Discrete Mathematics BCSC1010

Module 1

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Relations (Lecture 7)

Binary Relation

- Let A and B be non empty sets then any subset R of A x B is called a binary relation or relation from A to B.
- Thus, $R \subseteq AxB$
- Suppose R is a relation from A to B.
- Then R is a set of ordered pairs (a,b) where $a \in A$ and $b \in B$
- If $(a,b) \in R$; we then say "a is R-related to b", written aRb.
- If $(a,b) \notin R$; we then say "a is not R-related to b", written aRb.
- $R=\{(a,b):a\in A,\ b\in B \text{ and } aRb\}$

Example

- Let A={1,2,5} and B={2,4}
- Then ,
  AxB={(1,2),(1,4),(2,2),(2,4),(5,2),(5,4)}
- If Relation from A to B is expressed by statement “is less than” then
- R={(1,2),)(1,4),(2,4)}

Complement of a relation

- Complement of a relation will contain all the pairs where pair do not belong to relation but belongs to Cartesian product.

Example:

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A = { 1, 2}   B = { 3, 4}
R = { (1, 3) (2, 4) }
Then the complement of R
Rc = { (1, 4) (2, 3) }
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Ref:https://www.includehelp.com/basics/types-of-relation-discrete%20mathematics.aspx

Inverse of a Relation

- Let **R** be any relation from **A to B.**
- The inverse of **R** denoted by $\boldsymbol{R^{-1}}$ is the relation from **B to A** defined by:
- $\boldsymbol{R^{-1}}=\{(y,x): y \in B,\ x \in A, (x,y) \in R\}$

Example

- A={1,2,3} and B={3,4,5}
- AxB={(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)}
- If R={(2,3),(3,4)}
- Then $\boldsymbol{R}^{-1}$={(3,2),(4,3)}

Properties of Relations

- Reflexive Relation
- Irreflexive Relation
- Symmetric Relation
- Transitive Relation

Reflexive Relation

- A **reflexive relation** is the one in which every element maps to itself.
- A relation R on a set A is called reflexive if $(a, a) \in R$ for every element $a \in A$.

- **Example**
- Let $A = \{1, 2\}$
- If $R = \{(1, 1), (2, 2), (1, 2), (2, 1)\}$
- Then it will be a **Reflexive relation.**

Example

- Consider the following relations on {1, 2, 3, 4}:
- R1 = {(1, 1),(1,2),(2, 1),(2,2),(3,4),(4, 1), (4,4)}
- R2 = {(1, 1),(1,2),(2, 1)}
- R3 = {(1, 1),(1,2),(1,4),(2, 1),(2,2),(3,3),(4, 1), (4,4)}
- R4 = {(2, 1), (3,1), (3, 2), (4,1), (4, 2), (4, 3)}
- R5 = {(1, 1), (1, 2), (1, 3), (1,4), (2, 2), (2,3), (2, 4), (3, 3), (3, 4), (4, 4)}
- R6 = {(3, 4)}
- Which of these relations are reflexive?

Contd..

Solution

- The relations R3 and R5 are reflexive because they both contain all pairs of the form (a, a), namely, (1, 1), (2, 2), (3, 3), and (4, 4).
- The other relations are not reflexive because they do not contain all of these ordered pairs.
- In particular, R1, R2, R4, and R6 are not reflexive because (3, 3) is not in any of these relations.

Irreflexive Relation

- A binary **relation** is called **irreflexive**, or anti-reflexive, if it doesn't relate any element to itself.
- Example is the "greater than" **relation** $(x > y)$ on the real numbers.
- A **relation** is **irreflexive** if: $(a, a) \notin R, \ \forall a \in A$

Symmetric Relation

- A relation R on a set A is called symmetric if $(b, a) \in R$ whenever $(a, b) \in R$, for $\forall$ a, $b \in A$
- The relation R on the set A is symmetric if

$$\forall a \forall b((a, b) \in R \rightarrow (b, a) \in R)$$

Example

- Consider the following relations on {1, 2, 3, 4}:
- R1 = {(1, 1),(1,2),(2, 1),(2,2),(3,4),(4, 1), (4,4)}
- R2 = {(1, 1),(1,2),(2, 1)}
- R3 = {(1, 1),(1,2),(1,4),(2, 1),(2,2),(3,3),(4, 1), (4,4)}
- R4 = {(2, 1), (3,1), (3, 2), (4,1), (4, 2), (4, 3)},
- R5 = {(1, 1), (1, 2), (1, 3), (1,4), (2, 2), (2,3), (2, 4), (3, 3), (3, 4), (4, 4)}
- R6 = {(3, 4)}

- Here, R2 and R3 are symmetric