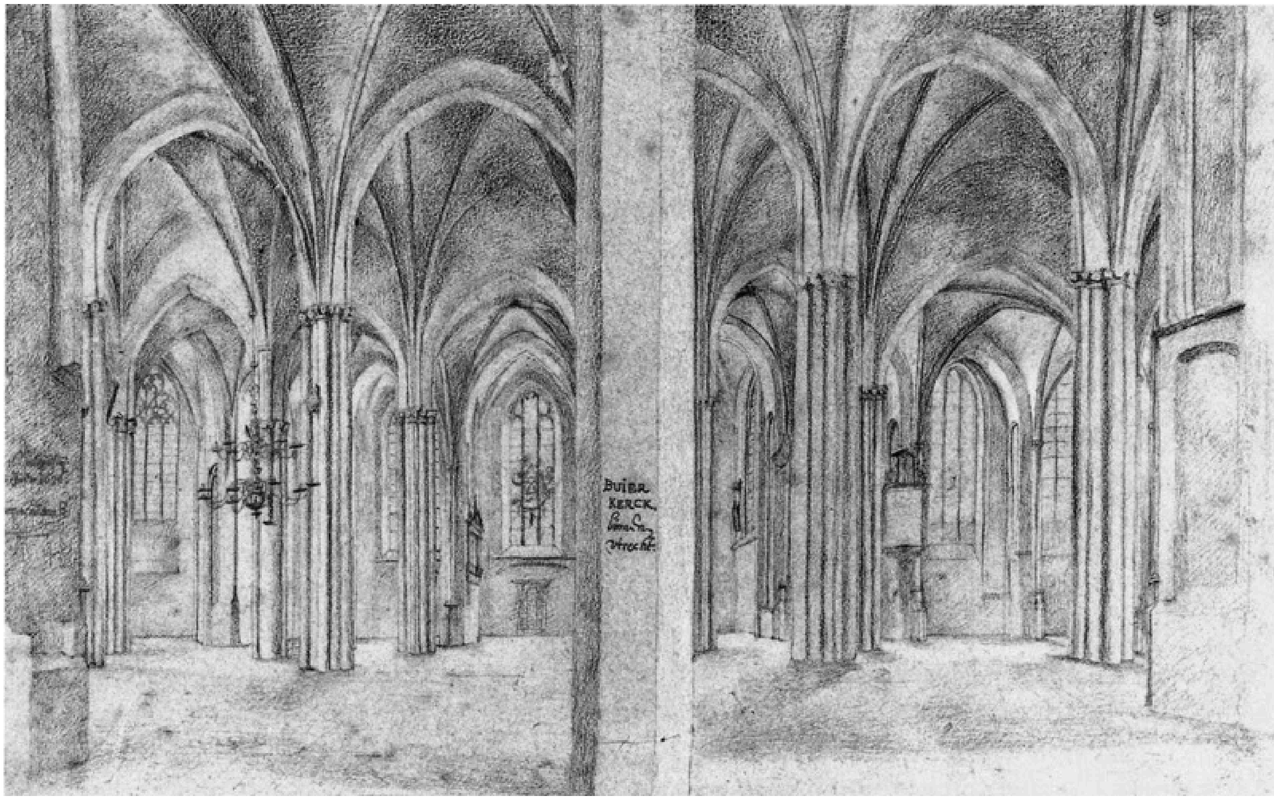

Figure 171: *The Interior of the Buurkerk at Utrecht* by Pieter Saenredam, preparatory drawings in pen and chalk on paper.

SAENREDAAM'S PAINTING of the St. Bavo Church in Haarlem in the Museum of Fine Arts in Boston depicts a pronounced departure from reality (Fig. 172). Saenredam painted this church over twenty times. Here, he purposely tilted converging perspective lines to heighten the nave.

Figure 172: *St. Bavo Church* by Pieter Saenredam, 1660. Link to MFA

SVETLANA ALPERS in her study of the unique character of the Dutch Golden Age, *The Art of Describing*, see Saenredaam's paintings of churches as one instance of the 'mapping impulse' of Dutch Art. Making accurate and beautiful maps is another explicit synthesis of art and science. The more detailed and reliable a depiction of the visible world, the more useful a map will be. During the 17th century, Dutch first began to use maps as decorative wall hangings. The deep interest in owing maps stems from the expeditions of the Dutch navy and merchant fleet, the source of wealth that underpinned Dutch achievements in art and science, wealth lavishly displayed in the newly genre of the still life.

Figure 173: *Young Woman with a Water Pitcher* by Vermeer, ca. 1662. Link to Google Arts and Culture.

VERMEER had an abiding interest in maps, used as props in many works. The only two lone male figures in his paintings – *The Astronomer* and *The Geographer* – are both map-makers in their professions. *The Art of Painting*, made near the end of Vermeer's career, also includes its largest and most extravagant map (Fig. 176). Vermeer paints its map of the Netherlands with exquisite detail, far more carefully than another contemporary painting that used the same map as a prop (Fig. 177, there is one surviving copy of the same 17^{th}-century map in Paris). Vermeer uniquely signs *The Art of Painting* by signing the map itself, asserting himself as both artist and maker of maps.

Figure 174: *Woman Reading a Letter* by Vermeer, c. 1663. Link to Google Arts and Culture.

Figure 175: *Young Woman with a Lute* by Vermeer, ca. 1662–63, Link to Google Arts and Culture

Figure 176: *The Art of Painting* by Vermeer, ca. 1666/68, Link to Google Arts and Culture

Figure 177: *The Music Lesson* by Jacob Ochtervelt (1634-1682), 1671. Link to Art Institute of Chicago

Painting

- Nard Kwast, Dutch reality TV star known for his recreation of Rembrandt's *Nightwatch* in the Rijksmuseum will lead a workshop to paint a Dutch Still Life. Link to slides

Reading

- Introduction of *The Art of Describing* Download entire book
- Review of *The Art of Describing* Download paper

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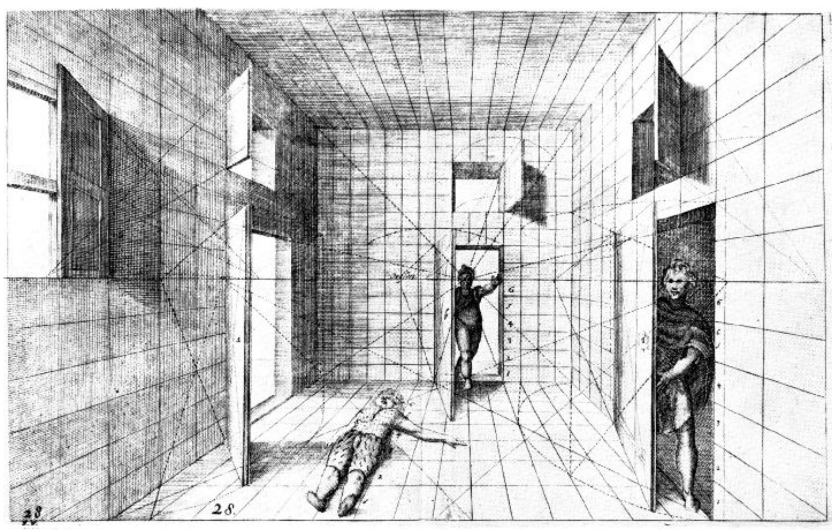
Updated: March 18, 2024

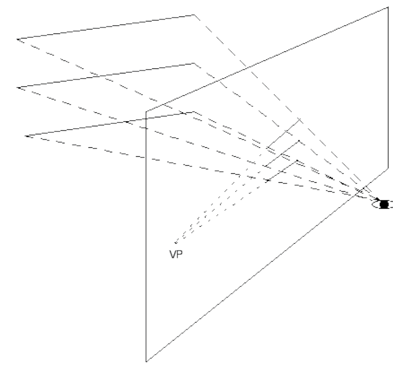
Appendix I - The Rules of Perspective by Chris W. Taylor

Linear perspective has a history going back at least to Aristarchus, a scene painter for Aeschylus in the 4th century BC who astonished his audience, including Plato, with his realistic depiction of depth by size reduction in the spatial layout of buildings. This Greek expertise was transmitted to the Roman Empire in the accurate central vanishing points in evidence in the wall-paintings of Pompeii, for example. The development of Renaissance one-point perspective may be traced over the 14th and 15th centuries. The elaboration of two-point perspective took another two centuries. Three-point perspective did not come into widespread use until well after the invention of photography, when the flexible availability of tilted camera angles revealed its visual effectiveness.



Jan Vredeman de Vries, *Perspective* (Leiden, 1604–5), plate 28. Courtesy, the Bancroft Library, Berkeley, California.

The following is an attempt to distill the rules of perspective into an elementary form that may be easily applied in practice. They are first stated in brief form, then elaborated to convey the rationale and significance of each of the rules.



THE RULES OF PERSPECTIVE are first stated in their most direct form, then elaborated.

1. There is only one geometry of perspective projection onto a fixed picture plane.

2. All straight lines in space project to straight lines (or points, if end on) in the picture plane.

3. The projections of all lines that are parallel in space either remain parallel in the picture plane or intersect at a single vanishing point.

4. All sets of parallel lines lying within a specified plane in space have vanishing points that fall along the horizon line defined by the orientation of that plane.

5. For two sets of parallel lines at some angle in the scene, the two vanishing points form that same angle at the viewer's eye, regardless of the orientation of the angle in space. In particular, the vanishing points for any 90° angle in space form a 90° angle at the viewer's eye.

6. Any planar figure in space is foreshortened in the direction of its slant from the observer (up to a 45° viewing angle).

7. Circles in the scene, if foreshortened, project to ellipses in the picture plane.

8. For correct projection of its perspective, a picture should be viewed from its center of projection in space.

9. When the eye is at the center of projection, the perspective geometry in the picture plane is independent of where in the plane the eye is looking.

Implications of the Rules of Perspective

1. There is only one geometry of perspective projection onto a fixed picture plane. Perspective is the geometry of projection from a scene through a plane to a point (or center of projection) corresponding to the pupil of the viewing eye. The plane is the picture plane on which the painter wishes to depict the scene. If the perspective is correct, the depiction on the plane will generate the same projective structure at the eye as did the scene behind it. The different forms of perspective construction concern the rules that apply to specific structures, allowing simplified forms of the projective geometry to be codified. But all are subcases of the same optical transform.
2. All straight lines in space project to straight lines (or points, if end on) in the picture plane. This fact is a simple consequence of the geometry of projection through a point in space (corresponding to the pupil of one eye). If a line is parallel to the picture plane, it must project to a straight line on that plane by virtue of similar triangles. Obviously, tilting the line within the plane of projection will not introduce any curvature, just a change in its extent within the line of projection. In the limit, the projected line may contract to a point in the picture plane when the line is viewed head on. Lines of any orientation can be described by this construction. Thus, all such point projections are to straight lines or points.
   (a) Introducing lens optics, as in the human eye, introduces the potential for curvature in the projection. Such curvature may consequently be a property of human perception at the extrema of the field, but the laws of perspective will be considered to be those of the point projection of pinhole optics, which permit no curvature.
   (b) Humans actually view scenes with two eyes, but the straight-line projections in each eye are both straight lines. The average, or binocularly-fused, projections is therefore also a straight line. No curvature is introduced by the geometry of binocular combination.
3. The projections of all lines that are parallel in space either remain parallel in the picture plane or intersect at a single vanishing point. This common intersection is valid for each entire set of parallels regardless of where in the visual field the lines arise. Each set of parallel lines intersects at a different vanishing point, of course. Thus, the first job in perspective projection is to identify all the lines in the scene that are parallel to a given line, then make sure that they are drawn so as to project to a common vanishing point.
   (a) Parallel lines in space that are also parallel to the picture plane remain parallel to each other in the projection. This leads to the particular case of central perspective, in which all the lines on the scene are either parallel with the line of sight or at right angles to it, parallel with the picture plane. The first set will be horizontal and receding from a viewer looking straight ahead. The vanishing point for this first set is directly in front of the viewer, making a central point of convergence for these horizontal receding lines. The second set consists of any lines at right angles to the first set, thus at any angle within the picture plane. These lines, such as the verticals of the sides of buildings, will all remain parallel within the picture plane if they were parallel in space. Note that what makes central perspective central is simply the choice of lines present in the scene. Perspective itself is universal, an optical projection of the light rays.
   (b) The corollary of the central perspective construction is that it is implicitly incorrect to set the "central" vanishing point is away from the viewing center of the picture. This modification was employed in the mid Renaissance, where the "central" vanishing point may have been moved even to a point beyond the edge of the picture. The "frontal" sides of all the squares nevertheless remained parallel

in such constructions (usually horizontal and vertical), so that the perspective is incorrect unless the picture is expected to be viewed from the unlikely position of directly front of the shifted vanishing point.

4. All sets of parallel lines lying within a specified plane in space have vanishing points that fall along the horizon line defined by the orientation of that plane. The particular case is the ground plane. All sets of parallel lines in the ground plane have vanishing points in the horizon line. (The fact that the earth is not flat means that it does not strictly conform to a ground plane, defined geometrically. The deviation is generally too small to be of consequence in art.)
   (a) Rules 3 and 4 may be combined to consider the vanishing points not just for lines within a single plane but within a sheaf or stack of parallel planes. All lines on all parallel planes still have vanishing points falling along the same line. For example, all lines on or parallel with the ceiling or floor have vanishing points in the line of the horizon, as do all horizontal edges of doors and casement windows. But all the lines at angles on the sides of a Ferris wheel, for example, would have vanishing points in a vertical line.
5. For two sets of parallel lines at some angle in the scene, the two vanishing points form that same angle at the viewer's eye, regardless of the orientation of the angle in space. In particular, the vanishing points for any 90° angle in space form a 90° angle at the viewer's eye. In particular, the vanishing points for any right angle in space form a 90° angle at the observer's eye. This result may be seen by considering the member of their respective parallel bundles, coming directly toward the viewer's eye. These lines form the same angle as any other pair from the two bundles. Their angle at the eye, and hence the viewing angle between the vanishing points, therefore match the angle of the lines in space.
   (a) A classic case of this rule is the diagonals of any square, which are always at 90° to each other. The vanishing points (or "distance points", Leonardo, 1492) for these diagonals should therefore form a 90° angle at the center of projection, regardless of their orientation in space. Twist the angle in any direction whatever in three-dimensional space (even to the point of complete foreshortening) and the vanishing points will nonetheless hold to a strict 90° angle at the viewer's eye. In terms of pictorial distance, this angle between the vanishing points corresponds to the same distance in the picture plane except for the tan transform for projection of the equal angles at the eye onto the plane.
   (b) The corollary of this principle is that, if the vanishing point in central perspective is displaced from the center of view, the simplicity of the central perspective construction has been violated and a second vanishing point arises at 90° from this displaced vanishing point (for lines in the same plane). In fact, an entire crescent of vanishing points is required to accommodate lines of all orientations, aligned diametrically opposite the direction of the displacement.
6. Any planar figure in space is foreshortened in the direction of its slant from the observer (up to a 45° viewing angle). In particular, any square in space must be foreshortened in the direction of its slant, even for the projection outside the frame up to the range defined by the vanishing points. Beyond that, the foreshortening becomes lengthening, but this will occur outside the range of almost any picture (unless its edges extend beyond a 45° angle from the line of sight).
   (a) The degree of foreshortening of a square of central perspective is defined by its diagonals, which should project to vanishing points at a 90° angle to the viewer. Thus, the vanishing point for the corners of horizontal squares in central perspective should be at 45° to (and at the same height as) the central vanishing point. The intersection of the diagonals with the cardinal grid defines the degree

of progressive foreshortening of the receding squares. This geometry of a second vanishing point to set the spacing of the horizontals corresponds to one version of the costruzione legittima, or distance point method, of Leonardo (1492).

(b) In general, foreshortening follows the construction of the multiple implicit vanishing points even in central perspective, which is conceptualized as having only a single vanishing point. As long as there are intersecting lines in the scene, as there will be in any piazza grid, the vanishing points for the construction lines through the intersections must obey rules 2, 3 and 5, lying in the same line at the horizon of the plane and at same angle to the viewer as the intersections of the lines themselves in space.

(c) The progressive foreshortening as equal divisions recede in space gives a sense of curvature to the perspective transform of a regular array. All the lines are straight, but the array elements change shape as they recede, violating one's expectation of self-similarity. On top of this gradient of shape, the line thickness in virtually all perspective diagrams does not vary as it should in true perspective. Thus, the lines form an increasing proportion of the element area and perhaps induce a sense of distortion in an otherwise correct transform.

7. Circles in the scene, if foreshortened, project to ellipses in the picture plane. Although perspective distorts rectangles to asymmetric trapezoids in general, the properties of circles are such that they always project to an ellipse of some orientation. If the circle is parallel to the picture plane, it projects to a circle, which is the limiting case of an ellipse with no bias.

   (a) The projection for a circle may be derived by inscribing it in a square, for which the previous rules define the perspective distortion. The requisite ellipse is then obtained by inscribing it within the trapezoid obtained from the projection of this square. In practice, this ellipse may be selected from a set of ellipse templates as the one that just touches all four sides of the trapezoid without crossing them at any point. These constraints uniquely define the correct ellipse, since an ellipse is defined by four points in the plane (as a circle is defined by three points). The four 'touches' define four points and the avoidance of crossing anywhere provides the fifth constraint.

   (b) The center of the requisite ellipse does not correspond with the center of the circle being projected, but it displaced away from the vanishing point and toward the observer. The projected center of the circle may be determined by drawing the diagonals for the trapezoid, which intersect at the center of the projected circle. The degree of displacement may be determined by drawing the major an minor axes of the ellipse. The intersection of these axes defines its geometric center, which will be displaced from the projected center of the circle.

   (c) Spheres in space also project to ellipses in the picture plane, although generally with much less distortion than circles because the roundness of the spheres means that they are never foreshortened. Spheres always project to circles when at the center of projection. The elongation arises only because of marginal distortion, the stretching of the image as the picture plane itself recedes from the viewer at increasing angles of view. The requisite ellipse may be obtained by first projecting the sphere to a circle in a plane at right angles to the line of sight through its center, then projecting this circle to the picture plane. The major axis of the resulting ellipse is thus aligned with the axis of rotation of this projection and its ellipticity from the cosine of the (dihedral) angle between the intermediate and final projection planes.

8. For correct projection of its perspective, a picture should be viewed from its center of projection in space. In terms of distance, Rule 5 implies that the vanishing points for lines at right angles should

be viewed so as to be orthogonal, forming 90° angle at the viewing position. Any parallel pair of right angles in the picture thus defines its correct viewing distance, by defining two orthogonal vanishing points. In terms of angle, the plane of the picture should be viewed at the angle of slant for which the perspective was designed. Other viewing angles, away from the center of projection in any direction, will result in perspective distortion.

(a) For the particular case of central perspective based on a square grid aligned with the line of sight, the 45° angle between the diagonals and the line of sight means that the distance points should have a 45° to the main vanishing point at the viewer's eye. The geometry between the eye, the central vanishing point and a distance point is therefore a 45° triangle, which means that the picture is correctly viewed at the distance that matches the span to each distance point. The physical distance between the vanishing points depends on the intended viewing distance, but a good rule of thumb is that it should correspond to a distance of at least twice the width of the picture. Leonardo recommended at least 20 times the height of the largest objects depicted.

(b) Telephoto distortion and its limiting case, orthographic projection, conversely, represent a strong magnification of the image in true perspective. If a tiny piece of the scene is magnified, the distance of its vanishing points is correspondingly magnified. The effect may be so extreme that the rear of the object looks as though it curls up toward the front of that object. The perceived distortion is so strong that it has been termed "reverse perspective", as though the lines which should be converging were in fact diverging. The effect is particularly strong in orthographic projection, where parallel lines in spaces are drawn as parallel in the picture, as though it were viewed from an infinite distance. Nevertheless, the perceived "reversal" is an illusion obtained simply from abnormally distant projection. Despite the distorted appearance, telephoto distortion is a valid perspective projection if viewed at the correct distance (although the scene may in practice be invisible at that distance).

9. Telephoto distortion and its limiting case, orthographic projection, conversely, represent a strong magnification of the image in true perspective. If a tiny piece of the scene is magnified, the distance of its vanishing points is correspondingly magnified. The effect may be so extreme that the rear of the object looks as though it curls up toward the front of that object. The perceived distortion is so strong that it has been termed "reverse perspective", as though the lines which should be converging were in fact diverging. The effect is particularly strong in orthographic projection, where parallel lines in spaces are drawn as parallel in the picture, as though it were viewed from an infinite distance. Nevertheless, the perceived "reversal" is an illusion obtained simply from abnormally distant projection. Despite the distorted appearance, telephoto distortion is a valid perspective projection if viewed at the correct distance (although the scene may in practice be invisible at that distance).

(a) The center of view in the picture should not be confused with the line of sight of the viewer. The center of view is the point in the picture plane closest to the viewer. The line of sight is the direction in which the viewer's eye is pointing, which may be anywhere within (or outside) the picture. Where the viewer samples the optic array does not affect the correctness of the perspective. The perspective is correct when the viewer's eye is placed at the center of projection in space for which the perspective was generated (regardless of the direction of the line of sight).

(b) This point is contentious because most authors agree that the perspective projection changes with viewing angle. Presumably this misconception arises because lines that are parallel in central projection of a scene appear to converge as the viewer looks to the side. This observation, however, implicitly assumes that the relevant picture plane is rotated with the rotation of the line of sight. Rule 9 refers instead to an observer viewing a fixed picture, projecting an optic array to the viewer's eye. In this case, if the observer's eye looks away from the center of view, the picture plane itself will project to the eye with a perspective transform. It is the perspective distortion of this picture plane that provides the convergence of the parallel lines in oblique view, just as in the physical scene itself. Thus the correct geometry for perspective in the picture plane is indeed independent of the direction in which the viewer is looking at this plane (assuming the eye stays at the center of projection).

(c) As long as the viewer is directly in front of the central vanishing point (and the picture plane is frontoparallel), the verticals project to parallel lines in the picture plane (Rule 3A). Once the main vanishing point is moved up or down from the viewer's eye position, the verticals are required to converge so as to make an angle of 90° to the direction of this vanishing point (Rule 5).

Appendix II - Essay by ChatGPT 3.5

Harold Cohen and Generative Art: A Contemporary Dialogue with Piero della Francesca

Generative art, a form of artistic expression that leverages algorithms and computational processes to produce unique visual compositions, has emerged as a significant facet of contemporary art. Among the pioneers of generative art stands Harold Cohen, an artist and computer scientist whose work transcends traditional boundaries, engaging with the computational realm to create compelling and thought-provoking pieces. This essay explores the life and work of Harold Cohen in the context of generative art, drawing connections to the Renaissance master Piero della Francesca and his painting *The Flagellation.*

Harold Cohen's Journey. Born in London in 1928, Harold Cohen began his artistic journey as a painter and eventually transitioned to explore the intersection of art and technology. In the late 1960s, Cohen started developing AARON, a computer program designed to autonomously generate artworks. AARON evolved over the years, showcasing an extraordinary capacity for creativity and a unique blend of human intuition and algorithmic precision.

Generative Art and AARON. Cohen's generative art challenges conventional notions of authorship and creativity. AARON, equipped with a set of rules and parameters defined by Cohen, autonomously generates visual compositions, demonstrating a form of artificial creativity. Cohen's intent was not to replace human artists but to collaborate with the machine, pushing the boundaries of what is conceivable in artistic creation.

Piero della Francesca's The Flagellation. To contextualize Cohen's work, it is crucial to revisit the Renaissance era and explore the art of Piero della Francesca. *The Flagellation*, a masterpiece created in the 15th century, is a poignant representation of the biblical event where Christ is flogged before his crucifixion. Piero's meticulous composition, geometric precision, and use of color convey a sense of harmony and order that resonates with mathematical beauty.

Connections between Cohen and Piero. The parallels between Cohen's generative art and Piero's meticulous approach are evident. Both artists engage with underlying structures—whether algorithmic or geometric—to create visually compelling pieces. While Cohen's AARON relies on computational processes, Piero's artistic practice is grounded in mathematical principles and the pursuit of harmony.

Cohen's work is not an attempt to replicate Piero's style but rather an exploration of the timeless relationship between art and structure. Both artists, separated by centuries and technological advancements, demonstrate the enduring pursuit of order, balance, and beauty in their respective works.

Conclusion. Harold Cohen's journey into generative art, as exemplified by AARON, offers a contemporary perspective on artistic creation. By exploring the intersection of art and technology, Cohen engages in a dialogue with historical masters like Piero della Francesca. The connections between Cohen's generative art and Piero's meticulous compositions underscore the enduring quest for harmony and structure in artistic expression, bridging the gap between tradition and innovation. As generative art continues to evolve, it stands as a testament to the timeless human desire to explore new avenues of creativity and redefine the boundaries of artistic practice.

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