MTH 100 : Worksheet 13

1. Find a diagonal matrix D and an orthogonal matrix P such that $A = PDP^T$ for the following matrix :
   $$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$

2. Find a singular value decomposition (SVD) for the matrix A given below.
   $$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}$$

3. Find a singular value decomposition (SVD) for the matrix A given below.
   $$A = \begin{bmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{bmatrix}$$

4. Let A be an $n \times n$ invertible square matrix.
   (a) Show that the eigen values of A^{-1} are the reciprocals of the eigen values of A.
   (b) Find a singular value decomposition of A^{-1}, assuming that you already have an SVD of A.

5. Let U be an $m \times n$ matrix with orthonormal columns, and suppose $\mathbf{x}$ and $\mathbf{y}$ are vectors in $\mathbf{R^n}$. Show that :
   (a) $U\mathbf{x} \cdot U\mathbf{y} = \mathbf{x} \cdot \mathbf{y}$
   (b) $\|U\mathbf{x}\| = \|\mathbf{x}\|$
   (c) $U\mathbf{x} \cdot U\mathbf{y} = \mathbf{0}$ if and only if $\mathbf{x} \cdot \mathbf{y} = \mathbf{0}$