Rubrics for Quiz 5

Total = 5 points

(a) Let us solve the homogeneous system

$$c_1 v_1 + c_2 v_2 + c_3 v_3 = \bar{0}$$

The coefficient matrix $A = [v_1, v_2, v_3]$

$$= \begin{bmatrix} 1 & -2 & 1 \\ -1 & 1 & -6 \\ 3 & -6 & 3 \\ 1 & -4 & -9 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1}]{} \begin{bmatrix} 1 & -2 & 1 \\ 0 & -1 & -5 \\ 0 & 0 & 0 \\ 0 & -2 & -10 \end{bmatrix}$$

$$\xrightarrow{R_2 \to (-1) R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & -2 & -10 \end{bmatrix} \xrightarrow{R_4 \to R_4 + 2R_2} \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 \to R_1 + 2R_2} \begin{bmatrix} 1 & 0 & 11 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \text{RREF matrix.}$$

(+2)

Since there are two pivot columns and one free variable the system of equation ($A\bar{x} = 0$) or $c_1 v_1 + c_2 v_2 + c_3 v_3 = \bar{0}$ $\left(\bar{x} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}\right)$ has a non trivial solution.

(+1)

Hence $\{v_1, v_2, v_3\}$ is linearly dependent.

(b) Let $\bar{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

Now if $\bar{b} \in \text{span}\{v_1, v_2, v_2\}$

the system of nonhomogeneous equation

$A\bar{x} = \bar{b}$ (or $c_1 v_1 + c_2 v_2 + c_3 v_3 = \bar{b}$ where $\bar{x} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$)

has a solution.

The Augmented matrix $= [A \vdots b]$

and let us perform the same set of row operations on

$$\bar{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1}]{} \begin{bmatrix} 1 \\ 1 \\ -3 \\ -1 \end{bmatrix} \xrightarrow{R_2 \to (-1)R_2} \begin{bmatrix} 1 \\ -1 \\ -3 \\ -1 \end{bmatrix} \xrightarrow{R_4 \to R_4 + 2R_2} \begin{bmatrix} 1 \\ -1 \\ -3 \\ -3 \end{bmatrix} \xrightarrow{R_1 \to R_1 + 2R_2} \begin{bmatrix} -1 \\ -1 \\ -3 \\ -3 \end{bmatrix}$$

+1

Thus the Augmented matrix $[A : b]$

becomes $\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & 5 & -1 \\ 0 & 0 & 0 & -3 \\ 0 & 0 & 0 & -3 \end{array}\right]$

Since the last column is a pivot column,

(There is a row $[0 \; 0 \; 0 \; -3]$)

the system $A\bar{x} = \bar{b}$ has no solution.

+1

Therefore $\boxed{\bar{b} \notin \text{span}\{v_1, v_2, v_3\}}$