AutoEncoder based generative models

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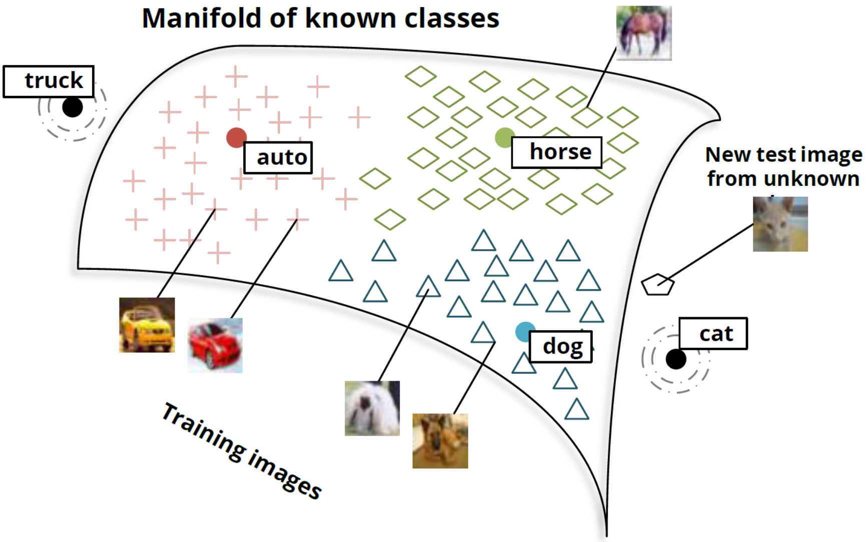
Generative model

- Data are given as $x \in X$
- Model is to learn the distribution of these data
- Trained model may be used to generate new data from the true probability distribution

Generative model – how to do it right



Generative model

Generative model performs two tasks at the same time

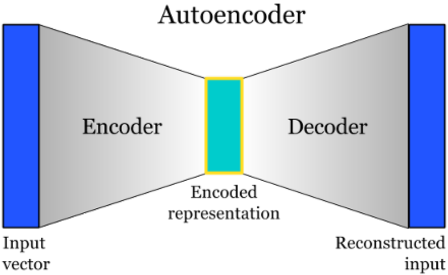
- selects the low dimensional manifold
- computes which data are more or less frequent

AutoEncoder

Generalization of PCA, idea based on compression of dataset $X = (x_i) \subset \mathbb{R}^N$ to a linear space Z of smaller dimension D (*latent space*).

We have an encoder $\mathbb{R}^N \ni x \to \mathcal{E}x \in Z$ and decoder $Z \ni z \to \mathcal{D}z \in \mathbb{R}^N$. We want to find such encoder and decoder which minimize reconstruction error:

$$MSE(X; \mathcal{E}, \mathcal{D}) = \frac{1}{n} \sum_{i=1}^{n} \|x_i - \mathcal{D}(\mathcal{E}x_i)\|^2.$$



Variational AutoEncoder

$$D_{KL} + RecErr = D_{KL}(N(\mu(X), \sigma(X))||N(0, I)) + MSE(X; \mathcal{E}, \mathcal{D}).$$

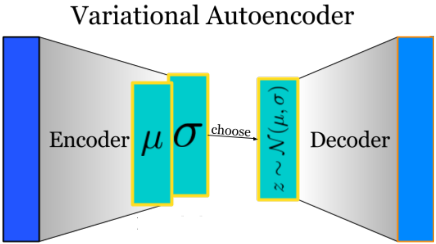


Figure: Variational AutoEncoder.

Wasserstein AutoEncoder - maximum mean discrepancy (MMD)

The main idea of WAE was based on the use of maximum mean discrepancy (MMD) distance as $d(P_{\mathcal{E}(X)}, P_{\mathcal{Z}})$, which required sampling from $P_{\mathcal{Z}}$.

$$COST(X; \mathcal{E}, \mathcal{D}) = MSE(X; \mathcal{E}, \mathcal{D}) + \lambda \cdot d(P_{\mathcal{E}(X)}, P_{\mathcal{Z}}).$$

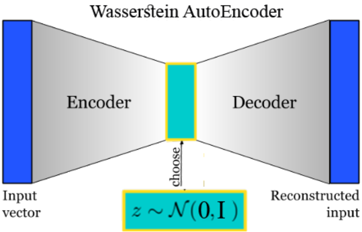


Figure: Wasserstein AutoEncoder.

Cramer-Wold AutoEncoder (CWAE)

The main idea of the Cramer-Wold AutoEncoder (CWAE) is to replace the Wasserstein distance by the newly introduced CW-distance between sample and distribution.

$$COST(X; \mathcal{E}, \mathcal{D}) = MSE(X; \mathcal{E}, \mathcal{D}) + \lambda \cdot d(P_{\mathcal{E}(X)}, N(0, I)).$$

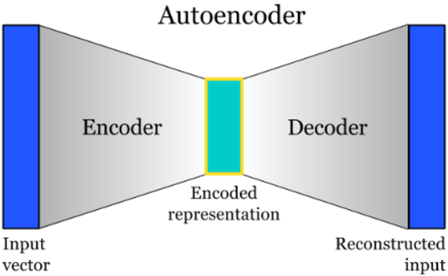


Figure: Cramer-Wold AutoEncoder.

Sliced-Wasserstein Autoencoder

- The modification introduced in SWAE relied on the use of the sliced Wasserstein distance to express $d(P_{\mathcal{E}(X)}, P_{\mathcal{Z}})$.
- The main idea was to take the mean of the Wasserstein distances between one-dimensional projections of $P_{\mathcal{E}(X)}$ and $P_{\mathcal{Z}}$ on a sampled collection of one-dimensional directions.

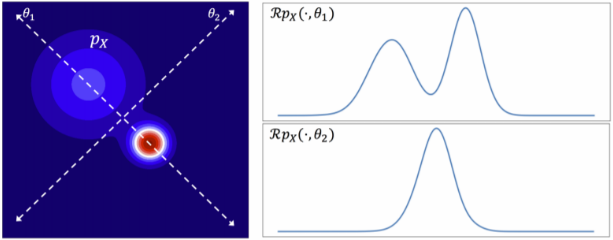


Figure: Sliced model.

Sliced-Wasserstein Autoencoder

- Consequently in SWAE two types of sampling were applied:
  - sampling over one-dimensional projections
  - sampling from the prior distribution $P_{\mathcal{Z}}$.

Thank you for your attention.