16ECT72-RF and Microwave Engineering

UNIT V RF AMPLIFIER DESIGN AND MATCHING NETWORKS

Course Handling Faculty

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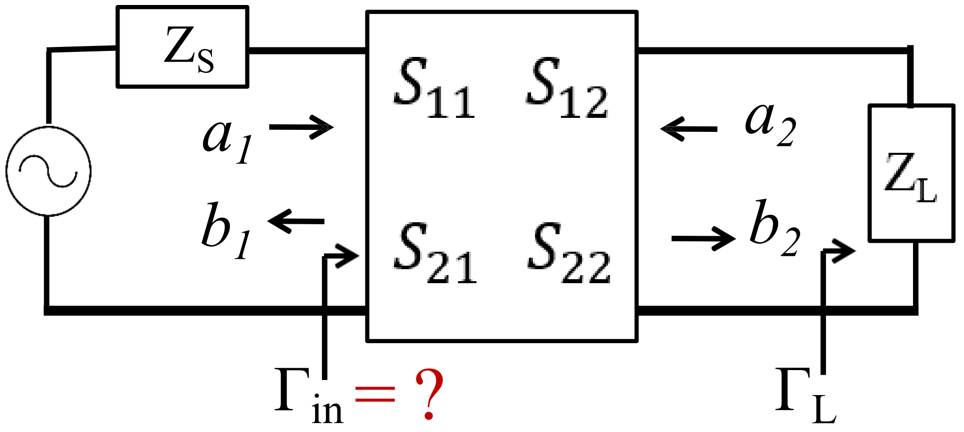
Derivation of Γ_{in} of a Device (Amplifier)



$$\Gamma_L = \frac{a_2}{b_2} \Rightarrow a_2 = \Gamma_L b_2 \quad \text{— } 3$$

S-Parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2 \quad \text{— } 1$$

$$b_2 = S_{21}a_1 + S_{22}a_2 \quad \text{— } 2$$

$$b_2 = S_{21}a_1 + S_{22}.\Gamma_L\, b_2$$

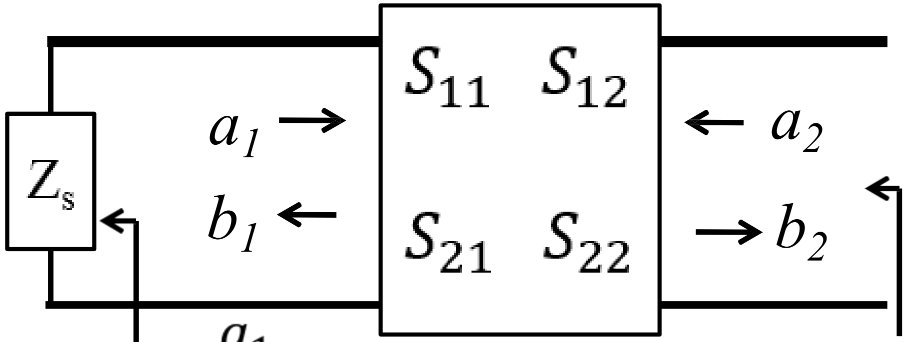
$$b_2 = \frac{S_{21}a_1}{1 - S_{22}\Gamma_L} \quad \text{— } 4$$

From 1, using 3 & 4

$$\boxed{\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}} \Rightarrow \boxed{\Gamma_{in} = \frac{S_{11} - \Delta.\Gamma_L}{1 - S_{22}\Gamma_L}}$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$

Derivation of Γ_{out} of a Device



$$\Gamma_s = \frac{a_1}{b_1}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = ?$$

S-Parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\Gamma_s = \frac{a_1}{b_1} \Rightarrow a_1 = \Gamma_s b_1$$

$$b_1 = S_{11}\Gamma_s b_1 + S_{12}a_2 \longrightarrow b_1 = \frac{S_{12}a_2}{1 - S_{11}\Gamma_s}$$

$$\boxed{\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}} \Rightarrow \boxed{\Gamma_{out} = \frac{S_{22} - \Delta.\Gamma_s}{1 - S_{11}\Gamma_s}}$$

Power Gain of an Amplifier

Power Gain	Symbol	Formula
Transducer Power Gain	G_t	$\frac{P_l}{P_{avs}}$
Available Power Gain	G_a	$\frac{P_{avn}}{P_{avs}}$
Operating Power Gain	G_p	$\frac{P_l}{P_{in}}$

$P_{in} = Input\ power$

P_l= $Power\ delivered\ to\ the\ load$

$$P_{avs} = Power\ available\ from\ source$$
$$= P_{in}, when\ \Gamma_{in} = \Gamma_s^*$$
$$P_{avn} = Power\ available\ from\ network$$
$$= P_l,\ \ when\ \Gamma_L = \Gamma_{out}^*$$

Power Gain of an Amplifier (contd.)

Transducer Power Gain:

$$G_t = \frac{P_l}{P_{avs}}$$

$$P_l = \frac{1}{2}(|b_2|^2 - |a_2|^2) = \frac{1}{2}|b_2|^2(1 - |\Gamma_L|^2)$$

$$P_{avs} = \frac{\frac{1}{2}|b_s|^2}{1 - |\Gamma_s|^2}$$

$$P_{avs} = \frac{1}{2}|b_s|^2, if\ |\Gamma_s| = 0$$

$$G_t = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{in}\Gamma_s|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22}\Gamma_L|^2}$$

$$G_t = \frac{1 - |\Gamma_s|^2}{|1 - S_{11}\Gamma_s|^2}|S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out}\Gamma_L|^2}$$

Three Cases of Amplifier Gain

Case 1: Matched Transducer Power Gain (G_{tm})

Both input and output ports are matched $\Gamma_s = 0$ $\Gamma_L = 0$ $G_t \rightarrow$ $\boxed{G_{tm} = |S_{21}|^2}$

Case 2: Unilateral Transducer Power Gain (G_{tu})

$|S_{12}| = 0$, Power flow in one direction

$$\boxed{G_{tu} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}$$

Case 3: Max. Uni. Transducer Power Gain ($G_{tu\,max}$)

$\Gamma_s = S_{11}^* \;\&\; \Gamma_L = S_{22}^*$ $\Rightarrow$ Maximum Gain

$$\boxed{G_{tu\,\max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}}$$

Stability of an Amplifier

1. Unilateral case: $S_{12} = 0$ → Unconditionally Stable

$$\Gamma_{\text{in}} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} = S_{11} \qquad \Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = S_{22}$$

2. Bilateral case: $S_{12} \neq 0$ → Check Stability of the amplifier

Stability Factor (K):

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2\,|S_{12}S_{21}|}$$

$|\Delta| < 1$ & $K > 1$ → Amplifier is unconditionally stable

Derivation of Stability Circles

Unconditional Stability $\rightarrow$ $\boxed{|\Gamma_{out}| \leq 1}$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} = \frac{S_{22} - \Delta\Gamma_s}{1 - S_{11}\Gamma_s} \rightarrow |\Gamma_{out}| = 1 \rightarrow \left|\frac{S_{22} - \Delta\Gamma_s}{1 - S_{11}\Gamma_s}\right|^2 = 1$$

$$(S_{22} - \Delta\Gamma_s)(S_{22} - \Delta\Gamma_s)^* = (1 - S_{11}\Gamma_s)(1 - S_{11}\Gamma_s)^*$$

$$|S_{22}|^2 - S_{22}\Delta^*\Gamma_s^* - \Delta\Gamma_s S_{22}^* + |\Delta|^2|\Gamma_s|^2 = 1 - S_{11}\Gamma_s - S_{11}^*\Gamma_s^* + |S_{11}|^2|\Gamma_s|^2$$

$$|\Gamma_s|^2(|S_{11}|^2 - |\Delta|^2) - \Gamma_s(S_{11} - \Delta S_{22}^*) - \Gamma_s^*(S_{11}^* - \Delta^* S_{22}) + (1 - |S_{22}|^2) = 0 \qquad (1)$$

Derivation of Stability Circles (contd.)

Equation of a circle: $|\Gamma_s - c_s|^2 = r_s{}^2$ $\quad c_s \rightarrow Center\,,\; r_s \rightarrow Radius$

$$(\Gamma_s - c_s)(\Gamma_s - c_s)^* = r_s{}^2 \Rightarrow |\Gamma_s|^2 - \Gamma_s c_s^* - c_s \Gamma_s^* + |c_s|^2 = r_s{}^2 \quad \text{— 2}$$

From eq. 1, dividing by $(|S_{11}|^2 - |\Delta|^2)$,

$$|\Gamma_s|^2 - \Gamma_s \frac{(S_{11} - \Delta S_{22}^*)}{|S_{11}|^2 - |\Delta|^2} - \Gamma_s^* \frac{(S_{11}^* - \Delta^* S_{22})}{|S_{11}|^2 - |\Delta|^2} + \frac{(1 - |S_{22}|^2)}{|S_{11}|^2 - |\Delta|^2} = 0 \quad \text{— 3}$$

Comparing 2 & 3

$$c_S = \frac{(S_{11} - \Delta S^*{}_{22})^*}{|S_{11}|^2 - |\Delta|^2} \qquad r_S = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right|$$

Stability circle center and radius for Source

Derivation of Stability Circles (contd.)

Equation of a circle for Load:

$$|\Gamma_L - c_l|^2 = r_l^{\,2} \qquad c_l \rightarrow Center\;, r_l \rightarrow Radius$$

By symmetry:

$$c_l = \frac{(S_{22} - \Delta S^*_{11})^*}{|S_{22}|^2 - |\Delta|^2} \qquad r_l = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$$

Stability circle center and radius for Load

Amplifier Stability Example

S-parameters of a transistor at 800 MHz are given. Determine the stability of the transistor and plot stability circles on Smith chart.

$$S_{11} = 0.65\angle{-95^0}$$
$$S_{12} = 0.035\angle 40^0$$
$$S_{21} = 5\angle 115^0$$
$$S_{22} = 0.8\angle{-35^0}$$

Find K and Δ for Stability Test

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.504\angle 249.6^0 \rightarrow \quad |\Delta| < 1$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2\,|S_{12}S_{21}|} = 0.547 \not> 1$$

Transistor is conditionally stable at 800 MHz

Stable region on Smith chart needs to be located to choose Γ_S and Γ_L

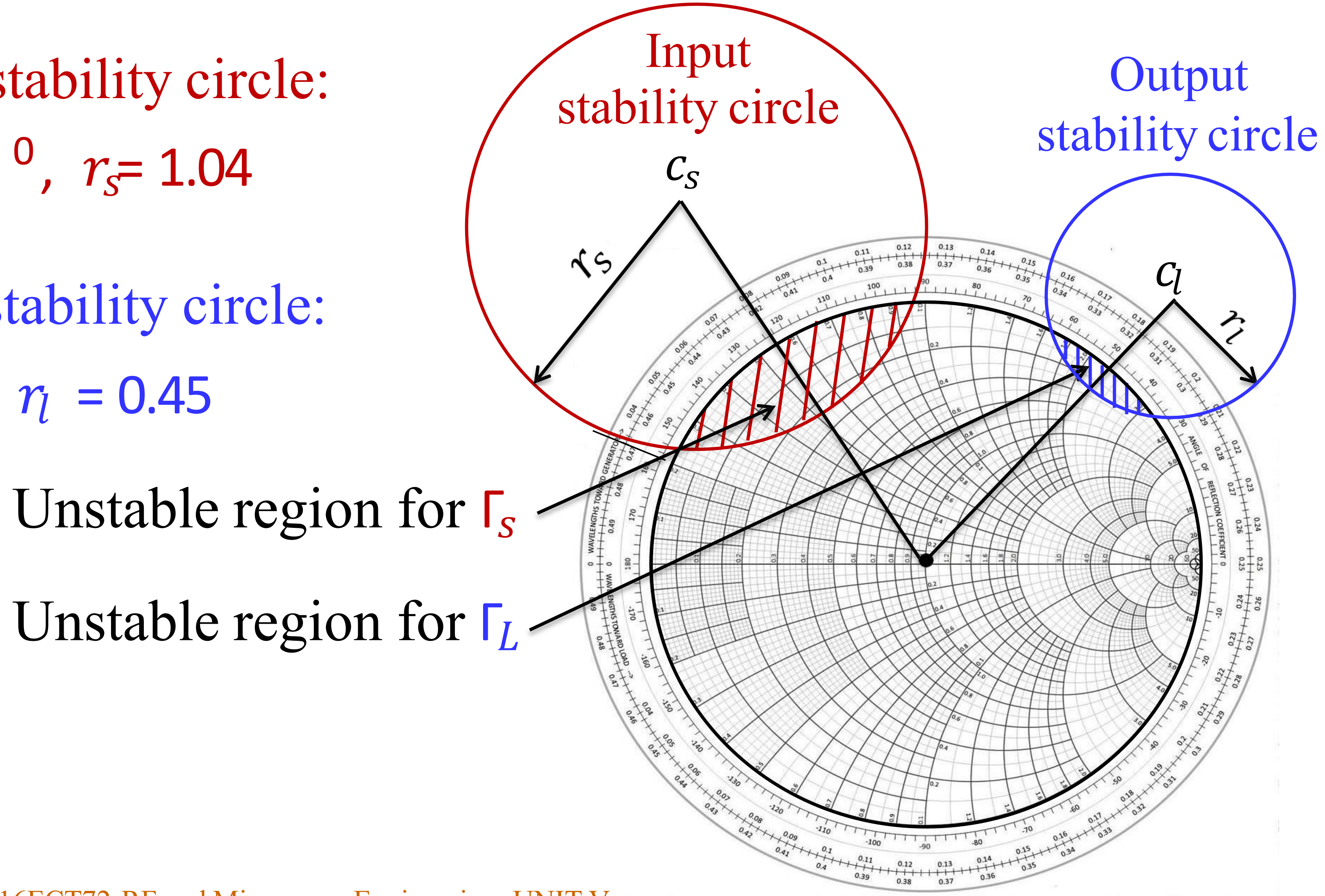
Amplifier Stability Example (Contd.)

Input (Source) stability circle:

$c_s = 1.79\angle 122^0$, $r_s = 1.04$

Output (Load) stability circle:

$c_l = 1.3\angle 48^0$, $r_l = 0.45$



Constant Gain Circles: Unilateral Case

$$G_{tu} = \underbrace{\frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}}_{g_s} |S_{21}|^2 \underbrace{\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}}_{g_l}$$

$$G_{tu\,\max} = \underbrace{\frac{1}{1-|S_{11}|^2}}_{g_{s\,max}} |S_{21}|^2 \underbrace{\frac{1}{1-|S_{22}|^2}}_{g_{l\,max}}$$

For desired G_{tu} gain choose g_s and g_l

Normalized $g_s = g_{ns} = \dfrac{g_s}{g_{s\,max}}$

$$\boxed{\begin{gathered} g_{ns} = g_s(1-|S_{11}|^2) \\[2ex] g_{ns} = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}(1-|S_{11}|^2) \end{gathered}}$$

Constant Gain Circles: Unilateral Case (Contd.)

Solving for Γ_s in $\left|\Gamma_s - c_{gs}\right|^2 = r_{gs}^{\ 2}$

$$c_{gs} = \frac{g_{ns} S_{11}^*}{1 - |S_{11}|^2 (1 - g_{ns})}$$

$$r_{gs} = \frac{\sqrt{1 - g_{ns}}\,(1 - |S_{11}|^2)}{1 - |S_{11}|^2 (1 - g_{ns})}$$

Center and radius of constant gain circle for Source

Similarly for Load

$$c_{gl} = \frac{g_{nl} S_{22}^*}{1 - |S_{22}|^2 (1 - g_{nl})}$$

$$r_{gl} = \frac{\sqrt{1 - g_{nl}}\,(1 - |S_{22}|^2)}{1 - |S_{22}|^2 (1 - g_{nl})}$$

Center and radius of constant gain circle for Load

For maximum gain, $g_{ns} = 1 \Rightarrow c_{gs} = S_{11}^* \quad r_{gs} = 0$

Unilateral Figure of Merit

Error when $|S_{12}| \neq 0$, but is very small and is assumed to be zero

$$\frac{G_t}{G_{tu}} = \frac{\dfrac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\dfrac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}}{\dfrac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\dfrac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}} = \frac{|1-S_{22}\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s}$$

$$\frac{G_t}{G_{tu}} = \frac{|1-S_{22}\Gamma_L|^2}{\left|1-\left(S_{22}+\dfrac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s}\right)\Gamma_L\right|^2}$$

$$= \frac{1}{\left|1-\dfrac{S_{12}S_{21}\Gamma_s\Gamma_L}{(1-S_{11}\Gamma_s)(1-S_{22}\Gamma_L)}\right|^2}$$

$$= \frac{1}{|1-X|^2}$$

$$\boxed{\frac{1}{(1+|X|)^2} < \frac{G_t}{G_{tu}} < \frac{1}{(1-|X|)^2}}$$

Unilateral Figure of Merit (contd.)

When $\Gamma_s = S_{11}^*$ & $\Gamma_L = S_{22}^*$ $G_{tu} \longrightarrow G_{tu\,max}$

Maximum error introduced when using $G_{tu\,max}$ is bounded by

$$\boxed{\frac{1}{(1+M)^2} < \frac{G_t}{G_{tu\,max}} < \frac{1}{(1-M)^2}}$$

$$M = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$$

Unilateral Figure of Merit M should be less than 0.05

Design of an Amplifier

S-parameters of a GaAs MESFET at 8 GHz biased at V_{ds} = 3 V and I_{ds}= 30 $m\,A$ with a 50 Ω reference are:

$$S_{11} = 0.52\angle -145^o$$
$$S_{12} = 0.03\angle 20^o$$
$$S_{21} = 2.56\angle 170^o$$
$$S_{22} = 0.48\angle -20^o$$

Design an amplifier for Gain = 10 dB.

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.168\angle 197^o \quad < 1$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2\,|S_{12}S_{21}|} = 3.53 \; > 1$$

Amplifier is unconditionally stable

$$G_{tm} = |S_{21}|^2 = 6.55 = 8.16\,\text{dB}$$

$$G_{tu\,max} = \frac{1}{1 - |S_{11}|^2}\,|S_{21}|^2\,\frac{1}{1 - |S_{22}|^2} = 11.67 = 10.67\text{ dB}$$

Design of an Amplifier (Contd.)

Maximum Gain Error:

$$M = \frac{|S_{12}||S_{21}||S_{11}||S_{22}|}{(1-|S_{11}|^2)(1-|S_{22}|^2)} = 0.04$$

$$\frac{1}{(1+M)^2} < \frac{G_t}{G_{tu\,max}} < \frac{1}{(1-M)^2}$$

$$0.92 < \frac{G_t}{G_{tu\,max}} < 1.09$$

$$-0.36\ dB < \frac{G_t}{G_{tu\,max}} < +0.37\ dB$$

Design of an Amplifier (Contd.)

$$G_{tu\,max} = \frac{1}{1-|S_{11}|^2}|S_{21}|^2\frac{1}{1-|S_{22}|^2}$$

$$= \underbrace{1.37}_{g_{smax}} \text{ x } 6.55 \text{ x } \underbrace{1.3}_{g_{lmax}} = 11.67 = 10.67 \text{ dB}$$

Design of an amplifier for Gain = 10 dB = 10

Choose $g_s \leq g_{smax}$

Let $g_s = 1.25$, then $g_l = 10 / (1.25 \text{ x } 6.55) = 1.22$

$g_{ns} = g_s(1 - |S_{11}|^2) = 1.25 \text{ x } (1 - 0.52^2) = 0.91$

$g_{nl} = g_l(1 - |S_{22}|^2) = 1.22 \text{ x } (1 - 0.48^2) = 0.94$

Design of an Amplifier (Contd.)

Calculate center and radius of constant gain circles:

$$c_{gs} = \frac{g_{ns} S_{11}^{*}}{1 - |S_{11}|^2 (1 - g_{ns})} = 0.485\angle 145^0$$

$$r_{gs} = \frac{\sqrt{1 - g_{ns}}\,(1 - |S_{11}|^2)}{1 - |S_{11}|^2 (1 - g_{ns})} = 0.224$$

$$c_{gl} = \frac{g_{nl} S_{22}^{*}}{1 - |S_{22}|^2 (1 - g_{nl})} = 0.457\angle 20^0$$

$$r_{gl} = \frac{\sqrt{1 - g_{nl}}\,(1 - |S_{22}|^2)}{1 - |S_{22}|^2 (1 - g_{nl})} = 0.19$$

Design of an Amplifier (Γ_s and Γ_L selection)

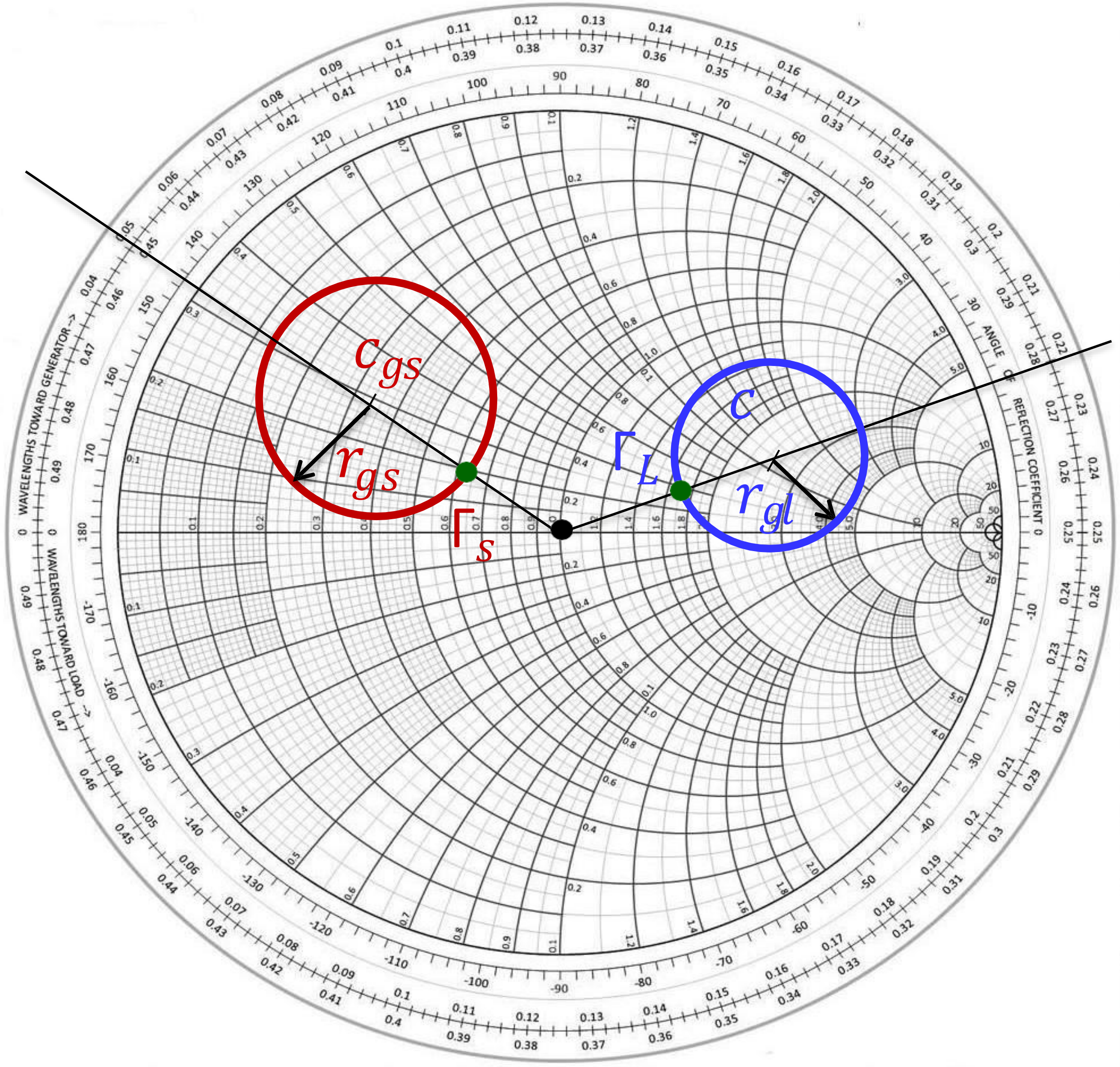
$c_{gs} = 0.485\angle 145^0$, $r_{gs} = 0.224$

$c_{gl} = 0.457\angle 20^0$, $r_{gl} = 0.19$

Since the transistor is unconditionally stable, any point on the constant gain circles can chosen for Γ_s and Γ_L

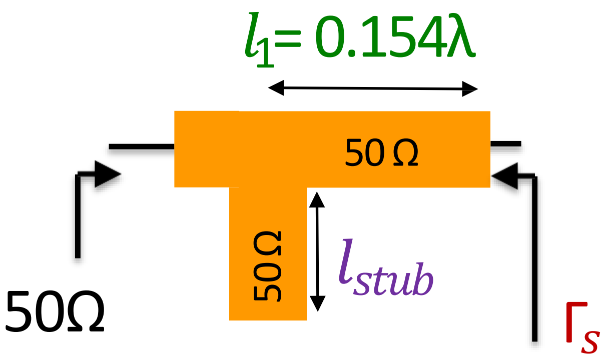
Chosen: $\Gamma_s = 0.261\angle 145^0$

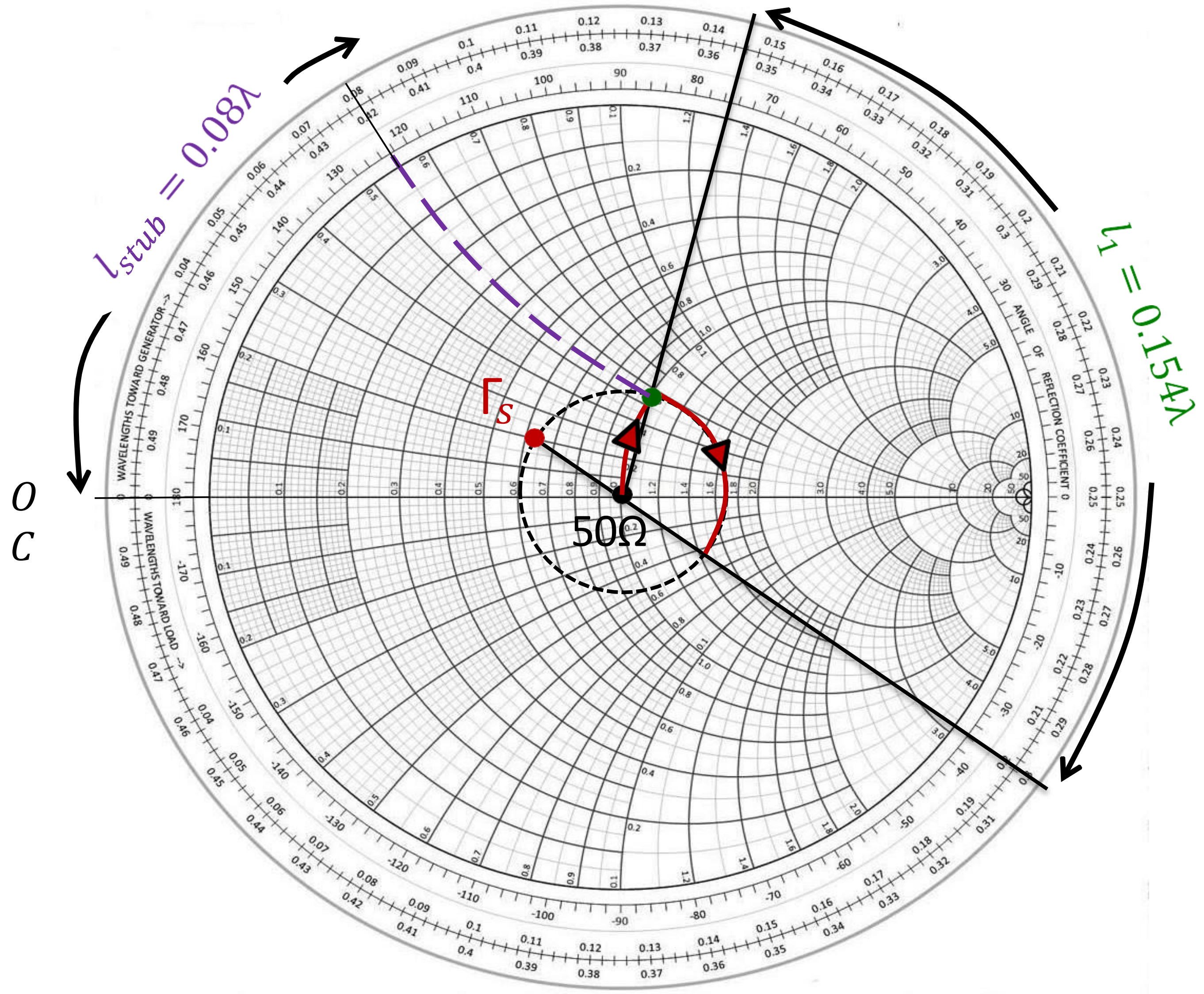
Chosen: $\Gamma_L = 0.267\angle 20^0$



Design of an Amplifier (IMN for Γ_s)

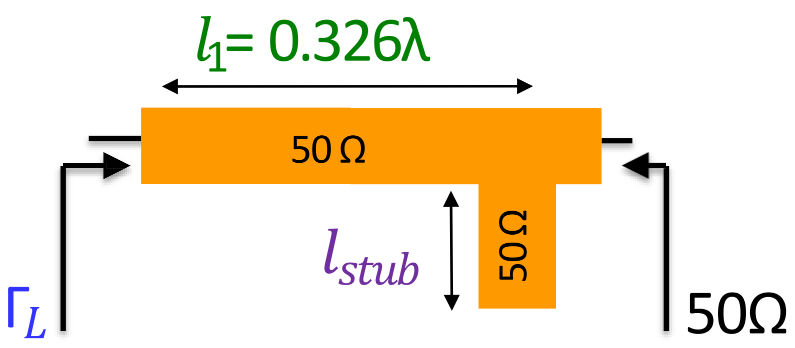
$\Gamma_s = 0.261\angle 145^0$

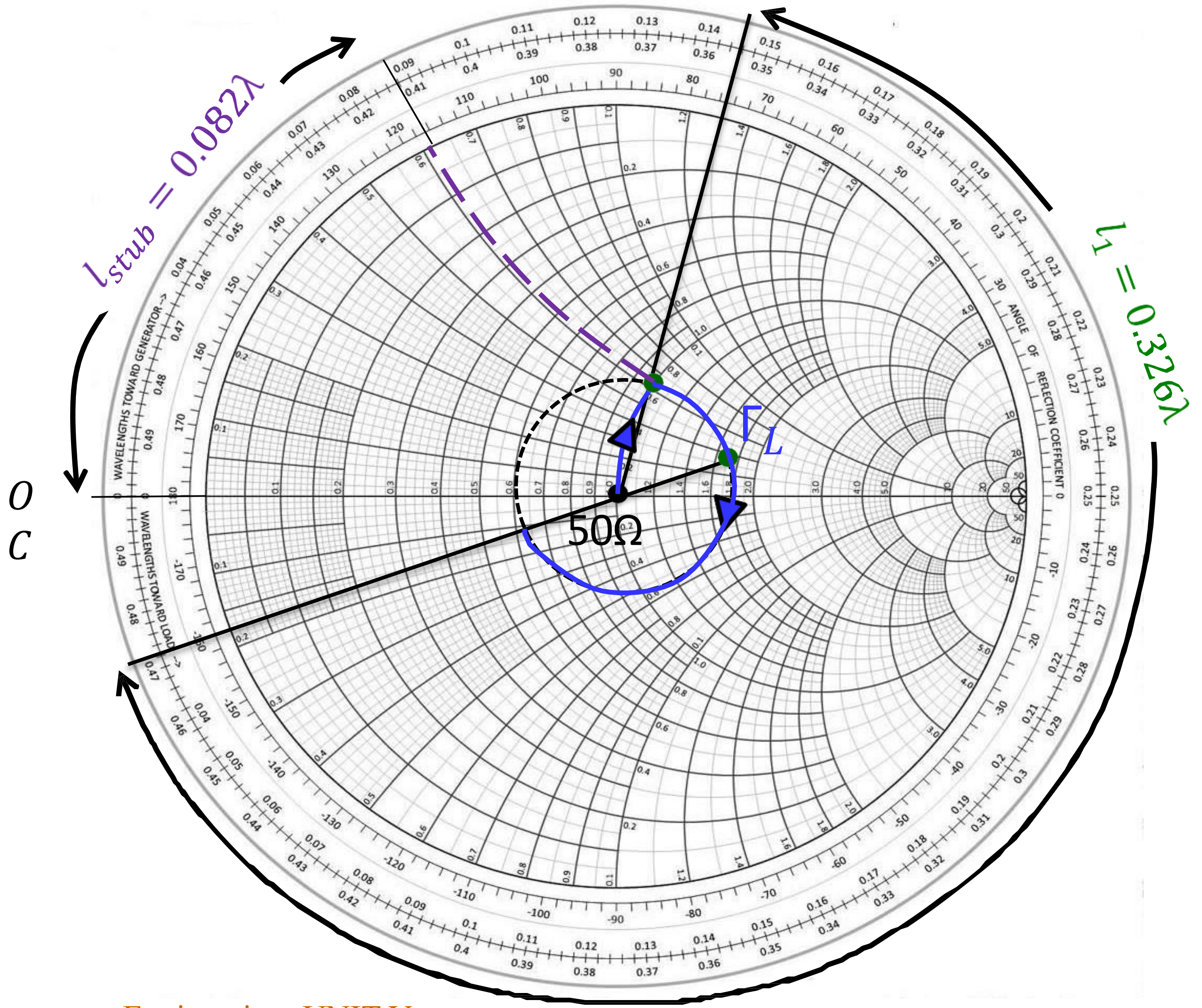




Design of an Amplifier (IMN for Γ_L)

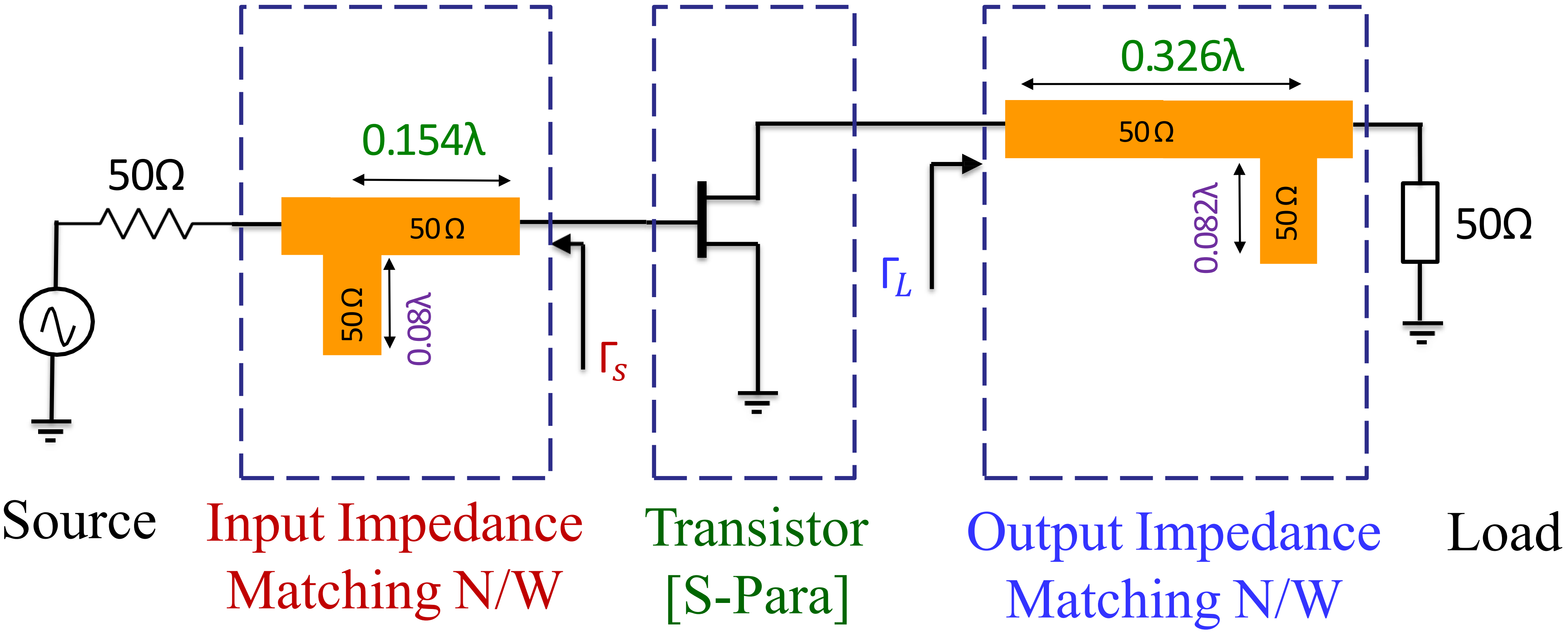
$\Gamma_L = 0.267\angle 20^0$





Design of an Amplifier (Final Circuit)

Final Amplifier Circuit Diagram



Noise figure.

- Noise figure F is defined as “the ratio of the input SNR to the output SNR”.

$$F = \text{Input SNR} / \text{Output SNR}$$

THANKYOU