COSC 4364 Spring 2018

Assignment 4

Out: 2018-02-15 Due: 2018-02-22

e-mail your answers and code to kasikeshav@gmail.com with copy to johnsson@cs.uh.edu. Hardcopy appreciated.

Points				
	a)	b)	c)	total
1	10			10
2	10			10
3	5			5
4	5			5
5	5			5
6	6	4		10
7	5			5
8	30			30
9	30			30
Total				110

Problem 1. (10 points) Write down the *cardinal polynomials* in the Lagrange interpolation polynomial for the points

x	0	2	3	4
y	7	11	28	63

and the Lagrange interpolating polynomial.

Problem 2. (10 points) Derive the coefficients in the Newton form of the interpolating polynomial for

x	4	2	0	3
y	63	11	7	28

using divided differences and write down the *Newton interpolating polynomial*. Organize the computation of the divided differences as a table as in slide 16, Lecture 8.

Problem 3. (5 points) Show that the polynomials in Problems 1 and 2 are the same.

Problem 4. (5 points) Show that the polynomials $p(x)=5x^3-27x^2+45x-21$ and $q(x)=x^4-5x^3+8x^2-5x+3$ both interpolates

x	1	2	3	4
y	2	1	6	47

Explain why this fact does not violate the uniqueness aspect of interpolating polynomials.

Problem 5. (5 points) Give an *error bound* for polynomial approximation of e^{-x} on [0,2] using 21 *equally spaced nodes* with $x_0=0$ and $x_{20}=2$.

Problem 6. (a) 6 points, b) 4 points) Let $f^{(3)(}x) =(1/2h^3)(f(x+2h)-2f(x+h)+2f(x-h)-f(x-2h))$.

a) Derive the *truncation error term* for $f^{(3)(}x)$. Hint: in deriving the formula for $f^{(3)}(x)$ consider introducing an expression for f(x+h)-f(x-h).
b) Derive the formula for *one step of Richard extrapolation* to eliminate the leading term in the truncation error. For which x values are f(x) evaluated?

Problem 7. (5 points) Derive the order of the error terms in Richardson extrapolation for D(3,0), D(3,1), D(3,2) and D(3,3) where $D(n,m) = L+O(h^{2(m+1)}/2^{2n(m+1)})$.

MATLAB problems

Problem 8. (30 points) For Runge's function $f(x) = 1/(1+25x^2)$ on the interval [-1,1] write a MATLAB program that interpolates the function with polynomials p(x) of order 5, 10, 20 and 40 using

a) Equally spaced nodes with $x^0=-1$, and $x^n=1$ for n = (5, 10, 20 and 40).
b) Nodes defined by $\cos(i\pi/n)$ for $0\le i\le n$ and n = (5, 10, 20 and 40).
c) The Chebyshev nodes $\cos((2i+1)\pi/(2n+2))$ with $0\le i\le n$ and n = (5, 10, 20 and 40)

Use Newton's formulation of the interpolating polynomial. For each of a), b) and c) compute the coefficients in the polynomial for each set of nodes (12 cases total) by Divided Differences based on the pseudocode on slide 20, Lecture 8 (or the book page 166).

Evaluate f(x) and p(x) at 200 equally spaced x values with $x_1=-1$ and $x_{200}=1$ for a), b) and c) for each set of nodes based on the pseudocode on slide 22, Lecture 8 (or the book page 166) and plot the error f(x)-p(x). Make three plots, one for a) one for b) and one for c) with each plot including plots of the error for the four sets of nodes.
What is the maximum positive error for each case?
What is the maximum negative error for each case?
What is the square root of the mean square error for each case? $(\mathrm{sqrt}(\sum_1^{200}(f(x_i)-p(x_i))^2/200))$
Compare and discuss the results.

Problem 9. (30 points) Compute the f'(x) by a) forward difference (1/h)(f(x+h)-f(x)) and b) centered difference (1/(2h))(f(x+h)-f(x-h)) for

i) cos(x) at x=0
ii) arctan(x) at x=1

Let h=10^{-2}, 10^{-4}, 10^{-6}, 10^{-8} and 10^{-10}. Determine the error by comparing the computed derivatives to evaluations of the analytic expressions for f'(x) at x=0 for i) and x=1 for ii).

Then, use two steps of Richardson extrapolation of a), and b) to get improved computed derivatives and determine the error by comparing the outcome of the Richardson extrapolation with evaluation of the analytic expressions for f'(x) at x=0 for i) and x=1 for ii).

The results can be captured in tables of the form below, one pair without Richardson extrapolation, one pair for two stages of Richardson extrapolation using the formula on slide 45 Lecture 9 (or the book page 193) (example pseudocode for center difference on slide 47, lecture 9 (or book page 193).

Formulas a) and b)		
Error cos(x) at x=0	f' forward	f' centered
h=10^{-2}		
h=10^{-4}		
h=10^{-6}		
h=10^{-8}		
h=10^{-10}		

Formulas a) and b)		
Error arctan(x) at x=1	f' forward	f' centered
h=10^{-2}		
h=10^{-4}		
h=10^{-6}		
h=10^{-8}		
h=10^{-10}		

Formulas a) and b) with two steps of Richardson extrapolation		
Error cos(x) at x=0	f' forward	f' centered
h=10^{-2}		
h=10^{-4}		
h=10^{-6}		
h=10^{-8}		
h=10^{-10}		

Formulas a) and b) with two steps of Richardson extrapolation		
Error arctan(x) at x=1	f' forward	f' centered
$h=10^{-2}$		
$h=10^{-4}$		
$h=10^{-6}$		
$h=10^{-8}$		
$h=10^{-10}$		