Problem Set 10.1

1. Evaluate the following
   (*i*) ${}^5P_3$ (*ii*) $P(5, 5)$ (*iii*) ${}^9P_5$ (*iv*) $P(10, 4)$
2. Find n, if
   (*i*) ${}^nP_2 = 30$
   (*ii*) $P(n, 4) = 20 \times P(n, 2)$
   (*iii*) $P(n, 6) = 3P(n, 5)$
   (*iv*) ${}^nP_4 : {}^{n+1}P_5 = 1 : 9$
   (*v*) ${}^{2n+1}p_{n-1} : {}^{2n-1}p_n = 3 : 5$.
   (*vi*) $P(11, n) = P(12, n-1)$, find n.
3. Show that
   (*i*) $P(n, n) = 2P(n, n-2)$
   (*ii*) ${}^{n+1}P_{r+1} = (n+1).{}^nP_r$
   (*iii*) ${}^nP_r = {}^{n-1}P_r + r.{}^{n-1}P_{r-1}$
4. An organizing committee requires one student representative from either the first year or the second year or the third year. If there are 60 first year, 40 second year and 20 third year students, how many different representatives are there ?
5. If E be the event of selecting a prime number less than 10 and F, the event of selecting an even number less than 10. Find the number of ways of happening E or F.
6. A house has 4 doors and 10 windows. In how many ways can a burglar rob the house by entering through a window and exiting through a door.
7. Twelve horses are in a race. The only results that matter are the first three finishers. How many possibilities are there ?
8. A student wishes to take a combination of three courses, one from each of three science departments. There are 4 Physics, 3 Chemistry and 2 Biology courses on offer. How many possible combinations are there ?
9. A person buying a personal computer system is offered a choice of four models of the basic unit, three models of key board, and two models of printer. How many distinct system can be purchased ?
10. There are 6 multiple choice questions in an examination. How many sequences of answers are possible if the first three question have four choices each and the next three have five each ?
11. In how many ways can 4 different prizes be awarded among 6 contestants so that a contestant may receive
    (*i*) atmost one prize (*ii*) any number of prizes
12. How many numbers of three digits can be formed with the digits 2, 3, 4, 5, 6, 7 no digit being repeated
13. How many numbers lying between 10 and 100 can be formed with digits 3, 4, 0, 5, 8 no digit being repeated.
14. How many three-digit numbers can be formed without using the digits 1, 1, 3?
15. How many numbers are there between 100 and 1000 such that 7 is in the unit's places.
16. In how many ways can 8 examination papers be arranged so that the least and the worst paper never come together?
17. From the digits 1, 2, 3, 4, 5, 6 how many three digit odd numbers can be formed when
    (*i*) the repetition of digits is allowed (*ii*) the repetition of digits is not allowed
18. How many permutations can be made out of the letters of the word TRIANGLE? How many of these will begin with *T*, and end with *E*? Also find how many permutations will begin with *T*, but will not end in *E*.

19. How many different words can be formed with the letters of the word ORDINATE
(*a*) so that the vowels occupy odd places (*b*) beginning with O
(*c*) beginning with O and ending with E ?
20. In how many ways can the letters of the word COMBINE be arranged so that
(*i*) the vowels are never separated (*ii*) all the vowels never come together
(*iii*) vowels occupy only the odd places?
21. How many signals can be produced with 6 flags of different column such that
(*i*) atmost three flags can be used for a signal. (*ii*) Exactly three flags are to be used for a signal.
(*iii*) at least three flags are to be used for a signal. (*iv*) any number of flags may be used for a signal.
22. Hexadecimal numbers are formed using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. How many hexadecimal numbers are there between (*a*) 10_{16} through FF_{16} (*b*) 60_{16} through FF_{16} (*c*) 40_{16} through BF_{16}.
23. How many different two digit positive integers can be formed from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (*i*) when repetition is not allowed (*ii*) when repetition is allowed.
24. Determine the number of odd three-digit positive integers that have no repeated digits.
25. A team consists of 6 boys and 4 girls and the other has 5 boys and 3 girls, how many single matches can be arranged between the two teams when a boy plays against a boy and a girl plays against a girl?
26. How many permutations can be made out of the letters of word 'COMPUTER' ? How many of these (*a*) begin with C ? (*b*) begin with R ? (*c*) begin with C and with R ? (*d*) C and R always together ?
27. How many permutations can be made out of the letters of word 'SUNDAY' ? How many of these (*a*) begin with S ? (*b*) end with Y ? (*c*) begin with S and end with Y ? (*d*) S and Y always together ?
28. Find the number of permutations that can be formed by the letters of the word 'DAUGHTER' (*i*) taking all the letters together without any restriction (*ii*) beginning with D (*iii*) beginning with D and ending with R (*iv*) vowels being always together (*v*) not all vowels together (*vi*) not even two vowels together (*vii*) vowels occupying even places.
29. A bit is either 0 or 1 ; a byte is a sequence of 8 bits. Find (*a*) the number of bytes (*b*) the number of bytes that being with 11 and end with 11 (*c*) the number of bytes that begin with 11 and do not end with 11, and (*d*) the number of bytes that being with 11 or end with 11.
30. Find the number of ways in which 5 ladies and 5 gentlemen may be seated in a row, so that no two ladies are together.
31. How many different numbers of six digits (without repetition) can be formed from the digits 3, 1, 7, 0, 9, 5 ?
(*i*) How many of them will have 0 in the unit place ? (*ii*) How many of them are divisible by 5 ?
(*iii*) How many of them are not divisible by 5 ?
32. Ten guests are to be seated in a row of which three are ladies. The ladies insist on sitting together while two of the gentlemen refuse to take consecutive seats. In how many ways can the guests be seated?
33. A computer code word is to consist of two distinct alphabets followed by two distinct integers between 1 and 9. (*a*) How many such code words are there ? (*b*) How many of them end with even integers ?
34. How many arrangements can be made of the letters of the word.
(*a*) APPLE (*b*) COMMERCE (*c*) PROGRAMMING
35. Find now many arrangements can be made with the letters of the word 'MATHEMATICS'. In how many of them the vowels occur together ?
36. In how many different ways can the letters of the word 'SALOON' be arranged if (*a*) the two O's must not come together (*b*) the consonants and vowels must occupy alternate places ?
37. In how many ways can the letters of the word "ARRANGE" be arranged ? If the two R's do not occur together, then how many arrangements can be made ? If besides the two R's, the two A's also do not occur together, how many permutations can be obtained ?
38. How many 4-digits numbers can be formed by using the digits 2, 4, 6, 8 when repetition of digits is allowed.
39. In how many ways can 4 prizes be distributed among 5 girls when
(*a*) no girl gets more than 1 prize. (*b*) a girl may get any number of prizes.
(*c*) no. girl gets all the prizes.
40. There are 6 gentlemen and 4 ladies to dine at a round table. In how many ways they can seat themselves so that no two ladies are together ?
41. A round table conference is to be held between delegates of 15 countries. In how many ways can they be seated if two particular delegates may wish to sit together ?

ANSWER 10.1

1. (*i*) 60 (*ii*) 120 (*iii*) 15120 (*iv*) 5040

2. (*i*) 6 (*ii*) 7 (*iii*) 8 (*iv*) 4

(*v*) 4 (*vi*) 9

4. 120 **5.** 7 **6.** 40 **7.** 1320

8. 24 **9.** 24 **10.** 8000

11. (*i*) 360 (*ii*) 1296 **12.** 120 **13.** 20

14. 216 **15.** 90 **16.** 30240

17. (*i*) 108 (*ii*) 60 **18.** 40320, 720, 4320

19. 576, 5040, 720 **20.** 720, 4320, 576

21. (*i*) 156 (*ii*) 120 (*iii*) 1920

22. 240, 160, 128 **23.** 90, 100 **24.** 320 **25.** 42

26. 4032, 5040, 5040, 720, 10080

27. 720, 120, 120, 24, 240

28. 40320, 5040, 720, 4320, 36000, 14400, 2880

29. 256, 16, 48, 112

30. 86400 **31.** 600, 120, 216, 384 **32.** 181440

33. 468000, 20800 **34.** 60, 5040, 9979200 **35.** 4989600, 120960

36. 240, 36 **37.** 1260, 900, 660 **38.** 256 **39.** 10, 625, 620

40. 43200 **41.** $13! \times 2!$ **42.** $\frac{1}{2}(14!)$ **43.** $\frac{1}{2}(n-1)!$