

## Module 3 Assignment 2

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March 4, 2023

## 1 Problem 1

Write the Boolean function implemented in Canonical Sum of Products format and in Canonical Product of Sums format.

$$\sum m(2, 4, 5) = \bar{x}y\bar{z} + x\bar{y}\bar{z} + x\bar{y}z$$

$$\prod M(0, 1, 3, 6, 7) = (x + y + z)(x + y + \bar{z})(x + \bar{y} + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$$

## 2 Problem 2

Write a simplified Boolean function for the function performed by the circuit below.

$$= \bar{C}\bar{D}\bar{A} + CD\bar{A} + A$$

## 3 Problem 3

Write a truth table for the outputs, then use Boolean identities to find the simplified Boolean function for the outputs  $S_a$  and  $S_b$ .

<i>Inputs</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>
0000	1	1	1	1	1	1	0
0001	0	1	1	0	0	0	0
0010	1	1	0	1	1	0	1
0011	1	1	1	1	0	0	1
0100	0	1	1	0	0	1	1
0101	1	0	1	1	0	1	1
0110	1	0	1	1	1	1	1
0111	1	1	1	0	0	0	0
1000	1	1	1	1	1	1	1
1001	1	1	1	0	0	1	1
1010	0	0	0	0	0	0	0
1011	0	0	0	0	0	0	0
1100	0	0	0	0	0	0	0
1101	0	0	0	0	0	0	0
1110	0	0	0	0	0	0	0
1111	0	0	0	0	0	0	0

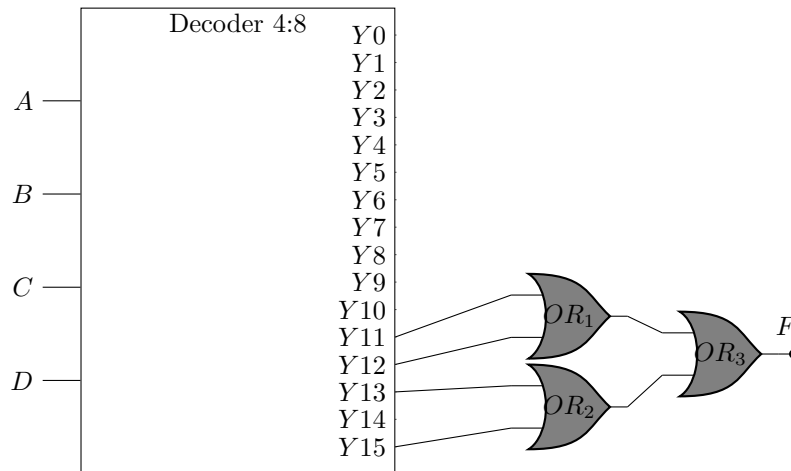
$$\begin{aligned}
S_a &:= \sum m(0, 2, 3, 5, 6, 7, 8, 9) = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}xy\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}z \\
&= \bar{w}(\bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz) + w(\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z) \\
&= \bar{w}(\bar{x}(\bar{y}\bar{z} + y\bar{z} + yz) + x(\bar{y}z + y\bar{z} + yz)) + w(\bar{x}\bar{y}) \\
&= \bar{w}(\bar{x}(\bar{z} + y) + x(z + y)) + w\bar{x}\bar{y} \\
&= \bar{w}(\bar{x}y + \bar{x}\bar{z} + xy + xz) + w\bar{x}\bar{y} \\
&= \bar{w}(y + \bar{x}\bar{z} + xz) + w\bar{x}\bar{y} \\
&= \bar{w}xz + \bar{w}\bar{x}\bar{z} + \bar{w}y + w\bar{x}\bar{y}
\end{aligned}$$

$$\begin{aligned}
S_b &:= \sum m(0, 1, 2, 3, 4, 7, 8, 9) = \bar{w}\bar{x}\bar{y}\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\bar{x}yz + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y}z \\
&= \bar{w}(\bar{x}(\bar{y}\bar{z} + \bar{y}z + y\bar{z} + yz) + x\bar{y}\bar{z} + xyz) + w\bar{x}\bar{y} \\
&= \bar{w}(\bar{x} + x\bar{y}\bar{z} + xyz) + w\bar{x}\bar{y} \\
&= \bar{w}\bar{x} + \bar{w}x\bar{y}\bar{z} + \bar{w}xyz + w\bar{x}\bar{y} \\
&= \bar{w}(\bar{x} + \bar{y}\bar{z} + yz) + w\bar{x}\bar{y} \\
&= \bar{w}\bar{x} + w\bar{x}\bar{y} + \bar{w}yz + \bar{w}\bar{y}\bar{z} \\
&= \bar{x}(\bar{w} + w\bar{y}) + \bar{w}yz + \bar{w}\bar{y}\bar{z} \\
&= \bar{x}(\bar{w} + \bar{y}) + \bar{w}yz + \bar{w}\bar{y}\bar{z} \\
&= \bar{w}\bar{x} + \bar{x}\bar{y} + \bar{w}yz + \bar{w}\bar{y}\bar{z}
\end{aligned}$$

## 4 Problem 4

Using a  $4 \times 16$  decoder module and a an OR gate to implement the Boolean function  $f(a, b, c, d) = ab\bar{c} + acd$

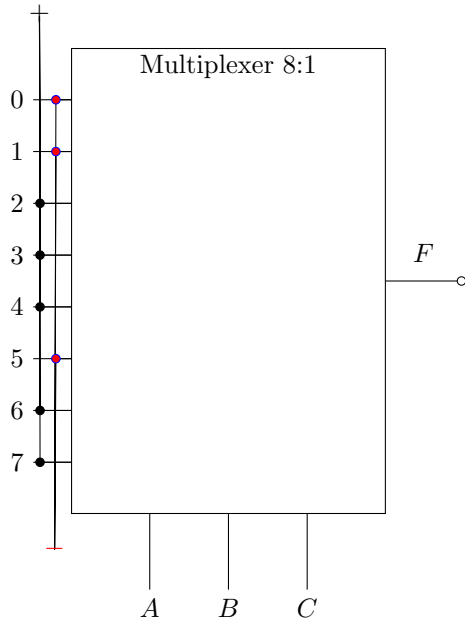
$$\sum m(11, 12, 13, 15)$$



## 5 Problem 5

Using an  $8 \times 1$  multiplexer module and a OR gate to implement the Boolean function  $f(a, b, c) = b + a\bar{c}$

$$\sum m(2, 3, 4, 6, 7)$$



## 6 Problem 6

Write Boolean functions for the circuit below in Canonical Sum of Product form.

$$\begin{aligned} Y : \sum m(1, 3, 5, 7, 9, 10, 11, 13, 15) = \\ = \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + ABC\bar{D} + ABCD \end{aligned}$$

$$\begin{aligned} Z : \sum m(5, 7, 9, 13, 15) = \\ = \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + AB\bar{C}D + ABCD \end{aligned}$$

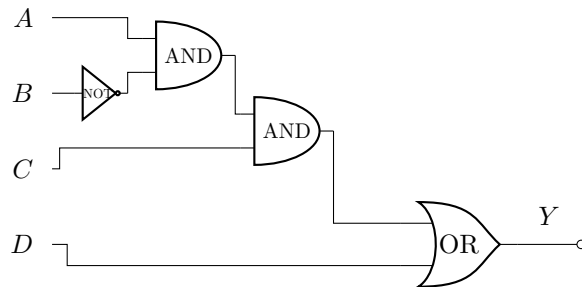
## 7 Problem 7

Simplify the Boolean functions from problem 6 and sketch the improved circuit with the same function.

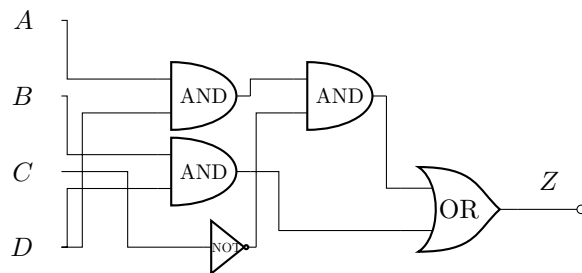
$$\begin{aligned}
 Y : \sum m(1, 3, 5, 7, 9, 10, 11, 13, 15) &= \\
 &= \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}B\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + A\bar{B}C\bar{D} + A\bar{B}CD + AB\bar{C}D + ABCD \\
 &= \bar{A}(\bar{B}\bar{C}D + \bar{B}CD + B\bar{C}D + BCD) + A(\bar{B}\bar{C}D + \bar{B}C\bar{D} + \bar{B}CD + B\bar{C}D + BCD) \\
 &= \bar{A}(\bar{B}D(1) + BD(1)) + A(\bar{B}D(1) + \bar{B}C\bar{D} + B\bar{C}D + BCD) \\
 &= \bar{A}(D) + A(D(\bar{B} + B) + \bar{B}C\bar{D} + B\bar{C}D) \\
 &= \bar{A}D + A(D(\bar{B} + B) + \bar{B}C\bar{D} + B\bar{C}D) \\
 &= \bar{A}D + A(\bar{B}D + CD + \bar{B}C\bar{D} + B\bar{C}D) \\
 &= \bar{A}D + A(C(D + \bar{B}\bar{D}) + \bar{B}D + B\bar{C}D) \\
 &= \bar{A}D + A(C(D + \bar{B}) + \bar{B}D + B\bar{C}D) \\
 &= \bar{A}D + A(CD + \bar{B}C + \bar{B}D + B\bar{C}D) \\
 &= \bar{A}D + ACD + A\bar{B}C + A\bar{B}D + AB\bar{C}D \\
 &= D(\bar{A} + AC) + A\bar{B}C + AD(\bar{B} + B\bar{C}) \\
 &= D(\bar{A} + C) + A\bar{B}C + AD(\bar{B} + \bar{C}) \\
 &= \bar{A}D + CD + A\bar{B}C + AD\bar{B} + AD\bar{C} \\
 &= D(\bar{A} + A\bar{C}) + A\bar{B}C + A\bar{B}D + CD \\
 &= D(\bar{A} + \bar{C}) + A\bar{B}C + A\bar{B}D + CD \\
 &= D(\bar{A} + A\bar{B}) + \bar{C}D + A\bar{B}C + CD \\
 &= D(\bar{A} + \bar{B}) + \bar{C}D + A\bar{B}C + CD \\
 &= \bar{A}D + \bar{B}D + \bar{C}D + A\bar{B}C + CD \\
 &= D(1) + \bar{B}D + \bar{A}D + A\bar{B}C \\
 &= D(1 + \bar{B}) + \bar{A}D + A\bar{B}C \\
 &= D + \bar{A}D + A\bar{B}C \\
 &= D(1 + \bar{A}) + A\bar{B}C \\
 &= D + A\bar{B}C
 \end{aligned}$$

$$\begin{aligned}
Z : \sum m(5, 7, 9, 13, 15) &= \\
&= \bar{A}\bar{B}\bar{C}D + \bar{A}BCD + A\bar{B}\bar{C}D + AB\bar{C}D + ABCD \\
&= B(\bar{A}\bar{C}D + \bar{A}CD + A\bar{C}D + ACD) + A\bar{B}\bar{C}D \\
&= B(CD(A + \bar{A}) + D\bar{C}(A + \bar{A})) + A\bar{B}\bar{C}D \\
&= B(CD + \bar{C}D) + A\bar{B}\bar{C}D \\
&= B(D(C + \bar{C})) + A\bar{B}\bar{C}D \\
&= BD + A\bar{B}\bar{C}D \\
&= D(B + A\bar{B}\bar{C}) \\
&= D(B + A\bar{C}) \\
&= BD + A\bar{C}D
\end{aligned}$$

Circuit for Y:



Circuit for Z:



## 8 Problem 8

Complete the truth table for the following sequential circuit.

$X$	$Q_A(t)$	$Q_B(t)$	$Q_A(t+1)$	$Q_B(t+1)$
0	0	0	1	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
1	0	1	1	1
1	1	0	1	0
1	1	1	0	0