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Sec- H 66

[MODULE-1]

BMAS0206:- Statistics and Complex Analysis-

Sol^n-1 $\mu_1 = 0$; $\mu_2 = 60$; $\mu_3 = -50$, $\mu_4 = 8020$

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{8020}{3600} = \underline{\underline{2.227}} \text{ Ans}$$

$$\gamma_2 = \beta_2 - 3$$
$$= \underline{\underline{-0.77}}$$

$\gamma_2 < 0 \leftarrow$ platykurtic

Sol^n-2

x	f	$(x_i - \bar{x})$	$(x_i-\bar{x})^2$	$(x_i-\bar{x})^3$	$(x_i-\bar{x})^4$
1	1	-4	16	-64	256
3	4	-2	4	-8	16
5	6	0	0	0	0
7	4	2	4	8	256
9	1	4	16	64	16
Σ 25	16	0	40	0.	544

$$\bar{x} = \frac{25}{5} = \underline{\underline{5}}$$

$$\mu_3 = \frac{0}{5} = 0$$

$$\mu_0 = 1$$

$$\mu_4 = \frac{544}{5} = \underline{\underline{1088}}$$

$$\mu_1 = 0$$

$$\mu_2 = \sum_{i=0}^{5} \frac{(x_i - \bar{x})^2}{x} = \frac{40}{5} = 8$$

$$\beta_1 = \frac{\mu_3^2}{(\mu_2)^2} = \frac{0}{64000} = 0$$

$$\gamma_1 = \sqrt{\beta_1} = \sqrt{0} = 0$$

$\gamma_1 = 0$ (Zero Skewness)

$$\beta_2 = \frac{\mu_4}{(\mu_2)^2} = \frac{108.8}{64} = 1.7$$

$$\gamma_2 = \beta_2 - 3 \Rightarrow -1.3$$

$\gamma_2 < 0 \rightarrow$ laptykurtic –

Ans 3

x_i	f_i	$x_i f_i$	$(x_i-\bar{x})^1$	$f_i(x_i-\bar{x})$	$f_i(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^3$	$f_i(x_i-\bar{x})^4$
12.5	1	12.5	-20.1	-20.1	404.01	-8120.601	163224
17.5	4	70	-15.1	-60.4	912.04	-13771.804	207948.
22.5	8	80	-10.1	-80.8	816.08	-8242.408	128538
27.5	19	522.5	-0.1	-96.9	494.19	-2520.369	0.003
32.5	35	1137.5	+4.9	-3.5	0.35	-0.035	1159.60
37.5	20	710	9.9	98	480.2	2352.98	67241
42.5	7	297.5	14.9	69.3	686.07	6792.059	24664
47.5	5	237.5	74.5	74.5	1110.05	16539.745	158239
52.5	1	52.5	19.9	19.9	396.01	7880.599	
	100	3260		0	5299	71910.2	949317

$$\bar{x} = \frac{3260}{100} = 32.6$$

$\mu_1 = 0$

$$\mu_2 = \frac{5299}{100} = 52.9$$

$$\mu_3 = \frac{71910.2}{100} = 719.102$$

$$\mu_4 = \frac{949317.97}{100} = 9493.$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{9493.1797}{(52.99)^2} = 3.38081$$

Sol^n-4 $n = 25$, $\Sigma x = 125$, $\Sigma x^2 = 650$, $\Sigma y = 100$

$\Sigma y^2 = 460$; $\Sigma xy = 508$; $r = ?$

replace in correct form –

$\Sigma x = 125 - 6 - 8 + 6 + 8 = 125$

$\Sigma y = 100 - 14 - 6 + 12 - 8 = 100$

$\Sigma x^2 = 650 - 36 - 64 + 64 - 36 = 650$

$\Sigma y^2 = 460 - 196 - 36 + 144 + 64 = 452$

$\Sigma xy = 508 - 84 - 48 + 96 + 48 = 520$

$$b_{yx} = \frac{25 \times 520 - 125 \times 100}{25 \times 650 - 15625} = \frac{500}{625} = 0.8$$

$$b_{xy} = \frac{500}{n\Sigma y_i^2 - (\Sigma y_i)^2} = \frac{500}{25 \times 452 - 10000} = \frac{500}{1300} = 0.3846$$

$$r = \sqrt{b_{xy} \cdot b_{yx}} \Rightarrow \sqrt{0.3076}$$

$$r = 0.55 \text{ Ans}$$

Ans 5 $n = 12$, $r_s = ?$

R_x	R_y	d_i	d_i^2
1	12	-11	121
2	9	-7	49
3	6	-3	9
4	10	-6	36
5	3	2	4
6	5	1	1
7	4	3	9
8	7	1	1

$$r_s = 1 - \frac{6\Sigma d_i^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 416}{12 \times 143}$$

$$= 1 - \frac{2496}{1716}$$

Ans-6

x	y	xy	x^2	y^2
25	08	200	625	64
30	10	300	900	100
32	15	480	1024	225
35	17	595	1225	289
37	20	740	1369	400
40	23	920	1600	529
42	24	1008	1764	576
45	25	1125	2025	625
286	142	5368	10532	2808

$\bar{x} = \frac{286}{8} = 35.75$

$n = 8$
$r = ?$

$r = \frac{8 \times 5368 - 286 \times 142}{\sqrt{8 \times 10532 - 81796}\sqrt{8 \times 2808 - 20164}}$

$r = \frac{2332}{49.558 \times 47.958} \Rightarrow \frac{2332}{2376.62}$

$r = 0.9804$ Ans

Ans-7 $\sigma_x^2 = 9$; $\sigma_x = 3$

$8x - 10y + 66 = 0$; $40x - 18y = 214$

replace $\Rightarrow x = \bar{x}$ & $y = \bar{y}$ for the same line occurrence

$8\bar{x} - 10\bar{y} = -66$

$40\bar{x} - 18\bar{y} = 214$ $\boxed{\bar{x} = 13}$ $\boxed{\bar{y} = 17}$

$8x - 10y = -66$

$-10y = -8x - 66$

$10y = 8x + 66$

$y = \frac{8}{10}x + \frac{66}{10}$

$b_{xy} = \frac{8}{10} = 0.8$

$40x - 18y = 214$

$40x = 214 + 18y$

$x = \frac{214}{40} + \frac{18}{40}y$

$b_{xy} = \frac{18}{40} = .535$

$r = \sqrt{b_{xy} \cdot b_{yx}} = r = \sqrt{\frac{8}{10} \times \frac{18}{40}}$

$r = 0.6$ Ans

$b_{yx} = r\frac{\sigma_y}{\sigma_x}$

$\frac{8}{10} = \frac{0.6}{3}\sigma_y$

$\sigma_y = \frac{8 \times 3}{6} = 4$ Ans

Ans-8

x	y	xy	x^2
46	40	1840	2116
42	38	1596	1764
44	36	1584	1936
40	35	1400	1600
43	39	1677	1849
41	37	1517	1681
45	41	1845	2025
301	266	11459	12,971

$\bar{x} = \frac{301}{7} = 43$

$\bar{y} = \frac{266}{7} = 38$

$b_{yx} = \frac{7 \times 11459 - 301 \times 266}{7 \times 12971 - 301 \times 301} = \frac{80213 - 80066}{90729 - 90601}$

$b_{yx} = \frac{147}{128} = 1.48$

$y - 38 = 1.48(x - 43)$

$y - 38 = 1.48x - 63.64$

$y = 1.48x - 25.64$

$x = 37$

$y = 1.48 \times 37 - 25.64$

$y = 32.5$ Ans

$n = 102$

As-9 → y = expense

x = duration

$\Sigma x = 510$; $\Sigma y = 7140$, $\Sigma xy = 54900$, $\Sigma x^2 = 4150$

$\Sigma y^2 = 740000$

① $b_{yx} = \frac{102 \times 54900 - 510 \times 7140}{102 \times 4150 - 260100} = \frac{1958400}{163200} = \underline{\underline{12}}$

② $b_{xy} = \frac{1958400}{24520800} = 0.0798$

$\bar{x} = \frac{\Sigma x}{n} = \frac{510}{102} = \underline{\underline{5}}$; $\bar{y} = \frac{7140}{102} = 70$

x on y

$(x-\bar{x}) = b_{xy}(y-\bar{y})$

$(x-5) = 0.07(y-70)$

$x = 0.079y - 0.53$

y on x

$(y-\bar{y}) = b_{yx}(x-\bar{x})$

$(y-70) = 12(x-5)$

$y = 12x + 10$

if $x = 7$

y on x

$y = 12x + 10$

$y = 12 \times 7 + 10$

$y = 84 + 10$

$y = 94$ Ans

As-10 $n = 8$

x	y	xy	x^2
0	120	0	0
0	360	0	0
0	60	0	0
0	180	0	0
80	1280	102400	6400
80	1120	89600	6400
80	1120	89600	6400
80	760	60800	6400
320	5000	342400	25600

$\bar{x} = \frac{\Sigma x}{n} = \frac{320}{8} = 40$

$\bar{y} = \frac{\Sigma y}{8} = \frac{5000}{8} = 625$

$b_{yx} = \frac{113200}{102400} = \underline{\underline{11.125}}$

11.125

$(y-\bar{y}) = b_{xy}(x-\bar{x})$

$(y-625) = 11.125(x-40)$

(a) $\boxed{y = 11.125x + 180}$

(b) if $x = 60$

$y = 11.125 \times 60 + 180$

$y = 847.5$

As-11 y on x

$y = 2.8x + 5$

x on y

$x = -0.5y + 3$

No, because b_{yx} is +ve but b_{xy} is -ve.

(12)

w	f	w^2	wf
50	12	2500	600
70	15	4900	1050
100	21	10000	2100
120	25	14400	3000
340	73	31800	6750

$\bar{f} = \frac{\Sigma f}{n} = \frac{73}{4} = 18.25$

$\bar{w} = \frac{\Sigma w}{n} = \frac{340}{4} = 85$

$b_{fw} = \frac{4 \times 6750 - 340 \times 73}{4 \times 318000 - 115600}$

$b_{fw} = \frac{2180}{11600} = 0.1879$

$(f - \bar{f}) = b_{fw}(w - \bar{w})$

$(f - 18.25) = 0.1879(w - 85)$

$\boxed{f = 0.1879w + 2.2785}$

when $w = 150$

$f = 0.1875 \times 150 + 2.2785$

$f = 30.4635$

Ans-13

$\bar{x} = 47.5 \quad \sigma_y = 10.8$

$\bar{y} = 39.5 \quad r = 0.95$

$\sigma_x = 16.8$

$b_{yx} = \frac{r\sigma_y}{\sigma_x}$

$= \frac{0.95 \times 109.8}{16.8}$

$= 0.6107$

$b_{xy} = \frac{r\sigma_x}{\sigma_y}$

$= \frac{0.55 \times 167.8}{105.8}$

$= 1.477$

Regression x on y

$(x - \bar{x}) = b_{xy}(y - \bar{y})$

$x - 47.5 = 1.477(y - 39.5)$

$x = 1.477y - 10.8415$

y on x

$(y - \bar{y}) = b_{yx}(x - \bar{x})$

$(y - 39.5) = 0.6107(x - 47.5)$

$\boxed{y = 0.6107x + 10.4918}$

if $x = 30$

$y = 0.6107 \times 30 + 10.4918$

$y = 18.321 + 10.4918$

$y = 28.81218$

-14

$p = \frac{4}{10} = 0.4$

$q = 1 - 0.4 = 0.6$

$P(x=r) \Rightarrow {}^nC_r \, p^r q^{n-r}$

$P(x=0) \Rightarrow {}^3C_0 (0.4)^0 (0.6)^3 = 1 \times 1 \times 0.216 = 0.216$

$P(x=1) \Rightarrow {}^3C_1 (0.4)^1 (0.6)^2 = 3 \times 0.4 \times 0.36 = 0.432$

$P(x=2) \Rightarrow {}^3C_2 (0.4)^2 (0.6)^1 = 3 \times 0.16 \times 0.6 = 0.288$

$P(x=3) \Rightarrow {}^3C_3 (0.4)^3 (0.6)^0 = 3 \times 0.64 \times 1 = 1.92$

Ans-15

$p = \frac{1}{4} = 0.25$

$q = \frac{3}{4} = 0.75$

$P(x=4) = 6C_4 (0.25)^4 (0.75)^2$

$= \frac{6!}{4! \times 2!} 0.003$

$= 15 \times 0.0031 \times 0.5625$

$= 0.033$ Ans

$P(x>=1) = 1 - P(x=0)$

$P(x=0) = 6C_0 (0.25)^0 (0.75)^6$

$= 1 \times 1 \times 0.822$

$= 0.822$ Ans

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Ans-16

$P = 0.1$

$q = 0.9$

$P(x=4) = 6C_4 \times (0.1)^4 \times (0.9)^2$

$= 15 \times 0.0001 \times 0.81$

$= 0.001215$ Ans

Ans-18

Mean $= \lambda = \frac{180}{60} = 3$ cars.

$P(x=k) = \frac{\lambda^k e^{-k}}{k!}$

$P(x>5) = 1 - P(x \le 5)$

$= 1 - (P(0) + P(1) + P(2) + P(3) + P(4) + P(5))]$

$P(x \le 5) = 0.049 + 0.149 + 0.224 + 0.2240 + 0.168 + 0.1008$

$= 0.91$

$P(x>5) = 1 - 0.91 = 0.085$ Ans

*

Ans-17

$P(n=k) = \frac{\lambda^k e^{-k}}{k!}$

mean $= np(\lambda) = 5$

$P\{x>5\} = 1 - P\{x \le 5\}$

$= 1 - [P(0) + P(1) + P(2) + P(3) + P(4) + P(5)]$

$= P(0) = \frac{5^0 e^{-5}}{0!} = 0.067$

$P(1) = \frac{5^1 e^{-5}}{1!} = 0.0336$

$P(2) = \frac{5^2 e^{-2}}{2!} = 0.084$

$P(3) = \frac{5^3 e^{-5}}{3!} = 0.140$

$P(4) = \frac{5^4 e^{-5}}{4!} = 0.175$

$P(5) = \frac{5^5 e^{-5}}{5!} = 0.175$

$P(x>5) = 1 - [P(0) \ldots P(5)]$

$= 1 - 0.6143$

$= 0.3857$

$P(x>5) = 0.3857$

$= 38.57\%$

Hence vehicle should be provided by council.

Ans-19 p = prob. of error on a page = $\frac{40}{600}$ = $1/15$

$n = 10$;

$\lambda = np$

$\lambda = 10 \times \frac{1}{15} = 0.67$

PD = $P(x=r) = \frac{e^{-\lambda}\lambda^r}{r!}$

$P(r=0) = \frac{e^{-0.67} \times 0.67^0}{0!} = 0.511$ Ans

Ans-20

x	f	xf
0	122	0
1	260	260
2	15	30
3	2	6
4	1	4
	400	300

mean $\lambda = \frac{\Sigma f_i x_i}{\Sigma f_i}$

$\lambda = \frac{300}{400} = 0.75$

$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} = \frac{e^{-0.75} \times (0.75)^r}{r!}$

Theoretical frequency $= N \cdot P(r) = N \times \frac{e^{-\lambda}\lambda^r}{r!}$

$= 400 \times \frac{e^{-0.75} \times 0.75^r}{r!}$

$= \frac{188.94 \times (0.75)^r}{r!}$

$r=0 \Rightarrow \frac{188.94 \times (0.75)^0}{0!} = 188.94$

$r=1 = \frac{188.94 \times (0.75)^1}{1!} = 141.70$

$r=2 = \frac{188.94 \times (0.75)^2}{2!} = 53.13$

$r=3 = \frac{188.94 \times (0.75)^3}{3!} = 13.28$

$r=4 = \frac{188.94 \times (0.75)^4}{4!} = 2.410$

Total = 188.94 + 141.70 + 53.13 + 13.28

= 400 verified

Ans-21 $N = 729$, $n = 6$,

p = getting 2 or 3 $= \frac{2}{6} = \frac{1}{3}$.

q = not getting 2 or 3 $= \frac{4}{6} = \frac{2}{3}$.

$P(x=r) = {}^nC_r\, p^r q^{n-r}$

$P(x \geq 3) = P(3) + P(4) + P(5) + P(6)$

$= {}^6C_3 \times (1/3)^3 (2/3)^3 + {}^6C_4 (1/3)^4 (2/3)^2 + {}^6C_5 (1/3)^5 (2/3)^1$

$+ {}^6C_6 (1/3)^6 (2/3)^0$

$= \frac{1}{(3)^6} (20 \times 8 + 15 \times 4 + 6 \times 2 + 1]$

$= \frac{233}{(3)^6} \times 729 = 233$ Ans

Ans-22 $n = 20$; $\Sigma x = 300$; $\Sigma x^2 = 5000$; median = 15

Mean $= \frac{\Sigma x}{n} = \frac{300}{20} = 15$

$\boxed{\text{Out of Syllabus}}$

Economic Condition	Intelligence Good	Bad	Row total
Rich	85	75	160
Poor	165	175	340
	250	250	500

Rich	$(160\times250)/500 = 80$	$(160\times250)/500 = 80$
Poor	$(340\times250)/500 = 170$	$(340\times250)/500 = 170$

$$\chi^2 = \sum \frac{(O_i-E_i)^2}{E_i} = \frac{25}{80}+\frac{25}{80}+\frac{25}{170}+\frac{25}{170} = \underline{0.9192}$$

degree of freedom = (row-1) × (col-1)
(2-1) × (2-1)
$\underline{1}$ Or

The critical value for a chi-square distribution with 1 degree of freedom at 0.05 significance level is $\underline{3.841}$.

Ans 24 $p = 1/2$ $n = 5$
$q = 1/2$

$P(x=0) = 5C_0 p^0 q^5 \times N$
$= 10$
$P(x=1) = 50$
$P(x=2) = 100$
$P(x=3) = 100$
$P(x=4) = 50$
$P(x=5) = 10$

O_i	E_i	$(O_i-E_i)^2$	$(O_i-E_i)^2/E_i$
18	10	64	6.4
56	50	36	0.72
110	100	100	1
88	100	144	1.44
40	50	100	2
8	10	4	0.4
320	320		11.96

$\chi^2 = 11.96$ dof = 6-1 = 5

$\chi^2_{calculated} > \chi^2_{tabulated}$

So we reject the binomial frequency.

Ans 25 ratio = 62:4:34

Total = 200 tonnes
Expected amount

NaCl = $\frac{62}{100}\times200 = 124$ tonnes
Mg = $4/100 \times 200 = 8$ tonnes.
other = $\frac{34}{100}\times200 = 68$ tons.

$$\chi^2 = \frac{36}{124}+\frac{4}{8}+\frac{16}{68} = \underline{1.025}$$

degree of freedom = 3-1 = 2

$\chi^2_{calculated} < \chi^2_{tabulated}$.

So, the data is acceptable.