Quiz 2

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Solve the following differential equation:

$$\left(3e^{3x}\ln\left(1+y^2\right)-\frac{x}{\sqrt[2]{x^2+1}}\right)dx=\left(2\sin\left(y\right)cos(y)-2e^{3x}\frac{y}{1+y^2}\right)dy \tag{1}$$

where $y(0)=0$

Exactness Condition:

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x} \tag{2}$$

$$M=3e^{3x}\ln\left(1+y^2\right)-\frac{x}{\sqrt[2]{x^2+1}} \tag{3}$$

$$N=-2\sin\left(y\right)cos(y)+2e^{3x}\frac{y}{1+y^2} \tag{4}$$

$$\frac{\partial M}{\partial y}=\frac{6ye^{3x}}{1+y^2}\qquad\frac{\partial N}{\partial x}=\frac{6ye^{3x}}{1+y^2} \tag{5}$$

Therefore, the differential equation is exact.

Solution:

$$\left(3e^{3x}\ln\left(1+y^2\right)-\frac{x}{\sqrt[2]{x^2+1}}\right)dx-\left(2\sin\left(y\right)cos(y)-2e^{3x}\frac{y}{1+y^2}\right)dy=0 \tag{6}$$

$$\left(3e^{3x}\ln\left(1+y^2\right)-\frac{x}{\sqrt[2]{x^2+1}}\right)dx+\left(-2\sin\left(y\right)cos(y)+2e^{3x}\frac{y}{1+y^2}\right)dy=0 \tag{7}$$

$$\frac{\partial F}{\partial x}=M\qquad\frac{\partial F}{\partial y}=N \tag{8}$$

$$\int\frac{\partial F}{\partial x}dx=\int\left(3e^{3x}\ln\left(1+y^2\right)-\frac{x}{\sqrt[2]{x^2+1}}\right)dx \tag{9}$$

$$
\begin{aligned}
&= \int \left(3e^{3x}\ln(1+y^2)\right)dx - \int \left(\frac{x}{\sqrt[2]{x^2+1}}\right)dx \\
&= e^{3x}\ln(1+y^2) - \int \left(\frac{x}{\sqrt[2]{x^2+1}}\right)dx
\end{aligned}
$$

$u = x^2 + 1$ so that $du = 2xdx$ and $dx = \frac{du}{2x}$

$$
\begin{aligned}
&= e^{3x}\ln(1+y^2) - \int \left(\frac{1}{2\sqrt[2]{u}}\right)du \\
&= e^{3x}\ln(1+y^2) - \sqrt[2]{u} + f'(y) \\
&= e^{3x}\ln(1+y^2) - \sqrt[2]{x^2+1} + f'(y)
\end{aligned}
$$

Hence, F equals:

$$
F(x,y) = e^{3x}\ln(1+y^2) - \sqrt[2]{x^2+1} + f'(y) \tag{10}
$$

Find the general solution to

$$(y - x)\, dx - (y + x)\, dy = 0 \tag{11}$$