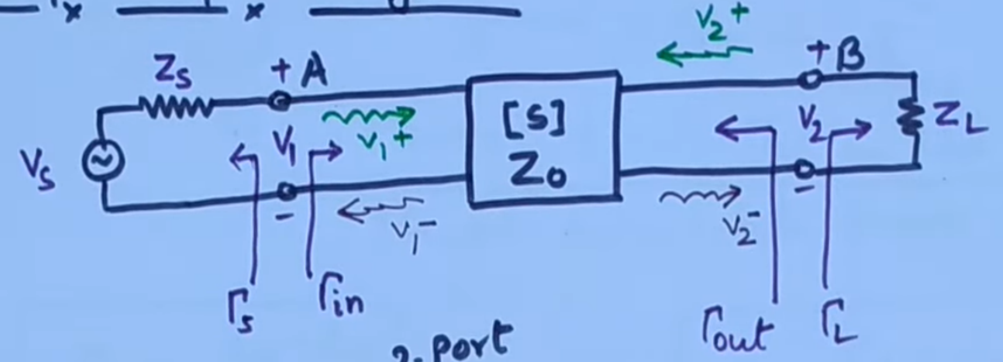
①

Two port power gain

MICROWAVE AMPLIFIER DESIGN.



Some definitions of 2-port power gain

A. Power gain $= G = \dfrac{P_L}{P_{in}} = \dfrac{\text{Power dissipated in } Z_L}{\text{Power delivered to input of the N/W}}$

B. Available power gain $= G = \dfrac{P_{avn}}{P_{avs}}$

$= \dfrac{\text{Power available from the N/W}}{\text{Power available from the source}}$

(2)

C. Transducer power gain $= G_T = \dfrac{P_L}{P_{avs}}$

$$= \frac{\text{Power dissipated to the load}}{\text{Power available from the source.}}$$

$$\boxed{\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}} \quad \text{and} \quad \boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

From signal flow graph or analytical method we can write.

$$\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12} S_{21} \Gamma_s}{1 - S_{11}\Gamma_s}$$

Note

$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$

$= S_{11}V_1^+ + S_{12}\Gamma_L V_2^-$

$\left(\because \Gamma_L = \dfrac{V_2^+}{V_2^-}\right)$

and

$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$

$= S_{21}V_1^+ + S_{22}\Gamma_L V_2^-$

3

$$V_2^- = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-$$

$$\Rightarrow V_2^-(1 - S_{22}\Gamma_L) = S_{21}V_1^+ \quad \text{i.e.} \quad \boxed{V_2^- = \left(\frac{S_{21}}{1 - S_{22}\Gamma_L}\right)V_1^+}$$

Apply voltage division at 'A'

$$V_1 = V_s\left(\frac{Z_{in}}{Z_s + Z_{in}}\right) \Rightarrow (V_1^+ + V_1^-) = V_s\left(\frac{Z_{in}}{Z_s + Z_{in}}\right)$$

$$(V_1^+ + \Gamma_{in}V_1^+) = V_s \frac{Z_0\left(\frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}{Z_0\left(\frac{1+\Gamma_s}{1-\Gamma_s} + \frac{1+\Gamma_{in}}{1-\Gamma_{in}}\right)}$$

[All impedance in reflection coefficient]

$$V_1^+(\cancel{1+\Gamma_{in}}) = V_s\left(\frac{\cancel{1+\Gamma_{in}}}{\cancel{1-\Gamma_{in}}}\right)\left(\frac{(1-\Gamma_s)(\cancel{1-\Gamma_{in}})}{1+\cancel{\Gamma_s}-\cancel{\Gamma_{in}}-\Gamma_s\Gamma_{in}+1-\cancel{\Gamma_s}+\cancel{\Gamma_{in}}-\Gamma_{in}\Gamma_s}\right)$$

$$V_1^+ = \frac{V_s}{2}\left(\frac{1-\Gamma_s}{1-\Gamma_s\Gamma_{in}}\right)$$

4

Avg. Power delivered to the network (P_{in})

→ $$P_{in} = \frac{|V_1^+|^2}{2Z_0}\left(1 - |\Gamma_{in}|^2\right)$$

$$\boxed{P_{in} = \frac{|V_s|^2 |1-\Gamma_s|^2}{8Z_0 |1-\Gamma_s \Gamma_{in}|^2}\left(1 - |\Gamma_{in}|^2\right)}$$

→ Power delivered to the load

$$P_L = \frac{|V_2^-|^2}{2Z_0}\left(1 - |\Gamma_L|^2\right)$$

$$= \left|\frac{S_{21} V_1^+}{1 - S_{22}\Gamma_L}\right|^2 \cdot \frac{1}{2Z_0}\left(1 - |\Gamma_L|^2\right) \quad \because \boxed{V_2^- = \frac{S_{21} V_1^+}{1 - S_{22}\Gamma_L}}$$

5

$$P_L = \frac{|V_s|^2}{8Z_0} \frac{(1-|\Gamma_L|^2)\,|1-\Gamma_s|^2\,|S_{21}|^2}{|1-S_{22}\Gamma_L|^2\,|1-\Gamma_s\Gamma_{in}|^2}$$

i.e. $\boxed{\text{Power gain } (G) = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)\,|1-S_{22}\Gamma_L|^2}}$

Power available from the source.

$$P_{avs} = P_{in}\Big|_{max} = P_{in}\Big|_{\Gamma_{in}=\Gamma_s^*}$$

$$= \frac{|V_s|^2\,|1-\Gamma_s|^2\,(1-|\Gamma_s|^2)}{8Z_0\,\left|1-|\Gamma_s|^2\right|^2} = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{(1-|\Gamma_s|^2)}$$

So, $\boxed{P_{avs} = \frac{|V_s|^2}{8Z_0}\frac{|1-\Gamma_s|^2}{(1-|\Gamma_s|^2)}}$

(6)

Power available from the network

$$P_{avn} = P_L\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|V_s|^2|S_{21}|^2(1-|\Gamma_{out}|^2)|1-\Gamma_s|^2}{8Z_0\cdot|1-S_{22}\Gamma_{out}^*|\,\underbrace{|1-\Gamma_s\Gamma_{in}|^2}}$$

$$|1-\Gamma_s\Gamma_{in}|^2\Big|_{\Gamma_L=\Gamma_{out}^*} = \frac{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)^2}{|1-S_{22}\Gamma_{out}^*|^2}$$

$$P_{avn} = \frac{|V_s|^2}{8Z_0}\,\frac{|S_{21}|^2|1-\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}$$

Available power gain $\boxed{(G_A) = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)}{|1-S_{11}\Gamma_s|^2(1-|\Gamma_{out}|^2)}}$

…sducer power gain $\boxed{(G_T) = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_s)^2(1-|\Gamma_L|^2)}{|1-\Gamma_s\Gamma_{in}|^2|1-S_{22}\Gamma_L|^2}}$

If $\Gamma_L = 0$ & $\Gamma_S = 0$ (i.e. Input and output are matched for zero reflection coefficient).

$$\boxed{G_T = |S_{21}|^2}$$

Unilateral transducer power gain (G_{TU})

if $S_{12} = 0$, $\boxed{\Gamma_{in} = S_{11}}$ and $\boxed{\Gamma_{out} = S_{22}}$

So, $$\boxed{G_{TU} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2 |1-S_{22}\Gamma_L|^2}}$$

G_T as separate components

$$G_T = \underbrace{\left(\frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2}\right)}_{G_S} \cdot \underbrace{\left(|S_{21}|^2\right)}_{G_0} \cdot \underbrace{\left(\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}\right)}_{G_L}$$