Module 3 Assignment 2

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1 Problem 1

Write the Booleean function implemented in Canonical Sum of Products format and in Canonical Product of Sums format.

$$\sum m(2,4,5) = \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z$$
$$\prod M(0,1,3,6,7) = (x+y+z)(x+y+\bar{z})(x+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(\bar{x}+\bar{y}+\bar{z})$$

2 Problem 2

Write a simplified Boolean function for the function performed by the circuit below.

$$= \bar{C}\bar{D}\bar{A} + CD\bar{A} + A$$

3 Problem 3

Write a truth table for the outputs,then use Boolean identities to find the simplified Boolean function for the outputs S_a and S_b.

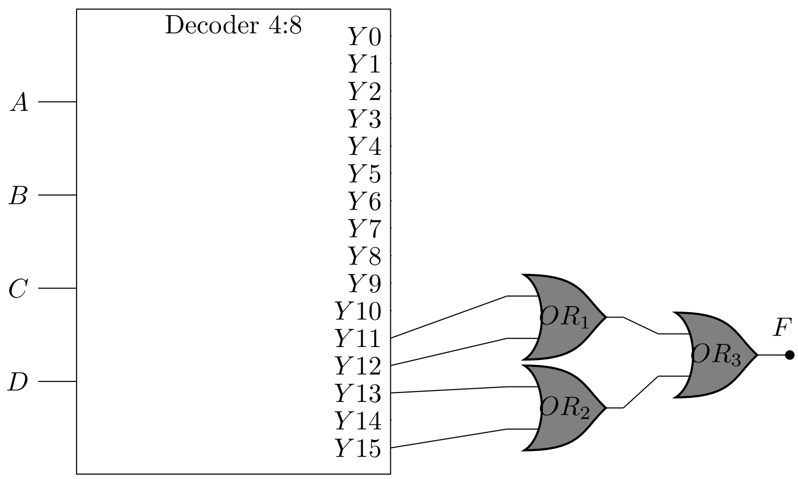
Inputs	a	b	c	d	e	f	g
0000	1	1	1	1	1	1	0
0001	0	1	1	0	0	0	0
0010	1	1	0	1	1	0	1
0011	1	1	1	1	0	0	1
0100	0	1	1	0	0	1	1
0101	1	0	1	1	0	1	1
0110	1	0	1	1	1	1	1
0111	1	1	1	0	0	0	0
1000	1	1	1	1	1	1	1
1001	1	1	1	0	0	1	1
1010	0	0	0	0	0	0	0
1011	0	0	0	0	0	0	0
1100	0	0	0	0	0	0	0
1101	0	0	0	0	0	0	0
1110	0	0	0	0	0	0	0
1111	0	0	0	0	0	0	0

$$
\begin{aligned}
S_a := \sum m(0,2,3,5,6,7,8,9) &= \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}z \\
&= \bar{w}(\bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz) + w(\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z) \\
&= \bar{w}(\bar{x}(\bar{y}\bar{z} + y\bar{z} + yz) + x(\bar{y}z + y\bar{z} + yz)) + w(\bar{x}\bar{y}) \\
&= \bar{w}(\bar{x}(\bar{z} + y) + x(z + y)) + w\bar{x}\bar{y} \\
&= \bar{w}(\bar{x}y + \bar{x}\bar{z} + xy + xz) + w\bar{x}\bar{y} \\
&= \bar{w}(y + \bar{x}\bar{z} + xz) + w\bar{x}\bar{y} \\
&= \bar{w}xz + \bar{w}\bar{x}\bar{z} + \bar{w}y + w\bar{x}\bar{y} \\
S_b := \sum m(0,1,2,3,4,7,8,9) &= \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}yz + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}z \\
&= \bar{w}(\bar{x}(\bar{y}\bar{z} + \bar{y}z + y\bar{z} + yz) + x\bar{y}\bar{z} + xyz) + w\bar{x}\bar{y} \\
&= \bar{w}(\bar{x} + x\bar{y}\bar{z} + xyz) + w\bar{x}\bar{y} \\
&= \bar{w}\bar{x} + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}\bar{y} \\
&= \bar{w}(\bar{x} + \bar{y}\bar{z} + yz) + w\bar{x}\bar{y} \\
&= \bar{w}\bar{x} + w\bar{x}\bar{y} + \bar{w}yz + \bar{w}\bar{y}\bar{z}
\end{aligned}
$$

4 Problem 4

Using a 4×16 decoder module and a an OR gate to implement the Boolean function $f(a, b, c, d) = ab\bar{c} + acd$

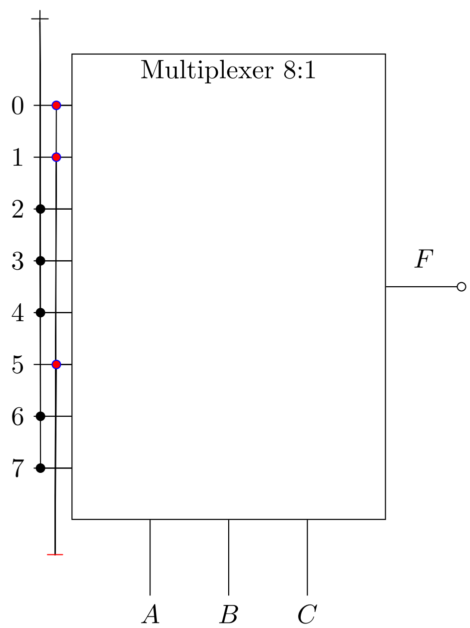
$$\sum m(11, 12, 13, 15)$$



5 Problem 5

Using an 8×1 multiplexer module and a OR gate to implement the Boolean function $f(a, b, c) = b + a\overline{c}$

$$\sum m(2, 3, 4, 6, 7)$$



6 Problem 6

Write Boolean functions for the circuit below in Canonical Sum of Product form.

$Y : \sum m(1, 3, 5, 7, 9, 10, 11, 13, 15) =$
$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}D + ABCD$

$Z : \sum m(5, 7, 9, 13, 15) =$
$= \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + AB\bar{C}D + ABCD$

7 Problem 7

Simplify the Boolean functions from problem 6 and sketch the improved circuit with the same function.

$Y : \sum m(1, 3, 5, 7, 9, 10, 11, 13, 15) =$

$= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}D + ABCD$

$= \bar{A}(\bar{B}\bar{C}D + \bar{B}CD + B\bar{C}D + BCD) + A(\bar{B}\bar{C}D + \bar{B}C\bar{D} + \bar{B}CD + B\bar{C}D + BCD)$

$= \bar{A}(\bar{B}D(1) + BD(1)) + A(\bar{B}D(1) + \bar{B}C\bar{D} + B\bar{C}D + BCD)$

$= \bar{A}(D) + A(D(\bar{B} + BC) + \bar{B}C\bar{D} + B\bar{C}D)$

$= \bar{A}D + A(D(\bar{B} + C) + \bar{B}C\bar{D} + B\bar{C}D)$

$= \bar{A}D + A(\bar{B}D + CD + \bar{B}C\bar{D} + B\bar{C}D)$

$= \bar{A}D + A(C(D + \bar{B}\bar{D}) + \bar{B}D + B\bar{C}D)$

$= \bar{A}D + A(C(D + \bar{B}) + \bar{B}D + B\bar{C}D)$

$= \bar{A}D + A(CD + \bar{B}C + \bar{B}D + B\bar{C}D)$

$= \bar{A}D + ACD + A\bar{B}C + A\bar{B}D + AB\bar{C}D$

$= D(\bar{A} + AC) + A\bar{B}C + AD(\bar{B} + B\bar{C})$

$= D(\bar{A} + C) + A\bar{B}C + AD(\bar{B} + \bar{C})$

$= \bar{A}D + CD + A\bar{B}C + AD\bar{B} + AD\bar{C}$

$= D(\bar{A} + A\bar{C}) + A\bar{B}C + A\bar{B}D + CD$

$= D(\bar{A} + \bar{C}) + A\bar{B}C + A\bar{B}D + CD$

$= \bar{A}D + \bar{C}D + A\bar{B}C + A\bar{B}D + CD$

$= D(\bar{A} + A\bar{B}) + \bar{C}D + A\bar{B}C + CD$

$= D(\bar{A} + \bar{B}) + \bar{C}D + A\bar{B}C + CD$

$= \bar{A}D + \bar{B}D + \bar{C}D + A\bar{B}C + CD$

$= D(1) + \bar{B}D + \bar{A}D + A\bar{B}C$

$= D(1 + \bar{B}) + \bar{A}D + A\bar{B}C$

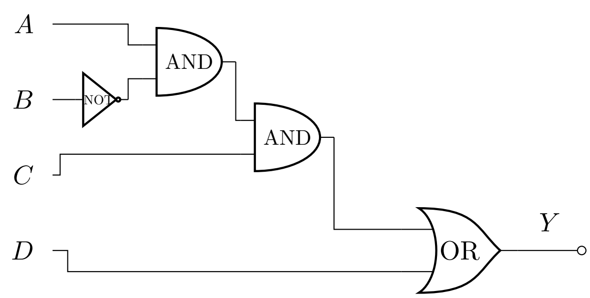
$= D + \bar{A}D + A\bar{B}C$

$= D(1 + \bar{A}) + A\bar{B}C$

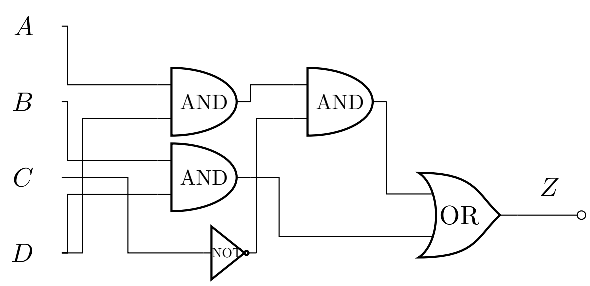
$= D + A\bar{B}C$

$$
\begin{aligned}
&Z : \sum m(5,7,9,13,15) = \\
&= \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + AB\bar{C}D + ABCD \\
&= B(\bar{A}\bar{C}D + \bar{A}CD + A\bar{C}D + ACD) + A\bar{B}\bar{C}D \\
&= B(CD(A+\bar{A}) + D\bar{C}(A+\bar{A})) + A\bar{B}\bar{C}D \\
&= B(CD + \bar{C}D) + A\bar{B}\bar{C}D \\
&= B(D(C+\bar{C})) + A\bar{B}\bar{C}D \\
&= BD + A\bar{B}\bar{C}D \\
&= D(B + A\bar{B}\bar{C}) \\
&= D(B + A\bar{C}) \\
&= BD + A\bar{C}D
\end{aligned}
$$

Circuit for Y:



Circuit for Z:



8 Problem 8

Complete the truth table for the following sequential circuit.

X	$Q_A(t)$	$Q_B(t)$	$Q_A(t+1)$	$Q_B(t+1)$
0	0	0	1	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0