

THE LIFE OF JOHANNES VERMEER is mostly mystery. Most evidence about Vermeer's methods are the paintings. These paintings reveal deep fascination with light and optics, refraction and reflection. Vermeer's *Soldier and a Laughing Girl* includes a deliberate and subtle use of perspective (Fig. 100). The soldier is painted as silhouette against bright background. Vermeer pays attention to linear perspective. The soldier is deliberately painted larger than the girl in proper geometric proportion with his position closer to the viewer. The exquisitely detailed map on the wall attests to his attention to 'photographic' realism.

VERMEER DELIBERATELY INCORPORATED OPTICAL EFFECTS that are characteristic of a camera obscura. In many paintings, Vermeer included 'circles of confusion', bright globules of coalesced highlights that simulate an out-of-focus effect caused by light diffraction. Circles of confusion appear on many objects in *The Lacemaker*, even on non-reflective matte surfaces that would not produce them in a camera obscura (Fig. 99). When the naked eye looks at a natural scene, every plane is in sharp focus because our lenses adapt. In a camera obscura with a glass lens, one image plane is in sharp focus, while nearer or farther image planes will have softer focus. Vermeer may have been mimicking this effect, either for aesthetics or optical realism.

PHILIP STEADMAN, architect and Emeritus Professor at University College London, was struck by the architectural consistency of room after room in painting after painting by Vermeer (Figs. 104, 102, 100, 103). Steadman conjectured that the same room was being translated from image to canvas, albeit with different decor from painting to painting. Vermeer might have used a camera obscura, not just to frame his subjects or to explore optical effects, but as a tool to translate the scene more accurately.

Steadman reconstructed the rooms in Vermeer's domestic interiors and worked out their viewing positions, where images of brightly lit interiors would occur if they were projected by a camera obscura in the back of the room. These reconstructions revealed remarkable consistency. Steadman proposed that Vermeer used a camera obscura, positioned in the same place at the back of the same room, to calculate the linear perspective in many paintings. Steadman envisions Vermeer's camera as a room, sectioned off and darkened, at one end of his studio. An aperture and lens cast an image of the scene onto a surface. This image, re-inverted and re-reversed, could be transferred to canvas (Fig. 105).



Figure 100: *Soldier and a Laughing Girl* by Johannes Vermeer. [Link to painting at Google Arts and Culture](#)



Figure 101: *Soldiers Playing Cards* by Pieter de Hooch 1657-1658. The heads of the figures are painted similar in size, not with different sizes as one would expect from perspective. From *Vermeer's Camera* by Steadman.



Figure 102: *The Girl with the Wine Glass*
by Vermeer. [Link to painting at Google Arts and Culture](#)

Updated: February 26, 2024



Figure 103: *The Glass of Wine* by Vermeer. [Link to painting at Google Arts and Culture](#)



Figure 104: *Lady Standing at a Virginal*
by Vermeer. [Link to painting at Google Arts and Culture](#)

Updated: February 26, 2024

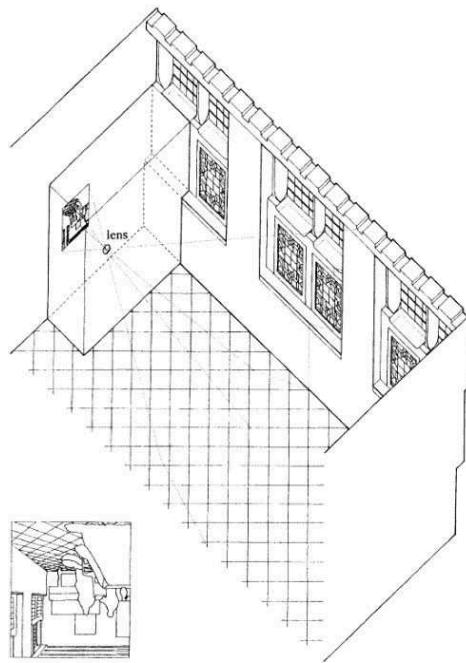


Figure 105: Did Vermeer use a camera to compose or create domestic scenes? *Left:* Lady at the Virginal with a Gentleman in *The Music Lesson* by Vermeer c.1662 - 1665. *Right:* Possible arrangement for Vermeer's *camera obscura* in the form of an enclosed booth, with the lens in the front wall projecting an image onto the back wall.

GIOVANNI ANTONIO CANAL (1697-1768), known as Canaletto, was not secretive about owing a camera obscura. He is best known for his cityscapes, like the view of Venice in the Harvard Art Museums, depicting its principal square of San Marco and the domes of The Basilica of Saint Mark. The sense of linear perspective is strongly enforced by the lines of architecture and patterned pavement.

GEOMETRIC ANALYSES of Harvard's Canaletto reveals both remarkable fidelity to perspective as well as deviations. The position of the vanishing point allows predicting the eye level of the painter, about 9 m above the Piazza. Canaletto would have needed to stand on a scaffold. The Basilica is painted too large, and has been artificially magnified. The geometric patterns on the Piazza are faithful to reality. Interestingly, the perspective lines converge to three different vanishing points, all on the horizon, so, if a camera obscura were used, this painting is a composite of three different views.

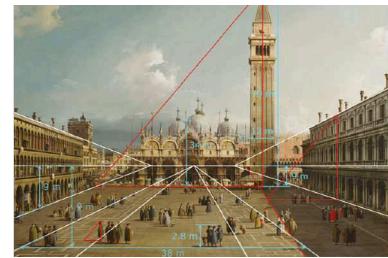


Figure 106: Analysis of Harvard's Canaletto by Erkelens (2019). White lines indicate perspective directions for the Procuratie Vecchie (left), geometric patterns (middle) and Procuratie Nuove (right). Red line and triangles are for analysis of shadows. Computed eye level, heights of the Campanile, Basilica and human figures, and outer width of the patterns are relative to heights of floors of buildings.



Figure 107: *Piazza San Marco, Venice* Canaletto, c. 1730-1734, [Link to painting](#)

PHILIP STEADMAN has compared modern photographs with Canaletto's sketches of extant buildings in Venice, uncovering remarkable fidelity. Most sketches have no ticks to indicate spacing. There are some guidelines, probably made with ruler. Elliptical curves of domes are drawn smoothly. None of this is definitive proof that Canaletto used a camera obscura. Suggestively, Canaletto's sketches take up entire pages, all the way to the edge. Buildings are cut off arbitrarily. One explanation might be that a standard size page in the sketchbook just caught what fell onto it from a camera obscura. If you were drawing freehand, you would probably automatically adjust the size of the image to fit nicely on the paper.



Figure 108: San Simeone Piccolo and adjoining buildings on the Grand Canal.



READING

- Martin Kemp. *Visualizations : the nature book of art and science*. Oxford University Press, Oxford ; New York, 2000. ISBN 0198564767
 - Basically Brunelleschian
 - Piero's Perspective
 - Vermeer's Vision
- 'The Camera Obscura' by Walter Liedtke In: Walter A Liedtke. *Vermeer : the complete paintings*. Harry N. Abrams, New York, 2008. ISBN 9789055447428 [Download paper](#)
- 'Canaletto's Camera' by Philip Steadman In: *Hockney's eye : the art and technology of depiction*. Paul Holberton Publishing Ltd., London, 2022. ISBN 1913645126 [Download paper](#)

LOOKING

- The Complete Virtual Vermeer Exhibit [Link to Pocket Gallery](#)

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- Martin Kemp. *The Science of Art*. Yale University Press, New Haven, Connecticut, 1990. ISBN 0-300-04337-6
- Martin Kemp. *Leonardo da Vinci, The Marvellous Works of Nature and Man*. Oxford University Press, 2006
- Antonio Criminisi, Martin Kemp, and Andrew Zisserman. Bringing pictorial space to life: Computer techniques for the analysis of paintings. pages 77–99, 2002

WEEK SIX - VIRTUAL MUSEUMS AND GENERATIVE ART

Thursday, 29 February 2024, 12:45 - 2:15 PM EST.

Harvard Art Museums, 600

OUR FIRST INTERACTIONS with the great works of art is usually through reproductions. Leo Steinberg (1920-2011), the American critic who transformed the way art history was written from rigorous formal exposition to approachable first-person narrative, first encountered art as a twelve-year old living with his Russian- émigré family in Berlin in 1923. Upon visiting a Socialist bookshop, he found the only book that wasn't about politics: *Die Früh-Renaissance der italienischen Malerei* (Fig. 109). The boy was enraptured by two hundred gray-and-reproductions of pictures that he could not stop looking at, and he would not stop looking for the rest of his life (Fig.). With this childhood memory, Steinberg disagrees with the "grown-up" view of Walter Benjamin, another Berliner and critic, who argued that distance from original works deprives them of their aura. The twelve-year old Steinberg discovered that the aura of an art work is "born in enchantment and nourished by reverence".¹⁵

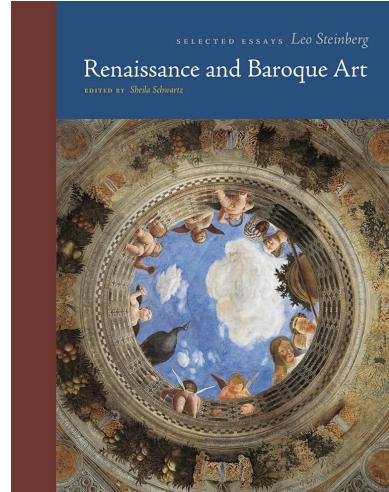
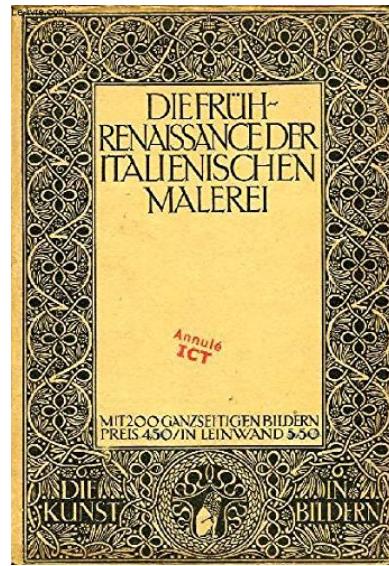
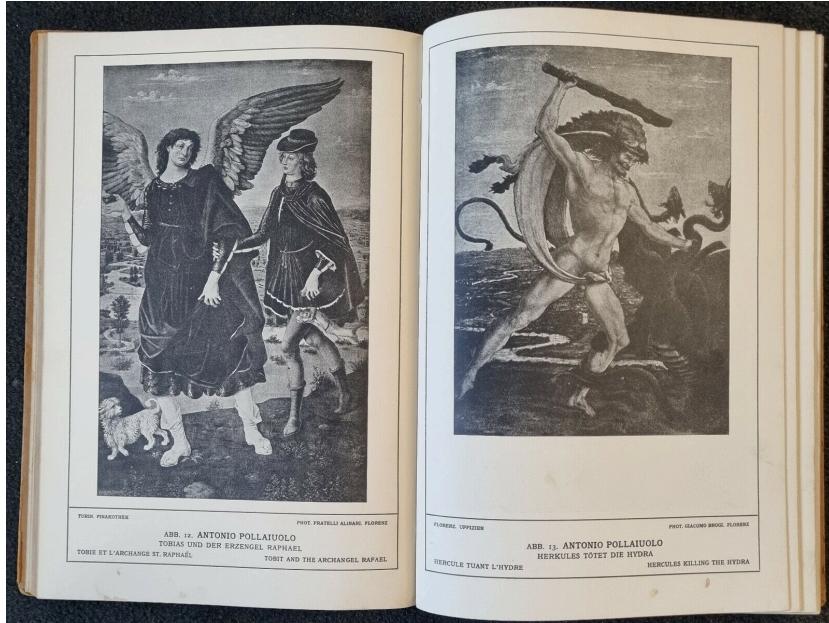


Figure 109: Top: Cover of *Die Früh-Renaissance der italienischen Malerei* that first enraptured Steinberg, pages reproduced in Fig. . Bottom: Cover of Steinberg's own lifelong writing on Renaissance and Baroque Art, depicting a ceiling painting by Mantegna (1431-1506) in the *Camera degli Sposi* of the Ducal Palace in Mantua that creates the illusion of looking upward to a blue sky with angels playing around a balcony, full of visual jokes like the boy angel threatening to drop an apple on the viewer.

¹⁵ Leo Steinberg. *Renaissance and Baroque Art : Selected Essays*. Steinberg, Leo, 1920-2011. Essays. Selections. 2018. The University of Chicago Press, Chicago, 2020

DAVID HOCKNEY HAD HIS FIRST ENCOUNTER WITH RENAISSANCE ART as an eleven-year old in 1948, seeing a poster reproduction of Fra Angelico's *Annunciation*. (Fig. 110). Hockney still thinks about the construction of linear perspective in Fra Angelico's *Annunciation*. In 2017, he responded to Fra Angelico with his own *Annunciation* where the perspective is 'reversed', instead of lines of perspectives converging to a central vanishing point, the lines of perspective expand outward, more like how we experience a wide vista with our visual perceptions (Fig. 111). As an eleven-year old, Hockney had a turning point:

At the age of eleven, I decided, in my mind, that I wanted to be an artist, but the meaning of the word "artist" to me then was very vague – the man who made Christmas cards was an artist, the man who painted posters was an artist, the man who did lettering for posters was an artist. Anyone was an artist who in his job had to pick up a brush and paint something... The idea of an artist just spending his time painting pictures, for himself, didn't really occur to me. Of course, I knew there were paintings you saw in books and in galleries, but I thought they were done in the evenings, when the artist had finished painting the signs or the Christmas cards or whatever they made their living from.



Figure 110: Fra Angelico, *The Annunciation*, 1440-1445, fresco, San Marco, Florence.



Figure 111: David Hockney, *The Annunciation*, 2017, acrylic on canvas, 121.9 cm x 243 cm.

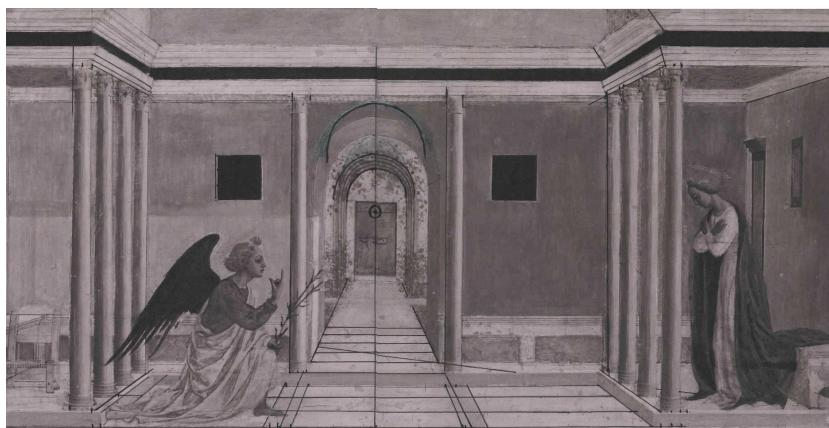
MARTIN KEMP'S FORMATIVE CAREER EXPERIENCE was with a real painting. The future art historian originally went to Cambridge University planning to study the natural science as an undergraduate. After arriving, he visited the Fitzwilliam Museum in Cambridge, where he discovered the *Annunciation* by Domenico Veneziano (1410-1461). This little picture, only 10" by 21", appealed to his scientific leanings with geometry incised into white gesso priming and passages of light that intrigued with religious and thematic meaning. This little picture marked Kemp's transition to art history.¹⁶

¹⁶ Martin Kemp. The annunciation by domenico veneziano. *Country Life*, page 44, 07 2018



Figure 112: Domenico Veneziano, *The Annunciation*, 1440s, Fitzwilliam Museum, [Link to painting](#)

Figure 113: Martin Kemp's analysis of linear perspective in the gesso priming of Veneziano's *Annunciation*.



IN OUR DIGITAL AGE, vast numbers of paintings and entire museums have been photographed, scanned, digitized, cataloged, and made available on the internet. During the pandemic, most art could only be seen as digital images. The Google Arts and Culture Program might be the largest repository of visual cultural heritage, with images of thousands of paintings from hundreds of museums worldwide taken with their traveling gigapixel cameras (although not the Louvre or the Prado). The Mauritshuis in The Hague, Netherlands – home of Vermeer's *The Girl with a Pearl Earring* and *The View of Delft* was an early adopter, fully digitizing seamless panoramic views of every brick-and-mortar gallery with gigapixel resolution, allowing a virtual reality tour through its rooms. This required collaboration between computer scientists and curators to solve an inverse perspective problem, creating the illusion of viewing actual two-dimensional paintings while virtually exploring a three-dimensional space.

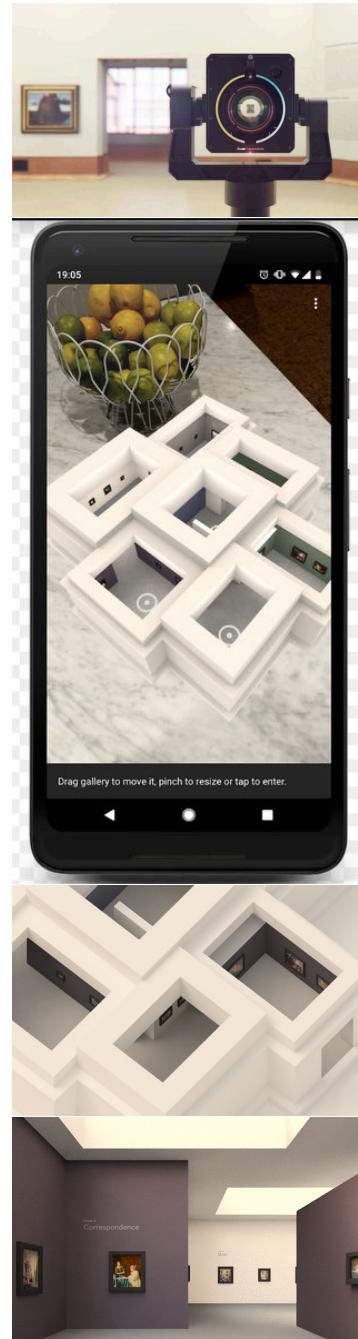


Figure 114: Top: Google's Art Camera captures gigapixel images of paintings, which can be used to build galleries with virtual reality in their "pocket gallery". artsandculture.google.com

Figure 115: *The Girl with a Pearl Earring*. In the Mauritshuis Museum, Netherlands. [Link to painting at Mauritshuis](#)

DIGITAL IMAGING AND ANALYSIS can reveal more than you can see with naked eye. Abbie Vandivere at the Mauritshuis led the most recent technical analysis of *The Girl with a Pearl Earring* with non-invasive imaging tools with non-visible wavelengths.¹⁷ This work revealed new clues. Vermeer used underlayers that varied in tone. He let these underlayers dry before applying surface paint, using thinner or thicker upper layers in different regions of the painting, thereby controlling how underlayers contribute to the appearance of light and shadow. High-resolution imaging reveals other preparatory steps such as fine black outlines. Analysis has also revealed *pentimenti*, subtle corrections and changes to the painting including relocations of the iris and ear. *The Girl With a Pearl Earring* was carefully planned and perfected at many levels.

DIGITAL IMAGING CANNOT provide the full experience of looking at a real painting that is more than a two-dimensional image. The pearl in *The Girl with a Pearl Earring* is not painted with flat pixels. The pearl in his most famous portrait is only made with a few brush-strokes, one stroke to create a reflection under another thick dab. The pearl only emerges as a reflectance of incident light from just two gobs of paint. Throughout the painting, Vermeer uses subtle reflectance to create effects. Highlights on the girl's lips and eyes reveal points of light reflecting off wet surfaces. Shadow evoke objects that are not actually drawn. Look closely. The bridge of the girl's nose is not directly defined. You 'see' her nose on the basis of its shadow and surface highlights. Digital images capture the aura of *The Girl with a Pearl Earring*, but to understand what Vermeer actually did with brush and paint, you need to see the real thing.

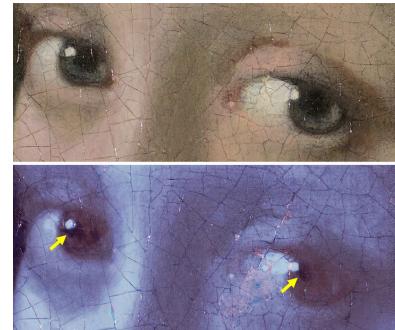


Figure 116: Evidence for pentimenti in the Girl's eyes. a Visible light photograph. b MS-IRR false colour detail. Dark marks indicate possible earlier iris locations (yellow arrows)

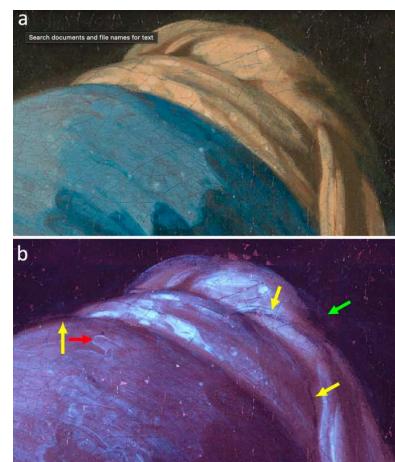


Figure 117: Evidence for contours and fine black outlines in the top of headscarf. a Visible light photograph. b MS-IRR false colour detail. Wavy brushstrokes at the surface (red arrow), fine black outlines applied in a preparatory phase (yellow arrows), back of the headscarf applied on top of an infrared-absorbing layer (green arrow).

¹⁷ Abbie Vandivere, Annelies van Loon, Kathryn A. Dooley, Ralph Haswell, Robert G. Erdmann, Emilien Leonhardt, and John K. Delaney. Revealing the painterly technique beneath the surface of Vermeer's Girl with a Pearl Earring using macro- and microscale imaging. *Heritage Science*, 7:64, 2019

BRICK-AND-MORTAR MUSEUMS are not being replaced by digital galleries, but are seeing resurgent interest. Museums are not only distant sites for celebrity artworks, but are civic monuments, secular temples, and mission-driven enterprises. Museums are integral components of cultural and national infrastructure.¹⁸ In the United States, 35,000 museums account for 750,000 jobs and \$50 billion annually to the economy.

THE ENCYCLOPEDIC MUSEUM is relatively new, an invention of the European Enlightenment, placing values in democratic ideas, public spaces, and intellectual improvement. The British Museum was the first public national museum to cover all fields of knowledge, created by an Act of Parliament in 1753. The Louvre opened in 1793, months after the execution of King Louis XVI. These museums had a head start over the first encyclopedic museums in the United States, the Metropolitan Museum of Art in NYC and the Museum of Fine Arts in Boston, both founded in 1870.

THE BRITISH MUSEUM grew with the depredations of the British Empire. The Louvre grew substantially during the Napoleonic Wars, when a river of artistic and cultural treasures flowed back to Paris. The important turning point that allowed the Metropolitan Museum of Art and the MFA to catch up was a change in American tax law: the War Revenue Act of 1917 permitted income-tax deductions for charitable contributions including artworks. Entire museums were created from estates. In NYC, the Frick Collection was established in 1935 to preserve the art collection of the industrialist Henry Clay Frick. Boston's Gardner Museum was built by Isabella Stewart Gardner (1840–1924) in the style of a 15th-century Venetian palace and has been open to the public since 1903. The Gardner Museum was also the victim of the biggest art heist in history in 1990, when several masterworks including a Vermeer – *The Concert*, now only seen in digital reproduction – were stolen from their gallery. The empty frames of the stolen paintings remain on the walls, awaiting their return.



Boston Thieves Loot a Museum Of Masterpieces

By FOX BUTTERFIELD
Special to The New York Times

BOSTON, March 18 — Dressed as police officers, thieves broke into the Isabella Stewart Gardner Museum here early this morning and made off with 12 priceless artworks, including paintings by Rembrandt, Vermeer, Degas and Manet.

The daring theft, which the museum said was not discovered until the cleaning crew arrived this morning, is believed to be one of the largest in the world, said the Federal Bureau of Investigation and a museum spokesman.

Corey Cronin, a spokesman for the Gardner Museum, said it was very difficult to place an exact value on the stolen paintings because "they were acquired by Mrs. Gardner at the turn of the century" and have never been offered for sale since then. But an official of the Federal Bureau of Investigation in Boston said he heard estimates today that the paintings could be worth anywhere from \$100 million to \$200 million.

Figure 118: Vermeer's *The Concert*, stolen from the Isabella Stewart Gardner Museum. NYTimes, 3/18/1990

¹⁸ Daniel H Weiss. *Why the museum matters. Why X matters*. Yale University Press, New Haven, 2022. ISBN 9780300275209

THOMAS HOVING (1931-2009), the former Director of the Metropolitan Museum of Art, started the tradition of international loans and rotating exhibitions that have become a main part of the programming at most art museums. Most museums now have special exhibition spaces or “white boxes” for blockbuster events and immersive experiences. This innovation started with the loan of the *Mona Lisa* to the Met that drew a million visitors in three weeks. Last year, the Rijksmuseum’s Vermeer exhibition drew 650,000 visitors in 16 weeks.



Figure 119: The courtyard of the Isabella Stewart Gardner Museum.

MUSEUMS ARE NOT REPOSITORIES OR VAULTS, but civic spaces and sanctuaries, works in progress that continuously adapt to stay relevant and serve public good.

DIGITAL ART arrived with digital imaging. David Hockney avoided digital art until recent improvements with the iPad and Apple Pencil. He spent lockdown during the pandemic with the App “Brushes”, depicting the arrival of Spring. With the iPad, Hockney particularly reveled in green. In painting, green is difficult. Pigments that produce pure greens were unstable (natural verdigris and copper resinate), weak (malachite), or poisonous (copper arsenic and copper acetoarsenite, also called ‘emerald’ green). Modern pigments produce varied greens with varied levels of toxicity. With an iPad, Hockney had free reign in experimentation with greens, both viewed on the iPad and the prints he made from the iPad. This requires experimentation. In a natural scene, variations from sunlight to shadow can range over $10,000\times$ in brightness. A digital monitor can produce about 1000 brightness levels. A color printout delivers only about 100 brightness levels. Yet, with controlled color and contrast, the difference between lit lawn and shadow in *20th April 2020 No. 1* renders strong contrast with only $1.2\text{--}1.5\times$ difference in brightness.



No. 219, 20th April 2020, iPad painting, © David Hockney

Figure 120: David Hockney, *20th April 2020 No. 1*, iPad drawing).

COMPUTER-GENERATED ART arrived with better computers. Generative AI can now produce meaningful written content that imitates human language. Artists have been programming computers to deploy ‘rules’ to generating art starting with the earliest programmable desktops. Harold Cohen (1928-2016) was a British-born artist who created AARON, a computer program designed to autonomously produce paintings and drawings. In 1968, he learned to program at UC San Diego and the AI labs at Stanford. In these early days, before color monitors and printers, he programmed mechanical plotters and hand-colored the results. He sought generative rules to guide the pen. The earliest images were abstract. Later, he found rules by which the computer could emulate the growth and structure of leaves and plants, not just their appearance. Cohen eventually discovered rule-based paradigms to generate other shapes and human figures. Eventually, his programs incorporated color. Unfettered by the human visual system, Cohen discovered that the computer created color combinations and contrasts with paint more freely than a human. A retrospective of Harold Cohen’s work is currently at the Whitney Museum of American Art in New York City: [Link to exhibit.](#)



Figure 121: Harold Cohen, *Untitled [Amsterdam Suitel]*, 1977/78. Colored pencil on lithograph.



Figure 122: Active plotters at the Whitney Museum drawing images from different versions of the AARON software.



Figure 123: Harold Cohen, *Susan with Plant*, 1991. Acrylic and plotter pen on canvas.

Figure 124: Harold Cohen, *AARON Gijon*, 2007. Screenshot. Artificial intelligence software.

HAROLD COHEN, in his article *Driving the Creative Machine*, makes direct reference to Piero della Francesca in thinking about rule-guided art. *The Flagellation* by della Francesca is a paradigm of linear perspective, where once the artist had defined the stage, the construction of the painting was governed by geometric rules. The notebooks of della Francesca contain numerous calculations and drawings, by which he calculated perspectival projections of complex shapes like the human head with various transformations to include in his paintings.¹⁹ Once the artist defines a seed in his painting, he follows rules to generate the final projections and pictures.

For amusement, I asked whether ChatGPT, the generative chatbot, had anything interesting to say about Piero della Francesca and Harold Cohen. The answer is in Appendix II.

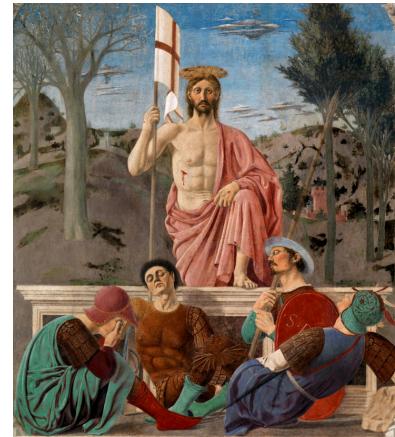
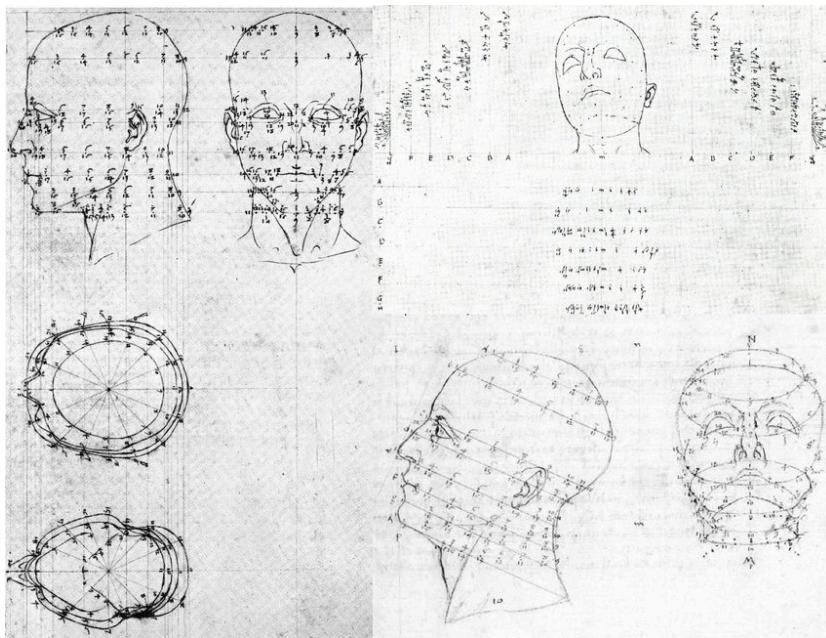


Figure 125: Piero della Francesca, *Resurrection of Christ*, 1465

Figure 126: from Piero della Francesca, *De Prospectiva pingendi*, studies that are incorporated in his *Resurrection of Christ*.

¹⁹ Martin Kemp. *The Science of Art*. Yale University Press, New Haven, Connecticut, 1990. ISBN 0-300-04337-6

I BROUGHT UP LEO STEINBERG at the beginning of this discussion, reminded of his illuminating essay “Mantegna: Did He Paint by the Book?”, where he paints a picture of Andrea Mantegna (1431-1506), a contemporary of Piero della Francesca, similarly enthralled by perspective.²⁰ Steinberg describes the many ways that Mantegna was guided by perspective but not imprisoned by it. The book of the title is Alberti’s *De Pictura* that describes the precepts of perspective, that Mantegna knew but did not slavishly follow.

²⁰ Leo Steinberg, Sheila Schwartz, and Stephen J Campbell. Mantegna: Did He Paint by the Book? In *Renaissance and Baroque Art*, pages 34–69. University of Chicago Press, United States, 2020. ISBN 022666872X



Figure 127: The fictive oculus, or opening to the sky, in the Camera degli Sposi in Mantua by Andrea Mantegna. Created with foreshortening and a strong sense of linear perspective (but looking upward), the oculus is ringed with figures looking down on the room below. A potted plant is perched on a wooden support, seemingly ready to fall. A cherub threatens to drop an apple. The painting is filled with visual jokes, generative figures based on verbalizable concepts, added as ornaments to the ringed balcony. The imaginative act of successively filling the balcony with a set of concept-to-figure illustrations – in the mind and by the hand of Andrea Mantegna – is perhaps not far from attaching text-to-image ornaments to any illustration using generative fill.

READING

- Abbie Vandivere, Annelies van Loon, Kathryn A. Dooley, Ralph Haswell, Robert G. Erdmann, Emilien Leonhardt, and John K. Delaney. Revealing the painterly technique beneath the surface of Vermeer's Girl with a Pearl Earring using macro- and microscale imaging. *Heritage Science*, 7:64, 2019 [Download paper](#)
- Harold Cohen. Driving the Creative Machine Renaissance art. *Orcas Center, Crossroads Lecture Series*, 2010 [Download paper](#)

LOOKING

- Use Google Pocket Galleries to visit a [virtual Vermeer gallery](#).
- Download Adobe Creative Cloud (free with Harvard HUID) and experiment with Adobe Firefly for Generative AI. [Link to Adobe Firefly](#). If you don't mind spending money, you can also try [DALL-E](#) or [Midjourney](#)

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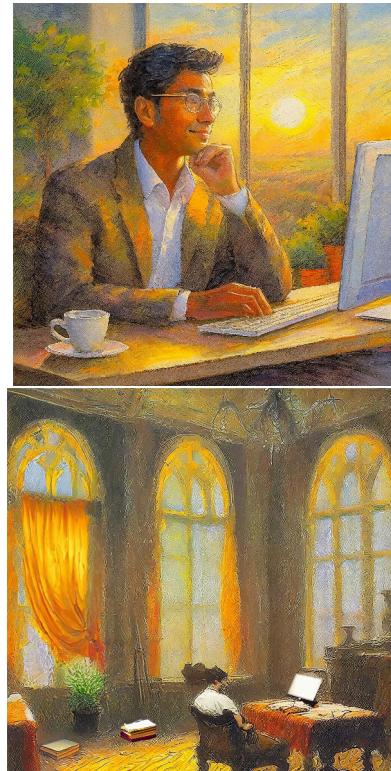
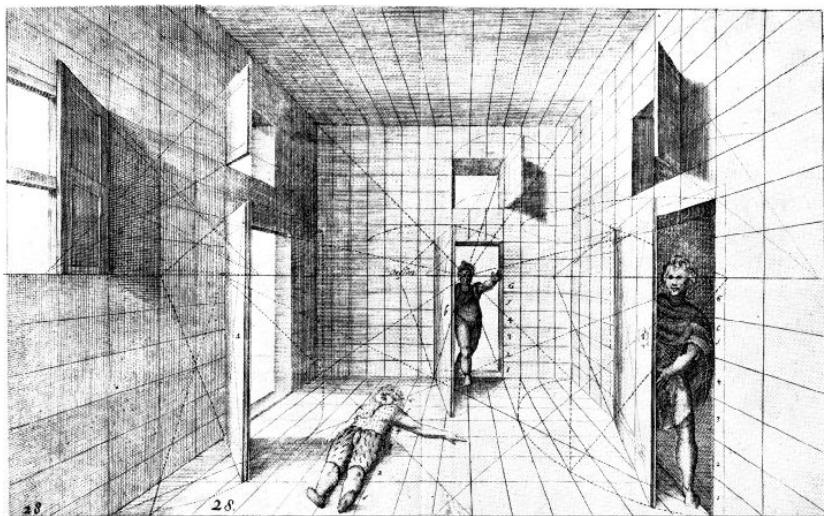


Figure 128: *Self-portraits in office*, Aravi Samuel, 2024, Generative AI

APPENDIX I - THE RULES OF PERSPECTIVE BY CHRIS W. TAYLOR

LINEAR PERSPECTIVE has a history going back at least to Aristarchus, a scene painter for Aeschylus in the 4th century BC who astonished his audience, including Plato, with his realistic depiction of depth by size reduction in the spatial layout of buildings. This Greek expertise was transmitted to the Roman Empire in the accurate central vanishing points in evidence in the wall-paintings of Pompeii, for example. The development of Renaissance one-point perspective may be traced over the 14th and 15th centuries. The elaboration of two-point perspective took another two centuries. Three-point perspective did not come into widespread use until well after the invention of photography, when the flexible availability of tilted camera angles revealed its visual effectiveness.

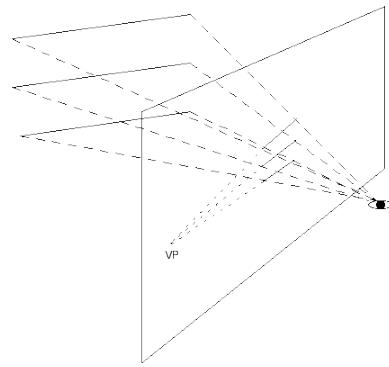


JAN VREDEMAN DE VRIES, *Perspective* (Leiden, 1604–5), plate 28. Courtesy, the Bancroft Library, Berkeley, California.

The following is an attempt to distill the rules of perspective into an elementary form that may be easily applied in practice. They are first stated in brief form, then elaborated to convey the rationale and significance of each of the rules.

THE RULES OF PERSPECTIVE are first stated in their most direct form, then elaborated.

1. There is only one geometry of perspective projection onto a fixed picture plane.
2. All straight lines in space project to straight lines (or points, if end on) in the picture plane.
3. The projections of all lines that are parallel in space either remain parallel in the picture plane or intersect at a single vanishing point.
4. All sets of parallel lines lying within a specified plane in space have vanishing points that fall along the horizon line defined by the orientation of that plane.
5. For two sets of parallel lines at some angle in the scene, the two vanishing points form that same angle at the viewer's eye, regardless of the orientation of the angle in space. In particular, the vanishing points for any 90° angle in space form a 90° angle at the viewer's eye.
6. Any planar figure in space is foreshortened in the direction of its slant from the observer (up to a 45° viewing angle).
7. Circles in the scene, if foreshortened, project to ellipses in the picture plane.
8. For correct projection of its perspective, a picture should be viewed from its center of projection in space.
9. When the eye is at the center of projection, the perspective geometry in the picture plane is independent of where in the plane the eye is looking.



IMPLICATIONS OF THE RULES OF PERSPECTIVE

1. There is only one geometry of perspective projection onto a fixed picture plane. Perspective is the geometry of projection from a scene through a plane to a point (or center of projection) corresponding to the pupil of the viewing eye. The plane is the picture plane on which the painter wishes to depict the scene. If the perspective is correct, the depiction on the plane will generate the same projective structure at the eye as did the scene behind it. The different forms of perspective construction concern the rules that apply to specific structures, allowing simplified forms of the projective geometry to be codified. But all are subcases of the same optical transform.
2. All straight lines in space project to straight lines (or points, if end on) in the picture plane. This fact is a simple consequence of the geometry of projection through a point in space (corresponding to the pupil of one eye). If a line is parallel to the picture plane, it must project to a straight line on that plane by virtue of similar triangles. Obviously, tilting the line within the plane of projection will not introduce any curvature, just a change in its extent within the line of projection. In the limit, the projected line may contract to a point in the picture plane when the line is viewed head on. Lines of any orientation can be described by this construction. Thus, all such point projections are to straight lines or points.
 - (a) Introducing lens optics, as in the human eye, introduces the potential for curvature in the projection. Such curvature may consequently be a property of human perception at the extrema of the field, but the laws of perspective will be considered to be those of the point projection of pinhole optics, which permit no curvature.
 - (b) Humans actually view scenes with two eyes, but the straight-line projections in each eye are both straight lines. The average, or binocularly-fused, projections is therefore also a straight line. No curvature is introduced by the geometry of binocular combination.
3. The projections of all lines that are parallel in space either remain parallel in the picture plane or intersect at a single vanishing point. This common intersection is valid for each entire set of parallels regardless of where in the visual field the lines arise. Each set of parallel lines intersects at a different vanishing point, of course. Thus, the first job in perspective projection is to identify all the lines in the scene that are parallel to a given line, then make sure that they are drawn so as to project to a common vanishing point.
 - (a) Parallel lines in space that are also parallel to the picture plane remain parallel to each other in the projection. This leads to the particular case of central perspective, in which all the lines on the scene are either parallel with the line of sight or at right angles to it, parallel with the picture plane. The first set will be horizontal and receding from a viewer looking straight ahead. The vanishing point for this first set is directly in front of the viewer, making a central point of convergence for these horizontal receding lines. The second set consists of any lines at right angles to the first set, thus at any angle within the picture plane. These lines, such as the verticals of the sides of buildings, will all remain parallel within the picture plane if they were parallel in space. Note that what makes central perspective central is simply the choice of lines present in the scene. Perspective itself is universal, an optical projection of the light rays.
 - (b) The corollary of the central perspective construction is that it is implicitly incorrect to set the “central” vanishing point is away from the viewing center of the picture. This modification was employed in the mid Renaissance, where the “central” vanishing point may have been moved even to a point beyond the edge of the picture. The “frontal” sides of all the squares nevertheless remained parallel

in such constructions (usually horizontal and vertical), so that the perspective is incorrect unless the picture is expected to be viewed from the unlikely position of directly front of the shifted vanishing point.

4. All sets of parallel lines lying within a specified plane in space have vanishing points that fall along the horizon line defined by the orientation of that plane. The particular case is the ground plane. All sets of parallel lines in the ground plane have vanishing points in the horizon line. (The fact that the earth is not flat means that it does not strictly conform to a ground plane, defined geometrically. The deviation is generally too small to be of consequence in art.)
 - (a) Rules 3 and 4 may be combined to consider the vanishing points not just for lines within a single plane but within a sheaf or stack of parallel planes. All lines on all parallel planes still have vanishing points falling along the same line. For example, all lines on or parallel with the ceiling or floor have vanishing points in the line of the horizon, as do all horizontal edges of doors and casement windows. But all the lines at angles on the sides of a Ferris wheel, for example, would have vanishing points in a vertical line.
5. For two sets of parallel lines at some angle in the scene, the two vanishing points form that same angle at the viewer's eye, regardless of the orientation of the angle in space. In particular, the vanishing points for any 90° angle in space form a 90° angle at the viewer's eye. In particular, the vanishing points for any right angle in space form a 90° angle at the observer's eye. This result may be seen by considering the member of their respective parallel bundles, coming directly toward the viewer's eye. These lines form the same angle as any other pair from the two bundles. Their angle at the eye, and hence the viewing angle between the vanishing points, therefore match the angle of the lines in space.
 - (a) A classic case of this rule is the diagonals of any square, which are always at 90° to each other. The vanishing points (or "distance points", Leonardo, 1492) for these diagonals should therefore form a 90° angle at the center of projection, regardless of their orientation in space. Twist the angle in any direction whatever in three-dimensional space (even to the point of complete foreshortening) and the vanishing points will nonetheless hold to a strict 90° angle at the viewer's eye. In terms of pictorial distance, this angle between the vanishing points corresponds to the same distance in the picture plane except for the tan transform for projection of the equal angles at the eye onto the plane.
 - (b) The corollary of this principle is that, if the vanishing point in central perspective is displaced from the center of view, the simplicity of the central perspective construction has been violated and a second vanishing point arises at 90° from this displaced vanishing point (for lines in the same plane). In fact, an entire crescent of vanishing points is required to accommodate lines of all orientations, aligned diametrically opposite the direction of the displacement.
6. Any planar figure in space is foreshortened in the direction of its slant from the observer (up to a 45° viewing angle). In particular, any square in space must be foreshortened in the direction of its slant, even for the projection outside the frame up to the range defined by the vanishing points. Beyond that, the foreshortening becomes lengthening, but this will occur outside the range of almost any picture (unless its edges extend beyond a 45° angle from the line of sight).
 - (a) The degree of foreshortening of a square of central perspective is defined by its diagonals, which should project to vanishing points at a 90° angle to the viewer. Thus, the vanishing point for the corners of horizontal squares in central perspective should be at 45° to (and at the same height as) the central vanishing point. The intersection of the diagonals with the cardinal grid defines the degree

of progressive foreshortening of the receding squares. This geometry of a second vanishing point to set the spacing of the horizontals corresponds to one version of the costruzione legittima, or distance point method, of Leonardo (1492).

- (b) In general, foreshortening follows the construction of the multiple implicit vanishing points even in central perspective, which is conceptualized as having only a single vanishing point. As long as there are intersecting lines in the scene, as there will be in any piazza grid, the vanishing points for the construction lines through the intersections must obey rules 2, 3 and 5, lying in the same line at the horizon of the plane and at same angle to the viewer as the intersections of the lines themselves in space.
 - (c) The progressive foreshortening as equal divisions recede in space gives a sense of curvature to the perspective transform of a regular array. All the lines are straight, but the array elements change shape as they recede, violating one's expectation of self-similarity. On top of this gradient of shape, the line thickness in virtually all perspective diagrams does not vary as it should in true perspective. Thus, the lines form an increasing proportion of the element area and perhaps induce a sense of distortion in an otherwise correct transform.
7. Circles in the scene, if foreshortened, project to ellipses in the picture plane. Although perspective distorts rectangles to asymmetric trapezoids in general, the properties of circles are such that they always project to an ellipse of some orientation. If the circle is parallel to the picture plane, it projects to a circle, which is the limiting case of an ellipse with no bias.
- (a) The projection for a circle may be derived by inscribing it in a square, for which the previous rules define the perspective distortion. The requisite ellipse is then obtained by inscribing it within the trapezoid obtained from the projection of this square. In practice, this ellipse may be selected from a set of ellipse templates as the one that just touches all four sides of the trapezoid without crossing them at any point. These constraints uniquely define the correct ellipse, since an ellipse is defined by four points in the plane (as a circle is defined by three points). The four 'touches' define four points and the avoidance of crossing anywhere provides the fifth constraint.
 - (b) The center of the requisite ellipse does not correspond with the center of the circle being projected, but it displaced away from the vanishing point and toward the observer. The projected center of the circle may be determined by drawing the diagonals for the trapezoid, which intersect at the center of the projected circle. The degree of displacement may be determined by drawing the major and minor axes of the ellipse. The intersection of these axes defines its geometric center, which will be displaced from the projected center of the circle.
 - (c) Spheres in space also project to ellipses in the picture plane, although generally with much less distortion than circles because the roundness of the spheres means that they are never foreshortened. Spheres always project to circles when at the center of projection. The elongation arises only because of marginal distortion, the stretching of the image as the picture plane itself recedes from the viewer at increasing angles of view. The requisite ellipse may be obtained by first projecting the sphere to a circle in a plane at right angles to the line of sight through its center, then projecting this circle to the picture plane. The major axis of the resulting ellipse is thus aligned with the axis of rotation of this projection and its ellipticity from the cosine of the (dihedral) angle between the intermediate and final projection planes.
8. For correct projection of its perspective, a picture should be viewed from its center of projection in space. In terms of distance, Rule 5 implies that the vanishing points for lines at right angles should

be viewed so as to be orthogonal, forming 90° angle at the viewing position. Any parallel pair of right angles in the picture thus defines its correct viewing distance, by defining two orthogonal vanishing points. In terms of angle, the plane of the picture should be viewed at the angle of slant for which the perspective was designed. Other viewing angles, away from the center of projection in any direction, will result in perspective distortion.

- (a) For the particular case of central perspective based on a square grid aligned with the line of sight, the 45° angle between the diagonals and the line of sight means that the distance points should have a 45° to the main vanishing point at the viewer's eye. The geometry between the eye, the central vanishing point and a distance point is therefore a 45° triangle, which means that the picture is correctly viewed at the distance that matches the span to each distance point. The physical distance between the vanishing points depends on the intended viewing distance, but a good rule of thumb is that it should correspond to a distance of at least twice the width of the picture. Leonardo recommended at least 20 times the height of the largest objects depicted.
 - (b) Telephoto distortion and its limiting case, orthographic projection, conversely, represent a strong magnification of the image in true perspective. If a tiny piece of the scene is magnified, the distance of its vanishing points is correspondingly magnified. The effect may be so extreme that the rear of the object looks as though it curls up toward the front of that object. The perceived distortion is so strong that it has been termed "reverse perspective", as though the lines which should be converging were in fact diverging. The effect is particularly strong in orthographic projection, where parallel lines in spaces are drawn as parallel in the picture, as though it were viewed from an infinite distance. Nevertheless, the perceived "reversal" is an illusion obtained simply from abnormally distant projection. Despite the distorted appearance, telephoto distortion is a valid perspective projection if viewed at the correct distance (although the scene may in practice be invisible at that distance).
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- (a) The center of view in the picture should not be confused with the line of sight of the viewer. The center of view is the point in the picture plane closest to the viewer. The line of sight is the direction in which the viewer's eye is pointing, which may be anywhere within (or outside) the picture. Where the viewer samples the optic array does not affect the correctness of the perspective. The perspective is correct when the viewer's eye is placed at the center of projection in space for which the perspective was generated (regardless of the direction of the line of sight).
 - (b) This point is contentious because most authors agree that the perspective projection changes with viewing angle. Presumably this misconception arises because lines that are parallel in central projection of a scene appear to converge as the viewer looks to the side. This observation, however, implicitly assumes that the relevant picture plane is rotated with the rotation of the line of sight. Rule 9

refers instead to an observer viewing a fixed picture, projecting an optic array to the viewer's eye. In this case, if the observer's eye looks away from the center of view, the picture plane itself will project to the eye with a perspective transform. It is the perspective distortion of this picture plane that provides the convergence of the parallel lines in oblique view, just as in the physical scene itself. Thus the correct geometry for perspective in the picture plane is indeed independent of the direction in which the viewer is looking at this plane (assuming the eye stays at the center of projection).

- (c) As long as the viewer is directly in front of the central vanishing point (and the picture plane is frontoparallel), the verticals project to parallel lines in the picture plane (Rule 3A). Once the main vanishing point is moved up or down from the viewer's eye position, the verticals are required to converge so as to make an angle of 90° to the direction of this vanishing point (Rule 5).

APPENDIX II - ESSAY BY CHATGPT 3.5

HAROLD COHEN AND GENERATIVE ART: A CONTEMPORARY DIALOGUE WITH PIERO DELLA FRANCESCA

GENERATIVE ART, a form of artistic expression that leverages algorithms and computational processes to produce unique visual compositions, has emerged as a significant facet of contemporary art. Among the pioneers of generative art stands Harold Cohen, an artist and computer scientist whose work transcends traditional boundaries, engaging with the computational realm to create compelling and thought-provoking pieces. This essay explores the life and work of Harold Cohen in the context of generative art, drawing connections to the Renaissance master Piero della Francesca and his painting *The Flagellation*.

Harold Cohen's Journey. Born in London in 1928, Harold Cohen began his artistic journey as a painter and eventually transitioned to explore the intersection of art and technology. In the late 1960s, Cohen started developing AARON, a computer program designed to autonomously generate artworks. AARON evolved over the years, showcasing an extraordinary capacity for creativity and a unique blend of human intuition and algorithmic precision.

Generative Art and AARON. Cohen's generative art challenges conventional notions of authorship and creativity. AARON, equipped with a set of rules and parameters defined by Cohen, autonomously generates visual compositions, demonstrating a form of artificial creativity. Cohen's intent was not to replace human artists but to collaborate with the machine, pushing the boundaries of what is conceivable in artistic creation.

Piero della Francesca's The Flagellation. To contextualize Cohen's work, it is crucial to revisit the Renaissance era and explore the art of Piero della Francesca. *The Flagellation*, a masterpiece created in the 15th century, is a poignant representation of the biblical event where Christ is flogged before his crucifixion. Piero's meticulous composition, geometric precision, and use of color convey a sense of harmony and order that resonates with mathematical beauty.

Connections between Cohen and Piero. The parallels between Cohen's generative art and Piero's meticulous approach are evident. Both artists engage with underlying structures—whether algorithmic or geometric—to create visually compelling pieces. While Cohen's AARON relies on computational processes, Piero's artistic practice is grounded in mathematical principles and the pursuit of harmony.

Cohen's work is not an attempt to replicate Piero's style but rather an exploration of the timeless relationship between art and structure. Both artists, separated by centuries and technological advancements, demonstrate the enduring pursuit of order, balance, and beauty in their respective works.

Conclusion. Harold Cohen's journey into generative art, as exemplified by AARON, offers a contemporary perspective on artistic creation. By exploring the intersection of art and technology, Cohen engages in a dialogue with historical masters like Piero della Francesca. The connections between Cohen's generative art and Piero's meticulous compositions underscore the enduring quest for harmony and structure in artistic expression, bridging the gap between tradition and innovation. As generative art continues to evolve, it stands as a testament to the timeless human desire to explore new avenues of creativity and redefine the boundaries of artistic practice.

