COSC 4364 Assignment 6

Austin Moreau

March 22, 2018

1 Problem 1

a) $Yes, This\ is\ a\ quadratic\ spline$

$Continuity\ of\ Q\ at\ 1$
$.3(1)^2 = 9(1)^2 - 17.4(1) + 8.7 = .3$

$Continuity\ of\ Q'\ at\ 1$
$.6(1) = 18(1) - 17.4$

$Polynomial\ of\ at\ most\ 2$

b) $Not\ a\ quadratic\ spline$

$Q'\ not\ continuous\ at\ 0$
$-2(0) \neq 1$

c) $Not\ a\ quadratic\ spline$

$Q'\ not\ continuous\ at\ 2$
$2(2) \neq 0$

2 Problem 2

$$z_1 = 0 + 2\left(\frac{3-2}{(-1/2)+1}\right) = 4$$
$$z_2 = -4 + 2\left(\frac{-1-3}{1/2+1/2}\right) = -12$$
$$z_3 = 12 + 2\left(\frac{1+3}{1/2}\right) = 28$$
$$z_4 = -28 + 2\left(\frac{2-1}{3/2-1}\right) = -24$$
$$z_5 = 24 + 2\left(\frac{3-2}{5/2-3/2}\right) = 26$$

$$Q_0(x)=\frac{4-0}{2(-1/2+(-1))}(x+1)^2+0+2=4(x+1)^2+2 \qquad (-1\leq x\leq -1/2)$$
$$Q_1(x)=\frac{-12-4}{2(1/2+1/2)}(x+1/2)^2+4(x-4)+3=-8(x+1/2)^2+4(x-4)+3 \qquad (-1/2\leq x\leq 1/2)$$
$$Q_2(x)=\frac{28+12}{2(1/2)}(x-1/2)^2-12(x-1/2)-1=40(x-1/2)^2-12(x-1/2)-1 \qquad (1/2\leq x\leq 1)$$
$$Q_3(x)=\frac{-24-28}{2(3/2-1)}(x-1)^2+28(x-1)+1=52(x-1)^2+28(x-1)+1 \qquad (1\leq x\leq 3/2)$$
$$Q_4(x)=\frac{26+24}{2(5/2-3/2)}(x-3/2)^2-24(x-3/2)+2=25(x-3/2)^2-24(x-3/2)+2 \qquad (3/2\leq x\leq 5/2)$$

3 Problem 3

No, the interval is undefined and therefore infinite. |x| is not a first degree spline.

4 Problem 4

$$\begin{array}{ll}
-8.2143\cdot 10^{-1}\cdot x^3+2.4643\cdot x^2+-6.4286\cdot 10^{-1}\cdot x+-1.0000, & \text{if } x\in[1,2] \\
1.1071\cdot x^3+-9.1071\cdot x^2+2.2500\cdot 10^1\cdot x+-1.6429\cdot 10^1, & \text{if } x\in(2,3] \\
3.9286\cdot 10^{-1}\cdot x^3+-2.6786\cdot x^2+3.2143\cdot x+2.8571, & \text{if } x\in(3,4] \\
-6.7857\cdot 10^{-1}\cdot x^3+1.0179\cdot 10^1\cdot x^2+-4.8214\cdot 10^1\cdot x+7.1429\cdot 10^1, & \text{if } x\in(4,5]
\end{array}$$

5 problem 5

a)
$$B_i^2(x)=\left(\frac{(x-t_i)^2}{(t_{i+2}-t_i)(t_{i+1}-t_i)}\right)B_i^0(x)+\left(\frac{(t_{i+3}-x)^2}{(t_{i+3}-t_{i+1})(t_{i+3}-t_{i+2})}\right)B_{i+2}^0(x)$$
$$+\left(\frac{(x-t_i)(t_{i+2}-x)}{(t_{i+2}-t_i)(t_{i+2}-t_{i+1})}+\frac{(x-t_{i+1})(t_{i+3}-x)}{(t_{i+2}-t_{i+1})(t_{i+3}-t_{i+1})}\right)B_{i+1}^0(x)$$

It is piecewise quadratic since the largest power of x is 2 and the derivative of B as well as B itself is continuous at point k, k+1, k-1, etc.

$$B_i^{2'}(x) = B_i^{1}(x) = \frac{x-t}{t_{i+1}-t_i} B_i^0(x) + \frac{t_{i+2}-x}{t_{i+2}-t_{i+1}} B_{i+1}^0(x)$$

$$= \begin{cases} (x-t_i)/(t_{i+1}-t_i) & t_i \le x < t_{i+1} \\ (t_{i+2}-x)/(t_{i+2}-t_{i+1}) & t_{i+1} \le x < t_{i+2} \\ 0 & \text{otherwise} \end{cases}$$

$$B_i^k(x) = \begin{cases} 0 & t_i = t_{i+1+k} \\ ((x-t_i)/(t_{i+k}-t_i))\, B_i^{k-1} = S_1(x) & t_i < t_{i+k} \ \& \ t_{i+1} = t_{i+1+k} \\ S_2(x) = ((t_{i+1+k}-x)/(t_{i+1+k}-t_{i+1}))\, B_{i+1}^{k-1}(x) & t_i = t_{i+k} \quad t_{i+1} < t_{i+1+k} \\ S_1(x) + S_2(x) & \text{otherwise} \end{cases}$$

b)