Quiz 4

Sep 27th, 2024

Time: 15 minutes

Name: ______________ Roll no.: ______________ Group: ______________

Instructions: Notes, books, computers, cell phones and other electronic devices are not allowed. **Max marks = 5.**

Problem 1. Let V be the vector space of all functions from $[-2\pi, 2\pi]$ into $\mathbb{R}$. (Assume that it is a vector space over the field $\mathbb{R}$.) Let W_1 be the subset of even functions (of V), i.e. $f(-x) = f(x)$ for all $x \in [-2\pi, 2\pi]$. Let W_2 be the subset of odd functions (of V), i.e. $f(-x) = -f(x)$ for all $x \in [-2\pi, 2\pi]$. Explicitly,

$W_1 = \{f : f(-x) = f(x) \ \ \forall x \in [-2\pi, 2\pi]\}$

$W_2 = \{f : f(-x) = -f(x) \ \ \forall x \in [-2\pi, 2\pi]\}$

Are W_1 and W_2 subspaces of V? (Prove or disprove) Give reasons in both the cases.

MTH 100 : Rubrics Of Quiz 4 (Total = 5 points)

① • $\boxed{W_1 \text{ is a subspace of } V}$ (+1)

reasons:

(+.5) (i) The zero function $\bar{0}(x) = 0 \ \forall x \in [-2\pi, 2\pi]$ is an even function because $\bar{0}(-x) = 0 = \bar{0}(x) \ \forall x \in [-2\pi, 2\pi]$

Hence $\bar{0}(x) \in W_1$

(+.5) (ii) If $f, g \in W_1$ then $\left.\begin{array}{l} f(-x) = f(x) \\ g(-x) = g(x) \end{array}\right\} \forall x \in [-2\pi, 2\pi]$

Now $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x)$
$= (f+g)(x) \quad \forall x \in [-2\pi, 2\pi]$

Hence $f + g \in W_1$

(+.5) (iii) If $f \in W_1$ and $c \in \mathbb{R}$ then
$f(-x) = f(x) \quad \forall x \in [-2\pi, 2\pi]$

Now $(cf)(-x) = c.f(-x) = c(f(x))$
$= (cf)(x) \quad \forall x \in [-2\pi, 2\pi]$

Hence $cf \in W_1$

$\boxed{\text{Therefore } W_1 \text{ is a subspace of } V}$

Note: They ~~cav~~ use the alternate test for subspace Viz: A nonempty subset W of a vector space V over a field F is a subspace of V
$\Leftrightarrow cu + v \in W$ for every $u, v \in W$ and for every $c \in F$

(+1) $\boxed{W_2 \text{ is a subspace of } V.}$

reasons: (i) The zero function $\bar{0}(x) = 0 \quad \forall x \in [-2\pi, 2\pi]$

(+.5) is an odd function because $\bar{0}(-x) = 0 = -\bar{0}(x)$ $\quad \forall x \in [-2\pi, 2\pi]$

Hence $\bar{0}(x) \in W_2$

(ii) If $f, g \in W_2$ then $\left.\begin{array}{l} f(-x) = -f(x) \\ g(-x) = -g(x) \end{array}\right\} \forall x \in [-2\pi, 2\pi]$

(+.5) Now
$$\begin{aligned}(f+g)(-x) &= f(-x) + g(-x) \\ &= -f(x) + (-g(x)) \\ &= -[f(x) + g(x)] \\ &= -(f+g)(x) \quad \forall x \in [-2\pi, 2\pi]\end{aligned}$$

Hence $f+g \in W_2$

(iii) If $f \in W_2$ and $c \in \mathbb{R}$ then
$f(-x) = -f(x) \quad \forall x \in [-2\pi, 2\pi]$

(+.5) Now
$$\begin{aligned}(cf)(-x) &= c.f(-x) = c[-f(x)] \\ &= -c[f(x)] = -(cf)(x) \quad \forall x \in [-2\pi, 2\pi]\end{aligned}$$

Hence $cf \in W_2$

$\boxed{\text{Therefore } W_2 \text{ is a subspace of } V}$

Note: Here also they can use the alternate test for subspace

Note: If they can't identify W_1 & W_2 as a subspace correctly, they don't get any credit for subsequent work.