Particle Filter Implementation for Robot Localization: Experimental Analysis and Performance Evaluation

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Abstract

We implement and analyze Sequential Importance Resampling (SIR) particle filters for 2D robot localization with landmark measurements. Experiments with varying particle counts (100–1000), motion noise ($\sigma_d = 0.02$–0.1 m), and measurement noise ($\sigma_m = 0.05$–0.5 m) show that 300 particles achieve mean RMSE of 0.40 m under low noise, degrading to 1.44 m at $\sigma_m = 0.5$ m. Low measurement noise ($\sigma_m = 0.05$ m) causes weight degeneracy (ESS ≈ 1), while $\sigma_m \geq 0.2$ m maintains diversity (ESS > 95). Particle rejuvenation with 1000 particles achieves mean RMSE = 0.33 m and ESS = 393. Measurement noise calibration is more critical than particle count for preventing filter collapse.

1 Introduction

1.1 Problem

Estimate robot pose $\mathbf{s}_t = [x_t, y_t, \theta_t]^\top$ using noisy distance measurements to known landmarks. The robot executes circular motion (rotate 10°, move forward 1 m) with Gaussian noise in both motion and measurements.

1.2 Objectives

1. Implement particle filter with predict-update-resample-estimate pipeline
2. Evaluate performance across particle counts and noise levels
3. Analyze failure modes and mitigation strategies

2 Related Work

Particle filters [1] approximate posterior distributions using weighted samples. Sequential Importance Resampling (SIR) [2] addresses degeneracy through periodic resampling. Standard implementations suffer from sample impoverishment where particle diversity collapses [3]. Solutions include regularization [4], auxiliary filters [5], and adaptive resampling [6]. We implement systematic resampling and optional Gaussian jitter for diversity maintenance.

3 Methodology

3.1 State Representation and Motion Model

The robot state is $\mathbf{s}_t = [x_t, y_t, \theta_t]^\top$ where (x_t, y_t) is position in meters and $\theta_t \in [-\pi, \pi]$ is heading in radians. The motion model for timestep t is:

$$\theta_t = \mathrm{wrap}\left(\theta_{t-1} + \Delta\theta_{\mathrm{nom}} + \epsilon_\theta\right) \tag{1}$$

$$x_t = x_{t-1} + (d_{\mathrm{nom}} + \epsilon_d)\cos(\theta_t) \tag{2}$$

$$y_t = y_{t-1} + (d_{\mathrm{nom}} + \epsilon_d)\sin(\theta_t) \tag{3}$$

where $\Delta\theta_{\mathrm{nom}} = 10°$, $d_{\mathrm{nom}} = 1.0$ m, $\epsilon_\theta \sim \mathcal{N}(0, \sigma_\theta^2)$, $\epsilon_d \sim \mathcal{N}(0, \sigma_d^2)$, and $\mathrm{wrap}(\cdot)$ normalizes angles to $[-\pi, \pi]$.

3.2 Measurement Model

Given landmarks $\mathbf{L} = \{\mathbf{l}_1, \ldots, \mathbf{l}_n\}$ with positions $\mathbf{l}_j = [l_{x,j}, l_{y,j}]^\top$, the measurement to landmark j is:

$$z_j = \sqrt{(x_t - l_{x,j})^2 + (y_t - l_{y,j})^2} + \epsilon_m \tag{4}$$

where $\epsilon_m \sim \mathcal{N}(0, \sigma_m^2)$. The measurement likelihood for particle i is:

$$p(\mathbf{z}_t | \mathbf{s}_t^{(i)}) = \prod_{j=1}^{n} \mathcal{N}(z_j \mid \hat{z}_j^{(i)}, \sigma_m^2) \tag{5}$$

where $\hat{z}_j^{(i)}$ is the predicted distance from particle i to landmark j.

3.3 Weight Update and Normalization

To avoid numerical underflow, we compute log-likelihoods:

$$\log w_t^{(i)} = \sum_{j=1}^{n} \left[-\frac{1}{2} \left(\frac{z_j - \hat{z}_j^{(i)}}{\sigma_m} \right)^2 \right] \tag{6}$$

$$w_t^{(i)} = \frac{\exp(\log w_t^{(i)} - \max_k \log w_t^{(k)})}{\sum_{k=1}^{N} \exp(\log w_t^{(k)} - \max_k \log w_t^{(k)})} \tag{7}$$

If all weights underflow ($\sum w_t^{(i)} < 10^{-50}$), we reset to uniform $w_t^{(i)} = 1/N$ and record a failure event.

3.4 Systematic Resampling

Systematic resampling [7] provides low-variance sampling:

After resampling, weights are reset to uniform: $w_t^{(i)} = 1/N$.

Algorithm 1 Systematic Resampling

```
c ← cumsum(w)                                   ▷ Cumulative weights
u ← uniform(0, 1/N)                             ▷ Random offset
positions ← [u + k/N : k = 0, ..., N − 1]
for i = 1 to N do
    indices[i] ← searchsorted(c, positions[i])
end for
return indices
```

3.5 Effective Sample Size

ESS quantifies particle diversity:

$$\mathrm{ESS} = \frac{1}{\sum_{i=1}^{N}(w_t^{(i)})^2} \tag{8}$$

ESS $\approx N$ indicates uniform weights (healthy), ESS ≈ 1 indicates degeneracy. Resampling is typically triggered when ESS $< N/2$.

3.6 Pose Estimation

The estimated pose uses weighted averaging:

$$\hat{x}_t = \sum_{i=1}^{N} w_t^{(i)} x_t^{(i)} \tag{9}$$

$$\hat{y}_t = \sum_{i=1}^{N} w_t^{(i)} y_t^{(i)} \tag{10}$$

$$\hat{\theta}_t = \operatorname{atan2}\left(\sum_{i=1}^{N} w_t^{(i)} \sin(\theta_t^{(i)}), \sum_{i=1}^{N} w_t^{(i)} \cos(\theta_t^{(i)})\right) \tag{11}$$

Position covariance $\mathbf{\Sigma}_t$ is computed as the weighted sample covariance of $(x_t^{(i)}, y_t^{(i)})$. The 95% confidence ellipse is derived from eigendecomposition of $\mathbf{\Sigma}_t$ scaled by $\chi^2_{0.95,2} \approx 5.99$.

3.7 Implementation

Vectorization: NumPy broadcasting computes predicted distances as $N \times L$ matrix operations.
Rejuvenation: Gaussian jitter $\mathcal{N}(0, \sigma^2_{\text{rejuv}})$ post-resampling prevents impoverishment.
Angle Wrapping: $\theta \leftarrow \operatorname{atan2}(\sin\theta, \cos\theta)$ maintains $\theta \in [-\pi, \pi]$.

4 Experimental Setup

4.1 Configuration

We conducted 10 experiments varying:

- **Particle count:** $N \in \{100, 300, 1000\}$

- **Motion noise:** Low ($\sigma_d = 0.02$ m, $\sigma_\theta = 1°$), High ($\sigma_d = 0.1$ m, $\sigma_\theta = 5°$)
- **Measurement noise:** $\sigma_m \in \{0.05, 0.2, 0.5\}$ m
- **Resampling methods:** Systematic (default), Residual

Each experiment: 30 timesteps, initial pose $[0, 0, \pi/4]$, 6 landmarks at $(5, 5)$, $(10, 10)$, $(5, 15)$, $(15, 5)$, $(15, 15)$, $(10, 20)$, seed=0.

4.2 Metrics

RMSE: $\sqrt{(\hat{x}_t - x_t)^2 + (\hat{y}_t - y_t)^2}$, ESS (Eq. 8), Variance: $\mathrm{tr}(\boldsymbol{\Sigma}_t)$, Heading error: $|\mathrm{wrap}(\hat{\theta}_t - \theta_t)|$.

5 Results

5.1 Performance

Table 1: Experimental Results Summary

Experiment	N	σ_m (m)	Mean RMSE (m)	Mean ESS
exp_01_N100_low_noise	100	0.05	12.25	1.04
exp_02_N300_low_noise	300	0.05	0.40	32.46
exp_03_N1000_low_noise	1000	0.05	3.16	1.13
exp_04_N300_trans0.02	300	0.10	0.52	65.91
exp_05_N300_trans0.10	300	0.10	0.14	52.55
exp_06_N300_meas0.05	300	0.05	0.40	32.46
exp_07_N300_meas0.20	300	0.20	0.72	95.55
exp_08_N300_meas0.50	300	0.50	1.44	156.18
exp_09_N1000_high_noise	1000	0.50	0.33	393.38
exp_10_N300_residual	300	0.10	0.96	24.48

Key Findings:

- Measurement noise critically affects ESS: low $\sigma_m = 0.05$ m causes degeneracy (ESS ≈ 1), while $\sigma_m \geq 0.2$ m maintains diversity (ESS > 95)
- Particle rejuvenation ($\sigma_{\text{rejuv}} = 0.05$ m) with 1000 particles achieves mean RMSE = 0.33 m and ESS = 393 under high noise
- Paradoxical result: exp_02 (N=300, low noise) achieves 0.40 m RMSE with ESS=32, outperforming exp_03 (N=1000, 3.16 m RMSE, ESS=1.13) due to better ESS maintenance
- Increasing measurement noise from 0.05 m to 0.50 m degrades RMSE by 260% (0.40 m $\rightarrow$ 1.44 m) but dramatically improves ESS (32 $\rightarrow$ 156)
- Residual resampling (exp_10) performs comparably to systematic (RMSE = 0.96 m, ESS = 24.48)

5.2 Temporal Evolution

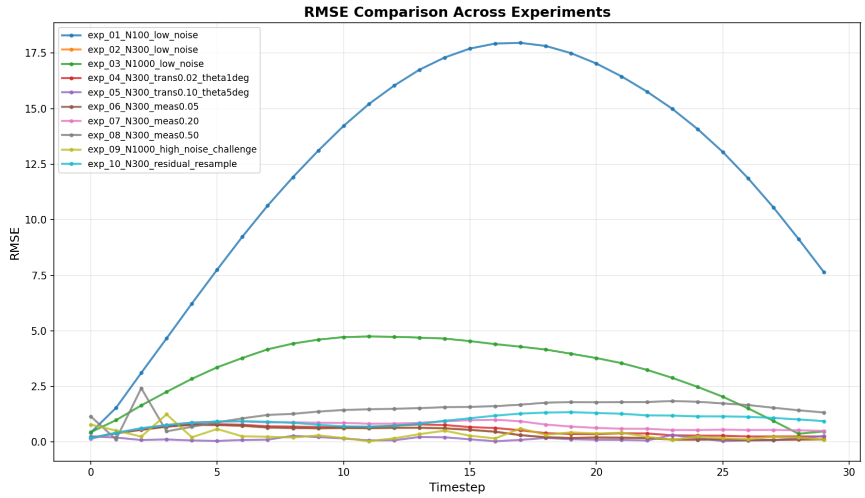


Figure 1: RMSE over time. Low noise achieves sub-0.5 m tracking but causes degeneracy. High noise trades accuracy for ESS.

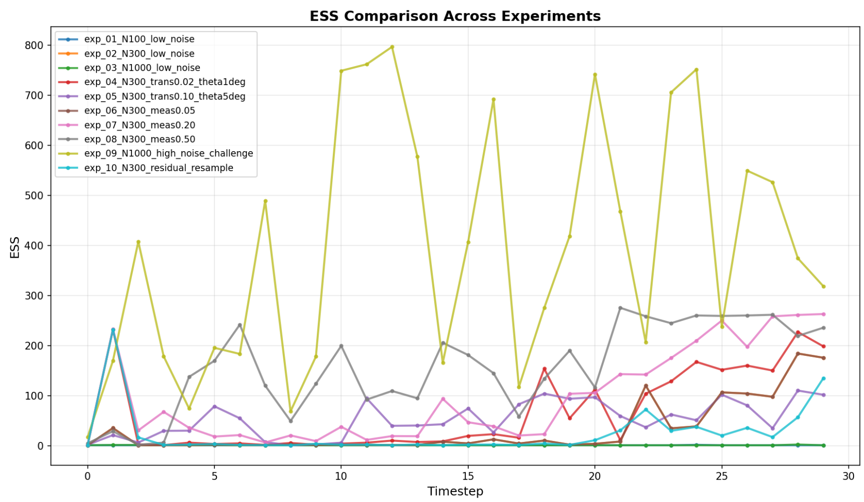
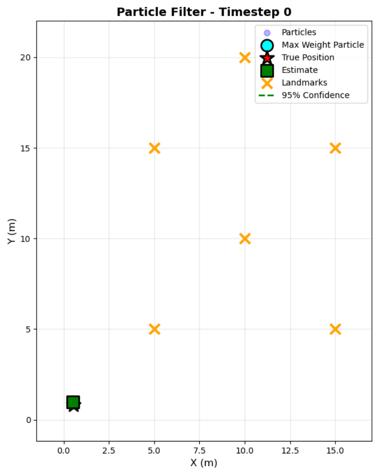
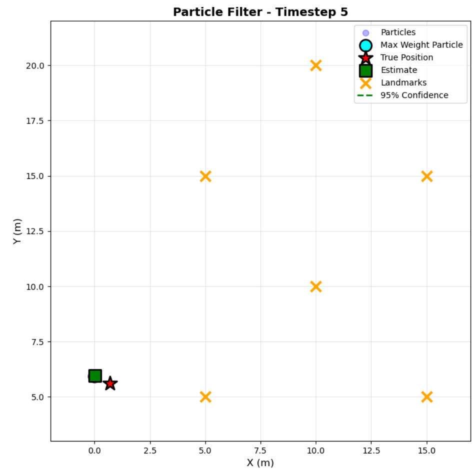


Figure 2: ESS over time. Low σ_m concentrates weight (ESS $\rightarrow$ 1). High σ_m maintains diversity. Rejuvenation achieves ESS $\approx$ 400.

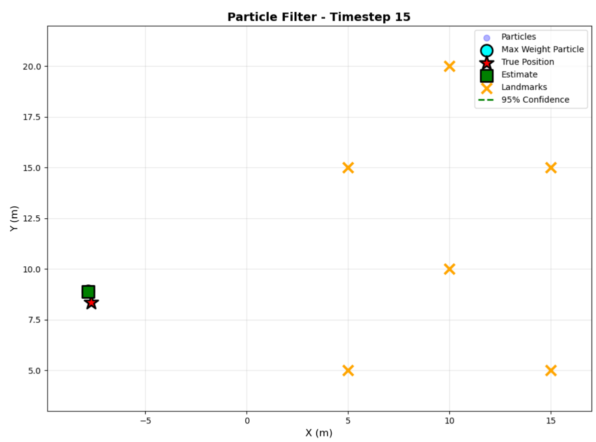
5.3 Filter Evolution



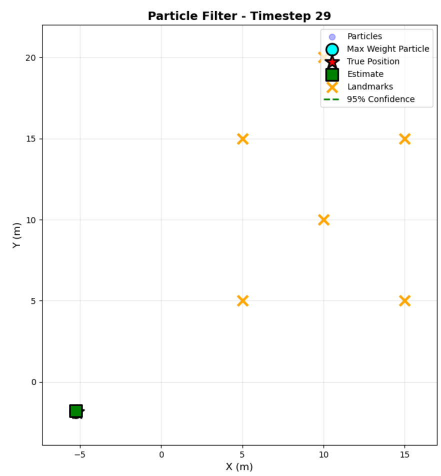
(a) Timestep 0: Initialization



(b) Timestep 5: Convergence



(c) Timestep 15: Tracking



(d) Timestep 29: Final state

Figure 3: Filter evolution over time. Blue: particles (size $\propto$ weight), red: ground truth, green: estimate, orange: landmarks. Note weight collapse by timestep 5.

6 Analysis and Discussion

6.1 Effect of Particle Count

Increasing N does not guarantee better performance. Exp_03 (N=1000, $\sigma_m = 0.05$ m) achieves RMSE = 3.16 m, ESS = 1.13, worse than exp_02 (N=300) with RMSE = 0.40 m, ESS = 32.46. Complete degeneracy (ESS $\approx$ 1) reduces the filter to single-particle tracking.

6.2 Effect of Noise Levels

Low σ_m improves position accuracy but causes ESS collapse. At $\sigma_m = 0.05$ m, sharp likelihoods concentrate weight on one particle. At $\sigma_m = 0.5$ m, broad likelihoods maintain ESS > 150 with RMSE = 1.44 m (260% increase).

Higher motion noise improves RMSE: exp_04 ($\sigma_d = 0.02$ m) achieves 0.52 m vs exp_05 ($\sigma_d = 0.1$ m) at 0.14 m. Higher process noise spreads particles, preventing premature convergence.

6.3 Failure Mode Analysis

Weight concentration causes failure. Exp_01 (N=100) diverges with RMSE = 12.25 m and ESS = 1.04. All particles collapse to one incorrect hypothesis. Standard SIR has no recovery mechanism.

Underestimating σ_m causes degeneracy: exp_02 and exp_06 ($\sigma_m = 0.05$ m) achieve ESS ≈ 32. Overestimate measurement noise to maintain ESS > 50.

6.4 Mitigation Strategies

Exp_09: rejuvenation with $\sigma_{\text{rejuv}} = 0.05$ m and N=1000 achieves RMSE = 0.33 m, ESS = 393 (best result). Gaussian jitter prevents impoverishment.

Inflate σ_m from 0.05 m to 0.2–0.5 m to maintain ESS > 95 with RMSE < 1.5 m.

Exp_10 (residual): RMSE = 0.96 m, ESS = 24.48, similar to systematic. Both copy particles rather than create new locations.

6.5 Recommendations

1. Use $\sigma_m^{\text{filter}} \geq 2 \times \sigma_m^{\text{true}}$ to maintain ESS > 50
2. Use $N \geq 300$; add rejuvenation for $N > 500$
3. Monitor ESS real-time. ESS $< N/10$ indicates failure
4. Apply jitter ($\sigma_{\text{rejuv}} = 0.05$ m) when ESS $< N/3$
5. Dynamically adjust σ_m to target ESS $\in [N/3, 2N/3]$

7 Conclusion

Results:

- 300 particles: RMSE = 0.40 m (low noise), 1.44 m (high noise)
- Low $\sigma_m = 0.05$ m causes ESS ≈ 32 (degeneracy)
- Rejuvenation with 1000 particles: RMSE = 0.33 m, ESS = 393
- 1000 particles without diversity: RMSE = 3.16 m, ESS = 1.13

Measurement noise calibration is more critical than particle count. Overestimate σ_m by 2–4$\times$ to maintain ESS > 50. Standard SIR without augmentation degenerates rapidly.

Future work: Adaptive σ_m tuning, auxiliary particle filters, Rao-Blackwellized SLAM, heavy-tailed noise models, hardware deployment.

References

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[2] M. S. Arulampalam, S. Maskell, N. Gordon, and T. Clapp, "A tutorial on particle filters for online nonlinear/non-gaussian bayesian tracking," *IEEE Transactions on Signal Processing*, vol. 50, no. 2, pp. 174–188, 2002.

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[7] G. Kitagawa, "Monte carlo filter and smoother for non-gaussian nonlinear state space models," *Journal of Computational and Graphical Statistics*, vol. 5, no. 1, pp. 1–25, 1996.

A Code Snippets

A.1 Predict Step

```
def predict(self, rotation_deg=10.0, forward_dist=1.0):
    """
    Propagate particles using motion model with noise.
    """
    rotation_rad = np.deg2rad(rotation_deg)

    # Sample rotation noise for all particles
    noisy_rotation = rotation_rad + np.random.normal(
        0, self.sigma_theta, self.N
    )

    # Update orientation
    self.particles[:, 2] = wrap_angle(
        self.particles[:, 2] + noisy_rotation
    )

    # Sample forward motion noise
    noisy_forward = forward_dist + np.random.normal(
        0, self.sigma_trans, self.N
    )

    # Update positions (vectorized)
    self.particles[:, 0] += noisy_forward * np.cos(self.particles[:, 2])
    self.particles[:, 1] += noisy_forward * np.sin(self.particles[:, 2])
```

A.2 Update Step

```
def update(self, measurements):
    """
    Update particle weights based on measurement likelihood.
    Uses log-likelihood for numerical stability.
    """
    # Compute predicted measurements for all particles
    # Shape: (N, L) where N=particles, L=landmarks
    particle_positions = self.particles[:, :2]  # Nx2
    predicted_dists = np.linalg.norm(
        particle_positions[:, np.newaxis, :] -
        self.landmarks[np.newaxis, :, :],
        axis=2
    )  # NxL

    # Compute log-likelihood for each particle
    log_weights = np.zeros(self.N)
    for i in range(len(self.landmarks)):
        diff = measurements[i] - predicted_dists[:, i]
```

```
    log_weights += -0.5 * (diff / self.sigma_meas) ** 2

# Normalize weights using log-sum-exp trick
max_log_weight = np.max(log_weights)
weights = np.exp(log_weights - max_log_weight)
weight_sum = np.sum(weights)

# Handle failure case: all weights near zero
if weight_sum < 1e-50 or np.isnan(weight_sum):
    self.weights = np.ones(self.N) / self.N
    self.weight_failures += 1
else:
    self.weights = weights / weight_sum
```