Reference Sheet for Formulas and Notations

- A separable first-order equation has the form $\dfrac{dy}{dx} = f(x)g(y)$, where the general solution is obtained from $\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$.
- Multiplying both sides of a first-order linear equation in the canonical form:
$$\frac{dy}{dx} + p(x)y = q(x) \tag{1}$$
with the integrating factor $\mu(x) = e^{\int p(x)dx}$ transforms (1) into the form:
$$\frac{d}{dx}\big(\mu(x)y\big) = \mu(x)q(x), \tag{2}$$
which can be solved for y by integrating both sides of (2) with respect to x.
- A first-order equation in the form
$$M(x,y)\,dx + N(x,y)\,dy = 0,$$
satisfying the exactness condition $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, may be written as $dF(x,y) = 0$, where F is computed from the system:
$$\frac{\partial F}{\partial x} = M \quad \text{and} \quad \frac{\partial F}{\partial y} = N.$$
- The substitution $u = \dfrac{y}{x}$ transforms the first-order homogenous equation in the form:
$$\frac{dy}{dx} = G\left(\frac{y}{x}\right)$$
into a separable equation with respect to u in the form:
$$u + x\frac{du}{dx} = G(u).$$
- The substitution $u = y^{1-n}$ transforms the Bernoulli equation in the form:
$$\frac{dy}{dx} + p(x)y = q(x)y^n \qquad (n \text{ a real number})$$
into a first-order linear equation with respect to u in the form:
$$\frac{1}{1-n}\frac{du}{dx} + p(x)u = q(x).$$