COSC 4364 Assignment 8

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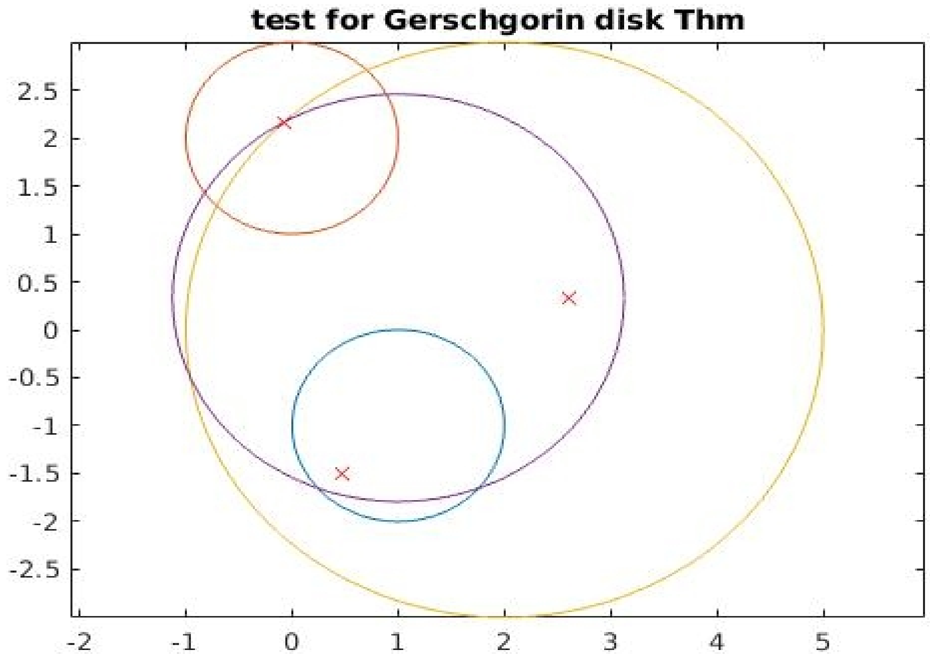
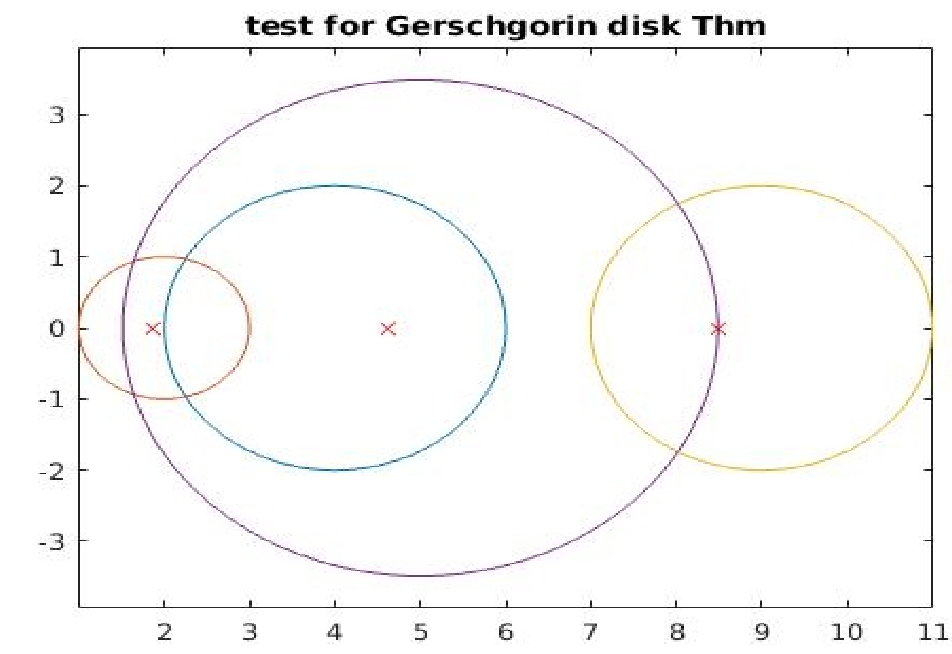
1 Problem 1

a)



b)



For the plots on the previous page, the x-axis represents the real value and the y-axis represents the imaginary value of the points. The red axes represent the eigenvalues of the Gershgorin circles. The purple circle is roughly the minimal circle containing the eigenvalue points.

2 Problem 2

a)

$$Ax=\lambda x \quad given$$
$$A^{(k+1)}x=A(A^k x)=A(\lambda^k)x=\lambda^k(Ax)=\lambda^{(k+1)}x \qquad for\ any\ k$$

b) Since a is true for any k including -k, b is also true

c) Not necessarily true as the characteristic polynomial, for example, would make the two sides not equivalent.

3 Problem 3

Assuming the question is asking what value of s makes the difference between I and svv* a unitary matrix, s would have to be 0, which is only half some of the answers so I will answer e which isn’t entirely correct either.

4 Problem 4

This pseudocode is for calculating the final eigenvalue and eigenvector, r and x respectively. It is the inverse power method.

5 Problem 5

The code for this problem is attached as Assignment8_5.m

Eigenvalues = 1.0000 2.0000 3.0000

Singular Values = 817.7597 2.4750 0.0030

Condition Number = 2.7585e+05

6 Problem 6

The code which produces a, b, and c is in the attached file Assignment8_6.m The output I encoded is below. For the described methods ||eigenvector x|| describes the normalized value of the xth eigenvector while lambda x describes the value of the xth eigenvalue.

The K value represents the iteration of the given algorithm performed. The code could be optimized to end more efficiently as even though the eigenvalue has been discovered the program will continue through the iterations and the eigenvector will steadily increase in value. The K I chose was the minimum K necessary to calculate the eigenvalue with an error smaller than 10^-5 for every algorithm. I designed these algorithms based off of the psuedocode provided for the power method.

POWER METHOD#####################

k = 1, ||eigenvector 1|| = 19.5192, lambda 1 = 8.72926
k = 2, ||eigenvector 1|| = 160.353, lambda 1 = 8.21512
k = 3, ||eigenvector 1|| = 1108.43, lambda 1 = 6.91245
k = 4, ||eigenvector 1|| = 7110.21, lambda 1 = 6.41466
k = 5, ||eigenvector 1|| = 44072.2, lambda 1 = 6.19844
k = 6, ||eigenvector 1|| = 268719, lambda 1 = 6.09725
k = 7, ||eigenvector 1|| = 1.62526e+06, lambda 1 = 6.04818
k = 8, ||eigenvector 1|| = 9.79057e+06, lambda 1 = 6.024
k = 9, ||eigenvector 1|| = 5.88607e+07, lambda 1 = 6.01198
k = 10, ||eigenvector 1|| = 3.53516e+08, lambda 1 = 6.00599
k = 11, ||eigenvector 1|| = 2.12216e+09, lambda 1 = 6.00299
k = 12, ||eigenvector 1|| = 1.27361e+10, lambda 1 = 6.0015
k = 13, ||eigenvector 1|| = 7.64262e+10, lambda 1 = 6.00075
k = 14, ||eigenvector 1|| = 4.58586e+11, lambda 1 = 6.00037
k = 15, ||eigenvector 1|| = 2.7516e+12, lambda 1 = 6.00019
k = 16, ||eigenvector 1|| = 1.65099e+13, lambda 1 = 6.00009
k = 17, ||eigenvector 1|| = 9.906e+13, lambda 1 = 6.00005
k = 18, ||eigenvector 1|| = 5.94362e+14, lambda 1 = 6.00002
k = 19, ||eigenvector 1|| = 3.56618e+15, lambda 1 = 6.00001
k = 20, ||eigenvector 1|| = 2.13971e+16, lambda 1 = 6.00001
k = 21, ||eigenvector 1|| = 1.28383e+17, lambda 1 = 6
k = 22, ||eigenvector 1|| = 7.70296e+17, lambda 1 = 6
k = 23, ||eigenvector 1|| = 4.62178e+18, lambda 1 = 6
k = 24, ||eigenvector 1|| = 2.77307e+19, lambda 1 = 6
k = 25, ||eigenvector 1|| = 1.66384e+20, lambda 1 = 6
k = 26, ||eigenvector 1|| = 9.98304e+20, lambda 1 = 6
k = 27, ||eigenvector 1|| = 5.98982e+21, lambda 1 = 6
k = 28, ||eigenvector 1|| = 3.59389e+22, lambda 1 = 6
k = 29, ||eigenvector 1|| = 2.15634e+23, lambda 1 = 6
k = 30, ||eigenvector 1|| = 1.2938e+24, lambda 1 = 6
k = 31, ||eigenvector 1|| = 7.76281e+24, lambda 1 = 6
k = 32, ||eigenvector 1|| = 4.65769e+25, lambda 1 = 6
k = 33, ||eigenvector 1|| = 2.79461e+26, lambda 1 = 6

DEFLATION METHOD################

k = 1, ||eigenvector 2|| = 20.1734, lambda 2 = 6.37939
k = 2, ||eigenvector 2|| = 61.3397, lambda 2 = 3.04062
k = 3, ||eigenvector 2|| = 185.67, lambda 2 = 3.02692
k = 4, ||eigenvector 2|| = 560.328, lambda 2 = 3.01787

k = 5, ||eigenvector 2|| = 1687.64, lambda 2 = 3.01188
k = 6, ||eigenvector 2|| = 5076.26, lambda 2 = 3.0079
k = 7, ||eigenvector 2|| = 15255.5, lambda 2 = 3.00526
k = 8, ||eigenvector 2|| = 45819.9, lambda 2 = 3.00351
k = 9, ||eigenvector 2|| = 137567, lambda 2 = 3.00234
k = 10, ||eigenvector 2|| = 412915, lambda 2 = 3.00156
k = 11, ||eigenvector 2|| = 1.23917e+06, lambda 2 = 3.00104
k = 12, ||eigenvector 2|| = 3.71837e+06, lambda 2 = 3.00069
k = 13, ||eigenvector 2|| = 1.11568e+07, lambda 2 = 3.00046
k = 14, ||eigenvector 2|| = 3.34739e+07, lambda 2 = 3.00031
k = 15, ||eigenvector 2|| = 1.00429e+08, lambda 2 = 3.0002
k = 16, ||eigenvector 2|| = 3.013e+08, lambda 2 = 3.00014
k = 17, ||eigenvector 2|| = 9.03926e+08, lambda 2 = 3.00009
k = 18, ||eigenvector 2|| = 2.71183e+09, lambda 2 = 3.00006
k = 19, ||eigenvector 2|| = 8.13561e+09, lambda 2 = 3.00004
k = 20, ||eigenvector 2|| = 2.44071e+10, lambda 2 = 3.00003
k = 21, ||eigenvector 2|| = 7.32216e+10, lambda 2 = 3.00002
k = 22, ||eigenvector 2|| = 2.19666e+11, lambda 2 = 3.00001
k = 23, ||eigenvector 2|| = 6.58999e+11, lambda 2 = 3.00001
k = 24, ||eigenvector 2|| = 1.977e+12, lambda 2 = 3.00001
k = 25, ||eigenvector 2|| = 5.93101e+12, lambda 2 = 3
k = 26, ||eigenvector 2|| = 1.7793e+13, lambda 2 = 3
k = 27, ||eigenvector 2|| = 5.33791e+13, lambda 2 = 3
k = 28, ||eigenvector 2|| = 1.60137e+14, lambda 2 = 3
k = 29, ||eigenvector 2|| = 4.80412e+14, lambda 2 = 3
k = 30, ||eigenvector 2|| = 1.44124e+15, lambda 2 = 3
k = 31, ||eigenvector 2|| = 4.32371e+15, lambda 2 = 3
k = 32, ||eigenvector 2|| = 1.29711e+16, lambda 2 = 3
k = 33, ||eigenvector 2|| = 3.89134e+16, lambda 2 = 3

INVERSE POWER METHOD################

k = 1, ||eigenvector 3|| = 1, lambda 3 = 0.986394
k = 2, ||eigenvector 3|| = 1, lambda 3 = 2.31753
k = 3, ||eigenvector 3|| = 1, lambda 3 = 2.59971
k = 4, ||eigenvector 3|| = 1, lambda 3 = 2.38506
k = 5, ||eigenvector 3|| = 1, lambda 3 = 2.033
k = 6, ||eigenvector 3|| = 1, lambda 3 = 1.8804
k = 7, ||eigenvector 3|| = 1, lambda 3 = 1.86855
k = 8, ||eigenvector 3|| = 1, lambda 3 = 1.89593
k = 9, ||eigenvector 3|| = 1, lambda 3 = 1.92534
k = 10, ||eigenvector 3|| = 1, lambda 3 = 1.94843
k = 11, ||eigenvector 3|| = 1, lambda 3 = 1.96498
k = 12, ||eigenvector 3|| = 1, lambda 3 = 1.97641
k = 13, ||eigenvector 3|| = 1, lambda 3 = 1.98418
k = 14, ||eigenvector 3|| = 1, lambda 3 = 1.98941
k = 15, ||eigenvector 3|| = 1, lambda 3 = 1.99293
k = 16, ||eigenvector 3|| = 1, lambda 3 = 1.99528
k = 17, ||eigenvector 3|| = 1, lambda 3 = 1.99685

k = 18, ||eigenvector 3|| = 1, lambda 3 = 1.9979
k = 19, ||eigenvector 3|| = 1, lambda 3 = 1.9986
k = 20, ||eigenvector 3|| = 1, lambda 3 = 1.99906
k = 21, ||eigenvector 3|| = 1, lambda 3 = 1.99938
k = 22, ||eigenvector 3|| = 1, lambda 3 = 1.99958
k = 23, ||eigenvector 3|| = 1, lambda 3 = 1.99972
k = 24, ||eigenvector 3|| = 1, lambda 3 = 1.99982
k = 25, ||eigenvector 3|| = 1, lambda 3 = 1.99988
k = 26, ||eigenvector 3|| = 1, lambda 3 = 1.99992
k = 27, ||eigenvector 3|| = 1, lambda 3 = 1.99995
k = 28, ||eigenvector 3|| = 1, lambda 3 = 1.99996
k = 29, ||eigenvector 3|| = 1, lambda 3 = 1.99998
k = 30, ||eigenvector 3|| = 1, lambda 3 = 1.99998
k = 31, ||eigenvector 3|| = 1, lambda 3 = 1.99999
k = 32, ||eigenvector 3|| = 1, lambda 3 = 1.99999
k = 33, ||eigenvector 3|| = 1, lambda 3 = 2