Quiz 6

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Problem 1 Find the inverse Laplace transform of the given function.

$$\frac{2s+16}{s^2+4s+13}$$

Solution:

We first simplify the given rational function.

$$\begin{aligned}
\frac{2s+16}{s^2+4s+13} &= \frac{2(s+8)}{(s+2)^2+9} \\
&= \frac{2(s+2+6)}{(s+2)^2+(3)^2} = \frac{2(s+2)}{(s+2)^2+(3)^2} + \frac{12}{(s+2)^2+(3)^2} \\
&= 2\left(\frac{s+2}{(s+2)^2+(3)^2}\right) + 4\left(\frac{3}{(s+2)^2+(3)^2}\right)
\end{aligned}$$

Now we find the inverse Laplace transform.

$$= 2\mathcal{L}^{-1}\left\{\frac{s+2}{(s+2)^2+(3)^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{3}{(s+2)^2+(3)^2}\right\}$$

$$= 2(e^{-2t}\cos 3t) + 4(e^{-2t}\sin 3t)$$

$$= 2e^{-2t}\cos 3t + 4e^{-2t}\sin 3t$$

Problem 2 Determine the partial fraction expansion for the given rational function.

$$\frac{4s^2 - 21s + 16}{s(s-2)^2}$$

Solution:

We first simplify the given rational function.

$$= \frac{4s^2 - 21s + 16}{s(s-2)^2} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{(s-2)^2}$$

$$= 4s^2 - 21s + 16 = A(s-2)^2 + Bs(s-2) + Cs$$

Now we find the coefficients.

$$\begin{aligned} s &= 2: \\ 4(2)^2 - 21(2) + 16 &= A(0)^2 + B(0) + 2C \\ 16 - 42 + 16 &= 2C \\ 2C &= -10 \\ C &= -5 \end{aligned}$$

$$\begin{aligned} s &= 0: \\ 4(0)^2 - 21(0) + 16 &= A(2)^2 \\ 16 &= A(4) \\ A &= 4 \end{aligned}$$

Solving by equation coefficients.

$$\begin{aligned} &= A(s^2 - 4s + 4) + B(s^2 - 2s) + Cs \\ &= As^2 - 4As + 4A + Bs^2 - 2Bs + Cs \\ &= s^2(A + B) + s(-4A - 2B + C) + 4A \end{aligned}$$

Hence:

$$\begin{aligned}
A + B &= 4 \\
-4A - 2B + C &= -21 \\
4A &= 16 \\
\text{Subsitute } A = 4 \text{ and } C &= -5 \\
4 + B &= 4 \\
-16 - 2B + (-5) &= -21 \\
-2B &= -21 + 16 + 5 \\
-2B &= 0 \\
B &= 0
\end{aligned}$$

Therefore,

$$\begin{aligned}
\frac{4s^2 - 21s + 16}{s(s-2)^2} &= \frac{4}{s} + \frac{0}{s-2} + \frac{-5}{(s-2)^2} \\
&= \frac{4}{s} - \frac{5}{(s-2)^2}
\end{aligned}$$

Problem 3 Determine the partial fraction expansion for the given rational function.

$$\frac{3s+5}{s(s^2+s-6)}$$

Problem 4 Determine $\mathcal{L}^{-1}\{F\}$

$$F(s) = \frac{5s^2 + 34s + 53}{(s+3)^2(s+1)}$$