I. CLASSICAL FRACTON MODEL

The Ising model with four-spin plaquette coupling has classical fracton excitations which may be partially deconfined along the crystallographic directions. This model has trivial thermodynamics but possesses a *dynamical* glass transition.

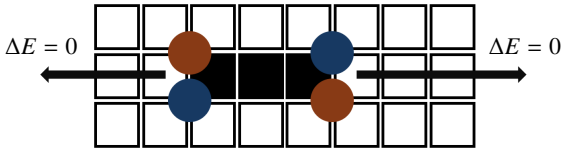
The four-spin Ising model has the energy

$$E = -K \sum_{\square} \sigma_i \sigma_j \sigma_k \sigma_\ell \tag{1}$$

where, $\square$ means only sum the products of four spins around plaquettes (squares) of the square lattice. The ground state degeneracy scales subextensively, $\Omega_0 = 2^{2L-1}$. This is because we may flip lines of spins without changing the energy.

A. Single Spin Flip Dynamics

A single spin flip changes the energy of its four connected plaquettes ($\Delta E = 8K$). These excited plaquettes are called fractons. A pair of fractons which are connected by a straight line along one of the crystallographic axes is called a fracton dipole. Each fracton is guaranteed to have two fracton pairs (fracton quadrupole). The charge of a fracton is always opposite of its two fracton pairs. Fractons are their own antiparticle regardless of charge. A single fracton may move away from its fracton pair at the cost of creating a fracton dipole ($\Delta E = 4K$). Because of this we have that the total fracton dipole moment is conserved and forces in the continuum theory can be represented by a rank-2 electric field tensor.



Fracton dipoles may be propagated along the direction perpendicular to their moment at zero energy cost by flipping the spins connecting their fracton pair. This means fracton dipoles are (partially) deconfined; they may be separated infinitely far away at finite energy cost in a particular direction. Despite this efficient but restricted mobility of excitations, the system freezes into a glassy state at some non-zero temperature.

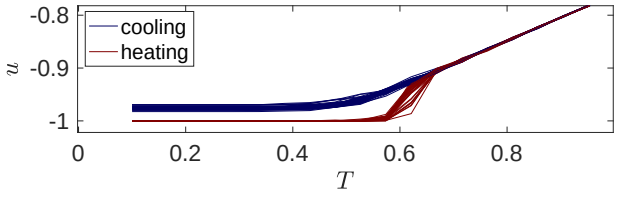


FIG. 1. Energy for 35 independent simulations with 10^4 measurements for (blue) cooling from $T = 1$, and (red) heating from the ground state. A glass transition occurs around $T \approx 0.6$. The lattice size is $L = 64$.