MA203 NUMERICAL METHODS

TUTORIAL SHEET 3
(Session: MO/23)

MODULE 2 **TOPIC: SYSTEM OF LINEAR EQUATIONS**

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1. Check whether the following system is diagonally dominant or not:
   (a) $10x_1 - 5x_2 + 7x_3 = -3; 3x_1 + 5x_2 = 4; 3x_1 + 2x_2 - 3x_3 = 0$
   (b) $5x_1 - x_2 + x_3 = 0; 3x_1 + 5x_2 = 2; x_1 + x_2 - 3x_3 = 0$

2. Compute the solution of the following system of given linear equations using Gauss Elimination Method.
$$3x + 2y + 3z = 18; x + 4y + 9z = 16; 2x + y + z = 10$$

3. Apply partial pivoting to determine the solution of the following system of linear equations with the help of Gauss-Elimination method:
$2x + y + 2z + w = 6;\quad 6x - 6y + 6z + 12w = 36;\quad 4x + 3y + 3z - 3w = -1; 2x + 2y - z + w = 10.$

4. Determine the solution of the following system using Gauss Jordan method:
$$3x + 4y + 5z = 40; 2x - 3y + 4z = 13; x + y + z = 9$$

5. Apply Gauss Jordan method to determine the value of unknowns from the following system using partial pivoting:
$2x + y + 2z + w = 6;\quad 6x - 6y + 6z + 12w = 36;\quad 4x + 3y + 3z - 3w = -1; 2x + 2y - z + w = 10.$

6. Decompose the matrix $A = LU$, where $A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & 2 \\ 3 & 2 & 4 \end{bmatrix}$, L is the lower triangular matrix, and U is the upper-triangular matrix with all diagonal elements as unity.

7. Using LU decomposition method, compute the solution of the following system:
$$2x + 3y + z = 9;\quad x + 2y + 3z = 6;\quad 3x + y + 2z = 8.$$
with $l_{ii} = 1, i = 1, 2, 3.$

8. Consider the equations:
$$x_1 + x_2 + x_3 = 1; 4x_1 + 3x_2 - x_3 = 6; 3x_1 + 5x_2 + 3x_3 = 4$$
Use the Crout's LU decomposition method to solve the system.

9. Show that the LU decomposition (Doolittle) method fails to solve the system of equations: $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 5 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$. Also, show that that it is possible to solve it through the same method by interchanging the first and last equations.

10. The following system of linear equations is given:
$$8x - 3y + 2z = 10; 6x + 3y + 12z = 35; 4x + 11y - z = 33$$
Perform three iterations of Jacobi method with suitable initial approximations. Also, execute Gauss-Seidel method to determine the approximate values of unknowns after third iteration.

11. Consider the linear system $AX = B$ as

$$\begin{bmatrix} 9 & -3 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$$

   (a) compare by solving the above systems using Jacobi and Gauss-Seidel methods starting with $X = (0, 0)$.

   (b) change the $(1, 1)$-th entry in matrix A from 9 to 1 so that the coefficient matrix is no longer diagonally dominant and check whether Gauss-Seidel method still works. Explain why or why not.

   (c) then change the $(2, 2)$-th entry from 8 to 1 as well and check. Again, explain the results.

12. Perform three iterations of the Gauss-Jacobi iteration method for solving the system of equations:

$$\begin{bmatrix} 6 & 1 & 2 \\ 1 & 4 & 3 \\ 2 & 1 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 8 \end{bmatrix}$$

   Take the components of the approximate initial vector as $\mathbf{x}^{(0)} = [1.3, -1.9, 0.8]^T$. Compare with the exact solution $x_1 = 1, x_2 = 2, x_3 = 1$.

13. Perform three iterations of the Gauss-Seidel iteration method for solving the system of equations:

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 2 \\ 5 & 4 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ 11 \end{bmatrix}$$

   Take the components of the approximate initial vector as $x_i^{(0)} = \frac{b_i}{a_{ii}}$, $i = 1, 2, 3$. Compare with the exact solution $\mathbf{x} = [1, -1, 1]^T$.

14. Obtain the solution of the following system by Gauss-Jacobi iteration method correct to two decimal places

$$20x_1 - x_2 + x_3 = 23.28; x_1 + 15x_2 - x_3 = 29.92; 2x_1 + x_2 - 20x_3 = -55.64$$

15. Solve by Gauss Seidel iterative method to determine the solution using Gauss-Seidel method correct up to three significant figures

$$x_1 + x_2 + 4x_3 = 9; 8x_1 - 3x_2 + 2x_3 = 20; 4x_1 + 11x_2 - x_3 = 33$$

16. *Use Power method to approximate dominant eigenvalue and a corresponding eigenvector of the matrix

$$\mathrm{A} = \begin{bmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$