COSC 4364 Spring 2018

Assignment 3

Given: 2018-02-08   Due: 2018-02-15

Upload your report and MATLAB codes to Blackboard.

		Points				
	a)	b)	c)	d)	e)	total
1	5					5
2	5					5
3	5	5				10
4	5					5
5	8	8	8	8	8	40
6	20	20				40
Total						106

Problem 1. (5 points) For a continuous function with a single root in the interval [-2,2] using the bisection method with the interval endpoints as starting points, derive how many steps are required to determine the root with an error of at most $1/2x10^{-15}$ (double precision).

Problem 2. (5 points) How many bisection steps are required for each decimal digit of accuracy?

Problem 3. (2x5=10 points) Let f(x)=x-R/x and g(x) = Rx-1/x.

a) Derive a Newton iteration formula for finding a root of f(x) that does not involve $1/x_n$. To which value does the Newton iterates x_n converge?
b) Derive a Newton iteration formula for finding a root of g(x) that does not involve $1/x_n$. To which value does the Newton iterates x_n converge?

Problem 4. (5 points) Derive a division free Newton iteration formula for computing the inverse square root of a positive number R.

MATLAB problems

Problem 5. (5x8=40 points) Write a MATLAB program not using built-in functions that finds the root of the polynomial $f(x) = 2x^3+7x^2+14x+3$ in the interval [-1,1] by the following methods.

a) Bisection
b) Regula Falsi
c) Modified Regula Falsi,
d) Newton
e) Secant.

Use a nested formula for the polynomial evaluation, double precision and an error tolerance for the root of $1/2x10^{-14}$.

Report the number of steps to reach the error tolerance for each method.

For the Newton and Secant methods make a Table showing the root approximations for each step until convergence. Try a few starting points. How sensitive is the number of iterations to the choice of starting point(s).

Problem 6. (2x20 points) Use Newton's method to compute sqrt(R) in double precision by

a) $x_{n+1} = ½(x_n+R/x_n)$ (20 points)
b) $x_{n+1} = [x_n(x_n^2+3R)]/[3x_n^2+R]$ (20 points)

For R = 0.001, 0.1, 10, and 1000 with a stopping criteria of 1) $|x_{n+1}-x_n|< 10^{-14}$ 2) $|x_{n+1}-x_n|< 10^{-6}$.

i) Record the number of iterations required to reach the stopping criteria for each of the two methods for each value of R.
ii) Plot the values of x_n for methods a) and b) in the same plot with separate plots for the different R values.
iii) How sensitive are the iteration counts to your selected starting value?
iv) Which method converge the fastest?
v) Why?