MTH 100 : Worksheet 8

1. V is a vector space with dim(V)=n. W_1 and W_2 are subspaces of V such that dim(W_1)=dim(W_2)=n-1 and $W_1 \cap W_2 = \{0\}$. Find n.

2. Given the vector space $\mathbf{R^3}$, let W_1 be the set of vectors of the form $(x, y, 0)$ and let W_2 be the set of vectors of the form $(0, a, b)$

   (a) Show that W_1 and W_2 are subspaces of $\mathbf{R^3}$.

   (b) Find the dimensions of W_1, W_2, $W_1 + W_2$ and $W_1 \cap W_2$.

   (c) Find two distinct subspaces U_1 and U_2 of $\mathbf{R^3}$ such that $\mathbf{R^3} = W_1 \bigoplus U_1 = W_1 \bigoplus U_2$ i.e. find two distinct complements of W_1. Justify your answer.

3. Given the matrix A below:

   (a) Find a basis for each of the spaces Nul A, Col A and Row A.

   (b) Find a basis for Row A, consisting of rows of the given matrix A, different from the one in previous part.

   (c) Is A invertible? Justify. $A = \begin{bmatrix} 2 & 6 & 3 \\ 4 & 12 & 5 \\ 13 & 39 & 17 \end{bmatrix}$

4. Given the matrices A and B below.

   (a) Find a basis for the row space of A a basis for the row space of B, showing your calculations.

   (b) Let $U =$ Span $\{(1, 2, -1, 3), (2, 4, -1, 2), (3, 6, 3, -7)\}$ and Let $W =$ Span $\{(1, 2, -4, 11), (2, 4, -5, 14)\}$ Is $U = W$? Justify your answer. $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 4 & 1 & -2 \\ 3 & 6 & 3 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -4 & 11 \\ 2 & 4 & -5 & 14 \end{bmatrix}$

5. Given any $m \times n$ matrix A, show that rank A$\leq \min\{m, n\}$.

   Given a non-trivial example in which equality is achieved and a non-trivial example in which strict inequality holds.