Gap Crossing Problem with CasADi

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Consider a robot with $n_c = 4$ contact points with the ground.
The quadruped is modeled as two compass bipeds linked by a prismatic joint. The bipeds have mass $m_1 = 50$ kg and $m_2 = 50$ kg, while the link between them is mass–less. We denote with $\boldsymbol{r}_{CoM,j}$ the position of the CoM of the $j-$th biped, where $j \in \{1(B),\, 2(F)\}$, and with $\boldsymbol{r}_{CoM,whole}$ the position of the CoM of the whole system, computed as

$$\boldsymbol{r}_{CoM,whole} = \frac{m_1\, \boldsymbol{r}_{CoM,1} + m_2\, \boldsymbol{r}_{CoM,2}}{m_1 + m_2}.$$

State: $\boldsymbol{x} = [\boldsymbol{r}^{[1]}_{CoM,j} \dots \boldsymbol{r}^{[n_{step}]}_{CoM,j}\ \boldsymbol{r}^{[1]}_{C,i} \dots \boldsymbol{r}^{[n_{step}]}_{C,i}\ \boldsymbol{F}^{[1]}_{C,i} \dots \boldsymbol{F}^{[n_{step}]}_{C,i}]$,
with $i \in \{1(BR),\, 2(BL),\, 3(FR),\, 4(FL)\}$. The superscript $[k]$ denotes the value of the variable at the $k-$th step. B: back, F: front, R: right, L: left.
We want to find the sequence $\boldsymbol{x}^*$ that moves the robot from $\boldsymbol{r}^{start}_{CoM,j}$ to $\boldsymbol{r}^{goal}_{CoM,j}$ solving the following NLP problem:

$$\min_{\boldsymbol{x}}\ w_{CoM} \sum_{k=1}^{n_{step}} \sum_{j=1}^{2} \|\boldsymbol{r}^{[k]}_{CoM,j} - \boldsymbol{r}^{goal}_{CoM,j}\|^2 + w_F \sum_{k=1}^{n_{step}} \|\boldsymbol{F}^{[k]}_{C}\|^2 \tag{1}$$

$$(m_1 + m_2)\, \boldsymbol{g} + \boldsymbol{G}_{CD} \boldsymbol{F}^{[k]}_c = \boldsymbol{0} \quad \text{centroidal dynamics} \tag{2}$$

$$\boldsymbol{f}_{env}(\boldsymbol{r}_{C,i}) = \boldsymbol{0} \quad \text{feet touch the floor} \tag{3}$$

$$\begin{cases} \boldsymbol{F}_{C,i} \cdot \boldsymbol{n}_{C,i} > F_{thr} \\ \|\boldsymbol{F}^t_{C,i}\| \le \mu_i (\boldsymbol{F}_{C,i} \cdot \boldsymbol{n}_{C,i}) \end{cases} \quad \text{friction cones} \tag{4}$$

$$\|\boldsymbol{r}^{[k]}_{C,i} - \boldsymbol{r}^{[k+1]}_{C,i}\|^2 \le 0.09 \quad \text{the step is inside a circle of radius 0.3, centered in } \boldsymbol{r}^{[k]}_{C,i} \tag{5}$$

$$1 \le \|\boldsymbol{r}^{[k]}_{C,BR} - \boldsymbol{r}^{[k]}_{C,FR}\|^2 \le 9 \quad \text{distance between feet is bounded} \tag{6}$$

$$1 \le \|\boldsymbol{r}^{[k]}_{C,BL} - \boldsymbol{r}^{[k]}_{C,FL}\|^2 \le 9 \quad \text{"} \tag{7}$$

$$0.25 \le \|\boldsymbol{r}^{[k]}_{C,BR} - \boldsymbol{r}^{[k]}_{C,BL}\|^2 \le 2.25 \quad \text{"} \tag{8}$$

$$0.25 \le \|\boldsymbol{r}^{[k]}_{C,FR} - \boldsymbol{r}^{[k]}_{C,FL}\|^2 \le 2.25 \quad \text{"} \tag{9}$$

$$\|\boldsymbol{r}^{[k]}_{C,i} - \boldsymbol{r}^{[k+1]}_{C,i}\|^2\ \|\boldsymbol{r}^{[k]}_{C,j} - \boldsymbol{r}^{[k+1]}_{C,j}\|^2 = 0,\ i \ne j \quad \text{only one foot can be moved} \tag{10}$$

$$\|\boldsymbol{r}^{[k]}_{CoM,j} - \boldsymbol{r}^{[k]}_{C,i}\|^2 = 0.5,\ \text{i = 1, 2 if j = 1 or i = 3, 4 if j = 2} \tag{11}$$

the distance between CoMs and feet is bounded

where:

$$
\begin{aligned}
\boldsymbol{G}_{CD} &= \begin{bmatrix} \boldsymbol{I}_3 & \dots & \boldsymbol{I}_3 \\ skewsym(\boldsymbol{r}_{C,1}^{[k]} - \boldsymbol{r}_{CoM,whole}^{[k]}) & \dots & skewsym(\boldsymbol{r}_{C,n_c}^{[k]} - \boldsymbol{r}_{CoM,whole}^{[k]}) \end{bmatrix} \\
\boldsymbol{F}_{C,i}^{n} &= (\boldsymbol{F}_{C,i} \cdot \boldsymbol{n}_{C,i})\boldsymbol{n}_{C,i} \\
\boldsymbol{F}_{C,i}^{t} &= \boldsymbol{F}_{C,i} - \boldsymbol{F}_{C,i}^{n} \\
\boldsymbol{n}_{C,i} &= -\frac{\nabla \boldsymbol{f}_{env}(\boldsymbol{r}_{C,i})}{\|\nabla \boldsymbol{f}_{env}(\boldsymbol{r}_{C,i})\|}.
\end{aligned}
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