Method of Undetermined Coefficients

To find a particular solution to the differential equation

$$ay'' + by' + cy = Ct^m e^{rt},$$

where m is a nonnegative integer, use the form

(14) $$y_p(t) = t^s(A_m t^m + \cdots + A_1 t + A_0)e^{rt},$$

with

(i) $s = 0$ if r is not a root of the associated auxiliary equation;
(ii) $s = 1$ if r is a simple root of the associated auxiliary equation; and
(iii) $s = 2$ if r is a double root of the associated auxiliary equation. To find a particular solution to the differential equation

$$ay'' + by' + cy = \begin{cases} Ct^m e^{\alpha t}\cos\beta t \\ \text{or} \\ Ct^m e^{\alpha t}\sin\beta t \end{cases}$$

for $\beta \neq 0$, use the form

(15) $$\begin{aligned} y_p(t) = {} & t^s(A_m t^m + \cdots + A_1 t + A_0)e^{\alpha t}\cos\beta t \\ & + t^s(B_m t^m + \cdots + B_1 t + B_0)e^{\alpha t}\sin\beta t\,, \end{aligned}$$

with

(iv) $s = 0$ if $\alpha + i\beta$ is not a root of the associated auxiliary equation; and
(v) $s = 1$ if $\alpha + i\beta$ is a root of the associated auxiliary equation.