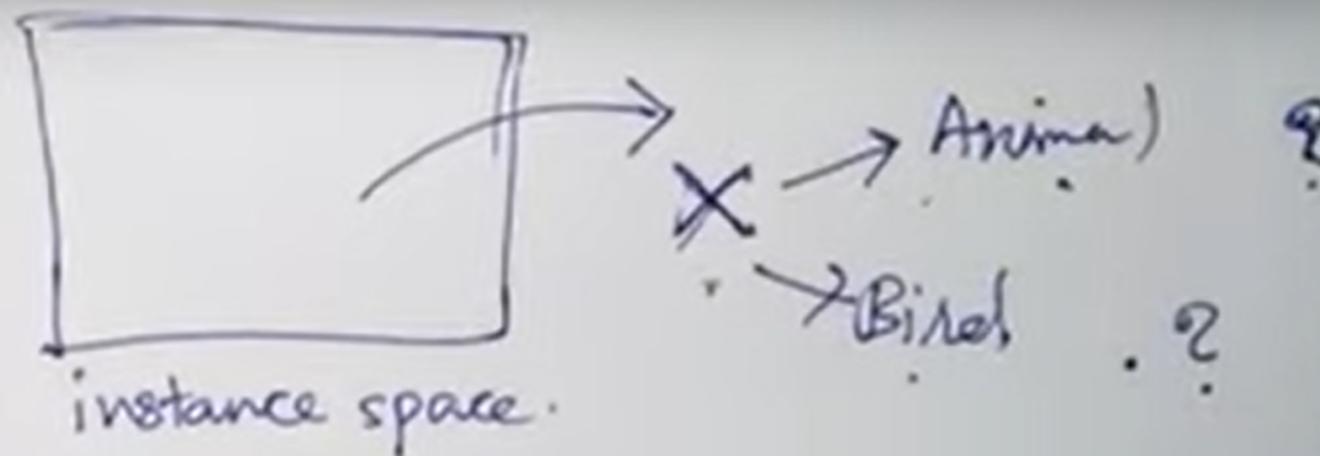
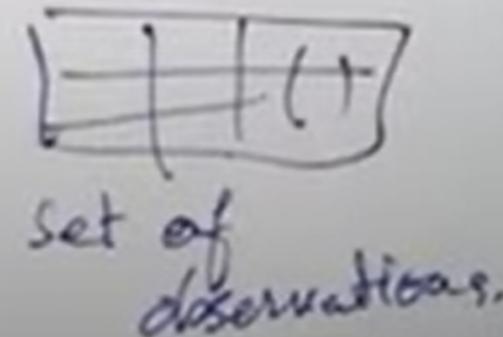


Maximum Likelihood Estimation

- Probability distribution function
- Likelihood function
- MLE
- Parameter estimation & Bernoulli density function.



MLE



$f(x, \theta)$

* MLE attempts to find the parameter values that maximize the likelihood function.

* The resulting estimate is called Maximum likelihood estimate (MLE).

* We need to estimate the parameters of probability distribution to test whether the given data set follow some particular distribution.

* MLE is a method to estimate the parameters of a probability distribution given observations.

$$P(\theta | \text{Data}) =$$

$$\frac{P(\text{Data} | \theta) P(\theta)}{P(\text{Data})}$$

$$P(x | \theta)$$

of parameters that appear in the function.

- * The likelihood of sample x is a function of the parameter θ . It can be defined as the chance that the parameter θ would generate the observed data.

$$l(\theta) = P(x|\theta)$$

$$(1-\lambda)P(y|\theta)$$

P(Data)

$$P(X|\theta) = l(\theta)$$

$$X = \{x_1, x_2, \dots, x_n\}$$

$$l(\theta) = P(X|\theta)$$

$$= P(x_1, x_2, \dots, x_n | \theta)$$

$$= P(x_1 | \theta) \cdot P(x_2 | \theta) \cdots \cdot P(x_n | \theta)$$

$$\log(a \cdot b) = \log a + \log b$$

$$\log(\ell(\Theta)) = \log [P(x_1|\Theta) P(x_2|\Theta) \dots P(x_n|\Theta)]$$

$$L(\Theta) = \log P(x_1|\Theta) + \log P(x_2|\Theta) + \dots + \log P(x_n|\Theta)$$

log likelihood

$$L(\theta) = \log P(x_1|\theta) + \log P(x_2|\theta) + \dots \\ \underbrace{\log \text{likelihood}}_{f^n} - \dots + \log P(x_n|\theta)$$

$$L(\theta) = \log f(x_1|\theta) + \log f(x_2|\theta) + \dots \\ \dots + \log f(x_n|\theta).$$

MLE Max. likelihood estimate (MLE)

Bernoulli density function

X → Success
→ Failure.

$x = 1$. p
 $x = 0$. $1 - p$.

\times $\rightarrow \text{Success}$ $x = 1.$ p $\rightarrow \text{Failure}.$ $x = 0.$ $1-p.$

$$f(x|p) = p^x (1-p)^{1-x}$$

 $\alpha = 1, 0$

parameter $\theta = p$

 $x = 1$

$$f(x|p) = p^x (1-p)^{1-x}$$

$$= p^1 (1-p)^0$$

$$= p \checkmark$$

$$f(x|p) = p^x (1-p)^{1-x}$$

$$L(\theta) = \log f(x_1|\theta) + \log f(x_2|\theta) + \dots + \log f(x_n|\theta)$$

$$L(p) = \log p^{x_1} (1-p)^{1-x_1} + \log p^{x_2} (1-p)^{1-x_2} + \dots + \log p^{x_n} (1-p)^{1-x_n}$$

$$\begin{aligned} &= \cancel{x_1 \log p + (1-x_1) \log(1-p)} + x_2 \log p + (1-x_2) \log(1-p) \\ &\quad + \dots + x_n \log p + (1-x_n) \log(1-p) \end{aligned}$$

$$= (x_1 + x_2 + \dots + x_n) \log p + [(1-x_1) + (1-x_2) + \dots + (1-x_n)] \log(1-p)$$