Quiz 1

Aug 30th, 2024

Time: 12 minutes

Name: _____ Roll no.: _____ Group: _____

Instructions: Notes, books, computers, cell phones and other electronic devices are not allowed. **Max marks = 5.**

Problem 1. 1. Find the reduced row echelon form of the augmented matrix of the system of equations:

$$\begin{aligned} \frac{2}{3}x_1 - \frac{8}{3}x_2 - \frac{2}{3}x_3 + \frac{2}{3}x_4 &= 2 \\ 2x_1 - 8x_2 + x_3 - 4x_4 &= 9 \\ -x_1 + 4x_2 - 2x_3 + 5x_4 &= -6 \end{aligned}$$

2. Is the above system consistent? (Give reasons for your answer)

3. Write the solution(s) of the above system in set notation.

4. How many solutions are there?

MTH100 | Rubrics of Quiz 1 | Total points = 5

(1) (a) The Augmented matrix of the given system of equations is:

$$\left[\begin{array}{cccc|c} \frac{2}{3} & -\frac{8}{3} & -\frac{2}{3} & \frac{2}{3} & 2 \\ 2 & -8 & 1 & -4 & 9 \\ -1 & 4 & -2 & 5 & -6 \end{array}\right] \xrightarrow{R_1 \to \frac{3}{2}R_1} \left[\begin{array}{cccc|c} 1 & -4 & -1 & 1 & 3 \\ 2 & -8 & 1 & -4 & 9 \\ -1 & 4 & -2 & 5 & -6 \end{array}\right]$$

$$\downarrow \; R_2 \to R_2 - 2R_1, \quad R_3 \to R_3 + R_1$$

$$\left[\begin{array}{cccc|c} 1 & -4 & -1 & 1 & 3 \\ 0 & 0 & 3 & -6 & 3 \\ 0 & 0 & -3 & 6 & -3 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{3}R_2} \left[\begin{array}{cccc|c} 1 & -4 & -1 & 1 & 3 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & -3 & 6 & -3 \end{array}\right]$$

$$\downarrow \; R_3 \to R_3 + 3R_2$$

$$\left[\begin{array}{cccc|c} 1 & -4 & -1 & 1 & 3 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \to R_1 + R_2} \underbrace{\left[\begin{array}{cccc|c} 1 & -4 & 0 & -1 & 4 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]}_{\text{This is the RREF matrix.}}$$

+2.5

Note: They can get the RREF matrix in different ways but the end result will be same.

(b) Since the last column $\begin{bmatrix}4\\1\\0\end{bmatrix}$ is not a Pivot column (or there is no row of the form $[0,0,0,0,b]$ with $b \neq 0$ or the last row is $[0,0,0,0,0]$) the system is consistent. (+1)

Note: • They can also write:
By proposition (4), the system is consistent.
• If they just write consistent (without any reason) they will get (+0.5)

(c) From the RREF matrix, we see that x_2 and x_4 are free variables and the system becomes: (+1)

$$\left.\begin{aligned} x_1 - 4x_2 - x_4 &= 4 \\ x_3 - 2x_4 &= 1 \end{aligned}\right\} \Rightarrow \begin{aligned} x_1 &= 4 + 4x_2 + x_4 \\ x_2 &= 0 + x_2 + 0.x_4 \quad \text{(dummy equation)} \\ x_3 &= 1 + 0.x_2 + 2x_4 \\ x_4 &= 0 + 0.x_2 + x_4 \quad \text{(dummy equation)} \end{aligned}$$

$$\Rightarrow \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}4\\0\\1\\0\end{bmatrix} + x_2\begin{bmatrix}4\\1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}1\\0\\2\\1\end{bmatrix}$$

So, in set notation, the solutions of the system is

$$S = \left\{ \begin{bmatrix}4\\0\\1\\0\end{bmatrix} + r\begin{bmatrix}4\\1\\0\\0\end{bmatrix} + s\begin{bmatrix}1\\0\\2\\1\end{bmatrix} : r, s \in \mathbb{R} \right\}$$

Note: They can use other symbols.

(d) There are infinitely many solutions (+0.5)