MA203 NUMERICAL METHODS

TUTORIAL SHEET 2
(Session: MO/23)

MODULE 1 **TOPIC: SOLNS. OF NONLINEAR EQUATIONS**

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1. Using $a = 0$ and $b = 1$ as initial approximations to the root, execute four iterations of Bisection method to find the positive real root of $x - \cos x = 0$ correct to four decimal places.

2. The nonlinear equation $x \log_{10} x = 1.2$ is given:

   (a) Obtain an interval of unit length which contains the positive real root of the equation.

   (b) Using end points of the above obtained interval as initial approximations, compute the value of root correct to four decimal places using Regula-Falsi method.

3. Apply False Position method to find the smallest positive root of the equation $x - e^{-x} = 0$ correct to three decimal places.

4. A real root of the equation $f(x) = x^3 - 5x + 1 = 0$ lies in the interval $(0, 1)$. Perform four iterations of the Secant method and Regula Falsi method to obtain the root.

5. Derive the Newton's iterative formula for finding $\sqrt[p]{N}$, where N is a positive real number and p is the p^{th} root . Hence, apply it for $N = 15$ and $p = 2$ for obtaining results correct to two decimal places.

6. Determine the smallest positive real root of the equation $3x = \cos x + 1$ correct to four decimal places using

   (a) Newton Raphson method.

   (b) direct (fixed point) method

   Compare the results obtained.

7. Use the Secant method to determine the root of the equation $\cos x - xe^x = 0$ with 0 and 1 as initial approximations up to four decimals. Compare with results obtained with Regula Falsi method with same initial approximations.

8. Identify a suitable representative of the equation $f(x) = x^3 + x^2 - 1 = 0$ in the form $x = \phi(x)$ for finding its root in the interval $(0, 1)$ by iterative (fixed point) method. Hence, using it, find the real root with an accuracy of 10^{-4}.

9. The smallest positive root of the equation $f(x) = x^4 - 3x^2 + x - 10 = 0$ is to be obtained:

   (a) Identify an interval of unit length which contains this root.

   (b) Using the end points of the above obtained interval as initial approximations, perform two iteration of Bisection method.

   (c) Taking the mid point of the last interval as the initial approximations, perform three iterations of Newton Raphson method.

10. The negative root of the smallest magnitude of the equation $f(x) = 3x^3 + 10x^2 + 10x + 7 = 0$ is to be computed

    (a) Find an interval of unit length which contains this root.

(b) Using the end points of the above obtained interval as initial approximations, perform two iteration of Bisection method.

(c) Taking the end points of the last interval (obtained through Bisection method) as initial approximations, perform three iterations of the Secant method.

11. Determine the smallest positive root of the equation: $10\int_0^x e^{-x^2}dt = 1$ correct to four decimal places using Newton Raphson method.

12. Consider the sequence $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$, $x_0 = 0.5$ obtained from the Newton Raphson method. Show that the sequence converges to 1.5.

13. Determine the absolute difference between the 4^{th} and 5^{th} approximation to the real root $f(x) = x^3 - 5x - 7 = 0$ using method of false position up to 4^{th} decimal place.

14. The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iterative (fixed point) method for the formula $x_{k+1} = -\frac{ax_k+b}{x_k}$ is convergent near $x = \alpha$ if $|\alpha| > |\beta|$.

15. Prove that the real sequence generated by iterative scheme $x_n = \frac{x_{n-1}}{2} + \frac{1}{x_{n-1}}$, $n \geq 1$ converges to fixed point $\sqrt{2}$ for $x_0 > \sqrt{\frac{2}{3}}$.

16. The equation $f(x) = x^3 - 7x^2 + 16x - 12$ has a double root at $x = 2$. Starting with initial approximation $x_0 = 1$, find the root correct up to 3- decimal place using
(i) Newton-Raphson method (ii) Modified Newton-Raphson method