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Assignment-2

Recurrence Relation and Generating Functions

Solution 1 → After observing the breeding pattern of rabits it is clear that they form a fabonacci series i.e the third term of series is the sum of the previous two terms.

i.e. $a_n = a_{n-1} + a_{n-2}$

with the basic conditions $a_0 = 0$ and $a_1 = 1$

Characteristic Equation :- $r^2 - r - 1 = 0$

$$r = \frac{1 \pm \sqrt{5}}{2}$$

$$a_n = C_1\left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

using Base conditions :-

$0 = C_1 + C_2$ — ①

$1 = C_1\left(\frac{1+\sqrt{5}}{2}\right) + C_2\left(\frac{1-\sqrt{5}}{2}\right)$

$1 = \frac{C_1 + \sqrt{5}C_1 + C_2 - \sqrt{5}C_2}{2}$

we know that

$C_1 = -C_2$

$$1 = \frac{-C_2 - \sqrt{5}C_2 + C_2 - \sqrt{5}C_2}{2}$$

$$1 = \frac{-2\sqrt{5}C_2}{2}$$

$$\boxed{C_2 = -\frac{1}{\sqrt{5}} \; ; \; C_1 = \frac{1}{\sqrt{5}}}$$

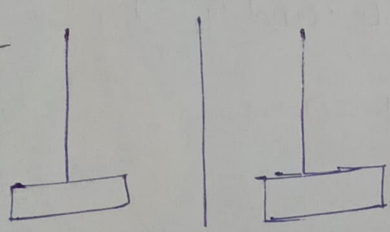
$$a_n = \left(\frac{1}{\sqrt{5}}\right)\cdot\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$a_n = \left(\frac{1}{\sqrt{5}}\right)\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$ Ans.

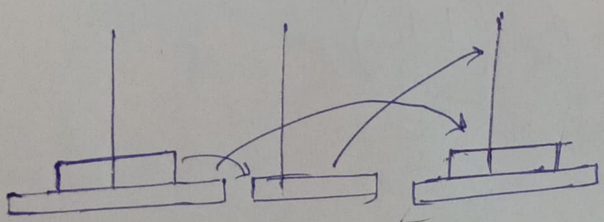
Ques2:- A popular puzzle of late 19th century ----

---------------- with the largest on the bottom.

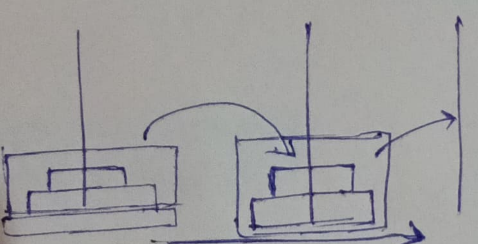
Solution 2:-



→ 1 step



→ 3 step



Here it req 3 step to move upper part then 1 step to move itself then again to 3 steps.

$$a_n = a_{n-1} + a_{n-1} + 1$$

$$a_n = 2a_{n-1} + 1$$

Hence the recurrence eq is formed.

$a_n = 2a_{n-1} + 1$

$a_n - 2a_{n-1} = 1$

Characteristic Equation

$r - 2 = 0$

$r = 2$

$a_n = C2^n$

R.H.S = Constant

$a_n = P$

$a_{n-1} = P$

$P - 2P = 1$

$\boxed{P = -1}$

$a_n = C2^n - 1$

we know that at 2nd step we need 3

$3 = C(2)^2 - 1$

$4 = 4C$

$C = 1$

Hence the solution is $2^n - 1$ Ans

Ques 3:> Find recurrence relation and give initial Condition

- - - - - - - - - - of length five?

Solution:- Let the f(n) be the function denoting the no. of two consecutive zero's together in a n-bit string.

for 1 bit string:-

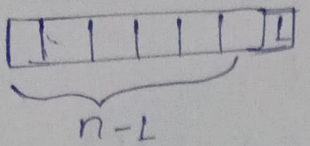
0 1 → 2 no. of 2 zero's not together

for 2 bit string:-

00 01 10 11 → 3 no are such type

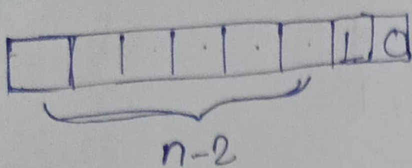
Now suppose of an n-bit string.

Case 1:- It ends with 1



Here we have to check for (n-1) bits.

Case 2:- It ends with 0



Because there is a zero in last it is mandatory for us to have 1 before it to make it non consecutive zero.

$$f(n) = f(n-1) + f(n-2)$$

And Base Conditions will be

$f(1) \longrightarrow 2$

$f(2) \longrightarrow 3$

For 5 bit string it would be 13 Ans.

$f(3) = f(1) + f(2)$

$\Rightarrow 2+3$

$f(3) = 5$

$f(4) = f(3) + f(2)$

$= 5+3 \Rightarrow 8$

$f(5) = f(4) + f(3)$

$= 13$,, Ans

Ques 4:> A computer system - - - - - - - - recursively.

Solution:- If the last digit of number is not 0, then the no. of valid codewords of length n is as same as of (n-1).

If the last digit of number is zero, it is necessary to have one more zero to make the condition true, here valid codewards of length n is a same as of (n-2).

Relation will be $a_n = a_{n-1} + a_{n-2}$

Initial Conditions:-

for a_1 we have choices from 9 digit except 0.

so, $a_1 = 9$

for a_2 we have 90 possible choices, after 00

$a_2 = 89$

Ques 5) Suppose that the roots ---------- general solu^n?

Solution:- $r = 2, 2, 2, 5, 5, 9$

$$a_n^{(H)} = (C_1 + C_2^n + C_3 n^2) 2^n + (C_4 + C_5 n) 5^n + C_6 9^n \text{ Ans.}$$