Problem: -

1.1 The S Parameter of the two port network is given. Find whether the network is reciprocal and lossless.

$$S = \begin{bmatrix} 0.2+j0.4 & 0.8-j0.4 \\ 0.8-j0.4 & 0.2+j0.4 \end{bmatrix}$$

Solution:-

(i) $[S] = [S]^T$ for Reciprocal N/W (OR) $S_{ij} = S_{ji}$ i.e.,

$$[S]^T = \begin{bmatrix} 0.2+j0.4 & 0.8-j0.4 \\ 0.8-j0.4 & 0.2+j0.4 \end{bmatrix} \Rightarrow [S] \quad S_{12} = S_{21} \text{ etc...}$$

Given network is Reciprocal.

(ii) $[S]^* [S]^T = U$ for lossless N/W (OR) $S_{11}S_{11}^* + S_{12}S_{12}^* = 1$ (or)

$|S_{11}|^2 + |S_{12}|^2 = 1$

$$[S]^* = \begin{bmatrix} 0.2-j0.4 & 0.8+j0.4 \\ 0.8+j0.4 & 0.2-j0.4 \end{bmatrix}$$

$\begin{pmatrix} 0.04 \\ +0.16 \end{pmatrix} + \begin{pmatrix} 0.64 \\ +0.16 \end{pmatrix} = 1$

$1 = 1$

Condition is Satisfied

$$U = \begin{bmatrix} 0.2+j0.4 & 0.8-j0.4 \\ 0.8-j0.4 & 0.2+j0.4 \end{bmatrix} \begin{bmatrix} 0.2-j0.4 & 0.8+j0.4 \\ 0.8+j0.4 & 0.2-j0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.04+0.16+0.64+0.16 & 0.16-0.16+0.16-0.16 \\ 0.16-0.16+0.16-0.16 & 0.64+0.16+0.04+0.16 \end{bmatrix}$$

$$U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$, condition is satisfied,

Given Network is Lossless.

1.2) 'S' parameter of a two port network is given by $S_{11} = 0.2\angle 0$, $S_{22} = 0.1\angle 0$, $S_{12} = 0.6\angle 90$, $S_{21} = 0.6\angle 90$. (1) Prove that Network is reciprocal but not lossless. (2) Find the return loss at port 1 when port 2 is short circuit.

Solution:-

(1)

$$[S] = \begin{bmatrix} 0.2\angle 0 & 0.6\angle 90 \\ 0.6\angle 90 & 0.1\angle 0 \end{bmatrix}$$

For reciprocal N/w, $S_{ij} = S_{ji}$ $\therefore$ $S_{12} = S_{21} = 0.6\angle 90$

both are same. $\therefore$ This is reciprocal Network (OR)

$[S] = [S]^T$, which also true for above matrix.

For lossless Network,

$$S_{11} S_{11}^* + S_{12} S_{12}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 1$$

$$(0.2)^2 + (0.6)^2 = 1$$

$$0.04 + 0.36 = 1$$

$$\Rightarrow 0.4 < 1$$

$\therefore$ The network is not lossless. $[S]^* [S]^T = [U]$ also ~~the~~ applicable.

(2) The input reflection co-efficient for mismatched load is given as,

$$\Gamma_1 = S_{11} + \frac{S_{12}^2 \Gamma_2}{1 - S_{22}\Gamma_2}$$

Here port 2 is short circuited, so that $\Gamma_2 = -1$, then

$$\Gamma_1 = S_{11} + \frac{S_{12}^2(-1)}{1 - S_{22}(-1)}$$

Substitute $S_{11}, S_{12}$ and $S_{22}$,

$$\Gamma_1 = 0.2 + \frac{(0.6\angle 90)^2(-1)}{1-(0.1)(-1)}$$

$$\Gamma_1 = 0.2 + \frac{(-0.36)(-1)}{1+0.1}$$

$$\Gamma_1 = 0.2 + \frac{0.36}{1.1} = 0.2 + 0.3273$$

$$\boxed{\Gamma_1 = 0.5273}$$

$$\text{Return Loss (dB)} = 20\log\left(\frac{1}{\Gamma_1}\right)$$

$$= 20 \times \log\left(\frac{1}{0.5273}\right)$$

$$\boxed{\text{Return loss (dB)} = 5.56\ dB}$$

1.3) The S parameters of 2 port N/w are given by $S_{11} = 0.2\angle 90$, $S_{12} = 0.5\angle 90$, $S_{22} = 0.2\angle 90$, $S_{21} = 0.5\angle 0$.

(1) Determine whether the network is lossy or not.

(2) Is the Network symmetrical and reciprocal find the insertion loss of the Network.

Solution:-

$$[S] = \begin{bmatrix} 0.2\angle 90 & 0.5\angle 90 \\ 0.5\angle 0 & 0.2\angle 90 \end{bmatrix}$$

For lossless,

$$S_{11} \cdot S_{11}^* + S_{12} \cdot S_{12}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 = 1$$

$$(0.2)^2 + (0.5)^2 = 1$$

$$0.04 + 0.25 = 1$$

$$0.29 = 1 \Rightarrow 0.29 < 1$$

$\therefore$ The network is not lossless.

For Reciprocal Network,

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21}$$

$0.5\angle 90^\circ = 0.5\angle 0^\circ$. The angle is not same. so it is not symmetrical.

Insertion Loss (dB) :-

$$= 20 \cdot \log \cdot \frac{1}{|S_{21}|}$$

$$= 20 \cdot \log \cdot \frac{1}{|S_{21}|}$$

$$= 20 \cdot \log \cdot \frac{1}{(0.5)(1)}$$

$$= 20 \times \log \frac{1}{(0.5)}$$

$$\boxed{IL = 6.02 \text{ dB}}$$

1.A) A four port Network has the following Matrix

$$[S] = \begin{bmatrix} 0.1\angle 90^\circ & 0.6\angle -45^\circ & 0.6\angle 45^\circ & 0 \\ 0.6\angle -45^\circ & 0 & 0 & 0.6\angle 45^\circ \\ 0.6\angle 45^\circ & 0 & 0 & 0.6\angle -45^\circ \\ 0 & 0.6\angle 45^\circ & 0.6\angle 45^\circ & 0 \end{bmatrix}$$

1.5) A four port network has the following.

$$[S] = \begin{bmatrix} 0.1\angle 90 & 0.8\angle -45 & 0.3\angle 45 & 0 \\ 0.8\angle -45 & 0 & 0 & 0.4\angle 45 \\ 0.3\angle 45 & 0 & 0 & 0.6\angle -45 \\ 0 & 0.4\angle 45 & 0.6\angle -45 & 0 \end{bmatrix}$$

1. Is the Network is lossless 2. Is the NW is reciprocal (3). Find the return loss at port 1 when all other ports are terminated with the matched load. (4). What is insertion loss b/w port 2 and port 4 when all other ports are terminated with matched load.
5) What is the reflection co-efficient seen at port 1 when short circuit is placed at port 3 and all other ports are terminated with the matched load.

Solution:-

(1) For lossless NW,

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$(0.1)^2 + (0.8)^2 + (0.3)^2 + 0 = 1$$

$$0.01 + 0.64 + 0.09 + 0 = 1$$

$$\underline{0.74 < 1}$$

The network is not lossless

(2) For Reciprocal,

$$S_{ij} = S_{ji}$$

$$S_{12} = S_{21} = 0.8\angle -45 \quad ; \quad S_{23} = S_{32} = 0$$

The network is Reciprocal.

1) Is the Network is lossless  2) Is the Network is reciprocal

3) What is the return loss at port 1 and all other ports are matched.

Solution :-

1. Lossless :-

$$\sum_{i=1}^{N} S_{ij} S_{ij}^* = 1$$

$$|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1$$

$$(0.1)^2 + (0.6)^2 + (0.6)^2 + (0)^2 = 1$$

$$0.01 + 0.36 + 0.36 = 1$$

$0.73 = 1$ ⇒ Not true, because $0.73 < 1$

The network is ~~lossless~~. Not lossless

2. Reciprocal :-

$S_{ij} = S_{ji}$ (or) $[S] = [S]^T$

$S_{12} = S_{21}$ (or) $S_{13} = S_{31}$ (or) $S_{41} = S_{14}$

All conditions are true here. the network is reciprocal.

3. If all the ports are Matched,

* Given is a four port N/w, then $a_2, a_3, a_4 = 0$,

so that $\Gamma_2 = 0$, $\Gamma_3 = 0$, $\Gamma_4 = 0$.

$$b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3 + S_{14} a_4$$

$$\therefore S_{11} = \frac{b_1}{a_1} = 0.1 \angle 90^\circ$$

$$\therefore \Gamma_1 = 0.1$$

Return Loss (dB) ⇒

$$RL = 20 \log \frac{1}{\Gamma_1}$$

$$= 20 \log \frac{1}{0.1}$$

$$\boxed{RL = 20dB}$$

Scanned with CamScanner

(3) Return loss

$$RL = 20 \log \frac{1}{\Gamma_1}$$

$$\Gamma_1 = |S_{11}| = \left|\frac{b_1}{a_1}\right|$$

Here $b_1 = S_{11} a_1 + S_{12} a_2 + S_{13} a_3 + S_{14} a_4$.

$P_2, P_3, P_4$ are matched, so that $a_2, a_3, a_4 = 0$

$$b_1 = S_{11} a_1 \Rightarrow S_{11} = 0.1 \angle 90$$

$$\Gamma = 0.1$$

$$\text{Return loss (dB)} = 20 \log \frac{1}{\Gamma_1}$$

$$= 20 \log \frac{1}{0.1}$$

$$\boxed{RL = 20\,dB}$$

(4) Insertion loss b/w Port 2 and Port 4.

$$IL = 20 \log \frac{1}{|S_{24}|}$$

$$= 20 \times \log \frac{1}{|0.4|}$$

$$\boxed{IL = 7.95\,dB}$$

(5) Reflection Coefficient :-

Here port 3 is short ckt'd, $\Gamma_3 = -1$

At Port 1,

$$\Gamma_1 = S_{11} + \frac{S_{13}^2 \Gamma_3}{1 - S_{33}\Gamma_3} \Rightarrow \frac{0.1\angle 90° + (0.3\angle 45)^2(-1)}{1-0}$$

$$= \cancel{0.1} \cancel{+} \frac{\cancel{(0.3)^2 \times (-1)}\ \cancel{(0.3\angle 45)^2(-1)}}{\cancel{1+(0)}} = 0.1\angle 90° + (0.3)\angle 45)^2(-1)$$

$$\cancel{\boxed{\Gamma_1 = 0.1 + 0.09j}} \quad = 0.1\angle 90° + 0.09j$$

$$\Gamma_1 = 0.1j + 0.09j = -0.19j$$

1.6f. A 5dB attenuator & having VSWR of 1.2 assuming that the device is reciprocal. Find 'S' parameter.

Solution:-

$S = 1.2$

$\Gamma = \frac{S-1}{S+1} \Rightarrow \Gamma = \frac{1.2-1}{1.2+1} = \frac{0.2}{2.2}$

$\boxed{\Gamma = 0.09}$

For matched networks, there is no i/p or o/p at Port 2, $a_2 = 0$

$S_{11} = S_{22} = 0.09$

$b_1 = S_{11} a_1 + S_{12} a_2$

$b_1 = S_{11} a_1$

Also $b_2 = a_1 + b_1$

$b_2 = a_1 + S_{11} \cdot a_1$

$b_2 = a_1 (1 + S_{11})$

$\frac{b_2}{a_1} = S_{21} = 1 + S_{11}$

$= 1 + 0.09$

$\boxed{S_{21} = 1.09}$

$\boxed{S_{12} = S_{21} = 1.09}$

$b_1 = S_{11} a_1 + S_{12} \cdot a_2$

$b_2 = S_{21} a_1 + S_{22} a_2$

$b_1 = S_{11} \cdot a_1 \Rightarrow \boxed{b_1 = 0.09 a_1}$

$b_2 = S_{21} a_1$

$b_1 = 0.09 \cdot (b_2 / S_{21})$

?

1.7. An Input of an amplifier has VSWR of 2 and output has VSWR of 3. Find 'S' parameters, $S_{11}$, $S_{12}$, under matched condition.

Solution:-

$S = 2$

$\Gamma_{i/p} = \frac{S-1}{S+1}$

$\Gamma_{i/p} = 0.33$

or

$\Gamma_1 = 0.33$

$S = 3$

$\Gamma_{o/p} = \frac{S-1}{S+1}$

$\Gamma_{o/p} = 0.5$

or

$\Gamma_2 = 0.5$

7. In a waveguide termination having VSWR of 1.1 is used to dissipiate 100 ~~watts~~ of power. Find the reflected Power $P_r$

Sol:-

$$\Gamma = \frac{S-1}{S+1} = \frac{1.1-1}{1.1+1} = \frac{0.1}{2.1} = 0.04762$$

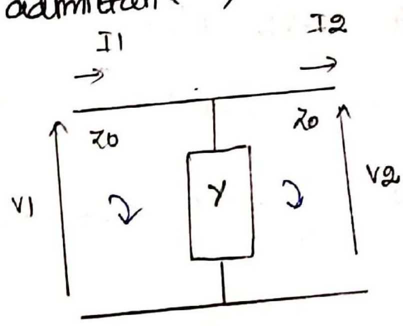
$$|\Gamma|^2 = \frac{P_r}{P_{in}}$$

$$\therefore P_r = |\Gamma|^2 . P_{in}$$

$$= (0.04762)^2 \times 100$$

$$\boxed{P_r = 0.2268 \text{ Watt}}$$

8. Find ABCD parameters of the shunt admittance Y in the transmission line shown in figure.




Solution:-

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$V_1 = AV_2 - BI_2 \rightarrow (1)$

$I_1 = CV_2 - DI_2 \rightarrow (2)$.

In a given Network, apply mesh analysis

$V_1 = I_1 Y - I_2 Y$ ; $V_2 = I_1 Y - I_2 Y$

Here $V_1 = V_2 \rightarrow (3)$ [ parallel Network ]

$A = \frac{V_1}{V_2} = 1 \Big|_{I_2=0}$ ; and $B = \frac{V_1}{-I_2} \Big|_{V_2=0}$ ; While P2 is sc'ted, $V_2=0$ then, $V_1=0$,

$\boxed{B=0}$

$C = -\frac{I_1}{V_2} = ? \Big|_{I_2=0}$ ; $D = \frac{I_1}{I_2} \Big|_{V_2=0}$

When P2 open ckted $I_2=0$,

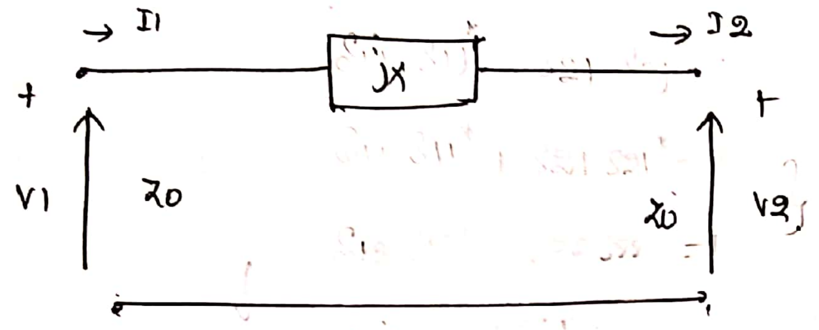
~~$V_1 = I_1 Y$~~

$V_2 = \frac{I_1 - I_2}{Y}$ , ie)

$\boxed{I_1 = V_2 Y + I_2}$ — ④ , compare ④ with ②

$C = Y$ , $D = 1$ .

1.9 Find the ABCD parameters of the series reactance $jX$ placed in the following transmission line.




**Solution:-**

Here $I_1 = I_2$ , series network,

$V_1 = AV_2 + BI_2$

$I_1 = CV_2 + DI_2$

$\therefore \boxed{D = 1}$ , $C = \frac{I_1}{V_2}\Big|_{I_2=0}$ , $\boxed{C = 0}$

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$ ; $B = \frac{V_1}{I_2}\Big|_{V_2=0}$.

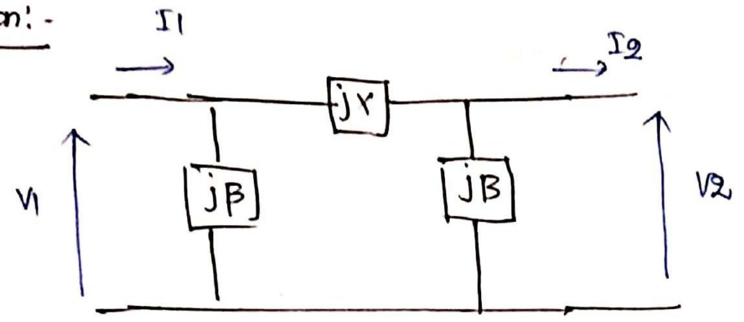
Here $I_2 = \frac{V_1 - V_2}{jX}$ $\Rightarrow$ $V_1 = I_2(jX) + V_2$

$A = 1$ $B = jX$

$$ABCD = \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix}$$

1.10. In a lossless transmission line of characteristic impedance $Z_0$, a series reactance $jX$ and two shunt susceptance $jB$ are placed as shown in figure. Find the ABCD parameters for the network.

Solution:-



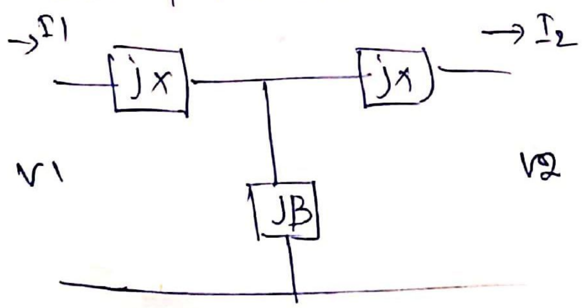

Here the network is symmetric & cascaded.

$$[ABCD] = [ABCD]_1 + [ABCD]_2 + [ABCD]_3$$

$$= \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix} \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix}$$

$$[ABCD] = \begin{bmatrix} 1-BX & jX \\ jB(2-BX) & (1-BX) \end{bmatrix}$$

1.11 Find ABCD parameter.




$$[ABCD] = \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix} \begin{bmatrix} 1 & jX \\ 0 & 1 \end{bmatrix}$$

$$[ABCD] = \begin{bmatrix} 1-BX & jX(2-BX) \\ jB & 1-BX \end{bmatrix}$$