# Maximum Likelihood Estimation

- → Probability distribution [illegible]ction
- → Likelihood function
- → MLE
- → Parameter estimation [illegible]f Bernoulli [illegible]nsity function.

instance space.

$X \rightarrow$ Animal ?

$X \rightarrow$ Bird ?

$f(x, \theta)$

$\theta$ ?

MLE

set of observations.

$\theta$

$f(x|\theta)$

* MLE attempts to find the parameter values that maximize the likelihood function.

* The resulting esti[illegible]te is called Maximum likelihood estimate (MLE).

* We need to estimate the parameters of probability distribution to test whether the given data set follow some particular distribution.

* MLE is a method to estimate the parameters of a probability distribution given observations.

Maximum likelihood estimation (MLE) / Parameter estimation of Bernoulli / KTU Machine learning

$$P(\Theta \mid Data) = \frac{P(Data \mid \Theta)\, P(\Theta)}{P(Data)}$$

$$P(X \mid \Theta)$$

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appear in the function.

* The likelihood of sample $x$ is a function of the parameter $\theta$. It can be defined as the chance that the parameter $\theta$ would generate the observed data.

$$l(\theta) = P(x|\theta)$$

Maximum likelihood estimation (MLE) / Parameter estimation of Bernoulli / KTU Machine learning

$P(\text{Data})$

$P(X|\theta) = l(\theta) \qquad X = \{x_1, x_2 \cdots x_n\}$

$$
\begin{aligned}
l(\theta) &= P(X|\theta) \\
&= P((x_1, x_2, \cdots x_n)|\theta) \\
&= P(x_1|\theta) \cdot P(x_2|\theta) \cdots \cdot P(x_n|\theta)
\end{aligned}
$$

$\log(a \cdot b) = \log a + \log b$

$$\log(l(\theta)) = \log\left[P(x_1|\theta)\, P(x_2|\theta) \cdots P(x_n|\theta)\right]$$

$$\boxed{L(\theta) = \log P(x_1|\theta) + \log P(x_2|\theta) + \cdots + \log P(x_n|\theta)}$$

log likelihood fn

$$L(\Theta) = \log P(x_1|\Theta) + \log P(x_2|\Theta) + \cdots \cdots + \log P(x_n|\Theta)$$

log likelihood fn

$$L(\Theta) = \log f(x_1|\Theta) + \log f(x_2|\Theta) + \cdots \cdots + \log f(x_n|\Theta).$$

~~MLE~~ Max. likelihood estimate (MLE)

# Bernoulli density function

$X$ → Success $\quad x=1. \quad p$

→ Failure. $\quad x=0. \quad 1-p.$

$X$ → Success $X=1$. $p$

→ Failure. $X=0$. $1-p$.

$$f(x|p) = p^x (1-p)^{1-x}$$

$x = 1, 0$

parameter $\theta = p$

$x=1$ $f(x|p) = p^x (1-p)^{1-x}$

$= p^1 (1-p)^0$

$= p$ ✓

density function using MLE method

$$f(x|p) = p^{x}(1-p)^{1-x}$$

$$L(\theta) = \log f(x_1|\theta) + \log f(x_2|\theta) + \cdots + \log f(x_n|\theta)$$

$$L(p) = \log p^{x_1}(1-p)^{1-x_1} + \log p^{x_2}(1-p)^{1-x_2} + \cdots + \log p^{x_n}(1-p)^{1-x_n}$$

$$= x_1 \log p + (1-x_1)\log(1-p) + x_2 \log p + (1-x_2)\log(1-p) + \cdots + x_n \log p + (1-x_n)\log(1-p)$$

$$= (x_1 + x_2 + \cdots + x_n)\log p + \left[(1-x_1) + (1-x_2) + \cdots + (1-x_n)\right]\log(1-p)$$