Pre Mid Sem Examination

Sep 20th, 2024

Time: 1 hour 15 minutes
Max Marks: 50

Instructions:

1. Attempt all the five questions. All questions carry equal marks (parts may have different marks).
2. All your intermediate steps and calculations must be clearly shown.
3. Marks for proof-type questions will depend on the logical progression of the steps. You may quote without proof any proposition or theorem covered in the lectures and tutorials but it must be clearly identified. You are not allowed to use determinant. Any other results used must be proved.
4. Write clearly and identify each part of your answers for the benefit of graders.

Problem 1. 1. Find if the system of equations $A\overline{x} = \overline{b}$ is consistent (show reasons), where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix} \quad \overline{b} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}.$$

2. If the answer is yes, find the solution(s).
3. How many solutions do you have?

Problem 2. 1. Find an LU factorization of $A = \begin{bmatrix} 3 & 1 & 3 & -4 \\ 6 & 4 & 8 & -10 \\ 3 & 2 & 5 & -1 \\ -9 & 5 & -2 & -4 \end{bmatrix}$

2. Use the LU factorization method to solve the linear system of equations $A\overline{x} = \overline{b}$ where A is given in previous part and $\overline{b}$ is the vector given below:

$$\overline{b} = \begin{bmatrix} 2 \\ 6 \\ 6 \\ 3 \end{bmatrix}$$

Problem 3. In each of the following parts, find whether the set W is a subspace of the vector space V. (Prove or disprove)

1. $V = \mathbb{R}^2$, W is the graph of the function $f(x) = x + 3x^2$.

   Recall the graph of a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is defined as the set of points $\{(x, y) : y = f(x)\}$.

2. $V = C[0, 1]$, the space of all real valued continuous functions defined on $[0, 1]$ and W is the set of all real valued continuous functions f defined on $[0, 1]$ such that $f(\frac{1}{2}) \in \mathbb{Q}$. ($\mathbb{Q}$ is the set of rational numbers.)

3. $V = \mathbb{R}^\infty$, the set of all real sequences and $W = C_0$, i.e., the set of all real sequences converging to zero.

4. $V = \mathbb{R}^n$, $W = \{(x_1, x_2, ..., x_n) \in \mathbb{R}^n : x_1 + x_2 + \ldots + x_n = 0\}$

Problem 4. 1. If A is a 4×4 matrix such that $5A^4 - 3A^3 + 8A^2 - 7A + 29I = 0$, find if A is invertible or not. If A is invertible write A^{-1} in terms of A.

2. Does $\mathbb{Z}_{21}$ contain any divisor of zero? Give reasons. Hence conclude if $\mathbb{Z}_{21}$ is a field or not.

Problem 5. 1. If a system of equations $A\overline{x} = \overline{b}$ is changed into a new system $C\overline{x} = \overline{d}$ by a row replacement operation, show that the two systems have the same set of solutions.

2. If A and B are $m \times n$ matrices over $\mathbb{R}$ and $A\overline{x} = B\overline{x}$ for all $\overline{x} \in \mathbb{R}^n$, show that $A = B$.

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(Total = 10 points in 1)

1 (1) $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ $\quad \bar{b} = \begin{bmatrix} 4 \\ 3 \\ 1 \\ 2 \end{bmatrix}$

The Augmented matrix

$$[A \mid b] = \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 3 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 2 & 2 \end{array}\right] \xrightarrow[R_4 \to R_4 - 2R_1]{R_3 \to R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & -3 \\ 0 & -1 & -2 & -6 \end{array}\right]$$

$$\xrightarrow{R_4 \to R_4 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -1 & -3 \end{array}\right] \xrightarrow{R_3 \to (-1) R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -1 & -3 \end{array}\right]$$

$$\xrightarrow{R_4 \to R_4 + R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_1 \to R_1 - 2R_3]{R_2 \to R_2 - R_3} \left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$=$ (RREF matrix R)

+6

+2 Since the right most column is not a pivot column, $\boxed{\text{the system of equations is Consistent}}$

(They can also write: there is no row of the form $[0, 0, \ldots 0, b]$ where $b \neq 0$)

② If $\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, the system of equation reduces to
$$\left.\begin{aligned} x_1 &= -2 \\ x_2 &= 0 \\ x_3 &= 3 \end{aligned}\right\}$$

+1

So, the solution is $\bar{x} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$

③ We have exactly one (ie unique) Solution.

+1

Total = 10 points

(2) (1)

(3)

$$A = \begin{bmatrix} 3 & 1 & 3 & -4 \\ 6 & 4 & 8 & -10 \\ 3 & 2 & 5 & -1 \\ -9 & 5 & -2 & -4 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - R_1 \\ R_4 \to R_4 + 3R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 3 & 1 & 3 & -4 \\ 0 & 2 & 2 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 8 & 7 & -16 \end{bmatrix}$$

$$\xrightarrow[R_4 \to R_4 - 4R_2]{R_3 \to R_3 - \frac{1}{2}R_2} \begin{bmatrix} 3 & 1 & 3 & -4 \\ 0 & 2 & 2 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -1 & -8 \end{bmatrix} \xrightarrow{R_4 \to R_4 + R_3} U = \begin{bmatrix} 3 & 1 & 3 & -4 \\ 0 & 2 & 2 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

+3

The row operations are:

e_1: $R_2 \to R_2 - 2R_1$

e_2: $R_3 \to R_3 - R_1$

e_3: $R_4 \to R_4 + 3R_1$

e_4: $R_3 \to R_3 - \frac{1}{2}R_2$

e_5: $R_4 \to R_4 - 4R_2$

e_6: $R_4 \to R_4 + R_3$

The inverse operations are

f_1: $R_2 \to R_2 + 2R_1$

f_2: $R_3 \to R_3 + R_1$

f_3: $R_4 \to R_4 - 3R_1$

f_4: $R_3 \to R_3 + \frac{1}{2}R_2$

f_5: $R_4 \to R_4 + 4R_2$

f_6: $R_4 \to R_4 - R_3$

+1

Then $L = \left(f_1 f_2 f_3\ f_4\ f_5\ f_6\right) I$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_4 \to R_4 - R_3} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_4 \to R_4 + 4R_2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 4 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + \frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ 0 & 4 & -1 & 1 \end{bmatrix} \xrightarrow{R_4 \to R_4 - 3R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & \frac{1}{2} & 1 & 0 \\ -3 & 4 & -1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 \to R_3 + R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & \frac{1}{2} & 1 & 0 \\ -3 & 4 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 \to R_2 + 2R_1} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{2} & 1 & 0 \\ -3 & 4 & -1 & 1 \end{bmatrix} = L$$

+2

So, $A = LU$ where L and U are [redacted]

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{2} & 1 & 0 \\ -3 & 4 & -1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 3 & 1 & 3 & -4 \\ 0 & 2 & 2 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

2(2) $\bar{b} = \begin{bmatrix} 2 \\ 6 \\ 6 \\ 3 \end{bmatrix}$

$L\bar{y} = \bar{b} \Rightarrow$

$y_1 = 2 \Rightarrow \boxed{y_1 = 2}$

$2y_1 + y_2 = 6 \Rightarrow 2\times 2 + y_2 = 6 \Rightarrow \boxed{y_2 = 2}$

$y_1 + \frac{1}{2}y_2 + y_3 = 6 \Rightarrow 2 + \frac{1}{2}\times 2 + y_3 = 6 \Rightarrow \boxed{y_3 = 3}$

$-3y_1 + 4y_2 - y_3 + y_4 = 3 \Rightarrow -3\times 2 + 4\times 2 - 3 + y_4 = 3$

$\Rightarrow \boxed{y_4 = 4}$

+2

Thus $\bar{y} = \begin{bmatrix} 2 \\ 2 \\ 3 \\ 4 \end{bmatrix}$

Now $U\bar{x} = \bar{y} \Rightarrow$

$$3x_1 + x_2 + 3x_3 - 4x_4 = 2$$
$$2x_2 + 2x_3 - 2x_4 = 2$$
$$x_3 + 4x_4 = 3$$
$$-4x_4 = 4$$

+2

$\Rightarrow \boxed{x_4 = -1}$ Then $x_3 + 4(-1) = 3 \Rightarrow \boxed{x_3 = 7}$

Now $2x_2 + 2\times 7 - 2(-1) = 2 \Rightarrow 2x_2 = -14$

$\Rightarrow \boxed{x_2 = -7}$

Now $3x_1 + (-7) + 3(7) - 4(-1) = 2 \Rightarrow 3x_1 = -16$

$\Rightarrow \boxed{x_1 = -\frac{16}{3}}$

So, $\boxed{\bar{x} = \begin{bmatrix} -\frac{16}{3} \\ -7 \\ 7 \\ -1 \end{bmatrix}}$

Note: There is an alternative way to find L (while looking at the reduction from A to U)

- The first column of L will be the first column of A divided by the pivot:

$$\begin{bmatrix} 3/3 \\ 6/3 \\ 3/3 \\ -9/3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ -3 \end{bmatrix}$$

(divide by 3)

The second column will be $\begin{bmatrix} 0 \\ 2/2 \\ 1/2 \\ 8/2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1/2 \\ 4 \end{bmatrix}$

(divide by 2)

The third column will be $\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$; The last column will be $\begin{bmatrix} 0 \\ 0 \\ 0 \\ -8/8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(divide by 1)

Thus $L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & \frac{1}{2} & 1 & 0 \\ -3 & 4 & -1 & 1 \end{bmatrix}$

- If they find L this way, they will get full credit for L i.e. +3 points

Total = $4 \times 2.5 = 10$ points | Please see (★★) on Page (9)

3 (1) Given $V = \mathbb{R}^2$ over the field $\mathbb{R}$

W = graph of the function $f(x) = x + 3x^2$

Now $(1, f(1)) = (1, 4) \in W$

$(2, f(2)) = (2, 14) \in W$

But $(1,4) + (2,14) = (3, 18) \notin W$

(because $3 + 3 \cdot 3^2 = 30 \neq 18$)

$1 + 1.5 = 2.5$

So, $\boxed{W \text{ is not a subspace of } \mathbb{R}^2}$

Note: (1) They can choose other points to show that W is not closed under vector addition.

(2) They can also show that W is not a subspace by showing that W is not closed under scalar multiplication.

e.g. $(1,4) \in W$, $2 \in \mathbb{R}$ but $2(1,4) = (2,8) \notin W$

(Since $2 + 3 \cdot 2^2 = 14 \neq 8$)

(2) Given $V =$ [redacted] $C[0,1]$ over the field $\mathbb{R}$

$W = \{ f \in C[0,1] \, ; \, f(\tfrac{1}{2}) \in \mathbb{Q} \}$

Let us take $f \in W$ and so $f(\tfrac{1}{2}) \in \mathbb{Q}$

Now $\sqrt{2} \in \mathbb{R}$ and $(\sqrt{2} f)(\tfrac{1}{2}) = \sqrt{2} f(\tfrac{1}{2}) \notin \mathbb{Q}$

$1 + 1.5 = 2.5$

So, $\boxed{W \text{ is not a subspace of } V = C[0,1]}$

Note: They can choose any other irrational number like $\sqrt{3}$, $\sqrt{5}$ or π etc.

3. $V = \mathbb{R}^\infty$ over $\mathbb{R}$

$W = C_0$

Now $\boxed{(0,0,0,\dots) \in C_0}$ because this is a real sequence converging to 0.

If $(x_n), (y_n) \in C_0$

then $(x_n), (y_n)$ are convergent and converges to zero

Then $(x_n + y_n)$ is also convergent and converges to zero

(because $\lim_{n\to\infty}(x_n + y_n) = \lim_{n\to\infty} x_n + \lim_{n\to\infty} y_n$
$= 0 + 0 = 0$)

So, $\boxed{(x_n) + (y_n) = (x_n + y_n) \in C_0}$

If $(x_n) \in C_0$ and $c \in \mathbb{R}$, then (cx_n) is also convergent and converges to zero

(because $\lim_{n\to\infty} cx_n = c \lim_{n\to\infty} x_n = c \times 0 = 0$)

So, $\boxed{c(x_n) = (cx_n) \in C_0}$

Therefore $\boxed{C_0 \text{ is a subspace of } \mathbb{R}^\infty}$.

1+1.5 =2.5

4. $V = \mathbb{R}^n$ over $\mathbb{R}$

1+1.5 =2.5

$W = \{(x_1, x_2, \dots, x_n) \in \mathbb{R}^n : x_1 + x_2 + \dots + x_n = 0\}$

Now $\boxed{(0,0,\dots 0) \in W}$ since $0 + 0 + \dots + 0 = 0$

Now, $(x_1, \dots x_n), (y_1, \dots, y_n) \in W \Rightarrow$ $x_1 + \dots + x_n = 0$
$y_1 + \dots + y_n = 0$

then $\boxed{(x_1, \dots x_n) + (y_1, \dots y_n) = (x_1 + y_1, \dots, x_n + y_n) \in W}$

(because $(x_1 + y_1) + \dots + (x_n + y_n)$
$= (x_1 + \dots + x_n) + (y_1 + \dots + y_n)$
$= 0 + 0 = 0$)

Now if $(x_1, \dots x_n) \in W$ and $c \in \mathbb{R}$,

then $\boxed{c(x_1, \dots x_n) = (cx_1, \dots, cx_n) \in W}$

because $cx_1 + \dots + cx_n = c(x_1 + \dots + x_n)$
$= c \times 0 = 0$

So, $\boxed{W \text{ is a subspace of } \mathbb{R}^n}$

Note: 1. Each part carries 2.5 points.

2. If they write correctly

Imp whether or not W is a subspace of V they get 1 points.

For correct reasons, they get 1.5 points

$\Big\}$ Total = 2.5 points

3. Instead of using the test (I have used in all the above examples (especially in (3) & (4))), they can use the alternate test

Viz: W is a subspace of a vector space V over a field $\mathbb{F}$

$$\Leftrightarrow \quad cu + v \in W \quad \forall u, v \in V \text{ and } \forall c \in \mathbb{F}$$

- If they use this test correctly, they get full credit.

Total = 10 points (6+4) (10)

4(1): Given $5A^4 - 3A^3 + 8A^2 - 7A + 29I = 0$

+4

$$\Rightarrow 5A^4 - 3A^3 + 8A^2 - 7A = -29I$$

$$\Rightarrow -\frac{1}{29}\left(5A^4 - 3A^3 + 8A^2 - 7A\right) = I$$

$$\Rightarrow A\left[-\frac{5}{29}A^3 + \frac{3}{29}A^2 - \frac{8}{29}A + \frac{7}{29}I\right] = I$$

Clearly the Left hand side is same as $\left[-\frac{5}{29}A^3 + \frac{3}{29}A^2 - \frac{8}{29}A + \frac{7}{29}I\right]A = I$

+2

Therefore A is invertible and

$$\boxed{A^{-1} = -\frac{5}{29}A^3 + \frac{3}{29}A^2 - \frac{8}{29}A + \frac{7}{29}I}$$

4(2) $\mathbb{Z}_{21} = \{0, 1, 2, \ldots, 20\}$

Now $3, 7 \in \mathbb{Z}_{21}$, $3 \neq 0$, $7 \neq 0$ in $\mathbb{Z}_{21}$

but $3 * 7 = 21 \pmod{21} = 0$

$\Rightarrow 3 * 7 = 0$ in $\mathbb{Z}_{21}$

+2

Hence $\boxed{\text{both 3 and 7 are divisors of zero in } \mathbb{Z}_{21}.}$

+2

Since a field can not have a zero divisor and $\mathbb{Z}_{21}$ has zero divisors,

$\boxed{\mathbb{Z}_{21} \text{ is not a field.}}$

(Total = 5 points)

5 1 Given that A and C are $m \times n$ matrices and a system of equation $A\bar{x} = \bar{b}$ is changed into a system $C\bar{x} = \bar{d}$ by a row replacement operation.

- Suppose the jth row (R_j) of A is replaced by the sum of jth row and λ-times the ith row (i.e. $R_j \longrightarrow R_j + \lambda R_i$)

Assume (without loss of generality) $i < j$

- Then all the rows of C (except the jth row) will be the same as that of A.

The entries of jth row will be

$a_{jk} + \lambda a_{ik}$ for $k = 1, 2, \ldots, n$ (where $A = [a_{ij}]_{m \times n}$)

+4

- Thus all the equations in $C\bar{x} = \bar{d}$ will be same as $A\bar{x} = \bar{b}$ <u>except</u> the jth equation.

The jth equation becomes

$$(a_{j1} + \lambda a_{i1})x_1 + \cdots + (a_{jn} + \lambda a_{in})x_n = (b_j + \lambda b_i) = d_j$$

Thus if $u = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ is a solution of $A\bar{x} = \bar{b}$,

it will satisfy <u>all the equations</u> of $C\bar{x} = \bar{d}$

and for the jth equation

$$(a_{j1} + \lambda a_{i1})u_1 + \cdots + (a_{jn} + \lambda a_{in})u_n$$
$$= (a_{j1}u_1 + \cdots + a_{jn}u_n) + \lambda(a_{i1}u_1 + \cdots + a_{in}u_n) = b_j + \lambda b_i$$
$$= d_j$$

+1

- Since <u>inverse of a replacement</u> operation $R_j \to R_j + \lambda R_i$ is also a replacement operation $R_j \to R_j - \lambda R_i$, A can be obtained from C by a replacement operation.

So, any solution of $C\bar{x} = \bar{d}$ will also be a solution of $A\bar{x} = \bar{b}$

(Total = 5 points)

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5(2): Given $A\bar{x} = B\bar{x} \ \forall \ \bar{x} \in \mathbb{R}^n$

Let $A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}$ $\quad B = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix}$

Let $\bar{x} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ Then $A\bar{x} = B\bar{x} \Rightarrow \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{pmatrix} = \begin{pmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{m1} \end{pmatrix}$

+5

Let $\bar{x} = \begin{pmatrix} 0 \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ Then $A\bar{x} = B\bar{x} \Rightarrow \begin{pmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{pmatrix} = \begin{pmatrix} b_{12} \\ b_{22} \\ \vdots \\ b_{m2} \end{pmatrix}$

$\vdots$

Similarly let $\bar{x} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{pmatrix}$ Then $A\bar{x} = B\bar{x} \Rightarrow \begin{pmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{pmatrix} = \begin{pmatrix} b_{1n} \\ b_{2n} \\ \vdots \\ b_{mn} \end{pmatrix}$

Combining we get $\boxed{A = B}$

Note: The students can give the proof in a more compact form:

If $e_i = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix}$ → ith place then $Ae_i = Be_i$ for $i = 1, 2, \ldots, n$

$\Rightarrow$ ith column of A = ith Column of B

for $i = 1, 2, \ldots, n$

Hence $\boxed{A = B}$

(Note This is acceptable & they will get full credit)