equations for all the ASL in the world

## Stratified

### If abundance is known without error

#### If there are proportions

The proportion of each age and/or sex category *z* will be estimated for each sampling stratum *t* as follows:

in which equals the number of fish sampled during sampling stratum classified as age and/or sex category , and equals the number of fish sampled for age and/or sex determination within sampling stratum .

The sampling variance of will be estimated as the following (Cochran 1977), in which represents the total abundance of fish in sampling stratum *t*:

if the finite population correction factor (FPC) is used; otherwise, as the following:

The total abundance by age and/or sex category in each sampling stratum will be estimated as follows:

with variance estimated as

The total abundance by age and/or sex category and its variance will then be estimated by summation as follows:

and

where equals the number of sampling strata.

Finally, the overall proportion by age and/or sex category and its variance will be estimated as follows:

and

where is the total abundance across all sampling periods.

The mean length by age and/or sex for each sampling stratum will be estimated as follows:

where is the length of the *i*th fish sampled of age and/or sex during sampling stratum .

The sampling variance of will be estimated as

if the finite population correction factor (FPC) will be used; otherwise, as the following:

The mean length by age and/or sex category will then be estimated as follows:

with its variance approximated using a Taylor’s series expansion (Mood et al. 1974):

#### If there are no proportions to estimate

The mean length for each sampling stratum will be estimated as follows, where is the length of the *i*th fish sampled within sampling stratum , and is the number of fish in stratum *t* sampled for length:

The sampling variance of will be estimated as

if abundance per stratum is known and if the finite population correction factor is used, otherwise as:

Stratified estimates of mean length will be calculated as follows, in which represents the abundance associated with sampling stratum , represents the total abundance, and *L* represents the number of sampling strata:

and

### If abundance is estimated with error

#### If there are proportions

The proportion of each age and/or sex category *z* will be estimated for each sampling stratum *t* as follows:

in which equals the number of fish sampled during sampling stratum classified as age and/or sex category , and equals the number of fish sampled for age and/or sex determination within sampling stratum .

The sampling variance of will be estimated as the following (Cochran 1977) in which is the estimated abundance of fish in sampling stratum :

if the finite population correction factor (FPC) is used; otherwise, as the following:

The total abundance by age and/or sex category in each sampling stratum will be estimated as follows:

with variance estimated as (Goodman 1960):

The total abundance by age and/or sex category and its variance will then be estimated by summation as follows:

and

where equals the number of sampling strata.

Finally, the overall proportion by age and/or sex category and its variance will be estimated as follows:

with variance estimated by the delta method (Casella & Berger 2002) as:

in which

The mean length by age and/or sex for each sampling stratum will be estimated as follows:

where is the length of the *i*th fish sampled of age and/or sex during sampling stratum .

The sampling variance of will be estimated as

if the finite population correction factor (FPC) is used; otherwise, as

The mean length by age and/or sex category will then be estimated as follows:

with its variance approximated using a Taylor’s series expansion (Mood et al. 1974):

#### If there are no proportions to estimate

The mean length for each sampling stratum will be estimated as follows, where is the length of the *i*th fish sampled within sampling stratum , and is the number of fish in stratum *t* sampled for length:

The sampling variance of will be estimated as

if the finite population correction factor (FPC) is used, otherwise as:

Stratified estimates of mean length will be calculated as follows, in which and represent the estimated abundance and mean length associated with stratum *t*, respectively:

and

in which

and

by means of the delta method (Casella & Berger 2002) and Goodman (1960), respectively.

### If abundance is unknown and sample weights are used

#### If there are proportions

The proportion of each age and/or sex category *z* will be estimated for each sampling stratum *t* as follows:

in which equals the number of fish sampled during sampling stratum classified as age and/or sex category , and equals the number of fish sampled for age and/or sex determination within sampling stratum .

The sampling variance of will be estimated as the following (Cochran 1977):

The overall proportion by age and/or sex category and its variance will be estimated as follows, in which represents the sampling weight associated with stratum *t* and *L* equals the number of strata. It is worth noting that weights are treated as constant (i.e. known without error), therefore all variance estimates must be interpreted as minima without further assumptions.

and

The mean length by age and/or sex for each sampling stratum will be estimated as follows:

where is the length of the *i*th fish sampled of age and/or sex during sampling stratum .

The sampling variance of will be estimated as:

The mean length by age and/or sex category will then be estimated as follows:

with its variance approximated using a Taylor’s series expansion (Mood et al. 1974):

#### If there are no proportions to estimate

The mean length for each sampling stratum will be estimated as follows, where is the length of the *i*th fish sampled within sampling stratum , and is the number of fish in stratum *t* sampled for length:

The sampling variance of will be estimated as:

Stratified estimates of mean length will be calculated as follows, in which and represent the sampling weight and average length associated with sampling stratum , respectively. It is worth noting that weights are treated as constant (i.e. known without error), therefore all variance estimates must be interpreted as minima without further assumptions.

and

## Pooled

### If abundance is known without error

#### If there are proportions

Proportions of each age and/or sex category will be estimated as follows (Cochran 1977):

and

if total abundance is known and the finite population correction factor (FPC) is used; otherwise as

in which denotes the number of fish sampled in age and/or sex category , denotes the total number of fish sampled, and *N* denotes the total abundance.

Total abundance for age and/or sex category will be estimated as

and

The mean length associated with age and/or sex category will be estimated as the following, in which represents the length of the *i*th fish in age and/or sex category *z* and represents the number of fish in age and/or sex category *z* with an associated length measurement:

and

if the finite population correction factor (FPC) is used, otherwise as:

#### If there are no proportions to estimate

The mean length of all fish will be estimated as the following, in which represents the length of the *i*th fish, represents the number of fish with an associated length measurement, and *N* represents the total abundance:

and

if the finite population correction factor (FPC) is used, otherwise as:

### If abundance is estimated with error

#### If there are proportions

Proportions of each age and/or sex category will be estimated as follows (Cochran 1977):

and

if the finite population correction factor (FPC) is used, otherwise as

in which denotes the number of fish sampled in age and/or sex category , denotes the total number of fish sampled, and denotes the estimated abundance.

Total abundance for age and/or sex category will be estimated as follows (Goodman 1960):

and

The mean length associated with age and/or sex category will be estimated as the following, in which represents the length of fish *i* within age and/or sex category category *z*, and denotes the number of fish within age and/or sex category *z* with an associated length measurement:

and

if the finite population correction factor (FPC) is used; otherwise as

#### If there are no proportions to estimate

The mean length of all fish will be estimated as the following, in which represents the length of the *i*th fish, represents the number of fish with an associated length measurement, and represents the estimated total abundance:

and

if the finite population correction factor (FPC) is used, otherwise as:

### If abundance is unknown

#### If there are proportions

Proportions of each age and/or sex category will be estimated as follows (Cochran 1977):

and

in which denotes the number of fish sampled in age and/or sex category , and denotes the total number of fish sampled.

The mean length associated with age and/or sex category will be estimated as the following, in which denotes the length measurement associated with the *i*th fish in age and/or sex category *z*:

and

#### If there are no proportions to estimate

The mean length will be estimated as the following, in which denotes the length measurement associated with the *i*th fish and denotes the number of fish with an associated length measurement:

and

## References

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