# MTH100 LINEAR ALGEBRA

## Worksheet 1

**Question 1.** Reduce the following matrix to an RREF matrix using elementary row operations.

$$A = \begin{bmatrix} 1 & 2 & -3 & 0 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -4 & 1 \end{bmatrix}.$$

**Question 2.** Reduce the following matrix to an RREF matrix using elementary row operations.

$$A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$

**Question 3.** Explicitly describe all non-zero $2 \times 2$ RREF matrices. You may also try to do this for $2 \times 3$ and $3 \times 3$ RREF matrices.

**Question 4.** Define a relation $T$ on the real number system $\mathbf{R}$ by $xTy$ if $y - x \in \mathbf{Z}$, the set of integers. Is $T$ an equivalence relation? Justify your answer. If yes, can you find a special representative in each equivalence class, just as we could do for row equivalence of matrices?

**Question 5.** Prove that row-reduction is an equivalence relation on the set $\mathbf{R}^{m \times n}$ of all m by n matrices with real entries.

**Question 6.** Show that if $E$ is an equivalence relation on a set $X$, then any two distinct equivalence classes must be disjoint.Also show that every element of $X$ has to belong to an equivalence class.

The equivalence class of any element $a \in X$ is the set of all elements of $X$ which are related to $a$, the formal definition is:

$[a] = \{x \in X : xEa, i.e. x \text{ is related to a under the ralation } E\}$

**Question 7.** Show that if $P$ is a partition of a set $X$, then there exists an equivalence relation $E$ on $X$ such that the equivalence classes correspond to the parts of the given partition $P$.($Q.7$ *is the converse of* $Q.6$)

**Question 8.** Find the solution set in the vector form for the homogeneous system $Ax = 0$ given $A$ below. NB: $A$ must be row-reduced to an RREF matrix in order to give the solution in standard form.

$$A = \begin{bmatrix} 1 & -2 & 3 & -1 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & 2 & 3 \end{bmatrix}$$