MTH 100 : Worksheet 2

1. (a) Row reduce the augmented matrix of the system given below to an RREF matrix using elementary row operations.

$$\begin{aligned} 3x + 2y + 7z + 9w &= 7 \\ 6x + 14y + 22z + 15w &= 13 \\ x + 4y + 5z + 2w &= 2 \end{aligned}$$

   (b) Is the system consistent or inconsistent? If consistent, express the solution in the form of a vector $\mathbf{u}$ which is a solution of the non-homogeneous system plus scalar multiples of vector(s) which are solutions of the associated homogeneous syatem.

2. (a) Row reduce the augmented matrix of the system given below to an RREF matrix using elementary row operations.

$$\begin{aligned} x + 5y - 3z &= -4 \\ -x - 4y + z &= 3 \\ -2x - 7y &= a \end{aligned}$$

   (b) For what values of a is the above system consistent and for what values of a is it inconsistent? Justify your answer.

3. For what values of a and b, the system

$$\begin{aligned} x + y + z &= 3 \\ 2x + 3y + 4z &= 9 \\ x - y + az &= b \end{aligned}$$

   has
   (a) Unuque solution
   (b) Infinitely many solutions
   (c) No solution

4. Is it possible for a non-homogeneous system $A\mathbf{x} = \mathbf{b}$, $\mathbf{b} \neq \mathbf{0}$, to be inconsistent when the associated homogeneous system $A\mathbf{x} = \mathbf{0}$ has a unique solution (i.e. only the trivial solution)? Answer YES or NO, and justify your answer. If YES, construct an example and verify. If NO, explain with reference to suitable propositions and theorems.

5. (a) Find the values of x for which the following matrix is an augmented matrix corresponding to a consistent system.

$$A = \begin{bmatrix} 1 & -2 & 1 & x \\ 0 & 5 & -2 & x^2 \\ 4 & -23 & 10 & x^3 \end{bmatrix}$$

(b) Find the RREF of the matrix formed by replacing x in A by π.

6. Prove that : If the matrix B has been obtained from the matrix A by an elementary row operation, then the vector $\mathbf{v}$ is a solution of the homogeneous system $A\mathbf{x} = \mathbf{0}$ if and only if $\mathbf{v}$ is a solution of the homogeneous system $B\mathbf{x} = \mathbf{0}$

(Note: Think about a similar version for a non-homogeneous system and prove it.)