



This worksheet demonstrates the functions defined in the CoolProp user function (CoolPropMathcadWrapper.dll). CoolProp is an open-source, cross-platform, thermophysical fluid properties library developed by Ian Bell et al. at the University of Liege, Liege, Belgium. The following URL provides full documentation of the CoolProp code:

<http://www.coolprop.org>

Revision History

2013-10-04: Creation and initial release of the CoolProp Mathcad Wrapper library. S. Polak
 2013-11-01: Incorporated CoolProp 4.0.0 pre-release code. S. Polak
 2013-11-21: Corrected typos and units. S. Polak
 2016-04-30: Props Functions name changes for CoolProp 5.x - J.P. Henning
 2016-05-07: Added low-level functions for get_fluid_param_string and set reference_state - J. P. Henning

Function Explanation & Examples

List of Fluids in the CoolProp Library (and Other Library Parameters)

The following low-level and high-level CoolProp functions are implemented in this Mathcad wrapper:

| | |
|-------------------------------------------------------------------------------------|--------------------------------------|
| get_global_param_string ("Property") | Extracts global CoolProp information |
| get_fluid_param_string ("Fluid", "Property") | Extracts fluid specific information |
| set_reference_state ("Fluid", "StateString") | Sets the reference state for a fluid |
| PropsSI ("OutProp", "InProp1", Val1, "InProp2", Val2, "Fluid") | State-dependent Properties |
| Props1SI ("Fluid", "PropName") | State-independent Properties |
| HAPropsSI ("OutProp", "InProp1", Val1, "InProp2", Val2, , "InProp3", Val3) | Humid Air Properties |

These functions will be explained in detail below and demonstrated with examples.

Low Level Information Functions

The function **get_global_param_string** is a wrapper for the CoolProp low level function *get_global_param_string*(). It can be used to extract parameters from the CoolProp library, such as the CoolProp version, and the list of fluids in the library using their acceptable name strings. Functions to parse the output of **get_global_param_string** are below (hidden for clarity - double click the arrow symbol in the left margin to expand the hidden/collapsed area).



The parameter "version" will output a single string identifying the version of CoolProp currently installed in the Mathcad installation directory.

`get_global_param_string("version") = "5.1.3dev"` Current version of the CoolProp library

The function parameter "FluidsList" will output a single string representing all the fluids available in the CoolProp library. For clarity, the string parsing functions defined above (hidden by default) splits that single string into a vector of strings, providing the scrollable list of fluids as shown below:

`strSplit(get_global_param_string("FluidsList"), ",") =`

| |
|------------|
| "1-Butene" |
| "Acetone" |
| "Air" |
| "Ammonia" |
| ... |

Click on the output vector (table) to the left to enable scrolling through the entire list

The parameter "predefined_mixtures" will output a single string representing all the predefined fluid mixtures available.

| | | |
|-----------------------------------------------------------------|-----------------|--------------------------------------------------------------------------------------------|
| strSplit(get_global_param_string("predefined_mixtures"), ",") = | "Air.mix" | Click on the output vector (table) to the left to enable scrolling through the entire list |
| | "Amarillo.mix" | |
| | "Ekofisk.mix" | |
| | "GulfCoast.mix" | |
| | ... | |

The function **get_fluid_param_string** is a wrapper for the CoolProp low level function get_fluid_param_string(). It can be used to extract information from the CoolProp library about a specific fluid such as the aliases, Chemical Abstract Service (CAS) registry number for that fluid, the (ASHRAE) standard 34 safety rating, the equivalent REFPROP_name, or a "pure" flag identifying if the fluid is a pure substance (true) or a mixture (false). Examples of the usage of **get_fluid_param_string** are shown below:

Aliases using the parameter "aliases"

```
get_fluid_param_string("1-Butene", "aliases") = "1Butene, 1BUTENE, 1-BUTENE, Butene"
```

```
get_fluid_param_string("CO2", "aliases") = "R744, co2, CO2, carbondioxide, CARBONDIOXIDE"
```

CAS numbers using the parameter "CAS"

```
get_fluid_param_string("1-Butene", "CAS") = "106-98-9"
```

```
get_fluid_param_string("CO2", "CAS") = "124-38-9"
```

ASHRAE standard 34 safety rating using the parameter "ASHRAE34"

```
get_fluid_param_string("H2", "ASHRAE34") = "A3"
```

```
get_fluid_param_string("CO2", "ASHRAE34") = "A1"
```

REFPROP equivalent name using the parameter "REFPROPName"

```
get_fluid_param_string("H2", "REFPROPName") = "HYDROGEN"
```

```
get_fluid_param_string("CO2", "REFPROPName") = "CO2"
```

Pure fluid flag using the parameter "pure"

```
get_fluid_param_string("air", "pure") = "false"
```

*Indicates that "air" is **not** a pure fluid, but a mixture.*

```
get_fluid_param_string("CO2", "pure") = "true"
```

*Indicates that "CO2" is a pure fluid, **not** a mixture.*

State Dependent Fluid Properties

The function **PropsSI** is used to extract state-dependent fluid properties. The calling structure of the function is as follows:

```
PropsSI( Output Property <string>,
          Input1 Property <string>,
          Input1 Value <scalar>,
          Input2 Property <string>,
          Input2 Value <scalar>,
          Fluid <string>)
```

Where "Fluid" is one of the fluid names from the FluidList table extracted above, and output and input properties are taken from the partial list of most common parameters below:

| Parameter | Units | Input/ Output | Trivial | Description [units] |
|--------------------------|-------------------|------------------|---------|---------------------------------------------------|
| D, DMASS, Dmass | kg/m ³ | IO | False | Mass density |
| H, HMASS, Hmass | J/kg | IO | False | Mass specific enthalpy |
| P | Pa | IO | False | Pressure |
| Q | mol/mol | IO | False | Mass vapor quality |
| S, SMASS, Smass | J/kg/K | IO | False | Mass specific entropy |
| T | K | IO | False | Temperature |
| U, UMASS, Umass | J/kg | IO | False | Mass specific internal energy |
| ACENTRIC, acentric | | O | True | Acentric factor |
| A, speed_of_sound | m/s | O | False | Speed of sound |
| CONDUCTIVITY, L, | W/m/K | O | False | Thermal conductivity |
| CPMASS, Cpmass, C | J/kg/K | O | False | Mass specific constant pressure specific heat |
| CVMASS, Cvmass, O | J/kg/K | O | False | Mass specific constant volume specific heat |
| GAS_CONSTANT | J/mol/K | O | True | Molar gas constant |
| I, SURFACE_TENSION | N/m | O | False | Surface tension |
| M, MOLARMASS | kg/mol | O | True | Molar mass |
| PCRIT, P_CRITICAL, | Pa | O | True | Pressure at the critical point |
| Pcrit, p_critical, pcrit | | | | |
| PMAX, P_MAX, | Pa | O | True | Maximum pressure limit |
| PMIN, P_MIN, | Pa | O | True | Minimum pressure limit |
| PRANDTL, Prandtl | [-] | O | False | Prandtl number |
| PTRIPLE, P_TRIPLE | Pa | O | True | Pressure at the triple point (pure only) |
| RHOCRIT, rhocrit | kg/m ³ | O | True | Mass density at critical point |
| TCRIT, Tcrit | K | O | True | Temperature at the critical point |
| TMAX, T_MAX, | K | O | True | Maximum temperature limit |
| T_max, Tmax | | | | |
| TMIN, T_MIN, | K | O | True | Minimum temperature limit |
| T_min, Tmin | | | | |
| TTRIPLE, T_TRIPLE, | K | O | True | Temperature at the triple point |
| T_triple, Ttriple | | | | |
| T_FREEZE, T_freeze | K | O | True | Freezing temperature for incompressible solutions |
| V, viscosity | Pa s | O | False | Viscosity |
| Z | [-] | O | False | Compressibility factor (= PV/RT) |

For a full list of valid Property parameters, please see

<http://www.coolprop.org/coolprop/HighLevelAPI.html#table-of-string-inputs-to-propssi-function>

Examples, PropsSI

$$h := \text{PropsSI}("H", "T", 300, "P", 500, "Helium") = 1563094.4$$

Enthalpy of Helium [J/kg] at a temperature of 300 K and a Pressure of 500 Pa

$$\text{PropsSI}("T", "H", h, "P", 500, "Helium") = 300$$

Using h and 500 Pa as the input pair, the input temperature above should be returned in [K].

$$\text{PropsSI}\left("V", "T", \frac{21 \text{ }^\circ\text{C}}{\text{K}}, "P", \frac{14.7 \text{ psi}}{\text{Pa}}, "Water"\right) = 0.000978$$

Dynamic viscosity of Water [Pa*s] at a temperature of 21 °C (converted to the CoolProp temperature units, K) and a Pressure of 14.7 psi (converted to the CoolProp pressure units, Pa)

Saturated Liquid / Vapor examples

Properties along the saturation curve can be evaluated using just one of the state variables (i.e. T, P, etc.) and the vapor quality, Q, with a value of 0 for liquid, or 1 for vapor.

$$\text{PropsSI}\left("T", "P", \frac{10 \cdot \text{MPa}}{\text{Pa}}, "Q", 0, "Water"\right) = 584.147$$

Saturation temperature [K] of Water at a pressure of 10 MPa and a Quality of 0 (which is arbitrary for temperature output).

$$\text{PropsSI}\left("D", "P", \frac{10 \cdot \text{MPa}}{\text{Pa}}, "Q", 0, "Water"\right) = 688.424$$

Saturated liquid density [kg/m³] of Water at a pressure of 10 MPa.

$$\text{PropsSI}\left("D", "P", \frac{10 \cdot \text{MPa}}{\text{Pa}}, "Q", 1, "Water"\right) = 55.463$$

Saturated vapor density [kg/m³] of Water at a pressure of 10 MPa.

Mixture examples

Properties can be determined for a number of pre-defined mixtures; taken from the list returned by the low-level function `get_global_param_string("predefined_mixtures")` as demonstrated above.

$$\text{PropsSI}("D", "T", 295.16, "P", 101325, "Air") = 1.1963$$

Density [kg/m³] of Air at room temperature and pressure.

User-defined mixtures can be specified in the last argument of `PropsSI()`. The mixture is specified by specifying each fluid[mole_fraction] separated by & between each fluid and mole fraction in a single string. For example, Air can be user defined using 21% O₂ and 79% N₂ as such:

$$\text{PropsSI}("D", "T", 295.16, "P", 101325, "O2[0.21]\&N2[0.79]") = 1.1916$$

Density [kg/m³] of Air at room temperature and pressure using the user defined mixture string.

Tertiary mixtures can be specified as well by appending another pure fluid and mole fraction to the last parameter string. To improve the mixture density calculation, Argon (Ar), the next largest component in air, can be included at the following mole fraction:

$$\text{PropsSI}("D", "T", 295.16, "P", 101325, "O2[0.20948]\&N2[0.78084]\&Ar[0.00934]") = 1.1957$$

CO₂ can be added to create a quaternary mixture and further improve the accuracy of the result:

$$\text{PropsSI}("D", "T", 295.16, "P", 101325, "O2[0.20948]\&N2[0.78084]\&Ar[0.00934]\&CO2[0.000314]") = 1.1963$$

Partial Derivatives for Single-Phase States

For some applications, it can be useful to have access to partial derivatives of thermodynamic properties. A generalized first partial derivative has been implemented into CoolProp, which can be obtained using the **PropsSI** function by encoding the desired derivative as a string. The format of the string is "d(OF)/d(WRT)|CONSTANT".

This example uses the following partial derivative of enthalpy as an alternate method for calculation the constant pressure specific heat.

$$c_p = \left. \frac{\partial h}{\partial T} \right|_P$$

Using the above encoding for the Output Property string in PropsSI:

$$\text{PropsSI}("d(H)/d(T)|P", "P", 101325, "T", 300, "Water") = 4180.635776556 \quad [\text{J/kg/K}]$$

As a check, using the built-in Property Name for constant pressure specific heat, we get a value of:

$$\text{PropsSI}("C", "P", 101325, "T", 300, "Water") = 4180.635776556 \quad [\text{J/kg/K}]$$

Second partial derivatives can be encoded into **PropsSI** as well. This is the second derivative of the above quantity, c_p , with respect to enthalpy, H , at constant pressure:

$$\text{PropsSI}("d(d(H)/d(T)|P)/d(H)|P", "P", 101325, "T", 300, "Water") = -7.7679894687 \times 10^{-5} \quad [1/\text{K}]$$

Warning: This second derivative formulation is currently only valid for homogeneous (single-phase) states. Two phase derivatives are not defined, and are for many combinations, invalid.

Backends

The CoolProp formulations use the Helmholtz Equation of State (HEOS). This formulation is the assumed default when specifying a fluid or mixture name. However, there are other backends that can be used within the CoolProp interface (if available) and they are specified by prefixing the fluid or mixture name with the Backend name followed by a double colon, i.e. "HEOS::".

| | |
|-----------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| $\text{PropsSI}("D", "T", 295.16, "P", 101325, "HEOS::Water") = 997.7712$ | Default HEOS backend for CoolProp |
| $\text{PropsSI}("D", "T", 295.16, "P", 101325, "IF97::Water") = 997.7706$ | IAPWS-IF97 backend for water and steam. NOTE: only T & P input pairs implemented at this time. |
| $\text{PropsSI}("D", "T", 295.16, "P", 101325, "INCOMP::DowQ") = 964.1091$ | INCOMP ressible backend - using <i>DowthermQ</i> pure fluid. |
| $\text{PropsSI}("D", "T", 295.16, "P", 101325, "INCOMP::MITSW[0.035]") = 1024.35$ | INCOMP ressible backend - using <i>MIT Seawater</i> as an aqueous solution at 3.5% salinity [gm/kg_w]. |
| $\text{PropsSI}("D", "T", 295.16, "P", 101325, "INCOMP::MITSW[0.0]") = 997.5942$ | INCOMP ressible backend - using <i>MIT Seawater</i> again. A mass fraction of zero for the solute indicates 100% water. |

For more information on the backends for CoolProp, please see <http://www.coolprop.org>. Note that support functions for *REFPROP* and *Tabular Interpolation* are not implemented in the Mathcad wrapper at this time.

Fluid Properties Not Dependent on State

The function **Props1SI** is used to extract fluid-specific parameters that are not dependent on the state. The calling structure of the function is as follows:

Props1SI(Fluid <string>, PropertyName <string>)

Where "Fluid" is one of the fluid names from the FluidList table above, and "PropertyName" is one of the property name abbreviations from the table below:

| | |
|----------|---------------------------------------|
| Tcrit | Critical temperature [K] |
| pcrit | Critical pressure [Pa] |
| rhocrit | Critical density [kg/m ³] |
| molemass | Molecular mass [kg/kmol] |
| Ttriple | Triple-point temperature [K] |
| Tmin | Minimum temperature [K] |
| ptriple | Triple-point pressure [Pa] |
| acentric | Acentric factor [-] |

For a full list, see the CoolProp Table of String Inputs at this address:

<http://coolprop.sourceforge.net/coolprop/HighLevelAPI.html#table-of-string-inputs-to-propssi-function>

All items in the table marked as Trivial (True) can be entered as a "PropertyName" into the PropsSI1 function.

Examples, Props1SI

| | |
|-------------------------------------------|-----------------------------------------------------------------|
| Props1SI("Argon" , "rhocrit") = 535.6 | Extracts the critical density of Argon, in [kg/m ³] |
| Props1SI("Water" , "Tcrit") = 647.096 | Extracts the critical temperature of Water, in [K] |
| Props1SI("CO2" , "ptriple") = 517964 | Extracts the critical pressure of CO2, in [Pa] |
| Props1SI("N2" , "acentric") = 0.0372 | Extracts the acentric factor for N2 [unitless] |
| Props1SI("1-Butene" , "molemass") = 0.056 | Extracts the molecular mass of 1-Butene, in [kg/kmol] |

NOTE: **Props1SI**("FluidName","PropertyName") is just a shortcut call to **PropsSI**("PropertyName","",0,"",0,"FluidName") where the two input parameters are not specified and given a value of zero. The same state-independent values can be found using **PropsSI()** as shown here:

| | |
|------------------------------------------------------------|-----------------------------------------------------------------|
| PropsSI("rhocrit" , "" , 0 , "" , 0 , "Argon") = 535.6 | Extracts the critical density of Argon, in [kg/m ³] |
| PropsSI("Tcrit" , "" , 0 , "" , 0 , "Water") = 647.096 | Extracts the critical temperature of Water, in [K] |
| PropsSI("ptriple" , "" , 0 , "" , 0 , "CO2") = 517964 | Extracts the critical pressure of CO2, in [Pa] |
| PropsSI("acentric" , "" , 0 , "" , 0 , "N2") = 0.0372 | Extracts the acentric factor for N2 [unitless] |
| PropsSI("molemass" , "" , 0 , "" , 0 , "1-Butene") = 0.056 | Extracts the molecular mass of 1-Butene, in [kg/kmol] |

Humid Air Fluid Properties

The function **HAPropsSI** is used to find fluid properties of humid air. The physics behind the HAPropsSI function is based on the analysis in ASHRAE RP-1845, which is available online:

<http://rp.ashrae.biz/page/ASHRAE-D-RP-1485-20091216.pdf>.

It employs real gas properties for both air and water, as well as the most accurate interaction parameters and enhancement factors. RP-1845 is based largely on the IAPWS-95 formulation for the properties of water. The calling structure of the function is as follows:

```
HAPropsSI( OutputProperty <string>,
            Input1 Property <string>,
            Input1 Value <scalar>,
            Input2 Property <string>,
            Input2 Value <scalar>,
            Input3 Property <string>,
            Input3 Value <scalar>)
```

Where "OutputParameter" and all three Input Parameters are variables from the table below:

| Parameter | Aliases | Input/Output | Description [units] |
|-----------|---------------------|--------------|-----------------------------------------------|
| B | Twb, T_wb, WetBulb | Input/Output | Wet-Bulb Temperature [K] |
| C | cp | Output | Mixture specific heat [J/kg dry air/K] |
| Cha | cp_ha | Output | Mixture specific heat [J/kg humid air/K] |
| D | Tdp, T_dp, DewPoint | Input/Output | Dew-Point Temperature [K] |
| H | Hda, Enthalpy | Input/Output | Mixture enthalpy [J/kg dry air] |
| Hha | | Input/Output | Mixture enthalpy [J/kg humid air] |
| K | k, Conductivity | Output | Mixture thermal conductivity [W/m/K] |
| M | Visc, mu | Output | Mixture viscosity [Pa-s] |
| P | | Input | Pressure [Pa] |
| R | RH, Relhum | Input/Output | Relative humidity in (0,1) [-] |
| S | Sda, Entropy | Input/Output | Mixture entropy [J/kg dry air/K] |
| T | Tdb, T_db, DryBulb | Input/Output | Dry-Bulb Temperature [K] |
| V | Vda | Input/Output | Mixture volume [m ³ /kg dry air] |
| Vha | | Input/Output | Mixture volume [m ³ /kg humid air] |
| W | Omega, HumRat | Input/Output | Humidity Ratio [kg water/kg dry air] |
| Z | | Output | Compressibility factor (pv/RT) |

Examples, HAPropsSI

$\text{HAPropsSI}("H", "T", 298.15, "P", 101325, "R", 0.5) = 5.042 \times 10^4$

Enthalpy of humid air [J kg⁻¹] at a dry-bulb temperature of 298.15 K, a Pressure of 101325 Pa, and relative humidity of 0.5

$\text{HAPropsSI}("T", "P", 101325, "H", h, "R", 0.5) = 371.24$

Temperature of saturated air (R=1.0) at the previous enthalpy value

Note that the humid air functions at a relative humidity of zero should give roughly the same values as Dry Air from the **PropsSI** function. This check can be easily demonstrated here.

$\text{HAPropsSI}("C", "T", 298.15, "P", 101325, "R", 0) = 1006.289371$ [J/kg/K]

$\text{PropsSI}("C", "T", 298.15, "P", 101325, "Air") = 1006.308143$ [J/kg/K]

Reference States

Enthalpy and entropy are relative properties! You should always be comparing differences in enthalpy rather than absolute values of the enthalpy or entropy. That said, it can be useful to set the reference state values for enthalpy and entropy to one of a few standard values. This is done by the use of the low-level **set_reference_state** function, which is a wrapper for the CoolProp set_reference_stateS() function.

A number of pre-defined reference states can be used:

- IIR: $h = 200 \text{ kJ/kg}$, $s = 1 \text{ kJ/kg/K}$ at 0C saturated liquid { IIR = International Institute of Refrigeration }
- ASHRAE: $h = 0$, $s = 0$ @ -40C saturated liquid
- NBP: $h=0$, $s=0$ for saturated liquid at 1 atmosphere { NBP = Normal Boiling Point }
- DEF: Go back to the default reference state for the fluid

as demonstrated below.

$\text{PropsSI}(\text{"H"}, \text{"T"}, 233.15, \text{"Q"}, 0, \text{"n-Propane"}) = 1.051 \times 10^5$ *Using the default reference state*

$\text{PropsSI}(\text{"H"}, \text{"T"}, 233.15, \text{"Q"}, 0, \text{"n-Butane"}) = 1.108 \times 10^5$

$\text{set_reference_state}(\text{"n-Propane"}, \text{"ASHRAE"}) = 0$ *Using the ASHRAE reference state.*

$\text{PropsSI}(\text{"H"}, \text{"T"}, 233.15, \text{"Q"}, 0, \text{"n-Propane"}) = 1.459 \times 10^{-11}$ *Value should be zero, or very close to it.*

$\text{PropsSI}(\text{"H"}, \text{"T"}, 233.15, \text{"Q"}, 0, \text{"n-Butane"}) = 1.108 \times 10^5$ *Note that value changed for n-Propane, as requested, but not for n-Butane..*

$\text{set_reference_state}(\text{"n-Propane"}, \text{"DEF"}) = 0$ *Back to default (DEF) reference state.*

$\text{PropsSI}(\text{"H"}, \text{"T"}, 233.15, \text{"Q"}, 0, \text{"n-Propane"}) = 1.051 \times 10^5$

$\text{PropsSI}(\text{"H"}, \text{"T"}, 233.15, \text{"Q"}, 0, \text{"n-Butane"}) = 1.108 \times 10^5$

Warning: The changing of the reference state should only be done at the very beginning of your worksheet or unexpected results may occur. It is not recommended to change the reference state during the course of making calculations as done here for demonstration purposes only. Further more, because of Mathcad's top-down calculation order, switching back and forth between reference states can lead to very unexpected results (real or apparent) and is not recommended.

Creating Specific Unit Functions & Plotting

Specialized Mathcad functions can be created that simplify input and correctly handle units. For example, the above saturation temperature call can be embedded in a user defined function:

$$T_{\text{sat}}(P, \text{fluid}) := \text{PropsSI}\left("T", "P", \frac{P}{\text{Pa}}, "Q", 0, \text{fluid}\right) \cdot \text{K}$$

This new function can then be used to easily calculate saturation temperature with units applied:

$$\text{fluid} := \text{"Water"} \quad P := 10 \cdot \text{MPa}$$

$$T_{\text{sat}}(P, \text{fluid}) = 310.997 \cdot ^\circ\text{C}$$

This is an extremely useful and compact way to call these functions over a range and create a plot of the results.

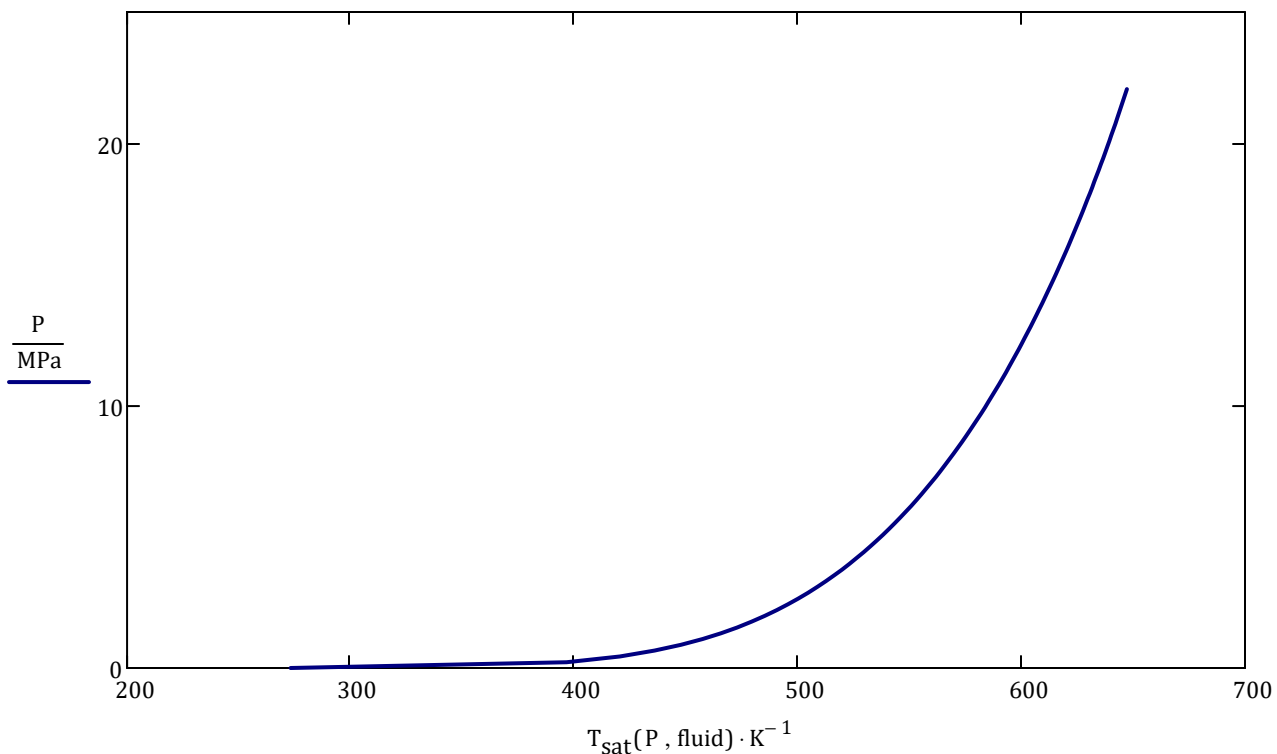
$$P_t := \text{PropsSI}(\text{"Water"}, \text{"ptriple"}) \cdot \text{Pa} = 611.655 \text{ Pa} \quad \text{Start range at the Triples Point Pressure.}$$

$$P_c := \text{PropsSI}(\text{"Water"}, \text{"pcrit"}) \cdot \text{Pa} = 22.064 \cdot \text{MPa} \quad \text{End range at the Critical Point Pressure.}$$

$$\Delta P := \frac{P_c - P_t}{100} = 0.221 \cdot \text{MPa} \quad \text{Set the range interval over 100 points.}$$

$$P := P_t, P_t + \Delta P .. P_c \quad \text{Set the Range}$$

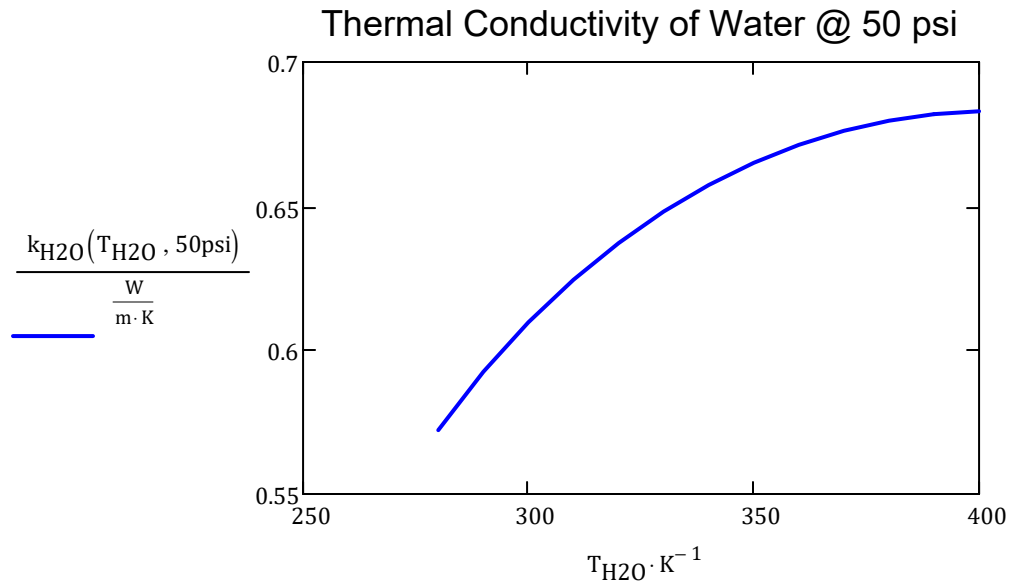
Plot Saturation Temperature-Pressure Curve for Water



Unit functions of other properties can be created as well.

$$T_{H2O} := 280 \cdot K, 290 \cdot K .. 400 \cdot K$$

$$k_{H2O}(T, P) := \text{PropsSI}\left("L", "T", \frac{T}{K}, "P", \frac{P}{Pa}, "Water"\right) \cdot \frac{W}{m \cdot K}$$



This is an example of plotting the wet bulb temperature of humid air at various dry-bulb temperatures and relative humidity ratios at a pressure of 1 atm.

$$T_{WB}(T, P, R) := \text{HAPropsSI}\left("B", "T", \frac{T}{K}, "P", \frac{P}{Pa}, "R", R\right) \cdot K$$

$$T_{HA} := 290 \cdot K, 300 \cdot K .. 370 \cdot K$$

