Worksheet 4

1. (a) Obtain a LU decomposition of the matrix A given below.

   (b) Solve the non-homogeneous system $Ax = b$ where b is given below (using the LU decomposition obtained in previous part)

$$A = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

2. Do the same problem with $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{bmatrix}$ $b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

3. Compute L and U for the symmetric matrix A

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

   Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

4. Find L and U for: $A = \begin{bmatrix} a & r & r & r \\ a & b & s & s \\ a & b & c & t \\ a & b & c & d \end{bmatrix}$

   Find the four conditions on a, b, c, d, r, s, t to get $A = LU$ with four pivots.

5. Solve $Lc = b$ to find c. Then solve $Ux = c$ to find x.

   What was A? ($A = LU$)

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$$

6. Factor the following tridiagonal matrices $A = LU$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } A = \begin{bmatrix} a & a & 0 \\ a & a+b & b \\ 0 & b & b+c \end{bmatrix}$$

7. Find L and U for $T = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 3 & 4 \end{bmatrix}$

8. (a) Find an LU factorization of the following matrix $A = \begin{bmatrix} 7 & -1 & 0 \\ 14 & 0 & 1 \\ 7 & -3 & 3 \end{bmatrix}$

   (b) Using the LU-factorization method solve the linear system $Ax = b$ where A is the matrix given in the previous part and b is the vector given below $b = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

9. Obtain a LU decomposition of the matrix A given below.

$$A = \begin{bmatrix} 2 & 4 & -1 & 5 & -2 \\ -4 & -5 & 3 & -8 & 1 \\ 2 & -5 & -4 & 1 & 8 \\ -6 & 0 & 7 & -3 & 1 \end{bmatrix}$$