A General Framework for Modeling Equipment Aging

Joel A. Nachlas, Ph. D., Virginia Tech

C. Richard Cassady, Ph. D., University of Arkansas

Key Words: equivalent age; ambient conditions; intensity of use; vintage; mathematical modeling

SUMMARY & CONCLUSIONS

For many types of devices and systems, age is not well described by duration of ownership. The most obvious example of the distinction between age and clock time occurs when a device may be turned off or is not used continuously. More interesting cases are those in which age is accumulated at a variable rate as a consequence of the operational profile the device experiences.

In this paper, we develop a general model that permits the analyst to portray three specific features of equipment use in the calculation of age accumulation. The three features are (1) variation in intensity of use, (2) variation in ambient operating conditions and (3) changes in device technology associated with different vintages of the equipment. The general model allows for any or all of these aspects of aging to be represented. Several analytical examples are constructed and analyzed. The analysis shows that the model provides greater sensitivity to operating profiles than previously used aging models and also that those previous models can be obtained as special cases of the model developed here. Thus, previous models are approximations for the model defined here. It is shown that an important implication of the more precise computation of equipment age is the ability to construct more effective preventive maintenance plans.

1 INTRODUCTION

Nearly all mathematical models of equipment reliability and/or preventive maintenance activities include equipment age, real time, or elapsed operating time as a variable. However, equipment age is rarely defined carefully and it is not commonly recognized that real time, elapsed operating time, and equipment age may be quite different quantities. In order to be precise and to emphasize the unique character of our research effort, define *equivalent age* to reflect the degree to which the life resource of a unit of equipment has been depleted. It is suggested here that unit life consumption (equipment aging) is governed by three classes of factors: the unit's *ambient operating conditions*, the unit's *service load* (intensity of use), and the unit's *vintage* (date of manufacture). To clearly define these factors and to put the idea of equivalent age in perspective, consider the analogy with a familiar unit of equipment, an automobile.

In the case of automobiles, as well as many other types of equipment, it is accepted that the ambient operating environment influences the rate of deterioration. In some cases, the influence is modest while in others, it is substantial. In the case of automobiles, the difference between the effects of temperature cycles, road treatment chemicals, and pollens in New York, Pennsylvania, and Michigan and the effects of the consistently warm temperatures and semi-arid conditions of California is immediately apparent. Comparable differences in deterioration are known to occur in manufacturing equipment subjected to different conditions of heat, humidity, dust and chemical solvents. Thus, the influence of ambient operating conditions is reasonably well established. In fact, the area of study known as accelerated life testing is based on this type of influence.

Service load is the intensity of equipment use. It is the manner in which a unit of equipment is operated over time (*task profile*, *operating profile*). For an automobile, use for commuting in an urban setting might be defined as relatively intense use while use for long-distance travel at uniform speeds is considered moderate or light use. Clearly, the rate of deterioration for the automobile differs under the two operating profiles.

Not so well recognized is the fact that two automobiles of the same type but different vintages (models) often have different sensitivities to environmental conditions and service loads. In the case of an automobile's surface finish, improved paint and coating formulas as well as new deposition processes have made the finishes less sensitive to adverse ambient conditions. Likewise, an automobile with disc brakes will experience slower brake deterioration than an automobile with drum brakes, even if the two automobiles are used in the exact same manner. Design evolution implies that the functional relationship between deterioration rate and operating conditions/task profile changes over time. Thus, even when considering identical operating conditions and task profiles, vintage differences imply distinct rates of unit aging.

The implication of the sensitivity of unit deterioration upon the three factor types is that the extent to which a unit has aged is not well represented by real time or elapsed operating time (or in the case of an automobile by the odometer). Consequently, efforts which require the measurement of equipment age, e.g. preventive maintenance planning, require a more representative measure of equipment status. This measure is labeled *equivalent age* and, in this paper, general models of the accumulation of equivalent age by units of equipment are defined. Furthermore, the use of these models to more effectively plan preventive maintenance activities for units of equipment is described.

978-1-4244-5103-6/10/$26.00

2 NOTATION

$X_i(t)$ the value of intensity of use measure i at time t
$\mathbf{X}(t) = \{X_1(t), X_2(t), \ldots, X_n(t)\}$
X_{0i} the nominal value of intensity of use measure i
$\mathbf{X_0} = \{X_{01}, X_{02}, \ldots, X_{0n}\}$
$Y_j(t)$ the value of ambient operating conditions measure j at time t
$\mathbf{Y}(t) = \{Y_1(t), Y_2(t), \ldots, Y_m(t)\}$
Y_{0j} the nominal value of ambient operating conditions measure j
$\mathbf{Y_0} = \{Y_{01}, Y_{02}, \ldots, Y_{0m}\}$
k index for equipment vintage
$\alpha_k(t)$ the equivalent equipment age at time t
α_{k0} a nominal or reference vintage
$q_k(t)$ function representing the effects of ambient operating and intensity of use conditions
δ_{ki} function representing the effects of intensity of use conditions
γ_{kj} function representing the effects of ambient operating conditions
e_k vintage effect parameter
Θ the equivalent age of the unit at failure
$F_\Theta(\theta)$ life distribution function
$\bar{F}_\Theta(\theta)$ reliability (survivor) function
$f_\Theta(\theta)$ life density function
$z_\Theta(\theta)$ hazard function
T_k time to failure
$\phi(\mathbf{X},\mathbf{Y})$ aging functions under ALT and PH models
β Weibull shape parameter
η Weibull scale parameter
T_r time to perform equipment repair
T_p time to perform PM
A limiting availability
τ^* optimal age replacement policy time

3 MODELING EQUIPMENT AGING

Suppose a unit of equipment is subjected to operating conditions that can be captured using two sets of variables: n measures of intensity of use (e.g. speed, vibration), and m measures of ambient operating conditions (e.g. temperature, relative humidity). Let $X_i(t)$ denote the value of intensity of use measure i at time t, and let $\mathbf{X}(t) = \{X_1(t), X_2(t), \ldots, X_n(t)\}$. Let X_{0i} denote the nominal value of intensity of use measure i, and let $\mathbf{X_0} = \{X_{01}, X_{02}, \ldots, X_{0n}\}$. Let $Y_j(t)$ denote the value of ambient operating conditions measure j at time t, and let $\mathbf{Y}(t) = \{Y_1(t), Y_2(t), \ldots, Y_m(t)\}$. Let Y_{0j} denote the nominal value of ambient operating conditions measure j, and let $\mathbf{Y_0} = \{Y_{01}, Y_{02}, \ldots, Y_{0m}\}$.

The unit's operating conditions govern its aging. Let $\alpha_k(t)$ denote the equivalent age of a unit of vintage k at time t. The instantaneous rate of aging at time t may be represented by

$$\alpha_k'(t) = \frac{d}{dt}\alpha_k(t) = (\alpha_{k0})^{q_k(t)} \tag{1}$$

where the subscript k denotes the vintage and α_{k0} is a nominal or reference vintage. Equation (1) thus represents the accumulation of equivalent age. Equation (2) is formulated to make the representation as general as possible and to reflect the dependence of equivalent age accumulation upon the ambient condition and intensity of use measures.

$$q_k(t) = \sum_{i=1}^{n} \delta_{ki}(X_i(t)) + \sum_{j=1}^{m} \gamma_{kj}(Y_j(t)) + e_k \tag{2}$$

Clearly, the specific form of equation (2) must be dictated by the equipment to which it is applied. The linear additive form may not be appropriate for all types of equipment but it should provide at least a reasonable approximation or bound for equipment having more complicated dependencies. Keep in mind that this linear additive function is an exponent. The functions $\{\delta_{k1}, \delta_{k2}, \ldots, \delta_{kn}\}$ and $\{\gamma_{k1}, \gamma_{k2}, \ldots, \gamma_{km}\}$ are vintage-dependent and such that

$$\delta_{ki}(X_{0i}) = 0 \qquad i = 1, 2, \ldots, n \tag{3}$$

and

$$\gamma_{kj}(Y_{0j}) = 0 \qquad j = 1, 2, \ldots, m \tag{4}$$

Note that e_k is a vintage-dependent parameter. In addition, it is assumed that the effects of individual measures of intensity of use and ambient operating conditions on equipment aging are independent (this assumption will be relaxed in future work). Therefore,

$$\alpha_k(t) = \int_0^t \alpha_k'(\tau)\, d\tau \tag{5}$$

The complexity of this function depends entirely upon the complexity of the functions $\{\delta_{k1}, \delta_{k2}, \ldots, \delta_{kn}\}$ and $\{\gamma_{k1}, \gamma_{k2}, \ldots, \gamma_{km}\}$.

3.1 Constant Operating Conditions

The equivalent age model expressed in equations (1) and (2) provides a generalization of the models of equipment aging that have been used in the past. It reflects the influence of variations in operating profiles on age accumulation. Before exploring the behaviors of the model, it is instructive to examine its simplified realizations and their correspondence to current models.

Suppose the unit of equipment is subjected to constant operating conditions, i.e. $\mathbf{X}(t) = \mathbf{X} = \{X_1, X_2, \ldots, X_n\}$ and $\mathbf{Y}(t) = \mathbf{Y} = \{Y_1, Y_2, \ldots, Y_m\}$ for all t. In this case, the instantaneous rate of aging is not time-dependent, i.e.

$$q_k(t) = q_k \tag{6}$$

$$\alpha_k'(t) = \alpha_k' \tag{7}$$

This implies

$$\alpha_k(t) = \alpha_k' t \tag{8}$$

So, the unit of equipment ages at a constant rate determined by the measures of intensity of use and ambient operating conditions. If these constant values are more severe than nominal, the model reduces to a form that corresponds to age acceleration such as is used in life testing.

If these measures maintain their nominal values, i.e. $\mathbf{X}(t) = \mathbf{X_0}$ and $\mathbf{Y}(t) = \mathbf{Y_0}$ for all t, then

$$q_k(t) = e_k \tag{9}$$

$$\alpha'_k(t) = (\alpha_{k0})^{e_k} \quad (10)$$

$$\alpha_k(t) = (\alpha_{k0})^{e_k}\, t \quad (11).$$

Assume that there exists some "nominal" (or reference) vintage ($k = v$) such that $e_v = 0$ and

$$\alpha'_k = 1 \quad (12).$$

If a unit of nominal vintage is subjected to constant, nominal operating conditions then

$$\alpha_v(t) = t \quad (13)$$

and the equipment ages at the same rate as real time.

4 MODELING EQUIPMENT FAILURE

Having constructed models of equivalent age, the next step is to capture the effect of these models on models of equipment failure. Let Θ denote the equivalent age of the unit at failure. Then the cumulative distribution function, survivor function, probability density function and hazard function for equivalent age at failure are given by

$$F_\Theta(\theta) = \Pr(\Theta \le \theta) \quad (14)$$

$$\bar{F}_\Theta(\theta) = 1 - F_\Theta(\theta) \quad (15)$$

$$f_\Theta(\theta) = \frac{d}{d\theta} F_\Theta(\theta) \quad (16)$$

and

$$z_\Theta(\theta) = \frac{f_\Theta(\theta)}{\bar{F}_\Theta(\theta)} \quad (17)$$

respectively.

Typically, analysis of the reliability and maintainability of equipment is focused on the time to failure rather than the age at failure. In addition, it is much simpler to monitor real time as opposed to equivalent age. Thus, let T_k denote the time to failure for a unit of vintage k where as a result of the model construction

$$T_k = \alpha_k^{-1}(\Theta) \quad (18)$$

The cumulative distribution function, survivor function, probability density function and hazard function for time failure are given by $F_k(t)$, $\bar{F}_k(t)$, $f_k(t)$, and $z_k(t)$ respectively.

4.1 Constant Operating Conditions

In the case of constant operating conditions, converting from age at failure to time to failure is quite simple. If $\mathbf{X}(t) = \mathbf{X}$ and $\mathbf{Y}(t) = \mathbf{Y}$ for all t, then

$$T_k = \frac{\Theta}{\alpha'_k} \quad (19)$$

and

$$F_k(t) = F_\Theta(\alpha'_k t) \quad (20).$$

As indicated above, this form represents the conventional time rescaling associated with accelerated life testing. Further simplification to nominal operating conditions yields a distribution that reflects only the effects of vintage. If $\mathbf{X}(t) = \mathbf{X_0}$ and $\mathbf{Y}(t) = \mathbf{Y_0}$ for all t, then

$$T_k = \Theta \big/ (\alpha_{k0})^{e_k} \quad (21)$$

and

$$F_k(t) = F_\Theta\left((\alpha_{k0})^{e_k}\, t\right) \quad (22).$$

If a unit of nominal vintage is subjected to constant, nominal operating conditions then

$$T_v = \Theta \quad (23)$$

and

$$F_v(t) = F_\Theta(t) \quad (24).$$

5 PROPORTIONAL HAZARDS AND ACCELERATED LIFE TESTING

Text It is important to note that the models presented in this paper are distinct from and much more general than the well-known proportional hazards model and the accelerated life testing models. As explained by Leemis [1], both the accelerated life testing and the proportional hazards models represent aging as a function of a set of variables called covariates that usually represent environmental operating conditions but may also include service load measures. Thus, the basic intent is similar to that of the models defined in this paper. However, important differences are that the accelerated life testing and proportional hazards models (i) are defined so that the life distributions are shape invariant (this is only true for the models defined here under constant operating conditions and intensity of use), and (ii) do not treat vintage. Furthermore, proportional hazards models treat environmental conditions and service loads as constant over time and express the effects of the environmental conditions and service loads as a linear function. Similarly, as described by Nelson [2], the accelerated life testing models are defined to represent the aging of a single population of devices when tested under controlled environmental conditions.

The general forms for both the proportional hazards model and the accelerated life testing model start with the definition of a function, say $\phi(\mathbf{X},\mathbf{Y})$ of the environmental conditions $\mathbf{Y}$ and the service load variables $\mathbf{X}$. Then, taking the base life distribution (on time to failure) to be $F_0(t)$ with survivor function , $\bar{F}_0(t)$, the proportional hazards model is:

$$\bar{F}_k(t) = \left(\bar{F}_0(t)\right)^{\phi(\mathrm{X},\mathrm{Y})} \quad (25)$$

and taking the derivative of this expression yields the system hazard function:

$$z_k(t) = \phi(\mathrm{X},\mathrm{Y}) z_0(t) \quad (26)$$

This form suggests the name proportional hazards.

In the case of the accelerated life-testing model, the function $\phi(\mathbf{X},\mathbf{Y})$ specifies a rate at which real time is rescaled so that the resulting model is

$$\bar{F}_k(t) = \bar{F}_0(\phi(\mathrm{X},\mathrm{Y})t) \quad (27)$$

For this model, the derivative yields:

$$z_k(t) = \phi(\mathrm{X},\mathrm{Y}) z_0(\phi(\mathrm{X},\mathrm{Y})t) \quad (28)$$

Note that both models imply a linear compression in the time scale with no change in the shape of the life distribution. Thus, neither model provides the capability to represent the general aging behaviors studied in this paper. On the other hand, both models can be obtained as simplified cases of the general models defined in this paper. Both models correspond to aging forms in which the time variable is multiplied by a constant. Thus they may be obtained as realizations of the equivalent age model in which either the intensity of use or the ambient condition measure is a constant that exceeds its value under nominal operation.

6 EXAMPLE BEHAVIORS

The model defined by equations (1) and (2) is quite general and may be tailored to apply to nearly any specific system. To illustrate the flexibility of the model, consider a unit of equipment for which there is a single primary measure of intensity of use and a single descriptor of the ambient operating conditions. Assume that the descriptor of the ambient operating conditions displays a cyclical pattern such as might be seen in diurnal temperature fluctuations. Represent this pattern as:

$$\gamma_k(Y_k(t)) = c_k \sin(d_k t) \tag{29}$$

In a similar manner, suppose the single intensity of use measure is appropriately represented by:

$$\delta_k(X_k(t)) = b_k t^{a_k} \tag{30}$$

Combining these definitions yields:

$$q_k(t) = b_k t^{a_k} + c_k \sin(d_k t) + e_k \tag{31}$$

and the implementation of equation (5) is:

$$\alpha_k(t) = \int_0^t \alpha_{k0}^{b_k u^{a_k} + c_k \sin(d_k u) + e_k} du \tag{32}$$

Suppose we wish to examine two vintages of the unit and that the model parameters for the two vintages are those given in Table 1. The values selected imply that the second vintage is less sensitive to the operating conditions and is used more intensively. The values of the parameter e_k are set to imply that (due to technology improvements) the second vintage ages more slowly than the first. Simplified cases are also specified to permit a comparison of model behaviors.

	k	α_{k0}	a_k	b_k	c_k	d_k	e_k
Case I – base	1	1.25	0.0	0.0	0.0	–	0.0
Case II – no intensity effect	1	1.25	0.0	0.0	7.5	0.25	0.0
	2	1.20	0.0	0.0	6.0	0.25	-0.8
Case III – time based intensity effect, no environmental effect	1	1.25	0.0	0.20	7.5	0.25	0.0
	2	1.20	0.0	0.25	6.0	0.25	-0.8
Case IV – time based intensity effect	1	1.25	0.6	0.20	7.5	0.25	0.0
	2	1.20	0.7	0.25	6.0	0.25	-0.8

Table 1. Example Aging Model Parameters

Graphical comparison of the behaviors portrayed by these model realizations are provided in Figures 1 and 2. In Figure 1, the aging patterns for vintage 1 for all four cases are shown. Observe that for the base case, age accumulates at a rate of one unit per unit of operating time. The effects of the other model terms are to increase the rate at which age is accumulated.

Figure 2 displays the effects of vintage for Cases III and IV. The vintage effect represented is a reduced rate of age accumulation for the second vintage, perhaps due to design improvements. At the same time, the figure highlights the types of environmental effects that the model can portray.

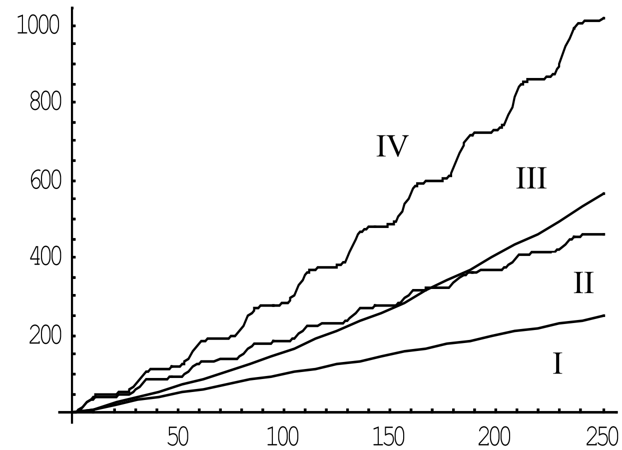

Figure 1. Example Equivalent Age Functions

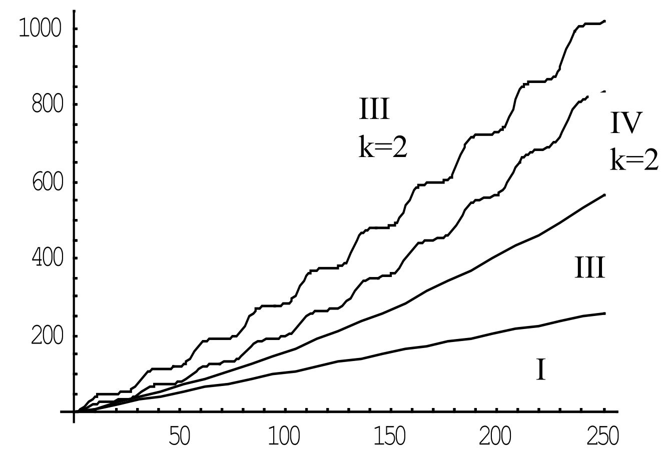

Figure 2. Vintage and Environmental Effects on Equivalent Age Functions

The impact of changes in vintage and/or operating conditions on the reliability of the equipment can also be analyzed. Suppose that the age at failure, Θ, is well modeled by a Weibull probability distribution having shape parameter β and scale parameter η, i.e.

$$F_\Theta(\theta) = 1 - \exp\left[-\left(\frac{\theta}{\eta}\right)^\beta\right] \tag{33}.$$

Suppose for the example above, that $\beta = 2$ and $\eta = 1000$.

If the unit of equipment ages according to Case I, the life distribution is Weibull. On the other hand, the derived reliability function for vintage 1 of the unit under the aging profiles of the other cases is no longer Weibull. Nevertheless, the equivalent age model permits the computation of density, distribution or reliability values for these cases. This is shown

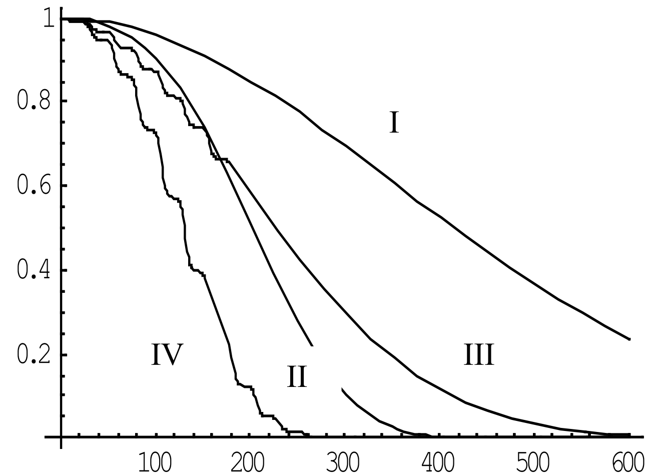

Figure 3. Reliability Functions Resulting from Equivalent Age Models

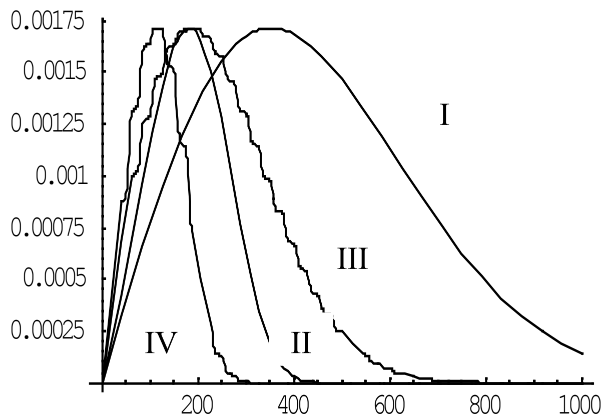

Figure 4. Failure Time Density Functions Resulting from Equivalent Age Models

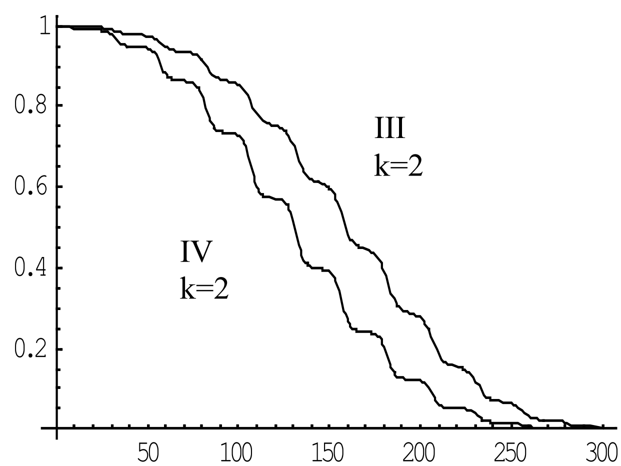

Figure 5. Vintage Effects on the Reliability Function

in Figures 3 and 4. The net effect of vintage on equivalent age accumulation and hence on unit reliability is shown for Case IV in Figure 5.

Observe that for both the proportional hazards model and the accelerated life testing model, the life distributions are Weibull for all four cases. The PH and ALT models cannot capture the consequences of irregular operating conditions.

7 UTILIZING EQUIVALENT AGE MODELS

The primary motivation for formulating equivalent age models is the potential for more effective preventive maintenance (PM) planning. The examples from Section 5 can be used to demonstrate this potential. Consider the four cases defined by Table 1 and their realizations for the behavior of a unit of equipment for which the Weibull life distribution having shape parameter $\beta = 2$ and scale parameter $\eta = 1000$ is an appropriate model.

Suppose that each unit is repaired upon failure and that repair restores the unit to a "good as new" condition. PM can be performed on each unit, and PM also restores the equipment to a "good as new" condition. The time required to perform repair is a random variable denoted by T_r, and the time required to complete PM is a random variable denoted by T_p. Suppose $E(T_r) = 50$ and $E(T_p) = 10$.

Suppose each unit is subjected to an age-based PM policy τ. In other words, if the unit operates without failure for a period of length τ, the unit is shut down and PM is performed. Under such a policy, the limiting availability of the unit is given by

$$A = \frac{\int_0^\tau t f_k(t)\,dt + \tau \bar{F}_k(\tau)}{\int_0^\tau t f_k(t)\,dt + \tau \bar{F}_k(\tau) + E(T_r)F_k(\tau) + E(T_p)\bar{F}_k(\tau)} \quad (34)$$

Numerical analysis can be used to compute the limiting unit availability for any PM policy and to identify the optimal policy τ^*.

For a unit of nominal vintage subjected to constant, nominal operating conditions (Case I), the optimal PM policy is τ^* = 511. If this policy were applied to all four units of equipment, the resulting unit limiting availability values would be the ones given in Table 2. However, under the more representative age accumulation patterns obtained from the equivalent age model, the PM policies can be tailored

Case	Vintage	*A* under Nominal Policy	Optimal Tailored Policy	*A* under Tailored Policy
I	1	0.9608	511	0.9608
II	1	0.9206	300	0.9306
	2	0.9546	453	0.9556
III	1	0.9161	300	0.9274
	2	0.9523	453	0.9536
IV	1	0.7439	152	0.8826
	2	0.6754	126	0.8957

Table 2. PM Optimization Example

individually to the unit. The resulting policies and the corresponding unit limiting availability values are also given in Table 2. As demonstrated in Table 2, tailoring the PM policy to the vintage and operating conditions of the unit may substantially improve equipment performance.

8 CONCLUSIONS

Accurate prediction of equipment reliability depends upon a reasonably clear understanding of the processes that drive the aging and thus the deterioration of the equipment. Similarly, effective maintenance planning must be based upon projections of equipment degradation rates. The models presented in this paper are intended to provide a more complete representation of the factors that comprise the aging process and to thereby increase failure model sensitivity to the variability in equipment operating conditions.

It is important that the models suggested here reduce to the models conventionally used in the past. This implies that for many devices, the previous models serve as rough approximations to the more representative models of equipment performance. Also, both the model forms can be useful during the study of equipment reliability but that ultimately, in any condition monitoring activity or maintenance planning effort, the models presented here should be used.

REFERENCES

1. Leemis, L. M. (1995). *Reliability: Probabilistic Models and Statistical Methods*. Prentice Hall, Englewood Cliffs.
2. Nelson, W. (1990). *Accelerated Testing: Statistical Models, Test Plans and Data Analysis*. Wiley, New York.

BIOGRAPHIES

Joel A. Nachlas, Ph. D.
Department of Industrial & Systems Engineering
250 Durham Hall
Virginia Tech
Blacksburg, VA 24061

e-mail: nachlas@vt.edu

Joel Nachlas has served on the faculty of Industrial and Systems Engineering at Virginia Tech since 1974. He has served and continues to serve as the coordinator for the department's Operations Research faculty and curricula and is also the coordinator of the department's international program. The foci of Dr. Nachlas research are the application of probability theory to reliability analysis and maintenance planning and of statistical methods to quality control. He earned the B.E.S. from Johns Hopkins University in 1970 and the M.S. and Ph. D. from the University of Pittsburgh in 1974 and 1976 respectinvely. All three of his degrees are in Industrial Engineering with a concentration in Operations Research. Dr. Nachlas has received numerous awards for his research including the 1991 P. K. McElroy Award and the 2004 Golomski Award. He is also the editor of the Proceedings of the Annual Reliability and Maintainability Symposium, a member of INFORMS, the Society of Reliability Engineers and a Fellow of the American Society for Quality. He also serves as head coach of the Virginia Tech men's lacrosse team and was selected in 2001 as th US Lacrosse MDIA national coach of the year.

C. Richard Cassady
Department of Industrial Engineering
4207 Bell Engineering Center
University of Arkansas
Fayetteville, AR 72701

e-mail: cassady@uark.edu

Richard Cassady is a Professor of Industrial Engineering and Director of Freshman Engineering at the University of Arkansas. His primary teaching interests are in reliability and maintainability engineering and probabilistic operations research. He also teaches courses in basic probability and statistics, statistical quality control, and deterministic operations research. He is an elected member of the University of Arkansas Teaching Academy, and he has received numerous teaching awards including the Charles and Nadine Baum Faculty Teaching Award from the University of Arkansas (2006), the inaugural Imhoff Outstanding Teacher Award from the College of Engineering (2005), the 2004 Alan O. Plait Award for Tutorial Excellence from the Annual Reliability and Maintainability Symposium, and the 2002 and 2003 Department of Industrial Engineering Outstanding Teacher Award. Dr. Cassady received his B.S., M.S. and Ph.D., all in Industrial and Systems Engineering, from Virginia Tech.