Discrete Mathematics BCSC1010 Module 1 Dr. Praveen Mittal Relations (Lecture 8)

Properties of Relations

- ► Reflexive Relation
- ► Irreflexive Relation
- ► Symmetric Relation
- ► Transitive Relation





Transitive Relation

- A relation R on a set A is called transitive if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.
- The relation R on a set A is transitive if we have $\forall a \forall b \forall c((a, b) \in R \text{ and } (b, c) \in R) \rightarrow (a, c) \in R).$

Example



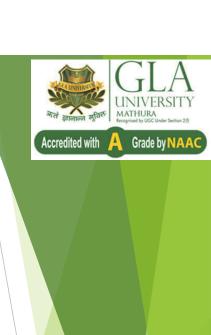
R = {(2, 1), (3,1), (3, 2), (4,1), (4, 2), (4, 3)}

▶ R is transitive

Summary

Let R is a relation on set X. It is

- Reflexive
 - •if $(a,a) \in \mathbb{R}$, for each $a \in X$.
- •Symmetric
 - •if $(a,b)\in R$ implies $(b,a)\in R$.
- Transitive
 - if (a,b)∈R and (b,c)∈R implies (a,c)∈R.
- Irreflexive
 - •if (a,a)∉R, for each a∈X.





Representing Relations Using Matrices

- A relation between finite sets can be represented using a zero-one matrix
- Suppose that R is a relation from $A = \{a_1, a_2, \dots, a_m\}$ to

$$B = \{b_1, b_2, ..., b_n\}.$$

The relation $[m_{ij}]$, where $m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$

nted by the matrix M_R



Example

Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Let R be the relation from A to B containing (a, b) if $a \in A$, $b \in B$, and a > b. What is the matrix representing R if $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$, and $b_1 = 1$ and $b_2 = 2$?

Solution: Because $R = \{(2, 1), (3, 1), (3, 2)\}$, the matrix for R is

$$\mathbf{M}_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

The 1s in M_R show that the pairs (2, 1), (3, 1), and (3, 2) belong to R. The 0s show that no other pairs belong to R.





Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4, b_5\}$. Which ordered pairs are in the relation R represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}?$$

Solution: Because R consists of those ordered pairs (a_i, b_j) with $m_{ij} = 1$, it follows that

$$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}.$$

Equivalence Relation



A relation on a set A is called an equivalence relation if it is reflexive, symmetric, and transitive.

Example

- The relation "is equal to", denoted "=", is an equivalence relation on the set of real numbers since for any $x, y, z \in R$:
- \triangleright 1. (Reflexivity) x = x,
- Lector Notes by Dr. (Symmetry) if x = y then y = x,
 - \triangleright 3. (Transitivity) if x = y and y = z then x = z.

Example

R be a relation on set of integers Z defined by $R = \{(x, y) \mid x-y \text{ is divisible by 3}\}$. Show that R is an equivalence relation.

Solution

Consider any a,b,c∈Z

- 1. Since $a-a=0=3.0 \Rightarrow (a-a)$ is divisible by 3.
 - \Rightarrow (a,a) \in R
 - \Rightarrow is reflexive.

Contd...



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2.Let (a,b) \in \mathbb{R} \Rightarrow (a-b) is divisible by 3.

\Rightarrow a-b=3q

\Rightarrow a-b=3q for some q \in \mathbb{Z}

\Rightarrow b-a=3(-q)

\Rightarrow (b-a) is divisible by

(\because q \in \mathbb{Z} \Rightarrow -q \in \mathbb{Z} \Rightarrow -q \in \mathbb{Z})
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Thus, $(a,b) \in R \Rightarrow (b,a) \in R \Rightarrow R$ is symmetric.



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3.Let (a,b) \in R and (b,c) \in R

\Rightarrow (a-b) is divisible by 3 and (b-c) is divisible by 3

\Rightarrow a-b=3q and b-c=3q' for some q,q' \in Z

\Rightarrow (a-b)+(b-c)=3(q+q')

\Rightarrow a-c=3(q+q')

\Rightarrow (a-c) is divisible by 3

(\because q,q' \in Z \Rightarrow q+q' \in Z)

\Rightarrow (a,c) \in R
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Thus, $(a,b) \in R$ and $(b,c) \in R \Rightarrow (a,c) \in R \Rightarrow R$ is transitive.

Therefore, the relation R is reflexive, symmetric and transitive, and hence it is an equivalence relation

Next Topic...

► Recurrence Relations

