) = -\Re(\mathbf{B}(x, y, z))$$
(5)

Note that unlike the electric fields and sources, the images of "magnetic" quantities H and B carry a sign opposite to that of the fields created by the reflection operator. That is, the reflection plane actually resembles a perfect magnetic conducting wall. In contrast to the above image fields, another set of fields can be created by simulating the reflection plane as a perfect electrical conducting wall with the signs of all "electric" quantities reversed, while maintaining the signs of magnetic quantities. However, these two sets of definitions lead to the same conclusions on the symmetry properties of polarimetric thermal radiations (section 4), polarimetric scattering coefficients (section 5), and criteria for a medium to be reflection symmetric (section 6). Thus we discuss only the case in which the reflection plane corresponds to a perfect magnetic conducting plane.

In the original problem the electric field **E**, magnetic field **H**, electric displacement **D**, and magnetic flux density **B** are related to the charge density ρ and current density **J** through the Maxwell equations,

$$\nabla \cdot \mathbf{D} = \rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$
(6)

Given the commutation relations of \Re with the divergence and curl operators, it can be shown that the fields and sources in the complementary problem satisfy Maxwell's equations. For Gauss's law of electric displacement, applying the commutation relation of the reflection operator with the divergence operator leads to

$$\nabla \cdot \mathbf{D}' = \nabla \cdot \Re(\mathbf{D}) = \Re(\nabla \cdot \mathbf{D}) = \Re(\rho) = \rho' \tag{7}$$

For the Faraday induction law, which involves the curl operator, making use of the commutation relation results in

$$\nabla \times \mathbf{E}' = \nabla \times \Re(\mathbf{E}) = -\Re(\nabla \times \mathbf{E}) = \frac{\partial \Re(\mathbf{B})}{\partial t} = -\frac{\partial \mathbf{B}'}{\partial t}$$
(8)

In a similar way it can be shown that **B** satisfies Gauss's law:

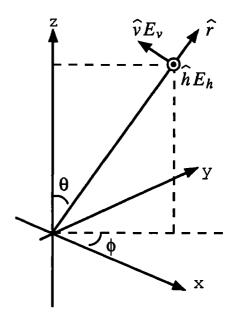


Figure 2. Illustration of horizontal and vertical polarization vectors \hat{h} and \hat{v} for the electric field propagating in the \hat{r} direction.

$$\nabla \cdot \mathbf{B}' = 0 \tag{9}$$

and the generalized Ampere's law holds for the image fields and currents:

$$\nabla \times \mathbf{H}' = \mathbf{J}' + \frac{\partial \mathbf{D}'}{\partial t} \tag{10}$$

Hence the image fields and sources obtained through the reflection operations satisfy Maxwell's equations. In other words, Maxwell's equations are invariant under the reflection operations.

3. Transformation of the Polarized Components of Electric Fields

To facilitate the symmetry analyses of polarimetric scattering and emission coefficients, this section shows how the horizontally and vertically polarized components of electric fields are transformed under the reflection operation. In polarimetric remote sensing an electric field in the far-field regime propagating in the direction $\hat{r} (= \hat{v} \times \hat{h})$ is decomposed into horizontal and vertical components (Figure 2),

$$\mathbf{E}(\theta, \, \phi) = E_h(\theta, \, \phi)\hat{h}(\theta, \, \phi) + E_v(\theta, \, \phi)\hat{v}(\theta, \, \phi) \tag{11}$$

where \hat{h} and \hat{v} represent the horizontal and vertical polarization vectors, respectively, and \hat{r} is the propagation direction,

$$\hat{h}(\theta, \, \phi) = \sin \, \phi \hat{x} - \cos \, \phi \hat{y}$$

$$\hat{v}(\theta, \phi) = -\cos \theta \cos \phi \hat{x} - \cos \theta \sin \phi \hat{y} + \sin \theta \hat{z}$$
 (12)

$$\hat{r}(\theta, \phi) = \sin \theta \cos \phi \hat{x} + \cos \theta \sin \phi \hat{y} + \cos \theta \hat{z}$$

Applying the reflection operator to the polarization vectors and propagating vector yields

$$\Re(\hat{h}(\theta, \phi)) = -\hat{h}(\theta, \phi)$$

$$\Re(\hat{v}(\theta, \phi)) = \hat{v}(\theta, \phi)$$

$$\Re(\hat{r}(\theta, \phi)) = \hat{r}(\theta, \phi)$$
(13)

Hence the electric field E' in the complementary problem obtained from the reflection operation is

$$\mathbf{E}'(\theta, \, \phi) = \Re(\mathbf{E}(\theta, \, \phi)) = -E_h(\theta, \, -\phi)\hat{h}(\theta, \, \phi)$$
$$+E_v(\theta, \, -\phi)\hat{v}(\theta, \, \phi) \tag{14}$$

However, since the electric field E' can also be expressed in terms of the polarization vectors

$$\mathbf{E}'(\theta, \, \phi) = E_h'(\theta, \, \phi)\hat{h}(\theta, \, \phi) + E_v'(\theta, \, \phi)\hat{v}(\theta, \, \phi) \tag{15}$$

we obtain the following relations by comparing the above two equations:

$$E'_{h}(\theta, -\phi) = -E_{h}(\theta, \phi)$$

$$E'_{v}(\theta, -\phi) = E_{v}(\theta, \phi)$$
(16)

for any θ and ϕ . The above results are essential to deriving the symmetry properties of passive emission and active scattering coefficients in the next two sections.

It should be noted that $E'_v(\theta, \phi)$ and $E'_h(\theta, \phi)$ are the polarization components of a wave propagating in the direction $\hat{r}(\theta, \phi)$, while $E_v(\theta, -\phi)$ and $E_h(\theta, -\phi)$ are those of an image wave propagating along the $\hat{r}(\theta, -\phi)$ direction. The propagation directions of these two waves are not parallel to each other.

4. Symmetry of Polarimetric Brightness Temperatures

Thermal emissions from geophysical media are electromagnetic radiations from fluctuating currents representing the random thermal motions of charged particles with their energy states changed by the absorption and scattering effects of the media. If the media have geometric directional features or anisotropic medium properties, emission from such media becomes partially polarized with its polarization state described by a Stokes vector I_s with four parameters, T_v , T_h , U, and V[Tsang et al., 1985]:

$$I_{s} = \begin{bmatrix} T_{v} \\ T_{h} \\ U \\ V \end{bmatrix} = c \begin{bmatrix} \langle E_{v}E_{v}^{*} \rangle \\ \langle E_{h}E_{h}^{*} \rangle \\ 2 \operatorname{Re} \langle E_{v}E_{h}^{*} \rangle \\ 2 \operatorname{Im} \langle E_{v}E_{h}^{*} \rangle \end{bmatrix}$$
(17)

where c is a constant and E_h and E_v are the horizontally and vertically polarized components of the radiated electric fields illustrated in Figure 2. The angle brackets denote the ensemble average, taking into consideration the random motion of charges or random distribution of medium parameters. The asterisk denotes the complex conjugate.

Before deriving the symmetry properties of thermally radiated electric fields, the implications of reflection symmetry on medium parameters and thermally excited electric currents J are discussed. Regarding medium parameters there are two types of media, deterministic and random. The medium parameters, such as permittivity and permeability, of a deterministic target are deterministic functions in space, while a random medium has to be realized by an ensemble of deterministic targets. Since deterministic targets can be considered a special case of random media with only one realization in the ensemble, it is not necessary to separately discuss the case of deterministic targets. For random media, reflection symmetry means that for any realization of the medium there is another realization. which is reflection symmetric to that realization. For example, the reflection image of an isotropic medium with $\varepsilon = \varepsilon(x, y, z)$ and $\mu = \mu(x, y, z)$ is a medium with $\varepsilon = \varepsilon(x, -y, z)$ and $\mu = \mu(x, -y, z)$. (The general conditions for two targets to be reflection symmetric to each other are detailed in section 6.) Besides a direct effect on the wave propagation and scattering, the symmetry of medium parameters implies the statistical symmetry properties of thermally induced electric currents and, consequently, the electromagnetic fields generated by the current sources. According to the fluctuation and dissipation theorem [Landau and Lifshitz, 1960;

Yueh and Kwok, 1993], the current density I due to the random motion of charges is random with zero mean and is spatially uncorrelated. In addition, its second moment is linearly proportional to the parameters characterizing the lossy effects of the medium, such as the imaginary parts of permittivity and permeability. Hence the random current can be characterized by a random process with the same symmetry properties as those of medium parameters. On the basis of the above discussion it is concluded that if we define an original problem representing a realization of the random medium and the random current, we can always find another realization, which forms the corresponding complementary problem defined in section 2, as long as the random medium is reflection symmetric.

For each realization of random currents and medium, (16) implies that the electric field E' radiated from the image current sources in the complementary problem must be related to the radiated electric field in the original problem by

$$|E'_h(\theta, -\phi)|^2 = |E_h(\theta, \phi)|^2$$

$$|E'_v(\theta, -\phi)|^2 = |E_v(\theta, \phi)|^2$$
(18)

$$E'_{v}(\theta, -\phi)E'_{h}*(\theta, -\phi) = -E_{v}(\theta, \phi)E'_{h}(\theta, \phi)$$

For media with reflection symmetry, which allows us to drop the superscript, ensemble averaging the above equation over all realizations of random fluctuating currents and medium parameters leads to

$$\langle |E_h(\theta, -\phi)|^2 \rangle = \langle |E_h(\theta, \phi)|^2 \rangle$$

$$\langle |E_v(\theta, -\phi)|^2 \rangle = \langle |E_v(\theta, \phi)|^2 \rangle \tag{19}$$

$$\langle E_v(\theta, -\phi) E_h^*(\theta, -\phi) \rangle = -\langle E_v(\theta, \phi) E_h^*(\theta, \phi) \rangle$$

Using (17), we can further translate the above equations into the symmetries of Stokes parameters:

$$T_{v}(\theta, \phi) = T_{v}(\theta, -\phi)$$

$$T_{h}(\theta, \phi) = T_{h}(\theta, -\phi)$$

$$U(\theta, \phi) = -U(\theta, -\phi)$$

$$V(\theta, \phi) = -V(\theta, -\phi)$$
(20)

Hence T_v and T_h of reflection symmetric media are even functions of ϕ , while the third and fourth Stokes parameters are odd functions.

Note that the relation shown in (20) is derived for the special case in which the x-z plane is the symmetry plane. However, following the same argument leading to (20) allows us to extend the result to the case in which the symmetry plane is perpendicular to the x-y plane and makes an azimuth angle ϕ_r with the x-z plane. The result is

$$T_{\nu}(\theta, 2\phi_{r} - \phi) = T_{\nu}(\theta, \phi)$$

$$T_{h}(\theta, 2\phi_{r} - \phi) = T_{h}(\theta, \phi)$$

$$U(\theta, 2\phi_{r} - \phi) = -U(\theta, \phi)$$

$$V(\theta, 2\phi_{r} - \phi) = -V(\theta, \phi)$$
(21)

for a medium reflection symmetric to the plane defined by the azimuth angle ϕ_r .

In the following, using the general results given in (21), we analyze the symmetry properties of Stokes parameters for two special cases: (1) a medium which is reflection symmetric with respect to both the x-z and y-z planes and (2) a medium that is reflection symmetric to any plane perpendicular to the x-y plane.

For case 1, in which both the x-z ($\phi_r = 0$) and y-z ($\phi_r = \pi/2$) planes are symmetry planes of the medium, substituting $\phi_r = \pi/2$ and replacing ϕ by $-\phi$ in (21) give

$$T_{v}(\theta, \pi + \phi) = T_{v}(\theta, -\phi)$$

$$T_{h}(\theta, \pi + \phi) = T_{h}(\theta, -\phi)$$

$$U(\theta, \pi + \phi) = -U(\theta, -\phi)$$

$$V(\theta, \pi + \phi) = -V(\theta, -\phi)$$
(22)

Further substituting (20) into (22) shows that the Stokes vector is symmetric to the origin

$$I_s(\theta, \phi) = I_s(\theta, \phi + \pi) \tag{23}$$

This is expected for media reflection symmetric to both the x-z and y-z planes.

The second special case corresponds to the situation in which the media do not have preferred azimuth-directional features or are azimuthally symmetric. This case is important because most natural media belong to this category. To be precise, the definition of azimuthal symmetry is that a medium is azimuthally symmetric if and only if the medium is reflection symmetric with respect to any plane perpendicular to the x-y plane. This allows a

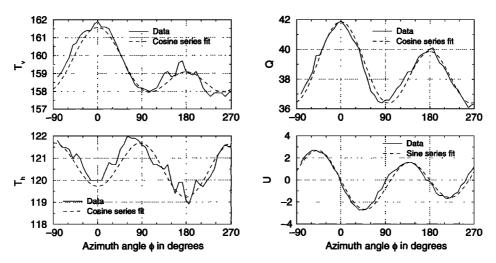


Figure 3. Directional dependence of the Stokes parameters of thermal emission from sea surfaces measured with an incidence angle of 40° using an aircraft K band multipolarization radiometer [Yueh et al., 1994c]. $Q = (T_v - T_h)$.

derivation of the symmetry properties of Stokes parameters using (21) along with the azimuthal symmetry assumption in two steps: First, letting $\phi_r = \phi/2$ shows that the Stokes parameters are not a function of ϕ . Next, substituting $\phi_r = \phi$ into (21) shows that U and V are zeros for an arbitrary azimuth angle ϕ . Hence the Stokes vector of thermal radiation from azimuthally symmetric targets is independent of azimuth observation direction, and furthermore the third and fourth Stokes parameters are zero.

The symmetry relations described in (20) are evident in Figure 3, illustrating the polarimetric brightness temperatures of sea surfaces measured with an incidence angle of 40° at K band (19.35 GHz) by Yueh et al. [1994c]. As shown, T_v , T_h , and $Q = T_v - T_h$ are symmetric with respect to the wind direction ($\phi = 0^{\circ}$), while the third Stokes parameter U is an odd function of ϕ . For easy comparison, second-harmonic cosine (sine) series fits of T_v , T_h , and Q(U) are also included. Note that because of the up/downwind asymmetric sea surface features, such as foams and hydrodynamically modulated capillary waves, the Stokes parameters are less symmetric with respect to the crosswind direction $\phi = 90^{\circ}$.

5. Symmetry of Polarimetric Scattering Coefficients

Recent interest in the symmetry of polarimetric scattering coefficients arise from the predictions of

theoretical scattering models [Borgeaud et al., 1987] and experimental observations for the polarimetric remote sensing of geophysical media [Nghiem et al., 1993a], indicating that the copolarized and cross-polarized components of backscattered fields are uncorrelated for media with azimuthal symmetry, i.e., that the media are reflection symmetric with respect to any plane perpendicular to the x-y plane. In particular, this relation has been successfully used to estimate the cross-polarization couplings existing in polarimetric radars using the responses from distributed targets with azimuthal symmetry [van Zyl, 1990]. Explanation of this complete decorrelation between copolarized and crosspolarized responses from azimuthally symmetric media was conducted by Nghiem et al. [1992], based on the condition given by (11) in their paper. In contrast, this paper directly derives from the Maxwell equations the symmetry relations of polarimetric bistatic and monostatic scattering coefficients of reflection symmetric media, and it shows that the complete decorrelation of copolarized and cross-polarized responses from targets with azimuthal symmetry is a special case of the general results.

Figure 4 depicts the scattering configuration, in which a target is illuminated by a plane wave with horizontally and vertically polarized components E_{hi} and E_{vi} incident from the direction θ_i and ϕ_i and scattered into the direction θ and ϕ with the horizontally and vertically polarized electric field com-

ponents denoted by E_{hs} and E_{vs} . The polarimetric scattering matrix elements $f_{\alpha\beta}$, with α and β being either h or v, relate the incident electric fields to the scattered electric fields by the following equation:

$$\begin{bmatrix} E_{hs}(\theta, \phi) \\ E_{vs}(\theta, \phi) \end{bmatrix} = \frac{e^{ikr}}{r} \begin{bmatrix} f_{hh}(\theta, \phi; \theta_i, \phi_i) & f_{hv}(\theta, \phi; \theta_i, \phi_i) \\ f_{vh}(\theta, \phi; \theta_i, \phi_i) & f_{vv}(\theta, \phi; \theta_i, \phi_i) \end{bmatrix}$$

$$\begin{bmatrix} E_{hi}(\theta_i, \phi_i) \\ E_{vi}(\theta_i, \phi_i) \end{bmatrix}$$
(24)

where r is the range from the target to the receiver. Similar notations are used with the addition of a prime indicating the electromagnetic fields in the complementary problem. For example, the scattering matrix elements in the complementary problem are denoted by $f'_{\alpha\beta}$.

According to (16), the electric fields in the original and complementary problems are related by

$$E'_{hs}(\theta, -\phi) = -E_{hs}(\theta, \phi)$$

$$E'_{vs}(\theta, -\phi) = E_{vs}(\theta, \phi)$$
(25)

and

$$E'_{hi}(\theta_i, -\phi_i) = -E_{hi}(\theta_i, \phi_i)$$

$$E'_{vi}(\theta_i, -\phi_i) = E_{vi}(\theta_i, \phi_i)$$
(26)

Because the above equations are valid for arbitrary excitations, the scattering matrix elements of the original and complementary media are therefore related by

$$f'_{hh}(\theta, -\phi; \theta_i, -\phi_i) = f_{hh}(\theta, \phi; \theta_i, \phi_i)$$

$$f'_{hv}(\theta, -\phi; \theta_i, -\phi_i) = -f_{hv}(\theta, \phi; \theta_i, \phi_i)$$

$$f'_{vh}(\theta, -\phi; \theta_i, -\phi_i) = -f_{vh}(\theta, \phi; \theta_i, \phi_i)$$

$$f'_{vv}(\theta, -\phi; \theta_i, -\phi_i) = f_{vv}(\theta, \phi; \theta_i, \phi_i)$$

$$(27)$$

For deterministic targets with reflection symmetry the complementary problem becomes the original problem. The prime can be dropped, leading to the results that copolarized scattering matrix elements are even functions with respect to x-z plane, while the cross-polarized components are odd functions. An interesting special case is that when $\phi = \phi_i = 0$, i.e., the incident and observation directions are on the reflection plane, the cross-polarized components f_{hv} and f_{vh} become zero, as expected.

For randomly distributed targets or rough sur-

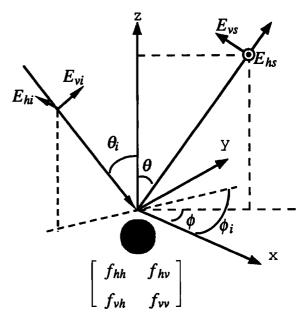


Figure 4. Bistatic scattering configuration.

faces their scattering properties are described by the polarimetric bistatic scattering coefficients:

$$\gamma_{\alpha\beta\mu\nu}(\theta, \phi; \theta_i, \phi_i) = \frac{4\pi \langle f_{\alpha\beta}(\theta, \phi; \theta_i, \phi_i) f_{\mu\nu}^*(\theta, \phi; \theta_i, \phi_i) \rangle}{A \cos \theta_i}$$
(28)

with α , β , μ , and ν being either h or v. The factor A is the illuminated area used to normalize the scattering cross sections. The scattering coefficient $\gamma_{\alpha\beta\mu\nu}$ characterizes the correlation between $f_{\alpha\beta}$ and $f_{\mu\nu}$ with the ensemble average, denoted by angle brackets, taken over the distributions of all the medium parameters, including the permittivity, permeability, and surface roughness.

For each realization of random medium or rough surface the polarimetric bistatic scattering coefficients, denoted by $\gamma'_{\alpha\beta\mu\nu}$, of the complementary problem can be shown to be related to the scattering coefficient of the original problem by

$$\gamma'_{\alpha\beta\mu\nu}(\theta, -\phi; \theta_i, -\phi_i) = \gamma_{\alpha\beta\mu\nu}(\theta, \phi; \theta_i, \phi_i)$$

$$\alpha = \beta \text{ and } \mu = \nu \quad \text{or} \quad \alpha \neq \beta \text{ and } \mu \neq \nu$$

$$\gamma'_{\alpha\beta\mu\nu}(\theta, -\phi; \theta_i, -\phi_i) = -\gamma_{\alpha\beta\mu\nu}(\theta, \phi; \theta_i, \phi_i)$$
otherwise

by substituting the symmetry relations given in (27) into (28). If the random medium is reflection symmetric with respect to the x-z plane, the comple-

mentary problem reduces to the original problem, meaning that (29) reduces to

$$\gamma_{\alpha\beta\mu\nu}(\theta, -\phi; \theta_i, -\phi_i) = \gamma_{\alpha\beta\mu\nu}(\theta, \phi; \theta_i, \phi_i)$$

$$\alpha = \beta \text{ and } \mu = \nu \quad \text{or} \quad \alpha \neq \beta \text{ and } \mu \neq \nu$$

$$\gamma_{\alpha\beta\mu\nu}(\theta, -\phi; \theta_i, -\phi_i) = -\gamma_{\alpha\beta\mu\nu}(\theta, \phi; \theta_i, \phi_i)$$
(30)

otherwise

Note that the scattering coefficients denoting the correlation between copolarized and cross-polarized responses have odd symmetry, while the others are symmetric.

In the case of backscattering, where transmitting and receiving antennas are colocated, the polarimetric backscattering coefficients are related to the bistatic scattering coefficients by

$$\sigma_{\alpha\beta\mu\nu}(\theta_i, \, \phi_i) = \cos \, \theta_i \gamma_{\alpha\beta\mu\nu}(\theta_i, \, \phi_i + \pi; \, \theta_i, \, \phi_i)$$
 (31)

In the following discussions, σ_{hhhh} , σ_{vvvv} , σ_{hvhv} , and σ_{vhvh} are represented by conventional notations σ_{hh} , σ_{vv} , σ_{hv} , and σ_{vh} . Because the factor $\cos \theta_i$ is not a function of the azimuth angle ϕ_i , the symmetry of polarimetric backscattering coefficients can be easily reduced from that of bistatic scattering coefficients. Explicitly, the conventional backscattering coefficients and the correlations between two copolarized or two cross-polarized responses are even functions of ϕ_i :

$$\sigma_{hh}(\theta_{i}, -\phi_{i}) = \sigma_{hh}(\theta_{i}, \phi_{i})$$

$$\sigma_{vv}(\theta_{i}, -\phi_{i}) = \sigma_{vv}(\theta_{i}, \phi_{i})$$

$$\sigma_{hhvv}(\theta_{i}, -\phi_{i}) = \sigma_{hhvv}(\theta_{i}, \phi_{i})$$

$$\sigma_{hv}(\theta_{i}, -\phi_{i}) = \sigma_{hv}(\theta_{i}, \phi_{i})$$

$$\sigma_{vh}(\theta_{i}, -\phi_{i}) = \sigma_{vh}(\theta_{i}, \phi_{i})$$

$$\sigma_{hvvh}(\theta_{i}, -\phi_{i}) = \sigma_{hvvh}(\theta_{i}, \phi_{i})$$

$$\sigma_{hvvh}(\theta_{i}, -\phi_{i}) = \sigma_{hvvh}(\theta_{i}, \phi_{i})$$

while the correlations between copolarized and cross-polarized backscatters are odd functions:

$$\sigma_{hhhv}(\theta_{i}, -\phi_{i}) = -\sigma_{hhhv}(\theta_{i}, \phi_{i})$$

$$\sigma_{hhvh}(\theta_{i}, -\phi_{i}) = -\sigma_{hhvh}(\theta_{i}, \phi_{i})$$

$$\sigma_{hvv}(\theta_{i}, -\phi_{i}) = -\sigma_{hvv}(\theta_{i}, \phi_{i})$$

$$\sigma_{hvv}(\theta_{i}, -\phi_{i}) = -\sigma_{hvv}(\theta_{i}, \phi_{i})$$

$$\sigma_{hvv}(\theta_{i}, -\phi_{i}) = -\sigma_{hvv}(\theta_{i}, \phi_{i})$$
(33)

For the special case in which the incidence direction is on the reflection plane ($\phi_i = 0$), (33) implies that

$$\sigma_{hhhv}(\theta_i, 0) = \sigma_{hhvh}(\theta_i, 0) = \sigma_{hvvv}(\theta_i, 0)$$

$$= \sigma_{vhvv}(\theta_i, 0) = 0$$
(34)

meaning that the copolarized and cross-polarized responses are uncorrelated. This has been observed in the results of many scattering models for geophysical media with reflection symmetry, such as the random medium model by *Borgeaud et al.* [1987], but has not proved to be exact until the study presented here.

Note that (32) shows that the backscattering coefficients σ_{hh} and σ_{vv} are even functions of the azimuth angle ϕ_i . This has been well known in the microwave backscattering coefficients of wind-generated sea surfaces, which are symmetric with respect to the wind direction. For example, the Seasat A satellite scatterometer (SASS) geophysical model function [Wentz et al., 1984] empirically relates the ocean wind vectors to the microwave backscattering coefficient σ_0 (σ_{hh} or σ_{vv}) by the following harmonics expansion:

$$\sigma_0 = A_0 + A_1 \cos \phi_i + A_2 \cos 2\phi_i \tag{35}$$

which is an even function of the azimuth angle ϕ_i . Figures 5a and 5b illustrate σ_{vv} and σ_{hh} , calculated by using the SASS geophysical model function, as a function of ϕ_i for the wind speed of 11.5 m/s. The plots also include the backscatters measured by NUSCAT during the Surface Wave Dynamics Experiment (SWADE) in 1991 [Nghiem et al., 1993b]. As shown, σ_{hh} and σ_{vv} are symmetric functions of ϕ_i . To study the symmetry properties of the other polarimetric backscattering coefficients, also included in Figure 5 are the theoretical polarimetric backscattering coefficients calculated by using a two-scale sea surface scattering model originally developed by Durden and Vesecky [1985] for σ_{vv} and σ_{hh} and generalized to polarimetric scattering by Yueh et al. [1993]. Figure 5e reveals a 180° phase change in ρ_{hhhv} and ρ_{hvm} at the upwind $(\phi_i = 0^\circ)$ and downwind (180°) directions, indicating that theoretical correlations between copolarized and crosspolarized responses from sea surfaces have odd symmetry—as proved in this paper based on reflection symmetry assumption. Note that as expected, all backscattering coefficients are less symmetric

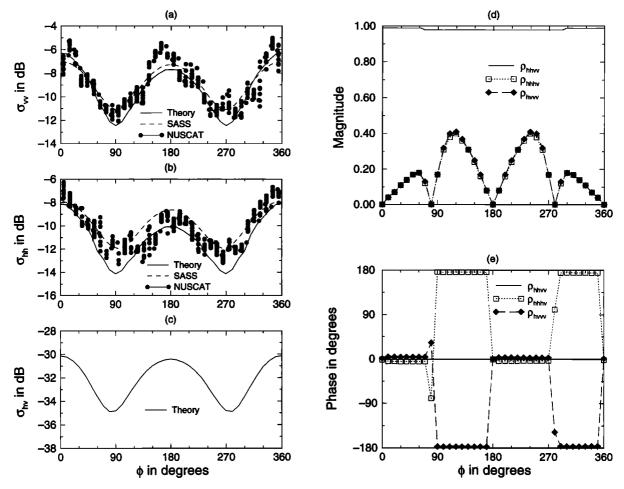


Figure 5. Sea surface backscattering coefficients versus azimuth angle for the wind speed of 11.5 m/s at the incidence angle $\theta_i = 30^\circ$: (a) σ_{vv} , (b) σ_{hh} , (c) σ_{hv} , (d) magnitudes of correlation coefficients ρ_{hhvv} , ρ_{hhhv} , and ρ_{hvvv} , and (e) phase angles of correlation coefficients ρ_{hhvv} , ρ_{hhhv} , and ρ_{hvvv} . The copolarized responses σ_{hh} and σ_{vv} include the data calculated from the SASS geophysical model function [Wentz et al., 1984], the data collected by NUSCAT during SWADE [Nghiem et al., 1993b], and a theoretical sea surface scattering model [Yueh et al., 1993]. The coefficients for the empirical sea surface spectrum [Durden and Vesecky, 1985] are used except that the spectrum amplitude is raised from 0.004 to 0.006 to better fit theoretical σ_{vv} and σ_{hh} with SASS and NUSCAT. The correlation coefficients are defined as follows: $\rho_{hhvv} = \sigma_{hhvv}/(\sigma_{hh}\sigma_{vv})^{1/2}$, $\rho_{hhhv} = \sigma_{hhhv}/(\sigma_{hh}\sigma_{hv})^{1/2}$, and $\rho_{hvvv} = \sigma_{hvvv}/(\sigma_{hv}\sigma_{vv})^{1/2}$.

with respect to the crosswind direction because of the up/downwind asymmetric features in wind-induced sea surfaces.

6. Conditions for Reflection Symmetry

Though reflection symmetry has been used frequently to verify solutions or simplify problems, the criteria for media to be reflection symmetric have not yet been addressed. In fact, this is a question that cannot be answered based on electromagnetics

alone, because the medium parameters, such as the permittivity and permeability, are determined by how the particles react under the influence of electromagnetic forces together with other mechanical forces present in the media. Hence the most general approach to decide whether a medium is reflection symmetric should be to show that all relevant physical laws, which govern particle motions, such as Lorentz force for charged particles and Schrödinger's equation in quantum mechanics, have to be

reflection symmetric. However, to avoid such complexity and to discuss cases as generally as possible without being constrained to specific physical mechanisms or governing laws, this paper investigates a general type of media, i.e., bianisotropic media [Kong, 1986].

In electromagnetics the macroscopic medium properties are described by a set of constitutive relations, which connect the vector fields E, H, D, and B. The constitutive relations of a bianisotropic medium are given by

$$\mathbf{D} = \boldsymbol{\varepsilon} \cdot \mathbf{E} + \boldsymbol{\chi} \cdot \mathbf{H}$$

$$\mathbf{B} = \boldsymbol{\zeta} \cdot \mathbf{E} + \boldsymbol{\mu} \cdot \mathbf{H}$$
(36)

where ε and μ are the permittivity and permeability tensors, χ describes how electric displacement reacts in the presence of a magnetic field, and ζ causes the magnetic flux density to respond to the electric field. Similarly, the constitutive relation of the medium in the complementary problem is described by

$$\mathbf{D}' = \mathbf{\varepsilon}' \cdot \mathbf{E}' + \chi' \cdot \mathbf{H}'$$

$$\mathbf{B}' = \zeta' \cdot \mathbf{E}' + \mu' \cdot \mathbf{H}'$$
(37)

For convenience, the tensors described above shall be represented by a three-by-three matrix in the Cartesian coordinate. For example, the matrix ε is denoted in the Cartesian coordinate as

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$
(38)

Similar notations are used for all the other tensors. Because the reflection operations defined by (4) and (5) hold for arbitrary field quantities, it is straightforward to show that ε' , μ' , χ' , and ζ' , evaluated at the position $\mathbf{r} = (x, y, z)$, are related to ε , μ , χ , and ζ , evaluated at $\mathbf{r} = (x, -y, z)$, by

$$\begin{bmatrix} \varepsilon'_{xx} & \varepsilon'_{xy} & \varepsilon'_{xz} \\ \varepsilon'_{yx} & \varepsilon'_{yy} & \varepsilon'_{yz} \\ \varepsilon'_{zx} & \varepsilon'_{zy} & \varepsilon'_{zz} \end{bmatrix}_{\mathbf{r}=(x,y,z)}$$

$$= \begin{bmatrix} \varepsilon_{xx} & -\varepsilon_{xy} & \varepsilon_{xz} \\ -\varepsilon_{yx} & \varepsilon_{yy} & -\varepsilon_{yz} \\ \varepsilon_{zx} & -\varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$
(39)

$$\begin{bmatrix} \mu_{xx} & \mu_{xy} & \mu_{xz} \\ \mu'_{yx} & \mu'_{yy} & \mu'_{yz} \\ \mu'_{zx} & \mu'_{zy} & \mu'_{zz} \end{bmatrix}_{\mathbf{r}=(x,y,z)}$$

$$= \begin{bmatrix} \mu_{xx} & -\mu_{xy} & \mu_{xz} \\ -\mu_{yx} & \mu_{yy} & -\mu_{yz} \\ \mu_{zx} & -\mu_{zy} & \mu_{zz} \end{bmatrix}_{\mathbf{r}=(x,-y,z)}$$
(40)

(36)
$$\begin{bmatrix} \chi'_{xx} & \chi'_{xy} & \chi'_{xz} \\ \chi'_{yx} & \chi'_{yy} & \chi'_{yz} \\ \chi'_{zx} & \chi'_{zy} & \chi'_{zz} \end{bmatrix}_{\mathbf{r}=(x,y,z)}$$

$$= \begin{bmatrix} -\chi_{xx} & \chi_{xy} & -\chi_{xz} \\ \chi_{yx} & -\chi_{yy} & \chi_{yz} \\ -\chi_{zx} & \chi_{zy} & -\chi_{zz} \end{bmatrix}_{\mathbf{r}=(x,-y,z)}$$
(41)

and

(37)
$$\begin{bmatrix} \zeta'_{xx} & \zeta'_{xy} & \zeta'_{xz} \\ \zeta'_{yx} & \zeta'_{yy} & \zeta'_{yz} \\ \zeta'_{zx} & \zeta'_{zy} & \zeta'_{zz} \end{bmatrix}_{\mathbf{r}=(x,y,z)}$$

$$= \begin{bmatrix} -\zeta_{xx} & \zeta_{xy} & -\zeta_{xz} \\ \zeta_{yx} & -\zeta_{yy} & \zeta_{yz} \\ -\zeta_{zx} & \zeta_{zy} & -\zeta_{zz} \end{bmatrix}_{\mathbf{r}=(x,-y,z)}$$
(42)

For homogeneous, bianisotropic media with reflection symmetry with respect to the x-z plane the above relations imply that

$$\varepsilon_{xy} = \varepsilon_{yx} = \varepsilon_{yz} = \varepsilon_{zy} = 0$$

$$\mu_{xy} = \mu_{yx} = \mu_{yz} = \mu_{zy} = 0$$

$$\chi_{xx} = \chi_{yy} = \chi_{zz} = \chi_{xz} = \chi_{zx} = 0$$

$$\zeta_{xx} = \zeta_{yy} = \zeta_{zz} = \zeta_{xz} = \zeta_{zx} = 0$$
(43)

Bianisotropic media satisfying the above symmetry conditions have been used by *Yang and Uslenghi* [1993] in their analysis of planar bianisotropic waveguides.

Special cases of bianisotropic media include isotropic media, biaxial media, chiral media, and gyrotropic media. For isotropic media with $\chi = \zeta = 0$ and ϵ and μ being a scalar, (39) and (40) reduce to

$$\varepsilon'(x, -y, z) = \varepsilon(x, y, z)$$
(44)

$$\mu'(x, -y, z) = \mu(x, y, z)$$

Therefore for deterministic, isotropic media the media are reflection symmetric if and only if ε and μ are even functions of y. In contrast, for random media with ε and μ described by spatial random processes, reflection symmetry means that the governing random processes for ε and μ are symmetric with respect to the reflection plane. In other words, if $\varepsilon = \varepsilon(x, y, z)$ and $\mu = \mu(x, y, z)$ are realizations of the random medium, then there exists another realization with $\varepsilon = \varepsilon(x, -y, z)$ and $\mu = \mu(x, -y, z)$.

For a homogeneous biaxial medium, χ and ζ are zero, the permeability is a scalar, and the permittivity is a diagonal tensor in the principal coordinate described by

$$\varepsilon = \varepsilon_x \hat{x}' \hat{x}' + \varepsilon_y \hat{y}' \hat{y}' + \varepsilon_z \hat{z}' \hat{z}'$$
 (45)

If the medium is rotated with respect to the y axis by an angle ψ , then the matrix representation of ε in the laboratory frame becomes

$$\varepsilon = \begin{bmatrix} \varepsilon_x \cos^2 \psi + \varepsilon_z \sin^2 \psi & 0 & (\varepsilon_z - \varepsilon_x) \cos \psi \sin \psi \\ 0 & \varepsilon_y & 0 \\ (\varepsilon_z - \varepsilon_x) \cos \psi \sin \psi & 0 & \varepsilon_x \sin^2 \psi + \varepsilon_z \cos^2 \psi \end{bmatrix}$$
(46)

Hence the medium is reflection symmetric with respect to the x-z plane but not the other vertical planes.

Unlike biaxial media, a gyrotropic medium is anisotropic but nonreciprocal [Kong, 1986], including either electrically gyrotropic media or magnetically gyrotropic media. The general form of ε for an electrically gyrotropic medium, such as electron plasma under the influence of an applied external constant magnetic field, with the optical axis pointing in the z direction is given by

$$\varepsilon = \begin{bmatrix} \varepsilon & i\varepsilon_g & 0 \\ -i\varepsilon_g & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$
 (47)

As discussed, the medium is reflection symmetric with respect to the x-y plane but not the x-z or y-z planes.

For chiral media, which have been receiving

significant attention in recent years, the constitutive relation is

$$\mathbf{D} = \varepsilon \mathbf{E} + i \chi \mathbf{H}$$

$$\mathbf{B} = -i \chi \mathbf{E} + \mu \mathbf{H}$$
(48)

Hence according to (43), homogeneous chiral media are not reflection symmetric with respect to any vertical plane. However, the reflection image of a right-handed chiral medium ($\chi > 0$) is a left-handed chiral medium ($\chi < 0$), and vice versa.

7. Summary

This paper has analyzed the symmetry properties of polarimetric active scattering and passive emission coefficients of media with reflection symmetry. For passive remote sensing it is shown that the first two Stokes parameters of thermal radiations from reflection symmetric objects are even functions with respect to the symmetry plane, while the third and fourth Stokes parameters are odd functions. For active scattering the correlation coefficients of copolarized and cross-polarized responses are antisymmetric, unlike the other polarimetric scattering coefficients. These symmetry relations are shown to agree with the azimuthal signatures of microwave backscattering and emission measurements of sea surfaces and artificially constructed surfaces with directional features reported in the literature.

Potential applications of the studied symmetry relations include the detection of geophysical directional features using polarimetric remote sensing measurements. For example, the symmetry property of each polarimetric scattering or emission parameter derived in this paper suggests an appropriate functional form for the geophysical model function which relates the radar or radiometer observations to the direction of geophysical features. That is, while a cosine series of the azimuth angle was appropriate for the HH and VV backscattering coefficients in the SASS model function [Wentz et al., 1984] and the brightness temperatures at horizontal and vertical polarizations in the SSM/I model function [Wentz, 1992] of sea surfaces, a sine series should be used in the geophysical model function of the other polarimetric scattering and emission coefficients that have odd symmetry [Yueh et al., 1994b, c].

Acknowledgment. This work was performed under contract with the National Aeronautics and Space Administration at the Jet Propulsion Laboratory, California Institute of Technology.

References

- Borgeaud, M., R. T. Shin, and J. A. Kong, Theoretical models for polarimetric radar clutter, *J. Electromagn.* Waves Appl., 1, 73-89, 1987.
- Durden, S. P., and J. F. Vesecky, A physical radar cross-section model for a wind-driven sea with swell, *IEEE J. Oceanic Eng.*, *OE-10*(4), 445–451, 1985.
- Dzura, M. S., V. S. Etkin, A. S. Khrupin, M. N. Pospelov, and M. D. Raev, Radiometers-polarimeters: Principles of design and applications for sea surface microwave emission polarimetry, paper presented at International Geoscience and Remote Sensing Symposium, Inst. of Electr. and Electron. Eng., Houston, Tex., 1992.
- Etkin, V. S., et al., Radiohydrophysical aerospace research of ocean (in Russian), *Rep. IIp-1749*, USSR, Space Res. Inst., Acad. of Sci., Moscow, 1991.
- Johnson, J. T., J. A. Kong, R. T. Shin, D. H. Staelin, K. O'Neill, and A. W. Lohanick, Third Stokes parameter emission from a periodic water surface, *IEEE Trans. Geosci. Remote Sens.*, 31(5), 1066-1080, 1993.
- Kong, J. A., Electromagnetic Wave Theory, Wiley-Interscience, New York, 1986.
- Landau, L. D., and E. M. Lifshitz, Electrodynamics of Continuous Media, Addison-Wesley, Reading, Mass., 1960.
- Nghiem, S. V., M. E. Veysoglu, R. T. Shin, J. A. Kong, K. O'Neill, and A. Lohanick, Polarimetric passive remote sensing of a periodic soil surface: Microwave measurements and analysis, J. Electromagn. Waves Appl., 5(9), 997-1005, 1991.
- Nghiem, S. V., S. H. Yueh, R. Kwok, and F. K. Li, Symmetry properties in polarimetric remote sensing, *Radio Sci.*, 27(5), 693-711, 1992.
- Nghiem, S. V., S. H. Yueh, R. Kwok, and D. T. Nguyen, Polarimetric remote sensing of geophysical medium structures, *Radio Sci.*, 28(6), 1111-1130, 1993a.
- Nghiem, S. V., F. K. Li, S. H. Lou, and G. Neumann, Ocean remote sensing with airborne Ku-band scatter-ometer, *Proc. Ocean Sympos.*, 1, pp. 20-24, 1993b.

- Tsang, L., J. A. Kong, and R. T. Shin, Theory of Microwave Remote Sensing, Wiley-Interscience, New York, 1985.
- van Zyl, J. J., Calibration of polarimetric radar images using only image parameters and trihedral corner reflectors responses, *IEEE Trans. Geosci. Remote Sens.*, 28(3), 337-348, 1990.
- Wentz, F. J., Measurement of oceanic wind vector using satellite microwave radiometers, *IEEE Trans. Geosci. Remote Sens.*, 30(5), 960-972, 1992.
- Wentz, F. J., S. Peteherych, and L. A. Thomas, A model function for ocean radar cross sections at 14.6 GHz, J. Geophys. Res., 89(C3), 3689-3704, 1984.
- Yang, H.-Y., and P. L. E. Uslenghi, Planar bianisotropic waveguides, *Radio Sci.*, 28(5), 919–927, 1993.
- Yueh, S. H., and R. Kwok, Electromagnetic fluctuations for anisotropic media and the generalized Kirchhoff's law, *Radio Sci.*, 28(4), 471–480, 1993.
- Yueh, S. H., R. Kwok, F. K. Li, S. V. Nghiem, W. J. Wilson, and J. A. Kong, Polarimetric passive remote sensing of wind-generated sea surfaces and ocean wind vectors, *Proc. Ocean Sympos.*, 1, pp. 31-36, 1993.
- Yueh, S. H., S. V. Nghiem, R. Kwok, W. J. Wilson, F. K. Li, J. T. Johnson, and J. A. Kong, Polarimetric thermal emission from periodic water surfaces, *Radio Sci.*, 29(1), 1994a.
- Yueh, S. H., R. Kwok, F. K. Li, S. V. Nghiem, W. J. Wilson, and J. A. Kong, Polarimetric passive remote sensing of ocean wind vectors, *Radio Sci.*, 29(4), 799–814, 1994b.
- Yueh, S. H., W. J. Wilson, S. V. Nghiem, F. K. Li, and W. B. Ricketts, Polarimetric measurements of sea surface brightness temperatures using an aircraft K-band radiometer, *IEEE Trans. Geosci. Remote Sens.*, in press, 1994c.
- R. Kwok, S. V. Nghiem, and S. H. Yueh, Jet Propulsion Laboratory, California Institute of Technology, Mail Stop 300-235, 4800 Oak Grove Drive, Pasadena, CA 91109. (e-mail: kwok@kahuna.jpl.nasa.gov; nghiem@malibu.jpl.nasa.gov; simon@malibu.jpl.nasa.gov)

(Received April 6, 1994; revised August 25, 1994; accepted August 25, 1994.)