Quiz 5

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$$f(t) = \begin{cases} t & \text{if } 0 < t < 1 \\ 1 & \text{if } t > 1 \end{cases}$$

Solution:

$$= \int_0^1 te^{-st}\,dt + \int_1^\infty e^{-st}\,dt$$

LHS integral:

$$= \int_0^1 te^{-st}\,dt = -\frac{te^{-st}}{s} + \int_0^1 \frac{e^{-st}}{s}\,dt$$

$$= \left[\left(-\frac{t}{se^{st}} - \frac{1}{s^2e^{st}}\right)\right]_0^1 = \left[-\frac{st-1}{s^2e^{st}}\right]_0^1 \quad = \left(\left(-\frac{s-1}{s^2e^s}\right) - \left(-\frac{1}{s^2}\right)\right)$$

$$= -\frac{s-1}{s^2e^s} + \frac{1}{s^2}$$

RHS integral:

$$= \int_1^\infty e^{-st}\,dt = \lim_{n\to\infty} \int_1^n e^{-st}\,dt$$

$$= \lim_{n\to\infty} \left[-\frac{1}{se^{st}}\right]_1^n = \lim_{n\to\infty} \left(-\frac{1}{se^{nt}} + \frac{1}{se^s}\right)$$

$$= \frac{1}{se^s}$$

Answer:

$$\mathcal{L}\{f\} = \frac{s-1}{s^2e^s} + \frac{1}{s^2} - \frac{1}{se^s} \text{ for } s > 0$$