Quiz 5

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$$f(t) = \begin{cases} t & \text{if } 0 < t < 1 \\ 1 & \text{if } t > 1 \end{cases}$$

Solution:

$$\mathcal{L}\{f\} = \int_0^1 te^{-st}\,dt + \int_1^\infty e^{-st}\,dt$$

LHS integral:

$$\begin{aligned} &= \int_0^1 te^{-st}\,dt = -\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\Big|_0^1 \\ &= \left(-\frac{1}{s} - \frac{1}{s^2}\right)e^{-s} - \left(0 - \frac{1}{s^2}\right) \\ &= \frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s} \end{aligned}$$

RHS integral:

$$\begin{aligned} &= \int_1^\infty e^{-st}\,dt = -\frac{e^{-st}}{s}\Big|_1^\infty \\ &= 0 - \left(-\frac{1}{s}e^{-s}\right) \\ &= \frac{1}{s}e^{-s} \end{aligned}$$

Answer:

$$\begin{aligned} \mathcal{L}\{f\} &= \frac{1}{s^2} - \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-s} + \frac{1}{s}e^{-s} \\ &= \frac{1}{s^2} - \frac{1}{s^2}e^{-s} \text{ for } s > 0 \end{aligned}$$