Assignment 4

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1 Exercise 1

Construct truth tables for the following XOR and XNOR gates.

a. XOR gate for a, b, c and d.:

a	b	c	d	XOR(a,b,c,d)
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

b. XNOR gate for a, b, c and d.:

a	b	c	XNOR(a,b,c)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

2 Exercise 2

Write the Boolean expression in Canonical Sum of Products and Canonical Product of Sums for the following truth table:

$$\sum m\,(0,1,3,4,7) = (\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + xyz)$$
$$\prod M\,(2,5,6) = (x + \bar{y} + z)(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)$$

3 Exercise 3

Use Boolean Identities to simplify the Canonical sum-of-product Boolean function obtained in problem 2.

$$\begin{aligned}
&\sum m\,(0,1,3,4,7) = (\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + xyz)\\
&= \bar{y}(\bar{x}\bar{z} + \bar{x}z + x\bar{z}) + y(\bar{x}z + xz)\\
&= \bar{y}(\bar{x}(\bar{z} + z) + x\bar{z}) + y(z(\bar{x} + x))\\
&= \bar{y}(\bar{x} + x\bar{z}) + yz\\
&= \bar{x}\bar{y} + x\bar{y}\bar{z} + yz\\
&= \bar{y}(\bar{x} + x\bar{z}) + yz\\
&= \bar{y}(\bar{x} + \bar{z}) + yz\\
&= \bar{x}\bar{y} + \bar{y}\bar{z} + yz
\end{aligned}$$

4 Exercise 4

Write the Boolean expression in Canonical sum-of-products and Canonical product-of-sum forms for the following truth table.

$$\sum m(0,1,3,4,7,8,12,13) = (\bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}yz + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + wx\bar{y}\bar{z} + wx\bar{y}z)$$

$$\begin{aligned}
\prod M(2,5,6,9,10,11,14,15) = &(w + x + \bar{y} + z)(w + \bar{x} + y + \bar{z})(w + \bar{x} + \bar{y} + z)\\
&(\bar{w} + x + y + \bar{z})(\bar{w} + x + \bar{y} + z)(\bar{w} + x + \bar{y} + \bar{z})\\
&(\bar{w} + \bar{x} + \bar{y} + z)(\bar{w} + \bar{x} + \bar{y} + \bar{z})
\end{aligned}$$

5 Exercise 5

Use Boolean identities to simplify the Canonical sum-of-product Boolean function obtained in problem 4.

$$\sum m(0,1,3,4,7,8,12,13) = (\bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}yz + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + wx\bar{y}\bar{z} + wx\bar{y}z)$$
$$= \bar{w}(\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}yz + x\bar{y}\bar{z} + xyz) + w(\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + x\bar{y}z)$$
$$= \bar{w}(\bar{x}\bar{y}(\bar{z} + z) + \bar{x}yz + x\bar{y}\bar{z} + xyz) + w(\bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + x\bar{y}z)$$
$$= \bar{w}(\bar{x}\bar{y} + y(\bar{x}z + xz) + x\bar{y}\bar{z}) + w(\bar{y}\bar{z}(\bar{x} + x) + x\bar{y}z)$$
$$= \bar{w}(\bar{x}\bar{y} + y(z(\bar{x} + x)) + x\bar{y}\bar{z}) + w(\bar{y}\bar{z} + x\bar{y}z)$$
$$= \bar{w}(\bar{x}\bar{y} + yz + x\bar{y}\bar{z}) + w(\bar{y}\bar{z} + x\bar{y}z)$$
$$= \bar{w}(\bar{y}(\bar{x} + x\bar{z}) + yz) + w(\bar{y}(\bar{z} + xz))$$
$$= \bar{w}(\bar{y}(\bar{x} + \bar{z}) + yz) + w(\bar{y}(\bar{z} + x))$$
$$= \bar{w}(\bar{x}\bar{y} + \bar{y}\bar{z} + yz) + w(\bar{y}\bar{z} + x\bar{y})$$
$$= \bar{w}\bar{x}\bar{y} + \bar{w}\bar{y}\bar{z} + \bar{w}yz + w\bar{y}\bar{z} + wx\bar{y}$$
$$= \bar{w}\bar{x}\bar{y} + \bar{y}\bar{z}(\bar{w} + w) + wx\bar{y} + \bar{w}yz$$
$$= \bar{w}\bar{x}\bar{y} + \bar{y}\bar{z} + wx\bar{y} + \bar{w}yz$$

6 Exercise 6

Find CSOP and CPOS forms for the following function:

a. $F(a, b, c) = ab + \bar{a}c$

$$\sum m(1,3,6,7) = ab + \bar{a}c$$
$$\prod M(0,2,4,5) = ab + \bar{a}c$$

b. $F(A, B, C, D) = A(\bar{B} + C\bar{D}) + \bar{A}B\bar{C}D$

$$\sum m(5,8,9,10,11,14) = A(\bar{B} + C\bar{D}) + \bar{A}B\bar{C}D$$
$$\prod M(0,1,2,3,4,6,7,12,13,15) = A(\bar{B} + C\bar{D}) + \bar{A}B\bar{C}D$$

7 Exercise 7

Use Boolean Identities to simplify the following Boolean functions:

a. $xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} + xyz$

$$
\begin{aligned}
&= x(y\bar{z} + \bar{y}\bar{z} + yz) + \bar{x}(yz + \bar{y}\bar{z}) \\
&= x(y(\bar{z} + z) + \bar{y}\bar{z}) + \bar{x}(yz + \bar{y}\bar{z}) \\
&= x(y + \bar{y}\bar{z}) + \bar{x}(yz + \bar{y}\bar{z}) \\
&= x(\overline{(y\bar{y} + \bar{y}z)}) + \bar{x}(yz + \bar{y}\bar{z}) \\
&= x(y + \bar{z}) + \bar{x}yz + \bar{x}\bar{y}\bar{z} \\
&= xy + x\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} \\
&= \bar{z}(x + \bar{x}\bar{y}) + y(x + \bar{x}z) \\
&= \bar{z}(\overline{(\bar{x}x + \bar{x}y)}) + y(\overline{(\bar{x}x + \bar{x}\bar{z})}) \\
&= \bar{z}(x + \bar{y}) + y(x + z) \\
&= x\bar{z} + \bar{z}\bar{y} + xy + yz \\
&= x\bar{z} + \bar{z}\bar{y} + yz
\end{aligned}
$$

b. $wx\bar{y}z + wxy\bar{z} + wx\bar{y}\bar{z} + w\bar{x}yz + w\bar{x}\bar{y}z + w\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

$$
\begin{aligned}
&= w(x\bar{y}z + xy\bar{z} + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z}) + \bar{w}(x\bar{y}z + \bar{x}yz + \bar{x}y\bar{z}) \\
&= w(x(\bar{y}z + y\bar{z} + \bar{y}\bar{z}) + \bar{x}(yz + \bar{y}z + \bar{y}\bar{z})) + \bar{w}(x\bar{y}z + \bar{x}(yz + y\bar{z})) \\
&= w(x(y\bar{z} + \bar{y}(z + \bar{z})) + \bar{x}(yz + \bar{y}(z + \bar{z}))) + \bar{w}(x\bar{y}z + \bar{x}(y(z + \bar{z}))) \\
&= w(x(y\bar{z} + \bar{y}) + \bar{x}(yz + \bar{y})) + \bar{w}(x\bar{y}z + \bar{x}y) \\
&= w(x\overline{(yz + y\bar{y})} + \bar{x}\overline{(y\bar{z} + y\bar{y})}) + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= w(x(\bar{y} + \bar{z}) + \bar{x}(\bar{y} + z)) + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= w(x\bar{y} + x\bar{z} + \bar{x}\bar{y} + \bar{x}z) + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= wx\bar{y} + wx\bar{z} + w\bar{x}\bar{y} + w\bar{x}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= \bar{y}(wx + w\bar{x}) + wx\bar{z} + w\bar{x}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= \bar{y}(w(x + \bar{x})) + wx\bar{z} + w\bar{x}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= \bar{y}w + wx\bar{z} + w\bar{x}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}y \\
&= \bar{y}(w + \bar{w}xz) + wx\bar{z} + w\bar{x}z + \bar{w}\bar{x}y \\
&= \bar{y}\overline{(\bar{w}w + \bar{w}\bar{x} + \bar{w}\bar{z})} + wx\bar{z} + w\bar{x}z + \bar{w}\bar{x}y \\
&= \bar{y}(w + xz) + wx\bar{z} + w\bar{x}z + \bar{w}\bar{x}y \\
&= w\bar{y} + x\bar{y}z + w\bar{x}z + \bar{w}\bar{x}y + wx\bar{z}
\end{aligned}
$$

c. $\bar{w}\bar{x}\bar{y} + \bar{w}xz + wxz + w\bar{x}\bar{y}\bar{z}$

$$
\begin{aligned}
&= \bar{x}(\bar{w}\bar{y} + w\bar{y}\bar{z}) + x(\bar{w}z + wz) \qquad (Distribution) \\
&= \bar{x}(\bar{y}(\bar{w} + w\bar{z})) + x(z(\bar{w} + w)) \qquad (Distribution) \\
&= \bar{x}(\bar{y}(\bar{w} + \bar{z})) + x(z) \qquad (Inverse \quad and \quad Absorption) \\
&= \bar{x}(\bar{y}\bar{w} + \bar{y}\bar{z}) + xz \qquad (Distribution) \\
&= \bar{x}\bar{y}\bar{w} + \bar{x}\bar{y}\bar{z} + xz \qquad (Distribution)
\end{aligned}
$$

d. $\left(X + Y + Z + \bar{W}\right)(V + X)\left(\bar{V} + Y + Z + \bar{W}\right)$

$$
\begin{aligned}
&Let \quad S = Y + Z + \bar{W} \\
&= (X + S)(V + X)\left(\bar{V} + S\right) \qquad (Distribution) \\
&= (X + (VS))\left(\bar{V} + S\right) \qquad (Distribution \quad and \quad Absorption) \\
&= X\bar{V} + XS + VS \qquad (Distribution) \\
&= X\bar{V} + X\left(Y + Z + \bar{W}\right) + V\left(Y + Z + \bar{W}\right) \qquad (Distribution) \\
&= X\bar{V} + XY + XZ + X\bar{W} + VY + VZ + V\bar{W} \qquad (Distribution) \\
&= X\bar{V} + XZ + X\bar{W} + YV + ZV + \bar{W}V \qquad (Consensus) \\
&= X\bar{V} + YV + ZV + \bar{W}V \qquad (Consensus)
\end{aligned}
$$