COSC 4364 Spring 2018

Assignment 9

Out: April 26. Due: May 4. e-mail your report and code.

	Points				
Problem	a)	b)	c)	d)	Total
1	3				3
2	3				3
3	3				3
4	6				6
5	3	6			9
6	5				5
7	30				30
8	40				40
Total					99

Problem 1. (3 points) Vector norms are used to create so-called subordinate matrix norms. Which of the following conditions, if any, is satisfied by every subordinate matrix norm?

a. $||\mathbf{A}x|| \geq ||\mathbf{A}||\ ||x||$ b. $||\mathbf{I}||=1$. c. $||\mathbf{AB}|| \geq ||\mathbf{A}||\ ||\mathbf{B}||$ d. $||\mathbf{A}+\mathbf{B}|| \geq ||\mathbf{A}||+||\mathbf{B}||$
e. Non of these

Problem 2. (3 points) Which of the following conditions, if any, defines diagonal dominance of a matrix A?

a. $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|$ b. $|a_{ii}| \geq \sum^{n}_{j=1,j\neq I} |a_{ij}|$ c. $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ji}|$ d. $|a_{ii}| \geq \sum_{j=1,j\neq i}^{n} |a_{ji}|$ e. None of these

Problem 3. (3 points) A general procedure for solving a system of equations $(\mathbf{I}-\mathcal{G})\mathbf{x}=\mathbf{b}$ by an iterative method is to form an iteration formula $\mathbf{x}^{k+1} = \mathcal{G}\mathbf{x}^{k}+\mathbf{b}$. What is a necessary and sufficient condition that guarantees that the sequence $\mathbf{x}^k$ converges to the solution to $(\mathbf{I}-\mathcal{G})\mathbf{x}=\mathbf{b}$?

a. $\mathcal{G}$ is diagonally dominant. b. The spectral radius of $\mathcal{G}<1$. c. None of these

Problem 4. (6 points) If a set of m+1 data points (x_k,y_k) k=0,1,…,m are to be represented by a least squares fit of $y=x^2-x+c$ derive an expression for c in terms of x_k and y_k.

Problem 5. (a) 3 points, b) 6 points) The Gram-Schmidt process is used on the vectors $\mathbf{x}_1 = [2,2,1]^T$, $\mathbf{x}_2 = [1,1,5]^T$ and $\mathbf{x}_3 = [-3,2,1]^T$ to create the orthonormal vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

a) Which one of the following vectors is the correct representation of $\mathbf{u}_1$?
i) $[2/3,2/3,1/3]^T$ ii) $[2,2,1]^T$ iii) $[2/5,2/5,1/5]^T$ iv) None of these

b) Which one of the following vectors is the correct representation of $\mathbf{u}_2$?
i) (1/sqrt(27))$[1,1,5]^T$ ii) (1/sqrt(18))$[-1,-1,4]^T$ iii) $[1,1,-4]^T$ iv) None of these

Problem 6. (5 points) A set of data points (x_k,y_k) are to be approximated by y=1/(a+bx). A direct use of the least squares method results in a non-linear problem. How would you reformulate the problem to form a linear least squares problem?

Matlab problems

Problem 7. (Exercise 8.4.11) (30 points) Write your own Matlab code for the conjugate gradient method to solve the system Ax=b where A and b are as below

$$A = \begin{bmatrix} 3 & -1 & & & & & & \tfrac{1}{2} \\ -1 & 3 & -1 & & & & \tfrac{1}{2} & \\ & -1 & 3 & -1 & & \tfrac{1}{2} & & \\ & & \ddots & \ddots & \ddots & & & \\ & & & -1 & 3 & -1 & & \\ & & & & \ddots & \ddots & \ddots & \\ & \tfrac{1}{2} & & & & -1 & 3 & -1 \\ \tfrac{1}{2} & & & & & & -1 & 3 \;\; -1 \\ \tfrac{1}{2} & & & & & & & -1 \;\; 3 \end{bmatrix} \qquad b = \begin{bmatrix} 2.5 \\ 1.5 \\ 1.5 \\ \vdots \\ 1.0 \\ 1.5 \\ \vdots \\ 1.5 \\ 2.5 \end{bmatrix}$$

(diag(A) = 3, 1st superdiag =-1, 1st subdiagonal =-1, antidiagonal=1/2 where it does not conflict with the diagonal and 1st super and subdiagonal. True solution is $x=(1,1,\ldots\ldots,1)^T$). For increasing values of the size *n* of the matrix compute the root mean square (RMS) error in your conjugate gradient solution by comparing to the true solution. Make a table and plot of how the number of iterations for convergence depend on *n* for an RMS error of 10^{-7}.

Problem 8. (Exercise 9.2.1). (40 points)
Step 1 - generating a noisy data set . For a 7th degree polynomial of your choice on the interval [-1,1] for 100 randomly selected values in [-1,1] compute the corresponding polynomial values. Then, find the maximum absolute value of the polynomial points, p_{max}. Then, perturb each polynomial value by randomly adding a value from $[-0.1p_{max}, 0.1p_{max}]$.

Step 2. Using the data set from step 1 find a least squares fit using the polynomial regression algorithm described on pages 440-443 in the book. Progressively specify an error tolerance 10^{-k} for k=3, 4, 5, 6, and 7. Report the polynomial you are getting for each error tolerance. Plot the polynomial you used to generate the noisy data and the polynomials generated for each error tolerance.