COSC 4364 Assignment 9

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9 Problem 1

b.

9 Problem 2

b.

9 Problem 3

c.

9 Problem 4

$$y = a_0 + a_1 x + a_2 x^2$$

$$Residual\, R^2 = \sum_{t=0}^{m} [y_t - (a_0 + a_1 x + a_2 x^2)]^2$$

To minimize the square residual we must have

$$\frac{\partial R^2}{\partial a_0} = 0,\ \frac{\partial R^2}{\partial a_1} = 0,\ \frac{\partial R^2}{\partial a_2} = 0$$

therefore

$$\frac{\partial R^2}{\partial a_0} = 0 \Rightarrow -2 \sum_{t=0}^{m} [y_t - (a_0 + a_1 x + a_2 x^2)]$$

$$\frac{\partial R^2}{\partial a_1} = 0 \Rightarrow -2 \sum_{t=0}^{m} [y_t - (a_0 + a_1 x + a_2 x^2)] x$$

$$\frac{\partial R^2}{\partial a_2} = 0 \Rightarrow -2 \sum_{t=0}^{m} [y_t - (a_0 + a_1 x + a_2 x^2)] x^2$$

Which lead to

$$\sum_{i=0}^{m} y_i = a_0(m+1) + a_1\sum_{i=0}^{m} x_i + a_2\sum_{i=0}^{m} x_i^2$$
$$\sum_{i=0}^{m} x_i y_i = a_0\sum_{i=0}^{m} x_i + a_1\sum_{i=0}^{m} x_i^2 + a_2\sum_{i=0}^{m} x_i^3$$
$$\sum_{i=0}^{m} x_i^2 y_i = a_0\sum_{i=0}^{m} x_i^2 + a_1\sum_{i=0}^{m} x_i^3 + a_2\sum_{i=0}^{m} x_i^4$$

and finally we get

$$\sum_{i=0}^{m} y_i = c(m+1) - \sum_{i=0}^{m} x_i + \sum_{i=0}^{m} x_i^2$$
$$\sum_{i=0}^{m} x_i y_i = c\sum_{i=0}^{m} x_i - \sum_{i=0}^{m} x_i^2 + \sum_{i=0}^{m} x_i^3$$
$$\sum_{i=0}^{m} x_i^2 y_i = c\sum_{i=0}^{m} x_i^2 - \sum_{i=0}^{m} x_i^3 + \sum_{i=0}^{m} x_i^4$$

and thus

$$c = \frac{1}{m+1}\left[\sum_{i=0}^{m} y_i + \sum_{i=0}^{m} x_i - \sum_{i=0}^{m} x_i^2\right]$$

$$OR \qquad = \frac{\sum_{i=0}^{m} x_i y_i + \sum_{i=0}^{m} x_i^2 - \sum_{i=0}^{m} x_i^3}{\sum_{i=0}^{m} x_i}$$

$$OR \qquad = \frac{\sum_{i=0}^{m} x_i^2 y_i + \sum_{i=0}^{m} x_i^3 - \sum_{i=0}^{m} x_i^4}{\sum_{i=0}^{m} x_i^2}$$

9 Problem 5

a) ii

b) iv

9 Problem 6

Change y to 1/y and (a+bx)^-1 to a+bx.
So you end up with the linear function 1/y = a+bx where you can solve for a and b using linear theory

9 Problem 7

The following results can be reproduced by opening the Assignment9_7.m file and hitting the run button.

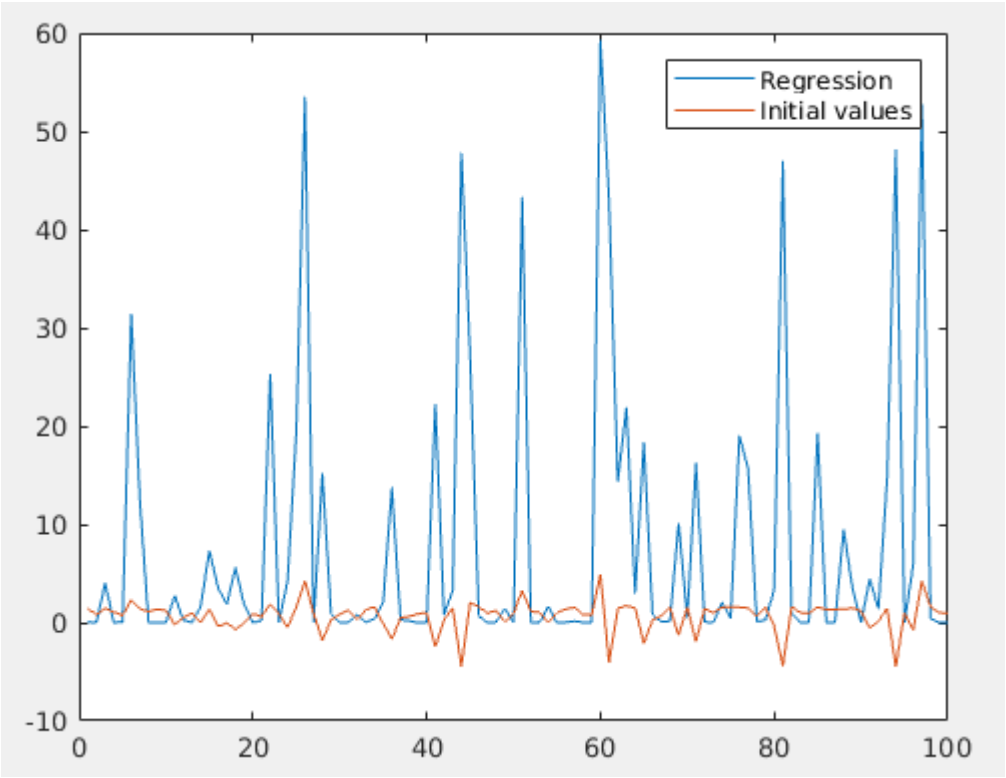
n	iterations	RMS
1.0e+02 *		
0.100000000000000	0.050000000000000	0.000000000000000
0.200000000000000	0.100000000000000	0.000000000000000
0.300000000000000	0.140000000000000	0.000000000370044
0.400000000000000	0.140000000000000	0.000000000381486
0.500000000000000	0.140000000000000	0.000000000346417
0.600000000000000	0.150000000000000	0.000000000193567
0.700000000000000	0.150000000000000	0.000000000197102
0.800000000000000	0.150000000000000	0.000000000192854
0.900000000000000	0.150000000000000	0.000000000186384
1.000000000000000	0.140000000000000	0.000000000575024

9 Problem 8

The following results can be reproduce by opening the Assignment9_8.m file and hitting the run button

for the following polynomial

```
y = @(x) 10.*x.^7+9.*x.^6-8.*x.^5-12.*x.^4+3.*x.^3+2.*x.^2+x+1;
```


Regression
Initial values
60
50
40
30
20
10
0
-10
0
20
40
60
80
100