COSC 4364 Spring 2018

Assignment 6 - Splines

Out: March 10 Due: March 27. e-mail code and report.

Points				
Problem	a)	b)	c)	Total
1	3	3	3	9
2	20			20
3	3			3
4	20			20
5	10	5		15
9	25	25		50
Total				117

Problem 1. (3 x 3 points) Are the functions below quadratic splines? Explain why or why not.

a)

$$Q(x) = \begin{cases} 0.3x^2 & (0 \le x \le 1) \\ 9x^2 - 17.4x + 8.7 & (1 \le x \le 1.3) \end{cases}$$

b)

$$Q(x) = \begin{cases} -x^2 & (-1 \le x \le 0) \\ x & (0 \le x \le 2)bb \end{cases}$$

c)

$$Q(x) = \begin{cases} x & (-5 \le x \le 1) \\ x^2 & (1 \le x \le 2) \\ 4 & (2 \le x \le 3) \end{cases}$$

Problem 2. (20 points) Find by hand-calculation a quadratic spline interpolant for

x	-1	-½	½	1	3/2	5/2
y	2	3	-1	1	2	3

assuming $z_0=0$

Problem 3. (3 points) Is |x| a first degree spline? Explain why or why not.

Problem 4. (20 points) Find by hand-calculation the natural cubic spline interpolant for

x	1	2	3	4	5
y	0	1	-1	-2	0

Problem 5. (a) 10 points, b) 5 points)

a) Find an expression for $B_i^2(x)$ and verify that it is piecewise quadratic.
   Hint: Use the recursive definition of B splines and properties of zero degree B splines.
b) Show that $B_i^2(x)$ is 0 at every knot except

$$B_i^2(t_{i+1})=(t_{i+1}-t_i)/(t_{i+2}-t_i) \text{ and } B_i^2(t_{i+2})=(t_{i+3}-t_{i+2})/(t_{i+3}-t_{i+1})$$

Matlab problem

Problem 6. (2x25 points) Matlab programming. Let S(x) be

a) a quadratic spline (Section 6.1 in the book)
b) a quadratic B-Spline (Section 6.3 in the book)

that interpolates $f(x)=1/(1+(5x)^2)$ on the interval at 41 equally spaced nodes in the interval [-1,1]. Evaluate f(x)-S(x) at 200 equally spaced x values with $x_1=-1$ and $x_{200}=1$ and plot the error f(x)-p(x).

What is the maximum positive error?
What is the maximum negative error?
What is the square root of the mean square error? $(\text{sqrt}(\sum_1^{200}(f(x_i)-p(x_i))^2/200))$

You should compare your results to those of Assignment 4 Problem 8.