CHAPTER 8

THREE DIMENSIONAL TRANSFORMATIONS

CONTENTS

8.0 OBJECTIVES:

The objectives of this chapter are:

- To discuss fundamental 3D transformations
- To explore the difference between 2D and 3D Transformations
- To Explore reflection and shear transformations
- To understand composite transformations

8.1 INTRODUCTION:

To do some effective thing in three dimensions, we need the manipulation of three dimensional objects and for successful manipulation of three dimensional objects we need three dimensional geometric and coordinate transformations.

Simple transformations like translations, rotation and scaling as well as composition of them are used in 3D object manipulations. In this chapter we shall learn how to manipulate 3D objects using basic transformations.

8.2. FUNDAMENTAL TRANSFORMATIONS:

We shall treat translation, rotation and scaling as the fundamental transformations. Translation means shifting the position of the object, rotation means moving the object about an axis and scaling is resizing the object. We shall first discuss translation.

8.2.1 TRANSLATION:

Let P(X, Y, Z) be a point in a space and the displacement vector is $V = t_x i + t_y j + t_z k$. So the new coordinate of be point will be

$P' = (X', Y', Z')$ where

$$X' = X + t_x$$
$$Y' = Y + t_y \qquad \text{……………………………………………8.1}$$
$$Z' = Z + t_z$$

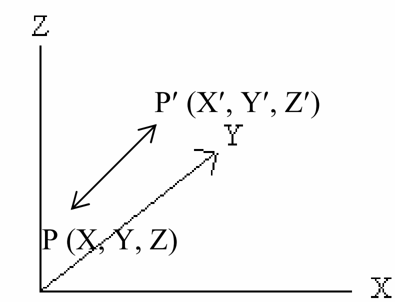

Fig-8.1: Translation of Object

In figure 8.1 the translation is shown. Here the vector V is in the direction PP′. To represent the transformation in matrix form we need to introduce homogeneous coordinates. The transformation can be expressed as

$$(X' \quad Y' \quad Z' \quad 1) = (X \quad Y \quad Z \quad 1)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{pmatrix} \quad \text{……… 8.2}$$

Or we may write

$$P' = P.T$$

where T is the translation matrix.

8.2.2 ROTATION:

Rotation in 3D is more complex than the rotation in 2D. In 2D, the angle of rotation and point of rotation was sufficient but in case of 3D we need an angle of rotation and an axis of rotation.

Canonical Rotation: A rotation in 3D is said to be canonical rotation if one of the positive X, Y or Z axis is chosen as the axis of rotation.

We shall discuss only canonical rotation.

Rotation about Z axis:

The rotation about Z axis can be given by

$$X' = X\cos\theta - Y\sin\theta$$
$$Y' = X\sin\theta + Y\cos\theta \qquad \ldots\ldots 8.3$$
$$Z' = Z$$

In homogeneous coordinate form we can describe the above rotation by

$$(X' \; Y' \; Z' \; 1) = (X \; Y \; Z \; 1)\begin{pmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad \ldots\ldots 8.4$$

i.e., $P' = P.\ R_z(\theta)$ ……………………………… 8.5

Rotation about Y axis:

This can be given by

$$X' = X\cos\theta + Z\sin\theta$$
$$Y' = Y$$
$$Z' = -X\sin\theta + Z\cos\theta \qquad \ldots\ldots 8.6$$

In homogeneous coordinate it can be given by

$$(X' \; Y' \; Z' \; 1) = (X \; Y \; Z \; 1) \begin{pmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots \; 8.7$$

i.e., $P' = P.\, R_y(\theta)$………………………………………………………… 8.8

Rotation about X axis:

If we rotate an object through an angleθ, then its equations can be given by

$X' = X$

$Y' = Y\cos\theta - Z\sin\theta$

$Z' = Y\sin\theta + Z\cos\theta$ ……………………………………………… 8.9

In homogeneous coordinates it can be given by

$$(X' \; Y' \; Z' \; 1) = (X \; Y \; Z \; 1) \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots \; 8.10$$

i.e., $P' = P.\, R_x(\theta)$………………………………………………………… 8.11

8.2.3 SCALING:

The scaling transformation changes the size of the object. The scaling factor determines that whether the object size will increase or decrease. If scale factor is greater than 1, then the object size will increase. If on the other hand, the scale factor is less than 1 the size of the object will decrease. We say them as the magnification of the object or reduction of the object respectively.

Scaling with respect to the origin where origin is the fixed point can be given for the point P(X Y Z) by

$P'(X' \; Y' \; Z')$ is

$X' = X.Sx$

$Y' = Y.Sy$ ……………………………………………………………….. 8 .12

$Z' = Z\,.\,Sz$

Where Sx, Sy and Sz are the scale factors along X-axis, Y-axis and Z-axis respectively.

In homogeneous coordinate it can be given by

$$(X' \ Y' \ Z' \ 1) = (X \ Y \ Z \ 1) \begin{pmatrix} Sx & 0 & 0 & 0 \\ 0 & Sy & 0 & 0 \\ 0 & 0 & Sz & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots\ldots 8.13$$

The Repositioning problem:

We have seen in two dimensional scaling that scaling not only resizes the object but also translate the object. If the scaling factor is less than 1, the object will come closer to the origin, if the scaling factor is greater than 1, the object will go away from the origin. This is called repositioning problem which we don't want in case of scaling. To keep the position of the object intact we will translate the object.

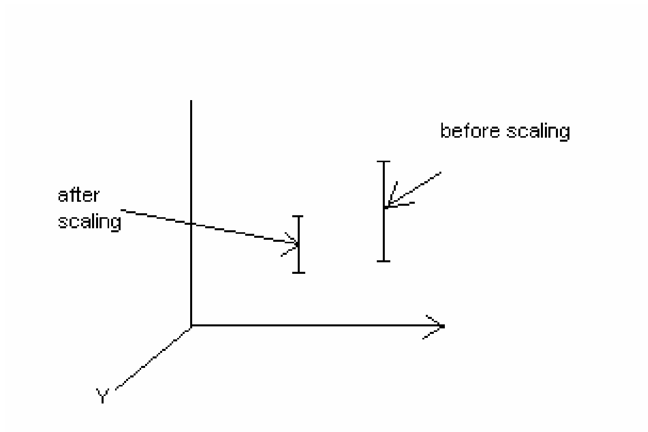

Fig-8.2: Reposition of Object after Scaling

For translation, first we have to find out a fixed point of the object. Fixed point is a point of the object whose coordinate will not change after scaling. We then translate the fixed point to the origin and the entre object accordingly. Then we shall perform scaling and then reverse transformation to shift the object back to the original position.

Let the coordinate of the fixed point is (h, k, m). Then the scaling can be given by

$$\begin{pmatrix} Sx & 0 & 0 & (1-Sx)h \\ 0 & Sy & 0 & (1-Sy)k \\ 0 & 0 & Sz & (1-Sz)m \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \ldots\ldots\ldots 8.14$$

8.3 OTHER TRANSFORMATIONS:

Some important transformations other the fundamental transformations are also frequently used in object transformations. These are reflection and shear transformations. In this section we shall discuss them.

8.3.1 REFLECTION:

We have discussed reflection in two dimensional transformation. Here also the concept is same. But here, these are two types of reflections

(i) Reflection with respect to an axis

(ii) Reflection with respect to a plane.

Reflection about an axis:

Reflection about a given axis means rotating the object 180^0 about that axis.

Reflection about a plane:

Rotation about a plane means rotating 180^0 in four dimensional space.

If the reflection planes are coordinate planes i.e., (XY, YZ, ZX), then we can think it as conversion left-hand system to right-hand system.

For example if we want a reflection with respect to the YZ plane, then all X coordinates will change their sign leaving Y and Z unchanged. The matrix representation for this can be given by

$$RFx = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Similarly reflection with respect to ZX plane can be given by the matrix

$$RFy = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

And the matrix of reflection about XY-plane can be given by

$$RFz = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \ldots$$

The reelection about other planes can be stated as the combination of rotation and the reflection about coordinate planes.

8.3.2 SHEAR TRANSFORMATIONS:

As stated in case of 2D, shearing will change the shape of the object. Changing the shape means slanting the object. X-shear maintains the value of y and Z but of changes the X-coordinates causing the slanting ness in the direction of X. whether the object will slant to the left or to the right that will be determined by the value of X-shear factor. Y-shear and Z-shear will also do the some thing but in the direction of Y and Z respectively. X and Y shear can be given by

$$\begin{pmatrix} 1 & 0 & Za & 0 \\ 0 & 1 & Zb & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

8.4 COMPOSITE TRANSFORMATIONS:

As in 2D, the idea of composite transformation is same. i.e., we can give the effect of composite transformation by multiplying the corresponding matrix of the individual operations. This concatenation is from right to left i.e., the rightmost operation will be applied at the very first and so on and the leftmost will be applied at the very last.

8.5 KEY WORDS:

- Homogeneous Coordinates
- Reflection
- Rotation
- Scaling
- Shearing
- Translation

8.6 SAMPLE QUESTIONS:

8.6.1 What do you mean by 3D translation?

8.6.2 Explain the concept of rotation in 3D and hence discuss the rotation about x-axis, y-axis and z-axis.

8.6.3 What is scaling in 3D? What is repositioning problem in scaling? How can it be eliminated?

8.6.4 Discuss Reflection in 3D.

8.6.5 What is shear transformation in 3D? Discuss x-shear transformation.