## I. FREE ENERGY

The Boltzmann weight of a configuration is

$$P(S_c) = \frac{e^{-\beta E_c}}{Z} \tag{1}$$

where $Z$ is the partition function

$$Z = \sum_c e^{-\beta E_c} \tag{2}$$

and $\beta = 1/T$.
The internal energy is

$$U = \sum_c E_c P(S_c) = \frac{1}{Z} \sum_c E_c e^{-\beta E_c} \tag{3}$$

The entropy is

$$S = -\sum_c P(S_c) \log P(S_c) \tag{4}$$

$$= -\sum_c \frac{e^{-\beta E_c}}{Z} \log\left(\frac{e^{-\beta E_c}}{Z}\right) \tag{5}$$

$$= -\sum_c \frac{e^{-\beta E_c}}{Z} (-\beta E_c - \log Z) \tag{6}$$

$$= \log Z \left(\sum_c \frac{e^{-\beta E_c}}{Z}\right) + \beta \left(\sum_c \frac{E_c e^{-\beta E_c}}{Z}\right) \tag{7}$$

$$= \log Z + \beta U \tag{8}$$

The free energy is

$$F = U - TS \tag{9}$$

$$= U - T(\log Z + \beta U) \tag{10}$$

$$= -T \log Z \tag{11}$$