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COLLEGE OF ENGINEERING AND TECHNOLOGY
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# 19ECCN1701 – RF and Microwave Engineering

# Unit I – Two Port Network Theory

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## Course Outcome 1:

Analyze the given High Frequency network using S parameters

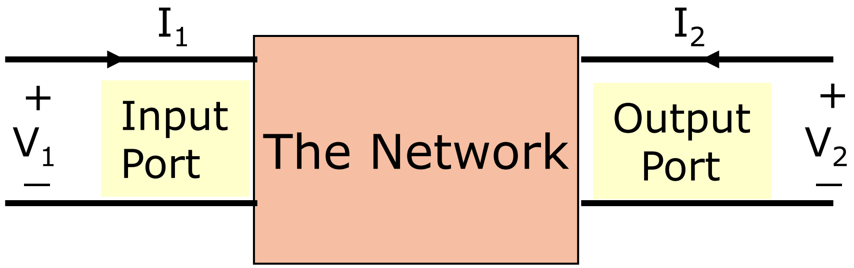
## Learning Outcome 1:

Distinguish Low frequency and High frequency circuit analysis

# Two Port Networks

## Generalities:

The standard configuration of a two port:



# Two Port Networks

## Network Equations

Impedance
Z parameters

$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Admittance
Y parameters

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Transmission
A, B, C, D
parameters

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Hybrid
H parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

# Two Port Networks

## Z parameters:

$$z_{11} = \frac{V_1}{I_1}\bigg|_{I_2=0}$$

$z_{11}$ is the impedance seen looking into port 1 when port 2 is open.

$$z_{12} = \frac{V_1}{I_2}\bigg|_{I_1=0}$$

$z_{12}$ is a transfer impedance. It is the ratio of the voltage at port 1 to the current at port 2 when port 1 is open.

$$z_{21} = \frac{V_2}{I_1}\bigg|_{I_2=0}$$

$z_{21}$ is a transfer impedance. It is the ratio of the voltage at port 2 to the current at port 1 when port 2 is open.

$$z_{22} = \frac{V_2}{I_2}\bigg|_{I_1=0}$$

$z_{22}$ is the impedance seen looking into port 2 when port 1 is open.

# Two Port Networks

## Y parameters:

$$y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

$y_{11}$ is the admittance seen looking into port 1 when port 2 is shorted.

$$y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

$y_{12}$ is a transfer admittance. It is the ratio of the current at port 1 to the voltage at port 2 when port 1 is shorted.

$$y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

$y_{21}$ is a transfer impedance. It is the ratio of the current at port 2 to the voltage at port 1 when port 2 is shorted.

$$y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

$y_{22}$ is the admittance seen looking into port 2 when port 1 is shorted.

# Two Port Networks

## Transmission parameters (A,B,C,D):

The defining equations are:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$A = \frac{V_1}{V_2}\Bigg|_{I_2 = 0} \qquad B = \frac{V_1}{-I_2}\Bigg|_{V_2 = 0}$$

$$C = \frac{I_1}{V_2}\Bigg|_{I_2 = 0} \qquad D = \frac{I_1}{-I_2}\Bigg|_{V_2 = 0}$$

# Two Port Networks

## Hybrid Parameters:

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The equations for the hybrid parameters are:

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2 = 0}$$

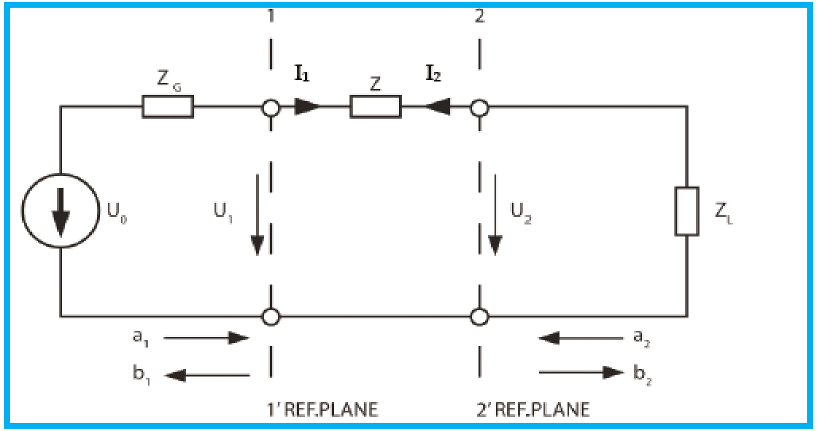
$$h_{12} = \frac{V_1}{V_2} \Big|_{I_1 = 0}$$

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2 = 0}$$

$$h_{22} = \frac{I_2}{V_2} \Big|_{I_1 = 0}$$

# Two Port Networks

## Scattering Parameters




$$a_1 = \frac{U_0}{2\sqrt{Z_0}} = \frac{\text{incident voltage wave (port 1)}}{\sqrt{Z_0}} \quad \frac{U_1^{\text{inc}}}{\sqrt{Z_0}}$$

$$b_1 = \frac{U_1^{\text{refl}}}{\sqrt{Z_0}} = \frac{\text{reflected voltage wave (port 1)}}{\sqrt{Z_0}}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=} \quad S_{21} = \frac{b_2}{a_1}$$

Note that **a** and **b** have the dimension $\sqrt{\text{power}}$

# Characteristics of Microwave

- Microwave Lengths are very small
- Microwave Pulses are very short so that they cab be used for distance or time measurement
- High frequency of microwave means very large bandwidth is available for communication
- Microwave Radiation penetrates fog and clouds, travels in straight lines and give reflections hence can be used for distance and direction measurement
- Microwaves are necessary for communication through satellite because they can pass through ionosphere which reflects low frequency waves
- Microwave Power is absorbed by water or another material containing water so that microwaves can be used for heating and drying

# Low frequency vs High Frequencies

- **At lower frequency**
  - Large wavelength, no phase variation over the devices' physical dimension, circuit theory, lumped-element, R, L, C.
- **At higher frequency**
  - Wavelength shorter than the device's physical dimension, transmission line theory needs to be introduced, distributed elements.

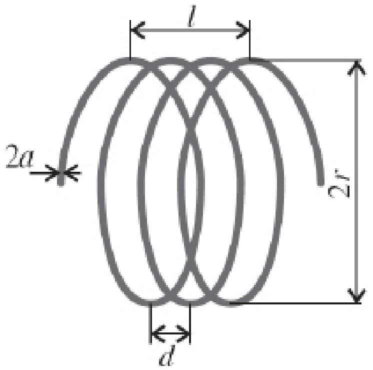
# Low frequency vs High Frequencies

- Lumped components (wires, resistors, capacitors, inductors, connectors etc.) behave differently at low and high frequencies.
- Why?
  - current and voltage vary spatially over the component size
  - Leads to the concept of distributed components!

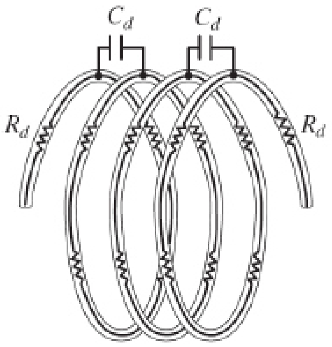
**The KCL and KVL are no more applicable**

# INDUCTORS AT HIGH FREQUENCY

**Low Frequency (Lumped)**



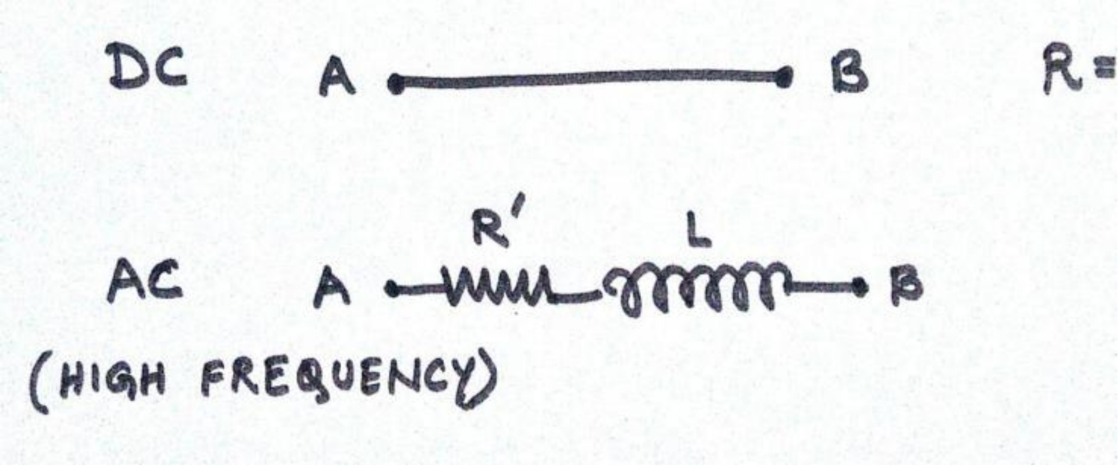

**High Frequency (Distributed)**




$$Z = R + j\omega L$$

$$Z = ?$$

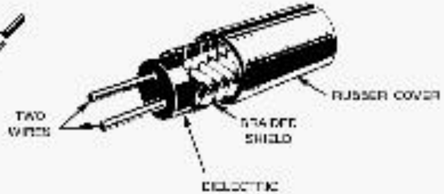
# WIRES AT HIGH FREQUENCY



# Common Types of Transmission Lines

Two-wire line



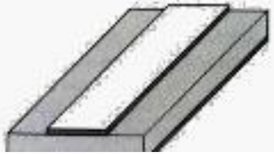



Coaxial





Microstrip




# Network Characterization

## At low frequencies

**H–Parameters**

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

**Y–Parameters**

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

**Z–Parameters**

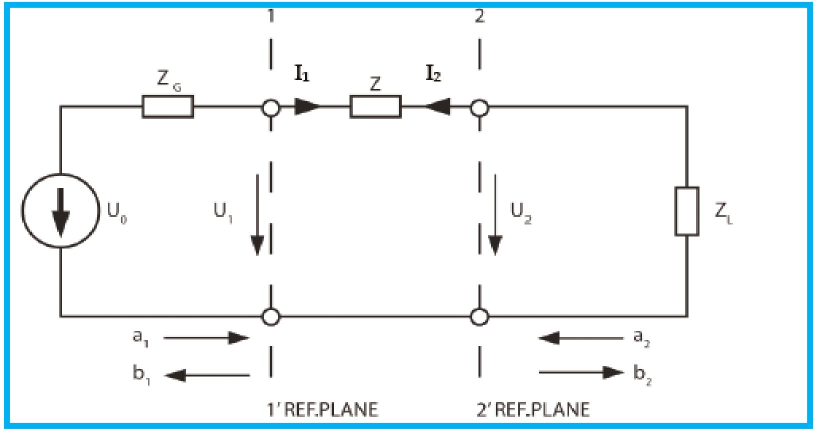
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

# Moving to higher frequencies

- Equipment is not readily available to measure total voltage and total current at the ports of the network.
- Short and open circuits are difficult to achieve over a broad band of frequencies.
- Active devices, such as transistors and tunnel diodes, very often will not be short or open circuit stable.

# Two Port Networks

## Scattering Parameters




$$a_1 = \frac{U_0}{2\sqrt{Z_0}} = \frac{\text{incident voltage wave (port 1)}}{\sqrt{Z_0}} \quad \frac{U_1^{inc}}{\sqrt{Z_0}}$$

$$b_1 = \frac{U_1^{refl}}{\sqrt{Z_0}} = \frac{\text{reflected voltage wave (port 1)}}{\sqrt{Z_0}}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2 =} \quad S_{21} = \frac{b_2}{a_1}$$

Note that **a** and **b** have the dimension $\sqrt{\text{power}}$

# Formulation of S Parameters for a two port network

# CO1:Analyze the given High Frequency network using S parameters.

| | |
|---|---|
| **LO1** | **Distinguish Low frequency and High frequency circuit analysis** |
| **LO2** | **Analyze the microwave network using S parameters.** |

# S Parameters (Scattering Parameters) – Introduction

The background needed for the study of S-parameters consists of two fundamental topics:

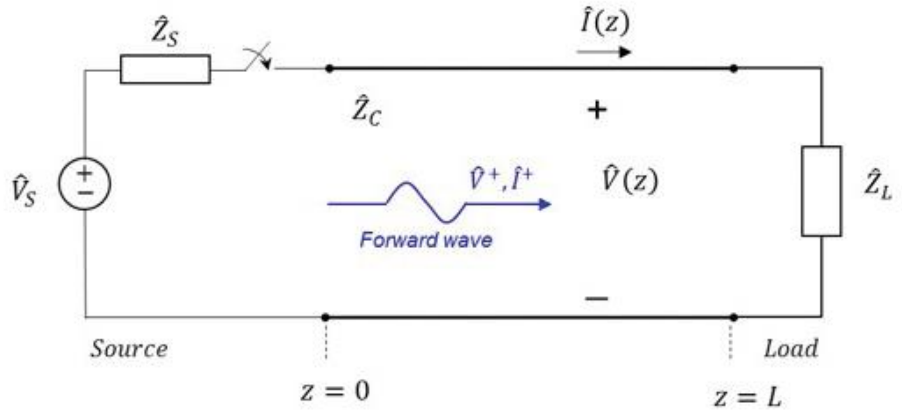
1. Two-port networks &
2. Reflections on transmission lines

# 1. Two-Port Network Theory:

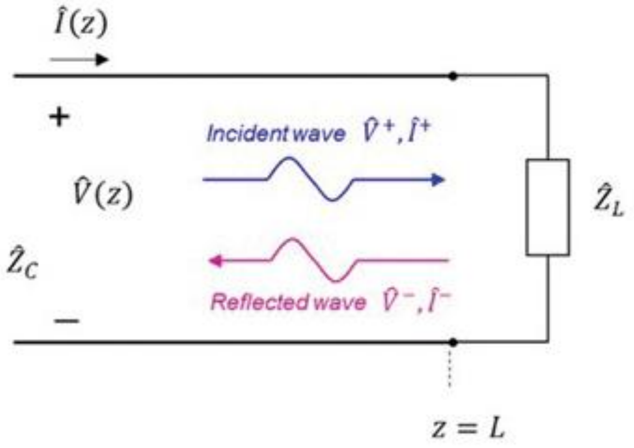
- Two-port network theory is a circuit analysis technique - different from the majority of other approaches.
- Most circuit analysis approaches (Kirchhoff's laws, node voltage/mesh current methods, superposition, and others) provide a way of calculating voltages and currents anywhere in the circuit.
- Thevenin or Norton theorems allow us to obtain an equivalent circuit model with respect to the specified pair of terminals (usually the output terminals, or the output port) of the network.

# 2. Reflections on transmission lines:

- Review of the reflections at the load and at the source, and then proceed to the reflections at a discontinuity along the transmission line.




(a) Transmission line circuit and forward wave




(b) Reflection at the load

# 2. Reflections on transmission lines – Contd..

- The voltage of the reflected wave is related to the voltage of the incident wave by $\hat{V}^{-} = \hat{\Gamma}_L \hat{V}^{+}$

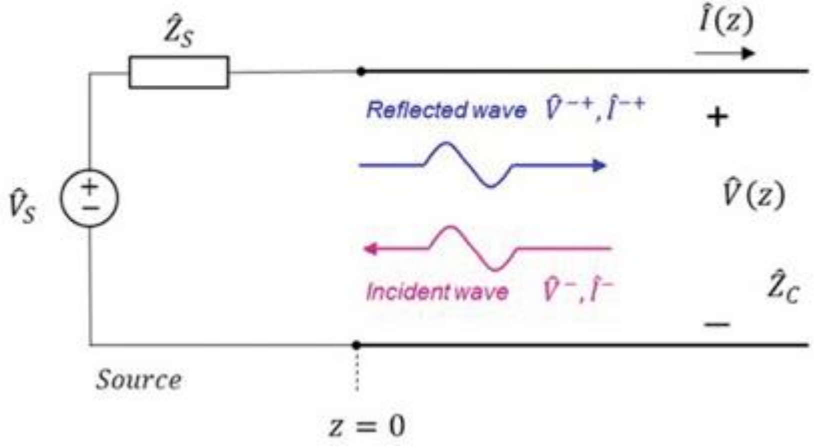
- Where $\hat{\Gamma}_L$ is the voltage reflection coefficient at the load, given by,

$$\hat{\Gamma}_L = \frac{\hat{Z}_L - \hat{Z}_C}{\hat{Z}_L + \hat{Z}_C}$$

- The total voltage at the load is the sum of the incident voltage and the reflected voltage. When the load is matched to the transmission line the reflection coefficient is zero, and therefore there is no reflected voltage.

# 2. Reflections on transmission lines – Contd..

- When the line is not matched at the load, a reflected wave, $V^{\wedge -}$ is created and travels back to the source. Upon the arrival at the source this wave gets reflected again, creating a forward voltage wave $V^{\wedge -+}$




**(c) Reflection at the source**

- ✓ So, now at the source $V^{\wedge -}$ is the incident wave from load and $V^{\wedge -+}$ is the reflected wave

# 2. Reflections on transmission lines – Contd..

- The voltage of the reflected wave, $\hat{V}^{-+}$ is related to the voltage of the incident wave, $\hat{V}^{-}$ by, $\hat{V}^{-+} = \hat{\Gamma}_S \hat{V}^{-}$
- where, $\hat{\Gamma}_S$ is the voltage reflection coefficient at the source, given by,

$$\hat{\Gamma}_S = \frac{\hat{Z}_S - \hat{Z}_C}{\hat{Z}_S + \hat{Z}_C}$$

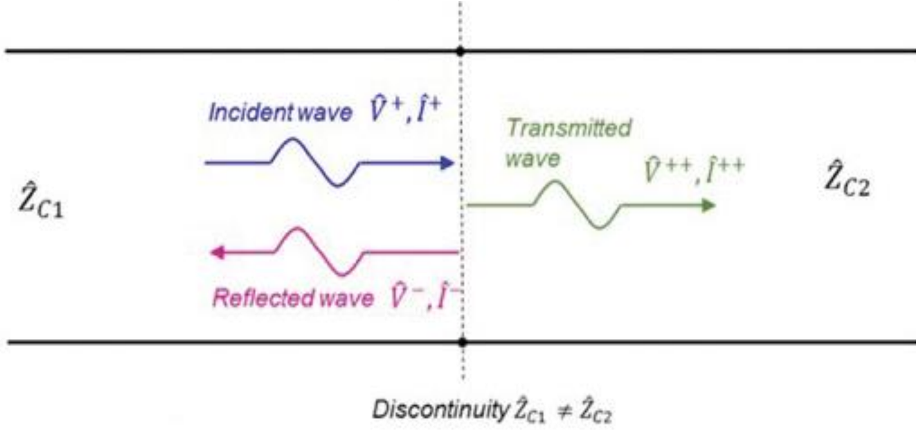
- When the source is matched to the transmission line, the reflection coefficient is zero, and therefore there is no reflected voltage at the source.

# 2. Reflections on transmission lines – Contd..

- Reflections along a transmission line discontinuity: Discontinuity along a transmission line can be caused by many different factors.

- The easiest case to consider is when the characteristic impedance of the transmission line changes (from $Z^\wedge{}_{c1}$ to $Z^\wedge{}_{c2}$)

**(c) Reflections at a discontinuity**



# 2. Reflections on transmission lines – Contd..

- When the incident wave traveling on transmission line 1 arrives at the junction it creates a reflected wave and a transmitted wave.

- The voltage of the reflected wave is related to the voltage of the incident wave by, $\hat{V}^{-} = \hat{\Gamma}_{12} \hat{V}^{+}$

- where, $\hat{\Gamma}_{12}$ is the voltage reflection coefficient, given by,

$$\hat{\Gamma}_{12} = \frac{\hat{Z}_{C2} - \hat{Z}_{C1}}{\hat{Z}_{C2} + \hat{Z}_{C1}}$$

# 2. Reflections on transmission lines – Contd..

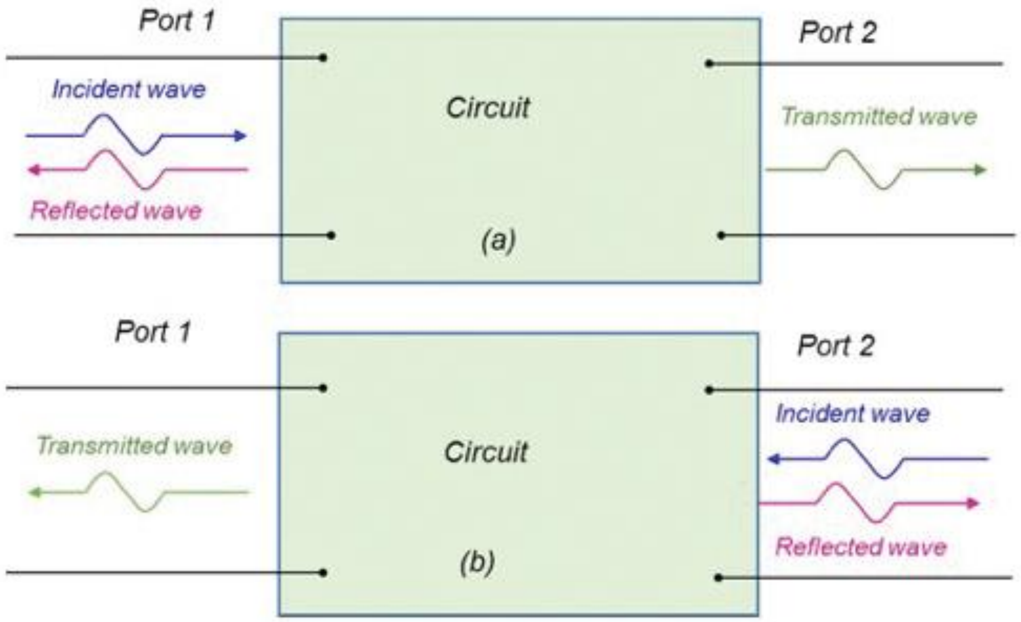
- The voltage of the transmitted wave is related to the voltage of the incident wave by, $\hat{V}^{++} = \hat{T}_{12}\hat{V}^{+}$

- where, $\hat{T}_{12}$ is the voltage transmission coefficient given by

$$\hat{T}_{12} = \frac{2\hat{Z}_{C2}}{\hat{Z}_{C2} + \hat{Z}_{C1}}$$

# S Parameters (Scattering Parameters)

- To characterize high-frequency circuits we use S parameters which relate traveling voltage waves that are incident, reflected and transmitted when a two-port network is inserted into a transmission line.




**Traveling waves impinging on:**

**(a) port 1 (b) port 2**

# S Parameters (Scattering Parameters) - Introduction

- S-parameters describe the input-output relationship between ports (or terminals) in an electrical system. It also describes the response of an N-port network to signal(s) incident to any or all of the ports
- S-parameters are complex numbers, having real and imaginary parts or magnitude and phase parts

# S Parameters (Scattering Parameters) - Introduction

- S-parameters are displayed in a matrix format (Scattering Matrix), with the number of rows and columns equal to the number of ports.
- The scattering matrix relates the voltage waves incident on the ports to those reflected from the ports.
- The scattering parameters can be calculated using network analysis techniques or it can be measured directly with a Vector Network Analyzer

# S Parameters (Scattering Parameters)

## Definition:

- The scattering matrix of an m-port junction is a square matrix of a set of elements which relate incident and reflected waves at the port of the junction.

# S Parameters (Scattering Parameters)

## Characteristics of S-matrix:

- It describes any passive microwave component.
- It exists for linear passive and time invariant networks.
- It gives complete information on reflection and transmission coefficients

# S Parameters (Scattering Parameters)

- Here are the S-matrices for one, two and three-port networks:

$$\begin{pmatrix} S_{11} \end{pmatrix} \quad \text{---- (one - port)}$$

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \quad \text{---- (two - port)}$$

$$\begin{pmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{pmatrix} \quad \text{---- (three - port)}$$

# S Parameters (Scattering Parameters)

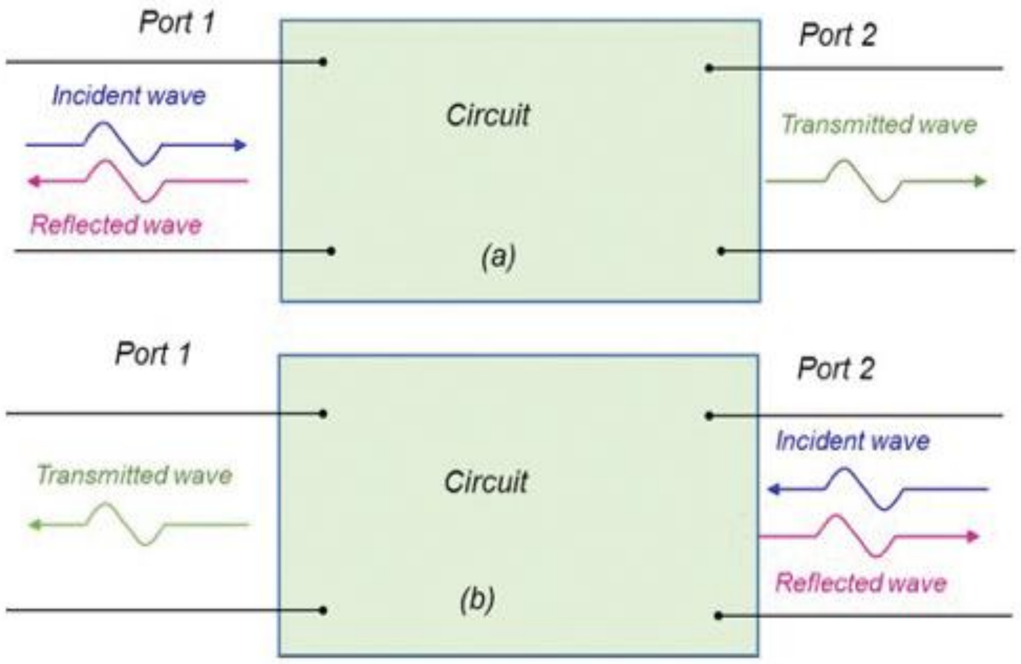
- For the S-parameter Sij the j subscript stands for the port that is excited (the input port), and the "i" subscript is for the output port.
- Thus $S_{12}$ refers to the ratio of the amplitude of the signal that reflects from port 1 to the amplitude of the signal incident on port 2.
- $\boldsymbol{S_{21}}$ means the response at port 2 due to a signal at port 1.

# S Parameters for a two port network

- Let's examine a two-port network

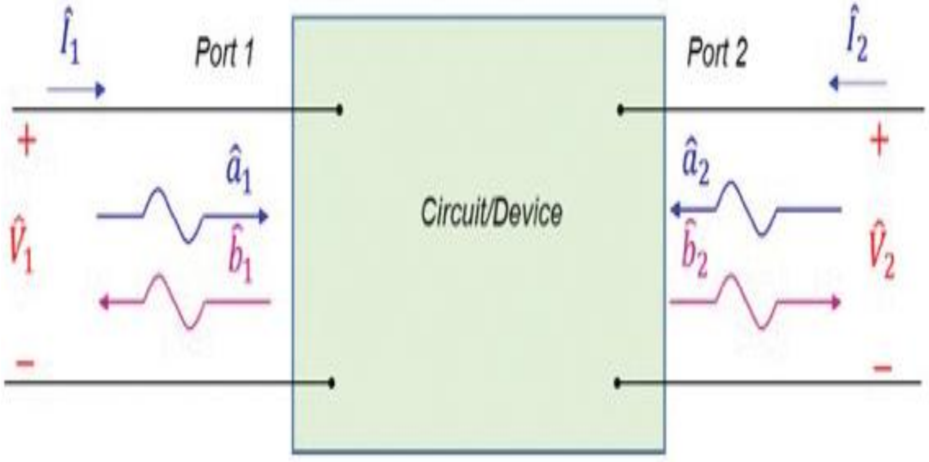
**Traveling waves impinging on:**

**(a) port 1 (b) port 2**



# S Parameters for a two port network

- **Two-port network**
- The incident waves (which give rise to the reflected and transmitted waves) can be impinging on either port 1 or port 2.
- Let's denote the wave incident on port 1 and port 2 by $a_1$ and $a_2$, respectively.
- These waves give rise to the reflected waves, $b_1$ and $b_2$ respectively.




**Incident and reflected waves at port 1 & port 2**

# S Parameters for a two port network

- A scattering matrix represents the relationship between the parameters $a_n$'s (incident wave amplitude) and $b_n$'s (reflected wave amplitude)

$$a_n = {v_n}^+/\sqrt{Z_0}\,;\, b_n = {v_n}^-/\sqrt{Z_0}$$

- where ${v_n}^+$ *and* ${v_n}^-$ represent incident and outgoing waves along the line connected to the nth port and

- $Z_0$ characteristic impedance of the line.

# S Parameters for a two port network

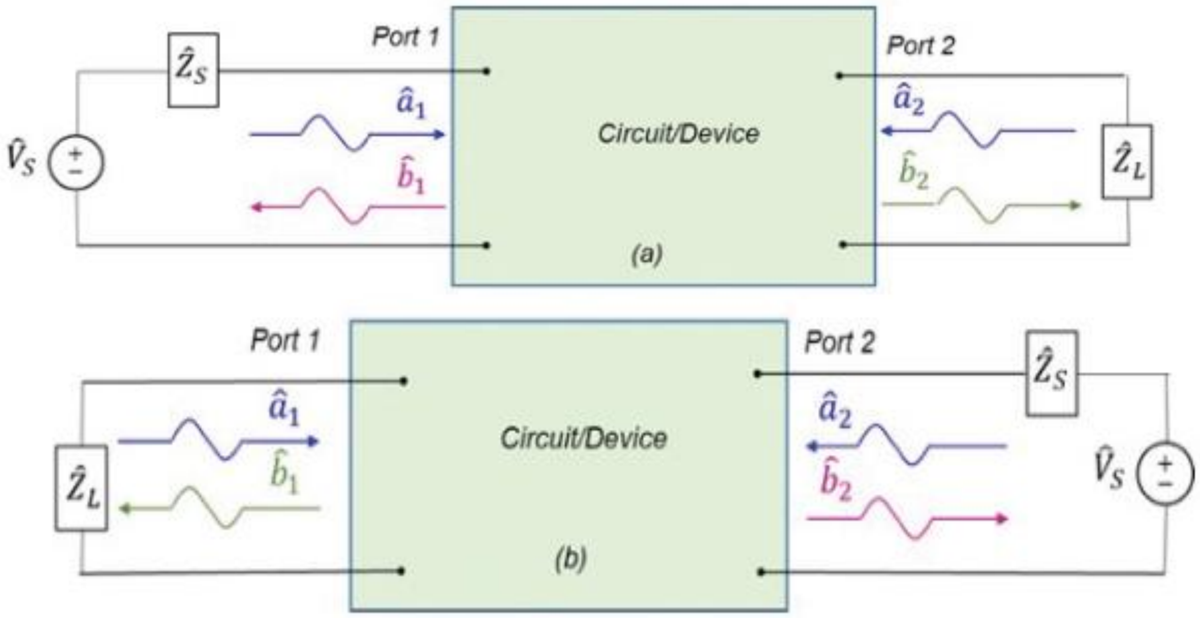
- The incident and reflected waves are used to define s parameters for a two port network.
- The linear equations describing the two-port network in terms of the S parameters are

$$b_1 = S_{11}\, a_1 + S_{12}\, a_2 \qquad (1)$$

$$b_2 = S_{21}\, a_1 + S_{22}\, a_2 \qquad (2)$$

- That is, s parameters define the reflected wave at a particular port in terms as of the incident wave at each port.
- Using Matrix notation, [b] = [S] [a]

# S parameters for a two port network




Typical two-port circuit application:

a) Circuit driven at port 1 and terminated by a load at port 2,
b) Circuit driven at port 2 and terminated by a load at port 1

# S parameters for a two port network

- $S_{11} = b_1/a_1|\ a_2 = 0$ => reflection coefficient at port 1 when the incident wave on port 2 is zero, which means that port 2 should be terminated in matched load to avoid reflections ($a_2$ =0).

- $S_{22} = b_2/a_2\ |\ a_1=0$ => reflection coefficient at port 2 when the incident wave on port 1 is zero, which means that port 1 should be terminated in matched load to avoid reflections ($a_1$=0).

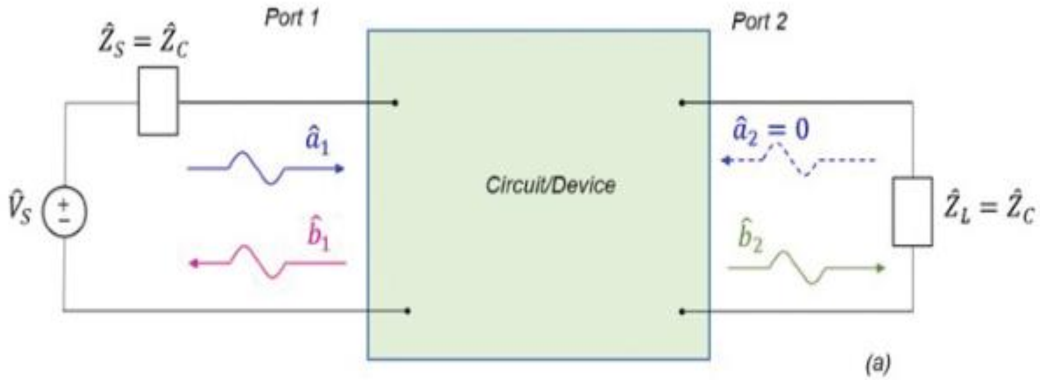
# S parameters for a two port network

- $S_{12} = b_1/a_2 \mid a_1 = 0$ => transmission coefficient from port 2 to port 1, with port 1 terminated in matched load

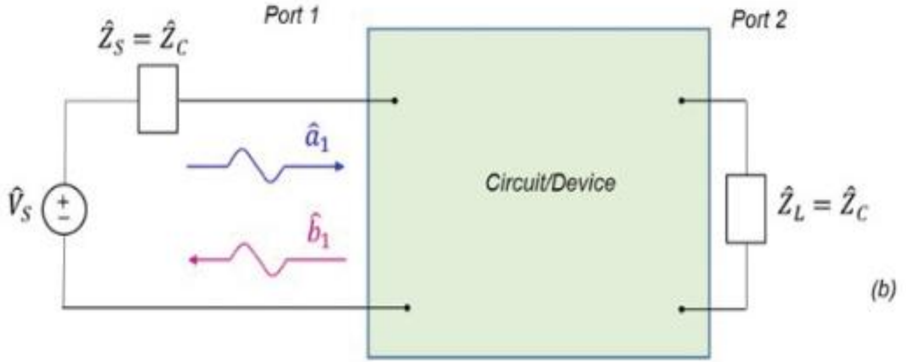
- $S_{21} = b_2/a_1 \mid a_2 = 0$ => transmission coefficient from port 1 to port 2, with port 2 terminated in matched load

# S parameters for a two port network

a) Circuit for determining $S_{11}$ or $S_{21}$

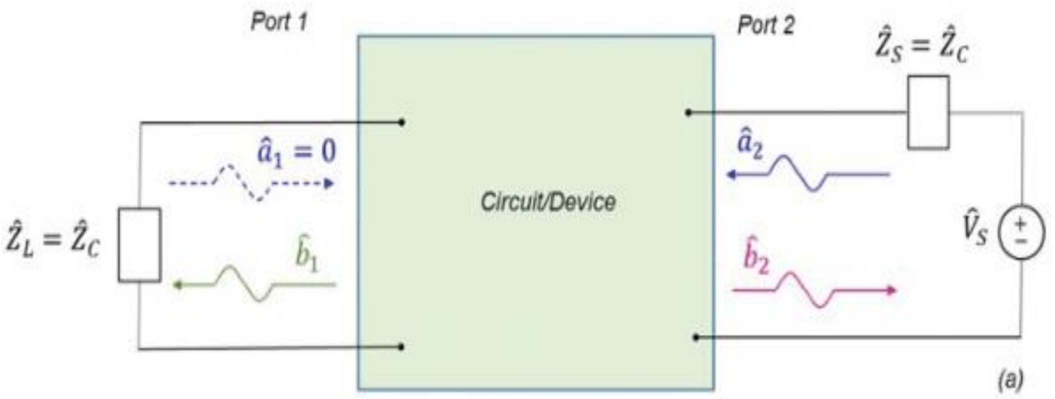



b) Alternative circuit for determining $S_{11}$

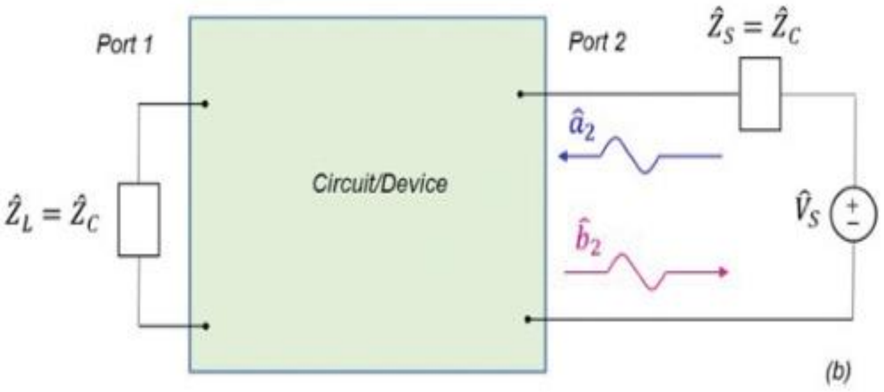


# S parameters for a two port network

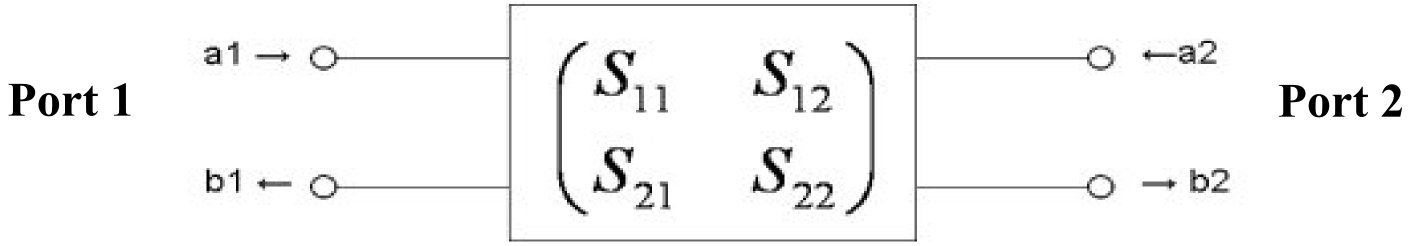
a) Circuit for determining $S_{22}$ or $S_{12}$

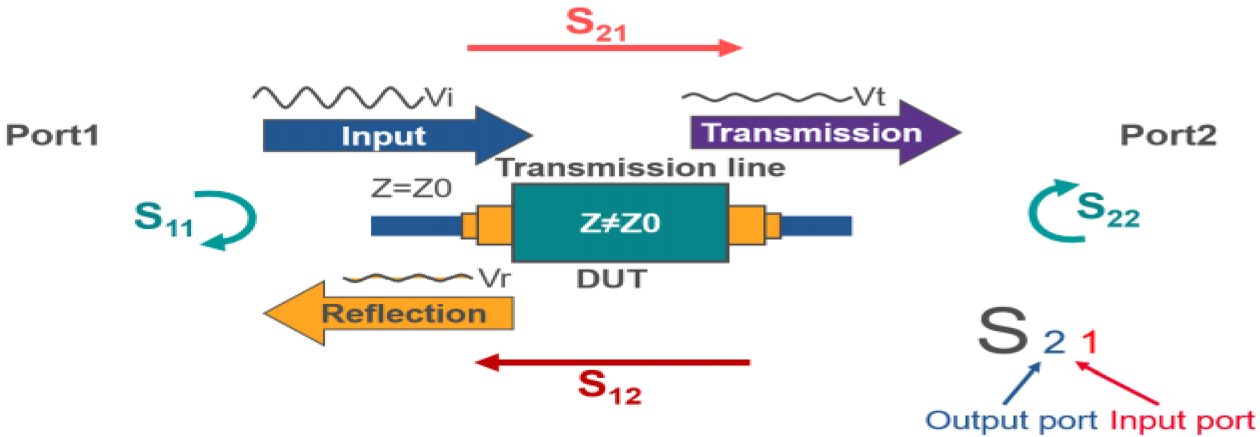



b) Alternative circuit for determining $S_{22}$



# S Parameters for a two port network




Reflection/Input = Reflection coefficient = $S_{11}$ , $S_{22}$

Transmission/Input = Transmission coefficient = $S_{12}$ , $S_{21}$

# S Parameters (Scattering Parameters)

- The incident and reflected amplitudes of microwaves at any port are used to characterize a microwave circuit.
- The amplitudes are normalized in such a way that the square of any of these variables gives the average power in that wave

# S Parameters (Scattering Parameters)

- For an n port network,

Input power at the nth port, $P_{in} = \frac{1}{2}|a_n|^2$

Reflected power at the nth port, $P_{rn} = \frac{1}{2}|b_n|^2$

- where $a_n$ and $b_n$ represent the normalized incident wave amplitude and normalized reflected wave amplitude at the nth port.
- The total or net power flow into any port is given by,

$$P = P_i - P_r = 1/2(|a|^2 - |b|^2)$$

# S Parameters (Scattering Parameters)

- We can relate the generalized s parameters to the powers as follows:

$$|s_{11}|^2 = \left.\frac{|b_1|^2}{|a_1|^2}\right|_{\hat{a}_2=0} = \frac{\text{Reflected power at port 1}}{\text{Incident power at port 1}}$$

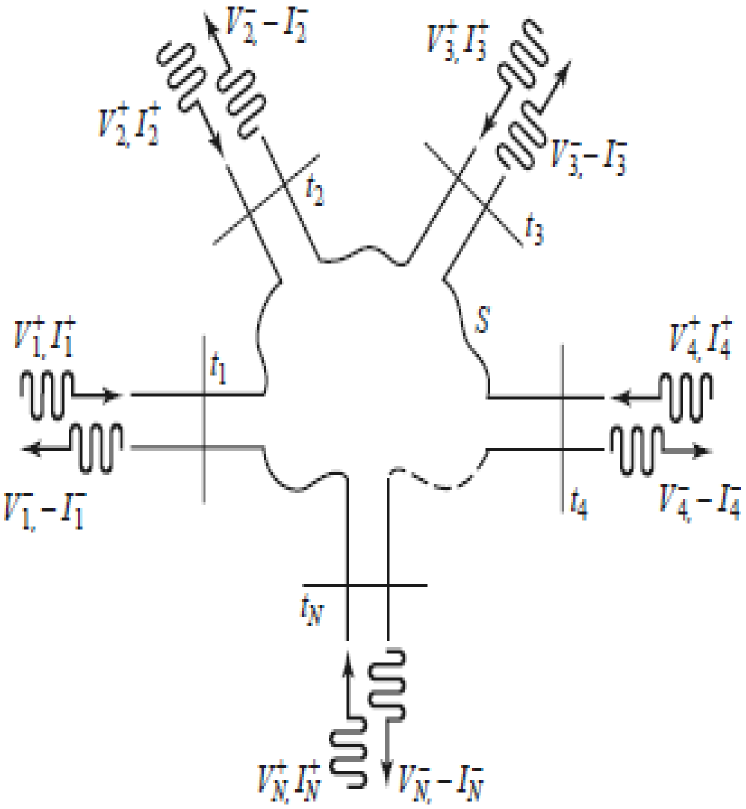
$$|s_{12}|^2 = \left.\frac{|b_1|^2}{|a_2|^2}\right|_{\hat{a}_1=0} = \frac{\text{Transmitted power to port 1}}{\text{Incident power at port 2}}$$

$$|s_{21}|^2 = \left.\frac{|b_2|^2}{|a_1|^2}\right|_{\hat{a}_2=0} = \frac{\text{Transmitted power to port 2}}{\text{Incident power at port 1}}$$

$$|s_{22}|^2 = \left.\frac{|b_2|^2}{|a_1|^2}\right|_{\hat{a}_2=0} = \frac{\text{Reflected power at port 2}}{\text{Incident power at port 2}}$$

# S parameters for an N port network

- For an N port network shown,
- The ports may be any type of transmission line
- At a specific point on the nth port, a terminal plane, $t_n$ is defined along with equivalent voltage and currents for the incident ($V_n^+$ , $I_n^+$) and reflected ($V_n^-, I_n^-$) waves.



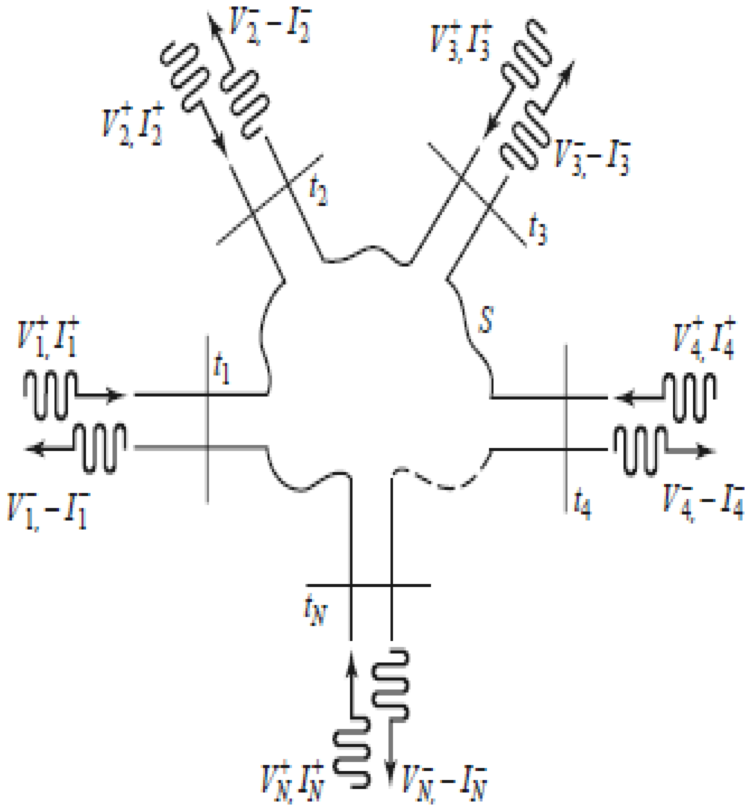
# S parameters for an N port network

- At the $n^{th}$ terminal plane, the total voltage and current is given by,

$$V_n = V_n^{\ +} + V_n^{\ -}$$

$$I_n = I_n^{\ +} - I_n^{\ -}$$

- $V_n^{\ +}$ is the amplitude of the voltage wave incident on port n and
- $V_n^{\ -}$ is the amplitude of the voltage wave reflected from port n.



# S parameters for an N port network

- The Scattering matrix, or [S] matrix is defined in relation to these incident and reflected voltage waves as,

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{pmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ S_{21} & & S_{2N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{bmatrix}$$

or

$$[V^-] = [S][V^+]$$

# S parameters for an N port network

- A specific element of the [S] matrix can be determined as,

$$S_{ij} = \frac{V_i^-}{V_j^+}, V_k^+ = 0 \text{ for } k \neq j$$

- The Scattering matrix or [S] matrix for n-port network,

$$\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{pmatrix} = \begin{bmatrix} S_{11} & \cdots & S_{1N} \\ S_{21} & & S_{2N} \\ \vdots & \ddots & \vdots \\ S_{N1} & \cdots & S_{NN} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix}$$

# Losses in Microwave Networks

- In a 2 port network if power fed at port 1 is $P_i$, Power reflected at the same port is $P_r$ and the output power at port is $P_o$ then following losses are defined in terms of S-parameters.

➢ Insertion loss (dB) $= 10 \log \frac{P_i}{P_o} \quad = 10 \log \frac{|a_1|^2}{|b_2|^2}$

$$= 20 \log \frac{1}{|S_{21}|} \quad = 20 \log \frac{1}{|S_{12}|}$$

# Losses in Microwave Networks

- Transmission loss or attenuation (dB) $= 10 \log \dfrac{P_i - P_r}{P_o}$

$$= 10 \log \frac{1 - |S_{11}|^2}{|S_{12}|^2}$$

- Reflection loss (dB) $= 10 \log \dfrac{P_i}{P_i - P_r} = 10 \log \dfrac{1}{1 - |S_{11}|^2}$

- Return loss (dB) $= 10 \log \dfrac{P_i}{P_r} = 20 \log \dfrac{1}{|\Gamma|} = 20 \log \dfrac{1}{|S_{11}|}$

# Course Outcome

| | |
|---|---|
| CO1 | Analyze the given High Frequency network using S parameters. |

# Learning Outcome

| | |
|---|---|
| LO2 | Analyze the microwave network using S parameters. |

# Properties of S Parameters:

a) Zero diagonal elements for perfect matched network

b) Symmetry of [S] for a reciprocal network

c) Unitary property for lossless junction

d) Phase shift property

# Properties of S Parameters:

a) Zero diagonal elements for perfect matched network

b) Symmetry of [S] for a reciprocal network

c) Unitary property for lossless junction

d) Phase shift property

# Properties of S Parameters:

## a) Zero diagonal elements for perfect matched network:

- For an ideal N-port network with matched termination, $S_{ii} = 0$, since there is no reflection from any port. Therefore under perfect matched conditions the diagonal elements of [S] are zero.

# Properties of S Parameters:

a) Zero diagonal elements for perfect matched network

b) Symmetry of [S] for a reciprocal network

c) Unitary property for lossless junction

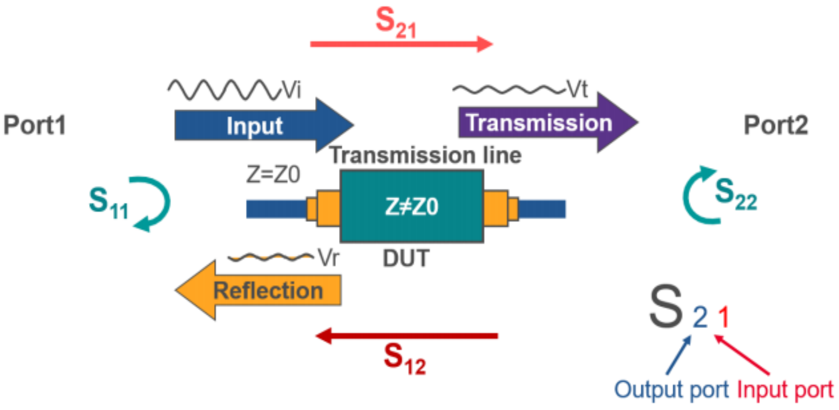
d) Phase shift property

# Properties of S Parameters:

## b) Symmetry of [S] for a reciprocal network

- A reciprocal device has the same transmission characteristics in either direction of a pair of ports and is characterized by a symmetric scattering matrix,

$$S_{ij} = S_{ji} \text{ (i} \neq \text{j)}$$




- which results in,

$$[S]_t = [S]$$

# b) Symmetry of [S] for a reciprocal network

## Proof:-

- In a multiport network the total voltage and current at the nth port can be written as,

$$V_n = {V_n}^+ + {V_n}^- \qquad (1)$$

$$I_n = {I_n}^+ - {I_n}^- = {V_n}^+ - {V_n}^- \qquad (2)$$

- By adding (1) & (2) we obtain

$${V_n}^+ = \frac{1}{2}(V_n + I_n)$$

- Impedance matrix equation,

$$[V^+] = \frac{1}{2}([Z] + [U])[I] \qquad (3)$$

# b) Symmetry of [S] for a reciprocal network: **Proof:-**

- Subtracting (2) from (1)

$$V_n^{\,-} = \frac{1}{2}(V_n - I_n)$$

- Impedance matrix equation,

$$[V^-] = \frac{1}{2}([Z] - [U])[I] \qquad (4)$$

- where, [U] is the identity matrix
- From (3)

$$[I] = \frac{2[V^+]}{[Z]+[U]} = 2[V^+]([Z] + [U])^{-1} \qquad (5)$$

- Substitute (5) in (4)

$$[V^-] = ([Z] - [U])([Z] + [U])^{-1}[V^+] \qquad (6)$$

## b) Symmetry of [S] for a reciprocal network: Proof:-

$$\frac{[V^-]}{[V^+]} = ([Z]-[U])([Z]+[U])^{-1} = [S]$$

$$[S] = ([Z]-[U])([Z]+[U])^{-1} \qquad (7)$$

- Taking transpose of (7)

$$[S]^t = ([Z]-[U])^t \{([Z]+[U])^{-1}\}^t \qquad (8)$$

- [U] is a diagonal matrix, so $[U]^t = [U]$, and if the network is reciprocal , [Z] is symmetric, so that $[Z]^t = [Z]$
- (8) reduces to, $[S]^t = ([Z]-[U])([Z]+[U])^{-1} \qquad (9)$
- When comparing (7) & (9),

$[S] = [S]^t$ **{Hence proved}**

# Properties of S Parameters:

a) Zero diagonal elements for perfect matched network

b) Symmetry of [S] for a reciprocal network

c) Unitary property for lossless junction

d) Phase shift property

# Properties of S Parameters:

## c) Unitary property for lossless junction:

- For any lossless network the sum of the products of each term of any one row or of any one column of the S-matrix multiplied by its complex conjugate is unity.

$$\sum_{k=1}^{N} S_{ki}\ S_{kj}^{*} = 1, for\ i = j \qquad \sum_{k=1}^{N} S_{ki}\ S_{kj}^{*} = 0, for\ i \neq j$$

# c) Unitary property for lossless junction: **Proof:-**

- $\frac{1}{2}[V^+]^t[V^+]^*$ represents the total incident power, while $\frac{1}{2}[V^-]^t[V^-]^*$ represents the total reflected power
- Therefore, $P_{av} = \frac{1}{2}[V^+]^t[V^+]^* - \frac{1}{2}[V^-]^t[V^-]^* = 0 \qquad (2)$
- So for a lossless junction, the incident and reflected powers are equal:

$$[V^+]^t[V^+]^* = [V^-]^t[V^-]^* \qquad (3)$$

- Using $[V^-] = [S][V^+]$ in (3)

$$[V^+]^t[V^+]^* = [V^+]^t[S]^t[S]^*[V^+]^*$$

$$\Rightarrow \quad [V^+]^t[V^+]^* = [V^+]^t[V^+]^*[S]^t[S]^*$$

$$\Rightarrow \quad \frac{[V^+]^t[V^+]^*}{[V^+]^t[V^+]^*} = [S]^t[S]^* \qquad (4)$$

# c) Unitary property for lossless junction: Proof:-

$$\frac{[V^+]^t[V^+]^*}{[V^+]^t[V^+]^*} = [S]^t[S]^*,$$

- for nonzero $[V^+]$, => $[S]^t[S]^* = [U]$ => $[S]^* = \{[S]^t\}^{-1}$ (5)
- A matrix that satisfies the condition (5) is called a unitary matrix.
- The matrix equation (5) can be written in summation form as

$$\sum_{k=1}^{N} S_{ki}S_{kj}^* = \delta_{ij}, \; for\ all\ i,j \qquad (6)$$

- where $\delta_{ij} = 1$ if i = j and $\delta_{ij} = 0$ if i ≠ j, the Kronecker delta symbol.
- If i = j, (6) reduces to $\sum_{k=1}^{N} S_{ki}S_{ki}^* = 1$ (7)
- while i ≠ j, (6) reduces to $\sum_{k=1}^{N} S_{ki}S_{kj}^* = 0$ (8)

# c) Unitary property for lossless junction: Proof:-

$$\sum_{k=1}^{N} S_{ki} S_{ki}^{*} = 1 ,\ \ \mathrm{i} = \mathrm{j} \qquad (7)$$

- (7) states that the dot product of any column of [S] with the conjugate of that column gives unity.

$$\sum_{k=1}^{N} S_{ki} S_{kj}^{*} = 0,\ \ \mathrm{i} \neq \mathrm{j} \qquad (8)$$

- (8) states that the dot product of any column with the conjugate of a different column gives zero (orthogonal).

**{Hence Proved}**

# Properties of S Parameters:

a) Zero diagonal elements for perfect matched network

b) Symmetry of [S] for a reciprocal network

c) Unitary property for lossless junction

d) Phase shift property

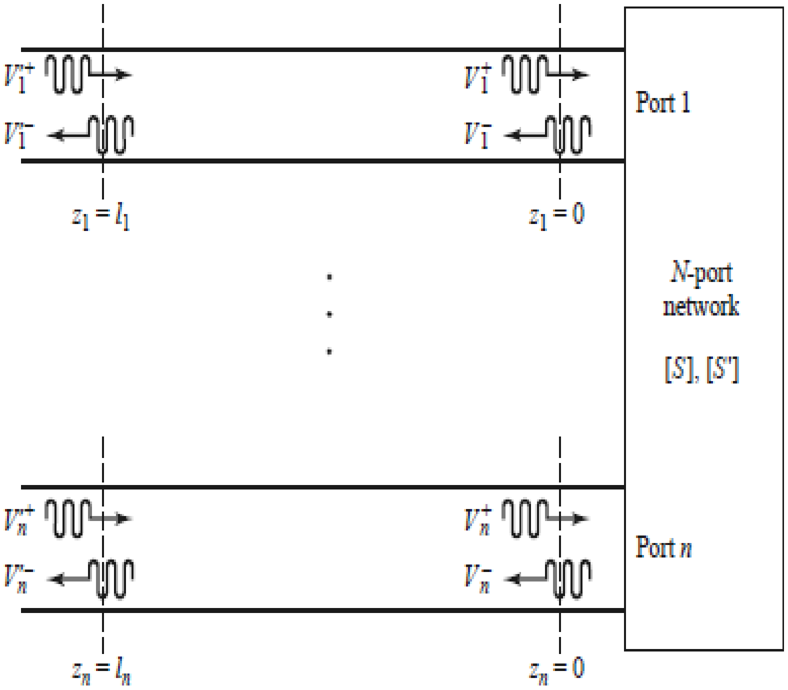
# Properties of S Parameters:

## d) Phase shift property:

- Complex S parameters of a network are defined with respect to the positions

- S parameters relate amplitudes (magnitude and phase) of traveling waves incident on and reflected from a microwave network, phase reference planes must be specified for each port of the network.

# d) Phase shift property:




**Shifting reference planes for an N port Network**

- Original terminal planes are at $z_n = 0$ for the $n^{th}$ port, where $z_n$ - arbitrary coordinate measured along the transmission line feeding the *n*th port.
- The scattering matrix for the network with this set of terminal planes is denoted by [*S*].
- For a new set of reference planes defined at $z_n = l_n$ for the $n^{th}$ port, the new scattering matrix be denoted as [*S'*].

# d) Phase shift property:

➢ The incident and reflected port voltages

$$[V^-] = [S][V^+] \qquad (1)$$

$$[V'^-] = [S'][V'^+] \qquad (2)$$

where,

- the unprimed quantities are referenced to the original terminal planes at $z_n = 0$ and
- the primed quantities are referenced to the new terminal planes at $z_n = l_n$

# d) Phase shift property:

- From the theory of traveling waves on lossless transmission lines we can relate the new wave amplitudes to the original ones as,

$$V_n'^{+} = V_n^{+} e^{j\theta_n} \qquad (3)$$

$$V_n'^{-} = V_n^{-} e^{-j\theta_n} \qquad (4)$$

- where $\theta_n = \beta_n l_n$ is the electrical length of the outward shift of the reference plane of port n
- (3) can be written as, => $V_n^{+} = V_n'^{+} e^{-j\theta_n}$ (5)
- (4) can be written as, => $V_n^{-} = V_n'^{-} e^{j\theta_n}$ (6)

# d) Phase shift property:

- (5) in matrix form, => $[V_n^{\,+}] = [V_n^{\prime +}]\begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix}$ (7)

- (6) in matrix form, => $[V_n^{\,-}] = [V_n^{\prime -}]\begin{bmatrix} e^{j\theta_1} & \cdots & 0 \\ 0 & e^{j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\theta_N} \end{bmatrix}$ (8)

- Substitute (7) & (8) in (1), (for $n^{th}$ port)

$$[V^-] = [S][V^+] \qquad (1)$$

# d) Phase shift property:

$$\begin{bmatrix} e^{j\theta_1} & \cdots & 0 \\ 0 & e^{j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{j\theta_N} \end{bmatrix} [V'^-] = [S] \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix} [V'^+] \qquad (7)$$

$$\Rightarrow [V'^-] = \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix} [V'^+] \qquad (8)$$

- On comparing (8) & (2)

$$[V'^-] = [S'][V'^+] \qquad (2)$$

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix} \qquad (9)$$

# d) Phase shift property:

$$[S'] = \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix} [S] \begin{bmatrix} e^{-j\theta_1} & \cdots & 0 \\ 0 & e^{-j\theta_2} & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-j\theta_N} \end{bmatrix}$$

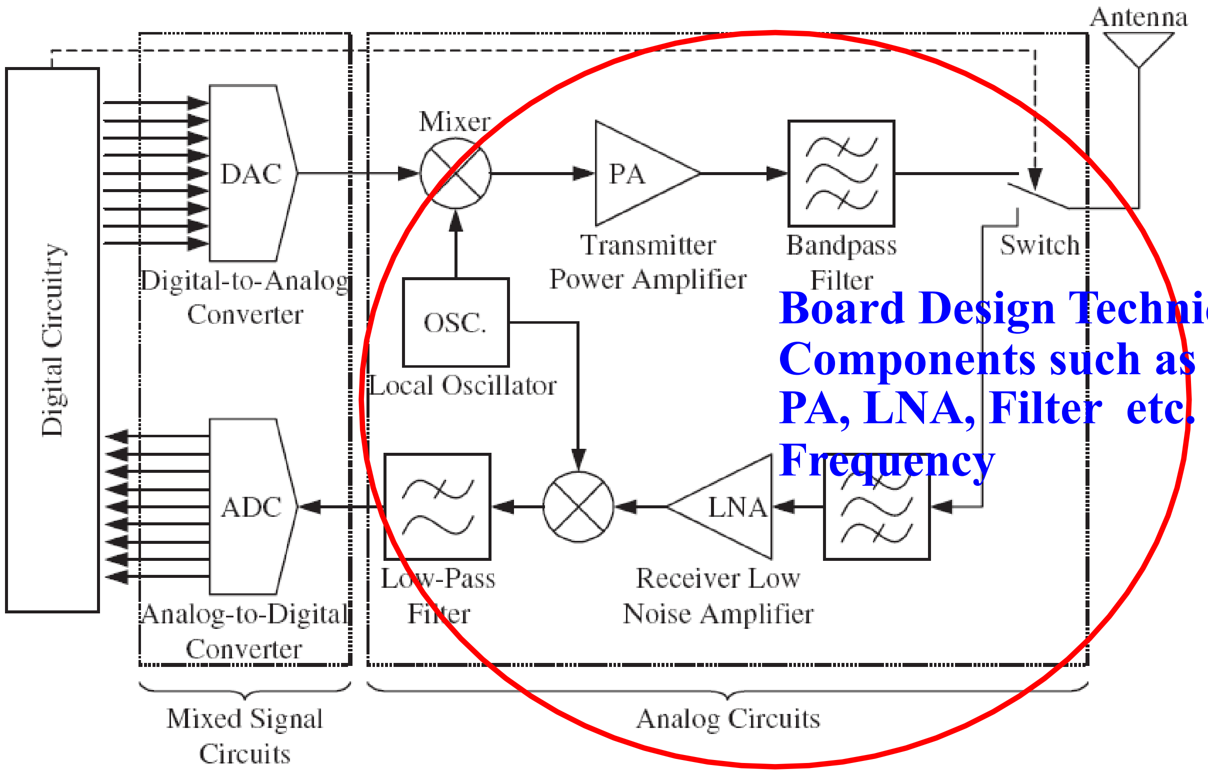
=> the desired result

**Note:**

$S_{nn}{}' = e^{-2j\theta_n} S_{nn}$ means, the phase of $S_{nn}$ is shifted by twice the electrical length of the shift in terminal plane n, because the wave travels twice over this length upon incidence and reflection.

# Components at High Frequencies

# RF Transceiver

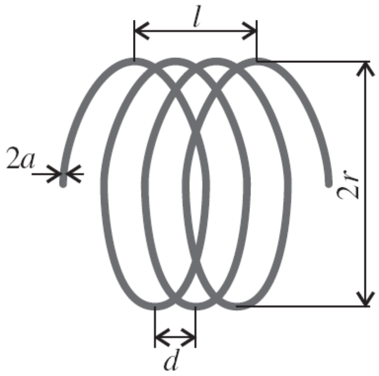



**Board Design Techniques of Components such as Interconnects, PA, LNA, Filter etc. at Cellular Frequency**

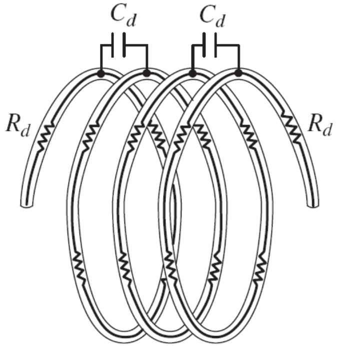
# What do we mean by distributed?

- Example – Inductor

**Low frequency (Lumped)**




**High Frequency (Distributed)**




$Z = R + j\omega L$

$Z = ?$

# RF Behavior of Passive Components

- ✓ Why do inductors, capacitors, and resistors behave differently at Radio Frequency?
- ✓ What is skin effect?
- ✓ Equivalent Circuit Model?

# RF Behavior of Passive Components (contd.)

For conventional AC circuit analysis:

- R is considered frequency independent
- Ideal Inductor (L) possesses an impedance ( $XL=j\omega L$)
- Ideal capacitor (C) possesses an impedance ($XC=1/j\omega C$)

Capacitor behaves as open circuit at DC and low frequency **whereas** an Inductor behaves as short circuit at DC and low frequencies

# RF Behavior of Resistors

## At low frequency:

- Resistances, inductances, and capacitances are formed by wires, coils, and plates etc.
- Even a single wire or a copper line on a PCB possesses resistance and inductance.
- This cylindrical copper conductor has a DC resistance:


$$R_{DC} = \frac{l}{\pi a^2 \sigma_{cond}}$$

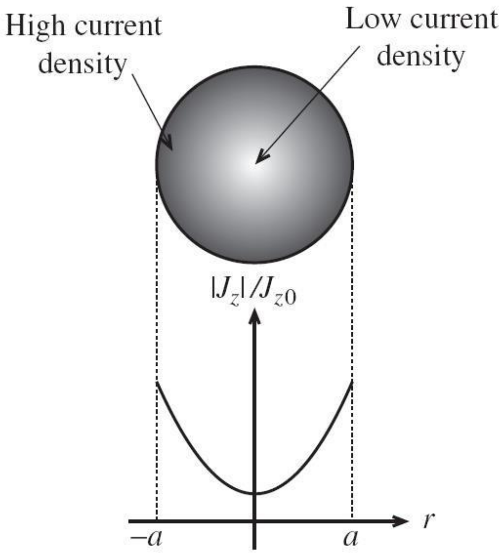
Length of cylinder

Radius of cylinder

conductivity

# RF Behavior of Resistors (contd.)

- At DC, current flows uniformly distributed over the entire conductor cross-sectional area.
- At AC, the alternating charge carrier flow establishes a magnetic field that induces an electric field (Faraday's Law) whose associated currenmt density opposes the initial current flow → this effect is very strong at the center (r=0) where the impedance is substantially increased → as a result the current flow resides at the outer periphery with the increasing frequency.




**Skin Effect**

DC Current Density: $\boxed{J = \dfrac{I}{\pi a^2}}$

# RF Behavior of Resistors (contd.)

- The current density at AC is given by:

$$J_z = \frac{pI}{2\pi a j\sqrt{r}} \exp\left( -(1+j)\frac{a-r}{\delta} \right)$$

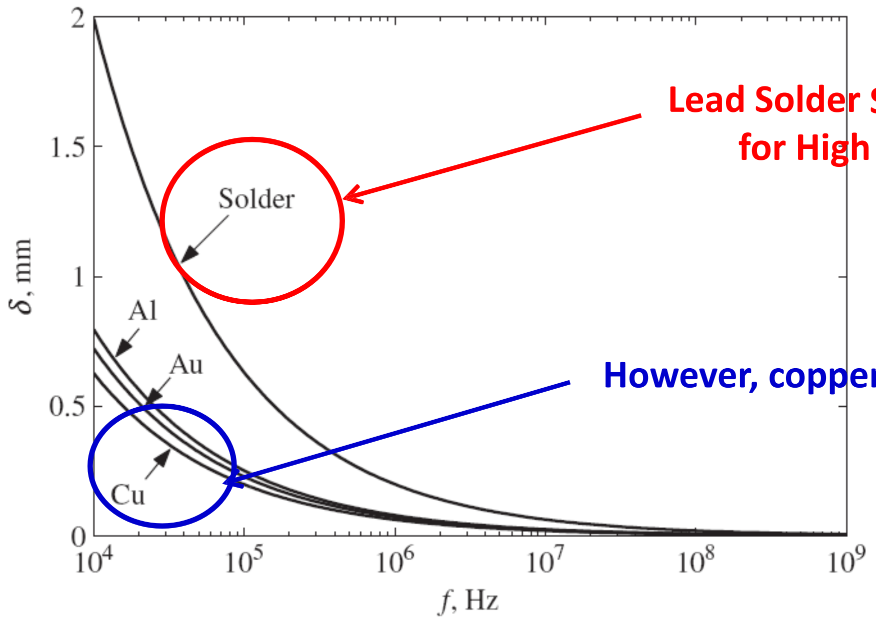
$$\boxed{p^2 = -j\omega\mu\sigma_{cond}}$$

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma_{cond}}}$$

**Skin Depth**

- $J_z$ drops with decrease in $r$ (proximity to the center)
- $\delta$ decreases with increase in frequency (skin depth from periphery reduces with increased frequency) → means the path for current conduction remains nearer to the periphery (skin effect) → means, current density towards center decreases with increase in frequency and increase in conductivity

# RF Behavior of Resistors (contd.)

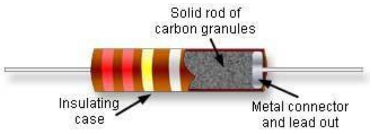



**Lead Solder Seems the Best for High Frequency**

**However, copper is still preferred**

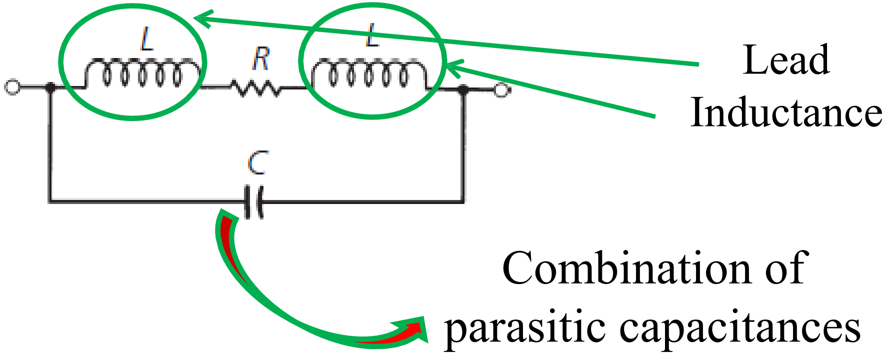
# Resistors at High Frequencies

1. Carbon-composition resistors:



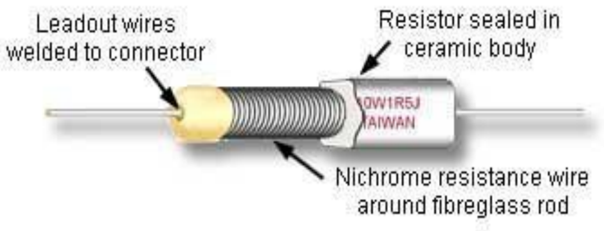

- Consists of densely packed dielectric particulates or carbon granules.
- **Between each pair of carbon granules <u>is</u> very small parasitic capacitor.**
- These parasitics, in aggregate, are significant → **primarily responsible for notoriously poor performance at high frequencies**

**<u>Equivalent Ckt Model:</u>**



# Resistors at High Frequencies

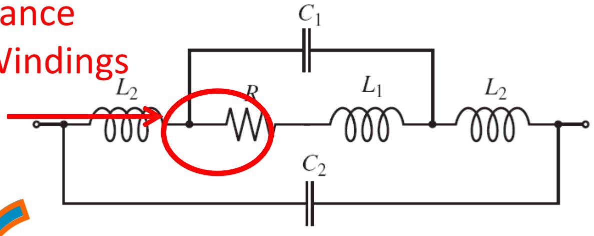
## 2. Wire-wound Resistors:




- Exhibit widely varying impedances over various frequencies.
- The inductor L is much larger here as compared to carbon-composition resistor.
- These resistors look like inductors → impedances will increase with increase in frequency.
- At some frequency $F_r$, the inductance will resonate with shunt capacitance →leads to decrease in impedance.

**Equivalent Ckt Model:**

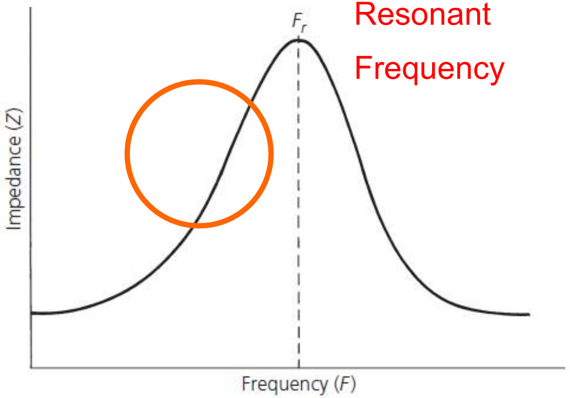
Capacitance between Windings




**$L_2$: lead inductance**

**$L_1$: inductance of resistive wires**

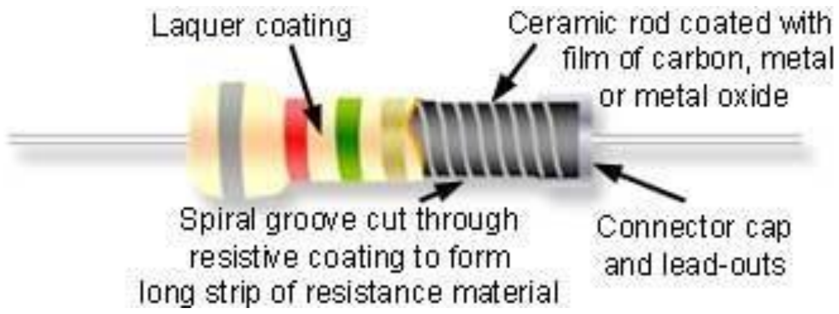
**$C_2$: Interlead Capacitance**



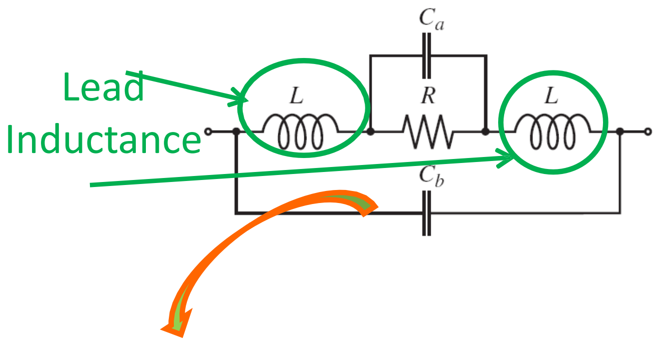

Resonant Frequency

# Resistors at High Frequencies

## 3. Metal-film Resistors




**Equivalent Ckt Model:**




$C_a$ models charge separation effects and $C_b$ models interlead capacitance

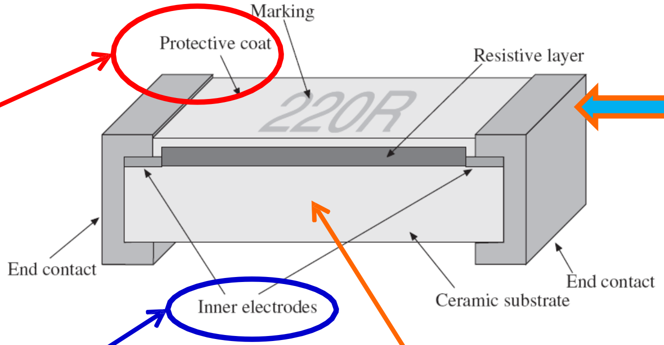
- Seem to exhibit very good characteristics over frequency.
- Values of L and C are much smaller as compared to wire-wound and carbon-composition resistors.
- It works well up to 10 MHz → useful up to 100 MHz

# Resistors at High Frequencies

## 4. Thin-film Chip Resistors:

- The idea is to eliminate or reduce the stray capacitances associated with the resistors
- Good enough upto 2 GHz

Protective coat prevents variations from any environmental interferences




The end contacts are required for soldering purposes

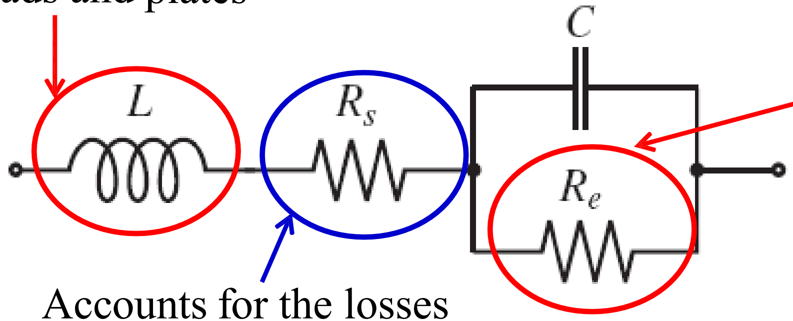
These electrodes are inserted after trimming the resistive layer to the desired value

A metal film (usually nichrome) layer is deposited on this ceramic substrate → this layer works as resistor

# Capacitors at High Frequencies

**Equivalent Circuit Representation of a capacitor → for a parallel plate**

Inductance of the leads and plates




Represents Insulation Resistance

Accounts for the losses in the leads

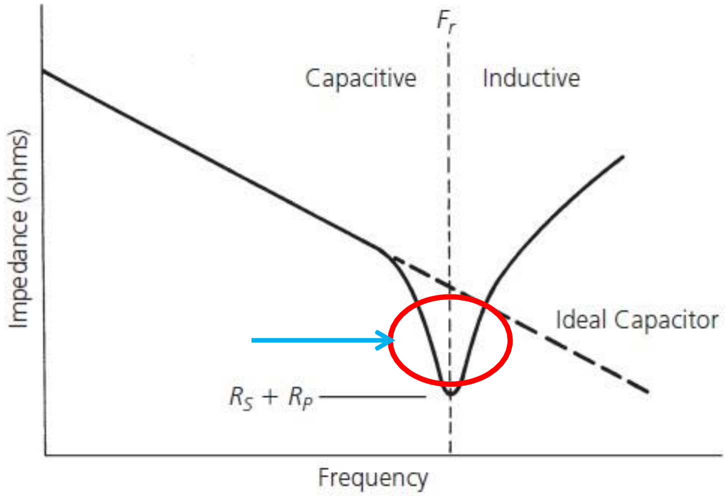
$$\boxed{C = \frac{\varepsilon A}{d} = \varepsilon_0 \varepsilon_r \frac{A}{d}}$$

At high frequency, the dielectric become lossy i.e., there is conduction current through it

Then impedance of capacitor becomes a parallel combination of C and conductance $G_e$

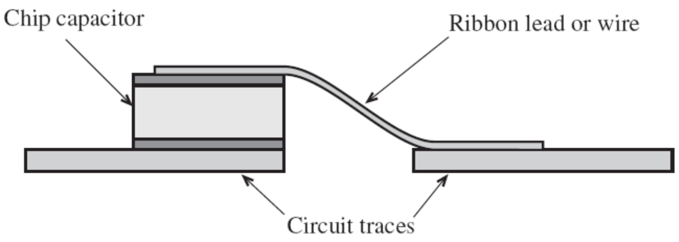
# Capacitors at High Frequencies (contd)

Presence of resonance due to dielectric loss and finite lead wires

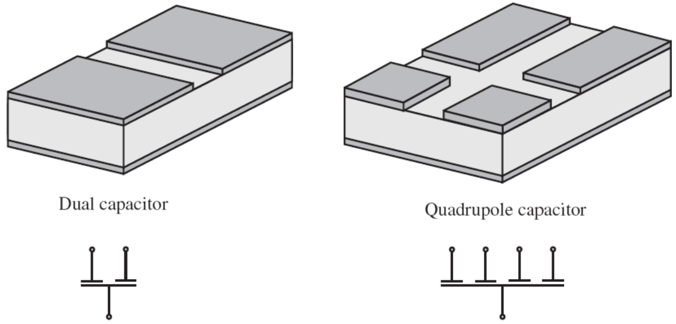



- Above $F_r$, the capacitor behaves as an inductor.
- In general, larger-value capacitors tend to exhibit more internal inductance than smaller-value capacitors.
- Therefore, it may happen that a $0.1\mu F$ may not be as a good as a $300pF$ capacitor in a bypass application at 250 $MHz$.
- The issue is due to significance of lead inductances at higher frequencies.

# Capacitors at High Frequencies (contd)



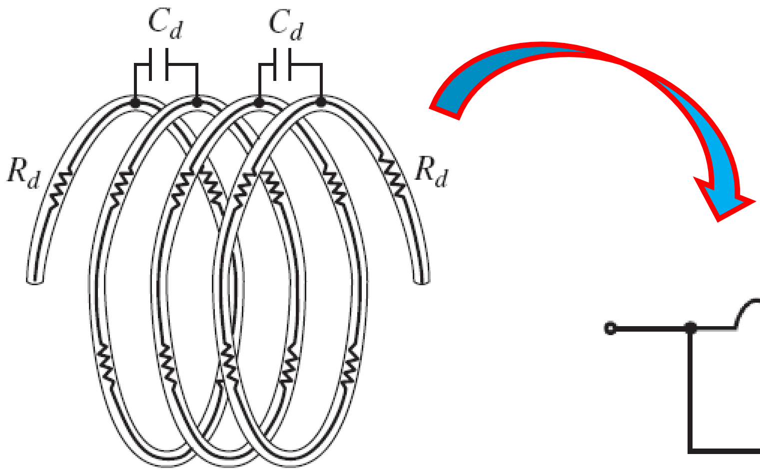

**Cross-section of a single-plate capacitor connected to the board**



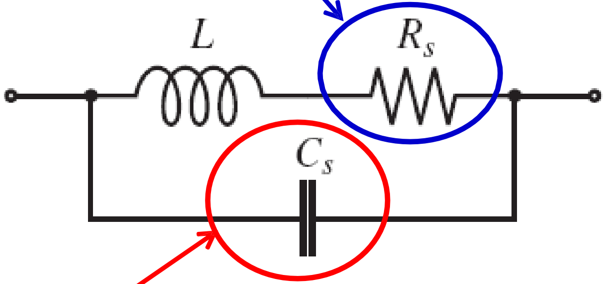
# Inductors at High Frequencies



**Equivalent circuit representation of an inductor → coil type**



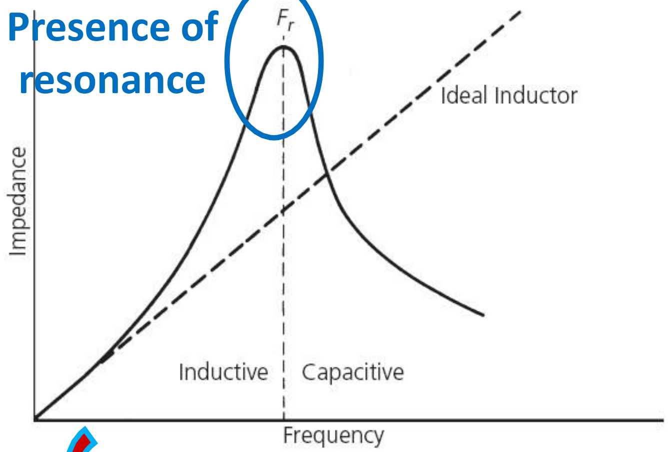

**Composite Effect of all the distributed Rd**




**Composite Effect of all the distributed $C_d$**

# Inductors at High Frequencies (contd)

Presence of resonance

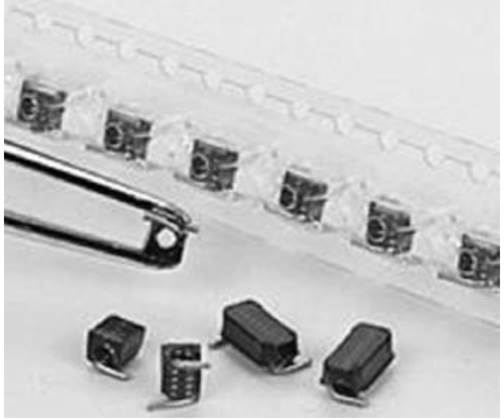



- Initially the reactance of inductor follows the ideal but soon departs from it and increases rapidly until it reaches a peak at the inductor's resonant frequency ($F_r$). **Why?**
- Above Fr, the inductor starts to behave as a capacitor.

**Implement this in MATLAB or ADS**

**HW#0**

# Chip inductors



Surface mounted inductors still come as wire-wound coil →these are comparable in size to the resistors and capacitors

# References: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# Thank you