Problem Set #3

Physics/Neuro 141

October 2023

Problem 1

The energy of activation for the *cis* to *trans* rhodopsin isomerization reaction can be determined from measuring the rate f (s^{-1}) of spontanous thermal rhodopsin isomerizations (dark events).

$$f = a \times e^{-\frac{E_a}{kT}} \tag{1}$$

a is the attempt frequency (s^{-1}), E_a is the energy of activation for the *cis* to *trans* transition, k is the Boltzmann constant, and T is the absolute temperature (Kelvin).
(a) Re-write the equation above by taking the natural logarithm.
(b) Generate an Arrhenius plot for the data below, and find the best linear fit.
(c) Determine the value of $\frac{E_a}{k}$. How does E_a compare to kT at room temperature.
(d) How does the E_a you determined compare to the energy of a blue photon?

	Temperature (Celcius)	Mean frequency (s^{-1})
1	10	0.005
2	13	0.01
3	20	0.017
4	25	0.035
5	30	0.045

Problem 2

Here we sketch the path of descent for a typical non-motile *E. coli* as it sediments in a test tube with 2 cm of water, at room temperature (the water does not stir convectively).
First, let's calculate the terminal velocity for the bacterium given *E. coli* 's density of $1.17\frac{g}{cm^3}$, and assuming that *E. coli* behaves as a sphere of similar size (1 micron diameter).
(a) How long does it take the bacterium to reach the bottom of the tube?

(b) What is its lateral displacement during the sedimentation process?

Problem 3

Consider a sphere of radius a in water. Due to random collisions, the sphere will rotationally diffuse. The diffusion law for rotational motion is analogous to the diffusion law for translational motion:

$$\langle \theta^2 \rangle = 2D_r t. \tag{2}$$

(a) What are the units of the rotational diffusion coefficient D_r? Use the Einstein relation to write down a formula for D_r, given that the rotational frictional drag coefficient for a sphere is $f_r = 8\pi\eta R^3$.

(b) Estimate the time it takes for an *E. coli* to diffuse through an angle of 1 radian. Assuming an *E. coli* has a diameter of 1 μm and a speed of 20 μm/s, what is the distance traveled by the cell during this time?

(c) What are the consequences of rotational diffusion on the navigational problem faced by *E.coli*?

Problem 4

Using a programming language of your choice, simulate an 1-D unbiased random walk for M= 1000 particles. Start each particle at position 0, and take N=1000 steps. At each step use a random number generator to assign a move to the right (+1), or a move to the left (-1) with equal probability. Plot the position vector for each particle as a function of N. At N=100, 300, 600, at 1000, plot a histogram for the positions of the M particles.
(a) What is the mean position of the particles?
(b) What is the standard deviation for the particles' position and how does the standard deviation relate to the number of steps?
(c) Now assume that the particle's random walk is biased, and that at each step the particle is 3 times more likely to go right than it is to go left. Plot a histogram for the positions of the M particles at N=100, 300, 600, 1000. How does the mean and standard vary as a function of N?

Problem 5

The shut-off of activated rhodopsins requires a number n of successive phosphorylation events. Each phosphorylation event is modeled as Poisson process with a constant probability per unit time (λ) of occurrence. Here, each step occurs with the same probability per unit time (λ). Using a programming language of your choice, let's simulate a Poisson process and determine the distribution of event occurrences (shut offs) as a function of n.

To begin, consider a probability per unit time $\lambda = 200\dfrac{events}{s}$ and the unit time interval ($1s$). Divide the unit time interval into a large number of smaller intervals N=20000, each with length $\dfrac{1}{N}s$. The probability of one phosphorylation event occurring in one $\dfrac{1}{N}s$ interval is very small, $\lambda \times \dfrac{1}{N} = 0.01$. Consider a large number of rhodopsin molecules (M=1000). "Step" each molecule through time. At each position use a random number generator to assign event occurrences and keep track where the first, second,..n^{th} event occurred for each molecules.
(a) Plot histograms for the time interval (small steps) that took the first n= 1, 3, 9, and 15 events to occur.
(b) How does the mean (μ), standard deviation (SD) and coefficient of variation ($CV = \dfrac{SD}{\mu}$) vary as a function of n, the number of events that need to occur for rhodopsins' shut off?

Problem 6

Fluorescein's molecular shape can be approximated to an ellipsoid with semi axes a= 7Å, and b= 2 Å. The transnational frictional drag coefficient for such an ellipsoid moving at random is given by $f = \dfrac{6\pi\eta a}{ln\dfrac{2a}{b}}$, where

η is the fluid's viscosity. $\eta_{water} 20°C = 10^{-2}\dfrac{g}{cm \times s}$. $\eta_{40\% sucrose} 20°C = 6.22 \times 10^{-2}\dfrac{g}{cm \times s}$
(a) What is fluorescein's diffusion coefficient at 20°C in water, and in a 40% sucrose solution?
(b) How long will it take for fluorescein to diffuse 1 cm in the liquids specified above? How about 1 m?