Discrete Mathematics (BCSC 1010)

Assignment-5

Mathematical Induction

Q1. Write out a formal proof by mathematical induction that the maximum number of regions the plane is divided into by n lines is $1/2\ (n^2 + n + 2)$.

Q2. By the mathematical induction, prove that

$$3^{2n+1} + (-1)^n 2 = 0(\text{mod } 5)$$

Q3. Prove using mathematical induction that for all $n \geq 1$,

$$1 + 4 + 7 + \cdot \cdot \cdot + (3n - 2) = n(3n - 1)/2$$

Q4. Use the Principle of Mathematical Induction to verify that, for n any positive integer, $6^n - 1$ is divisible by 6.

Q5. Verify that for all $n \geq 1$, the sum of the squares of the first 2n positive integers is given by the formula

$$1^2 + 2^2 + 3^2 + \cdot \cdot \cdot + (2n)^2 = (n(2n + 1)(4n + 1))/3$$

Q6. Use the Principle of Mathematical Induction to verify that, for n any positive integer $2^{2n} - 1$ is divisible by 3.

Q7. $2n + 1 < 2^n$, for all natual numbers $n \geq 3$.

Q8. An odd number of people stand in a yard at mutually distinct distances. At the same time each person throws a pie at their nearest neighbor, hitting this person. Use mathematical induction to show that there is at least one survivor, that is, at least one person who is not hit by a pie. (This problem was introduced by Carmony [Ca79].