Discrete Mathematics
BCSC1010

Module 1

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Recurrence Relations (Lecture9)

Introduction

- Here, we will discuss how recursive techniques can derive sequences and be used for solving counting problems.
- The procedure for finding the terms of a sequence in a recursive manner is called **recurrence relation**.
- We study the theory of linear recurrence relations and their solutions.
- Finally, we introduce **generating functions** for solving recurrence relations.

Definition

- A recurrence relation is an equation that recursively defines a sequence where the next term is a function of the previous terms (Expressing F_n as some combination of F_i with $i < n$).

- **Example** –

Fibonacci series – $F_n = F_{n-1} + F_{n-2}$; $F_1 = F_2 = 1$

Tower of Hanoi – $F_n = 2\,F_{n-1} + 1$; $F_1 = 1$

Linear Recurrence Relations

- A linear recurrence relation with constant coefficients is of the form:

  $\mathbf{C_0a_n + C_1a_{n-1} + C_2a_{n-2} + \ldots\ldots\ldots + C_ka_{n-k} = f(n)}$

  where C_i is a constant and $C_k \neq 0$

- It is of **order k** because a_n can be represented in terms of previous k elements of the sequence, so the order of a recurrence relation is **difference of highest and lowest subscript of a_i** .

Degree of Recurrence Relation

- The **degree** of a recurrence relation is defined as the highest power of $\mathbf{a_i}$**'s.**

- A recurrence relation of degree 1 is called as **linear recurrence relation**, otherwise non-linear recurrence relations.

- In linear recurrence relation, every subscripted term $\mathbf{a_i}$'s occurs to first power, i.e., it does not contain terms like $\mathbf{a_n^2}$, $\mathbf{a_n^3}$, $\mathbf{a_n.a_{n-1}}$ etc.

Homogeneous Recurrence Relation

- $\mathbf{C_0a_n + C_1a_{n-1} + C_2a_{n-2} + \ldots\ldots\ldots + C_ka_{n-k} = f(n)}$

  If f(n) is identically zero, it is called as **homogeneous** recurrence relation, otherwise non- homogeneous recurrence relation.

- $\mathbf{a_n = 2a_{n-1} + 3a_{n-2}}$ is a linear homogeneous recurrence relation of order 2 and degree 1.

- $\mathbf{a_n = a_{n-1} + n + 5}$ is a linear non-homogeneous recurrence relation of order 1 and degree 1.

Next Topic

- How to solve Recurrence Relations