Quiz 6

Oct 18th, 2024

Time: 20 minutes Max marks = 5

Name: ______________ Roll no.: ______________ Group: ______________

Instructions: Notes, books, computers, cell phones and other electronic devices are not allowed.

Problem 1. Find a basis of the null space, a basis of the column space and a basis of the row space of the given matrix. $A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix}$, Rubrics for Quiz 6

Total points = 5. ①

Solution of Quiz 6:

+2.5

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 + 3R_1 \\ R_4 \to R_4 - 4R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 0 & -3 & -6 & -1 & -13 \\ 0 & 5 & 10 & 1 & 19 \\ 0 & -3 & -6 & -3 & -21 \end{bmatrix}$$

$$\xrightarrow{R_2 \to \left(-\frac{1}{3}\right) R_2} \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\ 0 & 5 & 10 & 1 & 19 \\ 0 & -3 & -6 & -3 & -21 \end{bmatrix} \xrightarrow[R_4 \to R_4 + 3R_2]{R_3 \to R_3 - 5R_2} \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\ 0 & 0 & 0 & -\frac{2}{3} & -\frac{8}{3} \\ 0 & 0 & 0 & -2 & -8 \end{bmatrix}$$

$$\xrightarrow{R_3 \to \left(\frac{-3}{2}\right) R_3} \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -2 & -8 \end{bmatrix} \xrightarrow{R_4 \to R_4 + 2R_3} \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[R_1 \to R_1 - R_3]{R_2 \to R_2 - \frac{1}{3} R_3} \begin{bmatrix} 1 & 1 & 3 & 0 & 2 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2}$$

$$\text{RREF matrix } R \text{ (say)} = \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \\ 0 & 1 & 2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So, to solve the system of equations $A\bar{x} = \bar{0}$
we solve $R\bar{x} = 0$

$$\Rightarrow \left.\begin{aligned} x_1 + x_3 - x_5 &= 0 \\ x_2 + 2x_3 + 3x_5 &= 0 \\ x_4 + 4x_5 &= 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x_1 &= -x_3 + x_5 \\ x_2 &= -2x_3 - 3x_5 \\ x_3 &= x_3 \\ x_4 &= -4x_5 \\ x_5 &= x_5 \end{aligned}\right\}$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix}$$

So, a basis for null space (nul A) $= \left\{ \begin{bmatrix} -1 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \\ -4 \\ 1 \end{bmatrix} \right\}$

+1

A basis for Column space (Col A) $= \left\{ \begin{bmatrix} 1 \\ 2 \\ -3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 1 \end{bmatrix} \right\}$

+1

A basis for Row space (Row A) $= \left\{ \begin{bmatrix} 1 & 0 & 1 & 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 2 & 0 & 3 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 4 \end{bmatrix} \right\}$

+.5