Quiz 6

Oct 18th, 2024

Time: 20 minutes	Max marks = 5
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Instructions: Notes, books, computers, cell phones and other electronic devices are not allowed.

Problem 1. Find a basis of the null space, a basis of the column space and a basis of the row

space of the given matrix.
$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix}$$
,

Rubrice for Quiz 6

Total points = 5 1

Solution of Quiz 6:

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_4$$

$$R_3 \rightarrow R_3 + 3R_4$$

$$A = \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 2 & -1 & 0 & 1 & -1 \\ -3 & 2 & 1 & -2 & 1 \\ 4 & 1 & 6 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_4} \begin{bmatrix} 1 & 1 & 3 & 1 & 6 \\ 0 & -3 & -6 & -1 & -13 \\ 0 & 5 & 10 & 1 & 19 \\ 0 & 5 & 10 & 1 & 19 \\ 0 & -3 & -6 & -3 & -21 \\ 0 & -3 & -6 & -3 & -21 \end{bmatrix}$$

$$R_{2} \rightarrow \left(-\frac{1}{3}\right) R_{2}$$

$$\begin{bmatrix}
1 & 1 & 3 & 1 & 6 \\
0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\
0 & 0 & 0 & -\frac{2}{3} & -\frac{8}{3}
\end{bmatrix}$$

$$\begin{bmatrix}
1 & 1 & 3 & 1 & 6 \\
0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\
0 & 0 & 0 & -\frac{2}{3} & -\frac{8}{3} \\
0 & 0 & 0 & -2 & -8
\end{bmatrix}$$

$$\begin{array}{c}
R_3 \rightarrow R_3 - 5R_2 \\
R_4 \rightarrow R_4 + 3R_2
\end{array}$$

$$\begin{bmatrix}
1 & 1 & 3 & 1 & 6 \\
0 & 1 & 2 & \frac{1}{3} & \frac{13}{3} \\
0 & 5 & 10 & 1 & 19 \\
0 & -3 & -6 & -3 & -21
\end{array}$$

$$R_3 \longrightarrow \left(\frac{3}{2}\right) R_3$$

$$R_2 \rightarrow R_2 - \frac{1}{3}R_3$$

$$R_1 \rightarrow R_1 - R_3$$

So, to solve the system of equations $A\bar{x}=0$

we solve
$$R\bar{\chi} = 0$$

$$\Rightarrow \begin{array}{c} x_1 + x_3 - x_5 = 0 \\ x_2 + 2x_3 + 3x_5 = 0 \\ x_4 + 4x_5 = 0 \end{array} \Rightarrow \begin{array}{c} x_1 = -x_3 + x_5 \\ x_2 = -2x_3 - 3x_5 \\ x_3 = x_3 \\ x_4 = -4x_5 \\ x_5 = x_5 \end{array}$$

$$\begin{array}{c}
\chi_{1} = -\lambda_{3} + \lambda_{5} \\
\chi_{2} = -2\chi_{3} - 3\chi_{5}
\end{array}$$

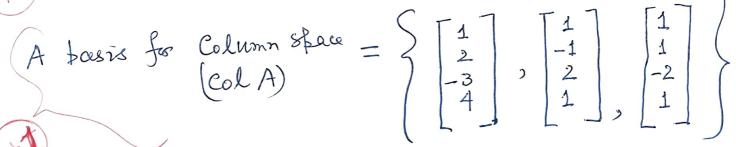
$$\chi_{3} = \chi_{3}$$

$$\chi_{4} = -4\chi_{5}$$

$$\chi_{5} = \chi_{5}$$

$$\Rightarrow \begin{bmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \\ \chi_4 \\ \chi_5 \end{bmatrix} = \begin{bmatrix} \chi_3 \\ -\frac{1}{2} \\ -\frac{1}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \chi_5 \\ -\frac{3}{2} \\ -\frac{4}{2} \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$30$$
, a basis for and space = $\begin{cases} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \\ 0 \end{cases}$, $\begin{cases} -\frac{1}{3} \\ 0 \\ -\frac{4}{1} \\ 1 \end{cases}$



A basis for Row space =
$$\begin{cases} [1010-1], [0.1203], \\ [00014] \end{cases}$$