Gap Crossing Problem with CasADi

Francesco Roscia, Matteo Sodano

Consider a robot with $n_c = 4$ contact points with the ground.
The quadruped is modeled as two compass bipeds linked by a prismatic joint. We denote with $\boldsymbol{r}_{CoM,j}$ the position of the CoM of the $j-$th biped, where $j \in \{1(B),\, 2(F)\}$.
State: $\boldsymbol{x} = [\boldsymbol{r}^{[1]}_{CoM,j} \dots \boldsymbol{r}^{[n_{step}]}_{CoM,j}\; \boldsymbol{r}^{[1]}_{C,\,i} \dots \boldsymbol{r}^{[n_{step}]}_{C,\,i}\; \boldsymbol{F}^{[1]}_{C,\,i} \dots \boldsymbol{F}^{[n_{step}]}_{C,\,i}]$,
with $i \in \{1(BR),\, 2(BL),\, 3(FR),\, 4(FL)\}$. The superscript $[k]$ denotes the value of the variable at the $k-$th step. B: back, F: front, R: right, L: left.
We want to find the sequence $\boldsymbol{x}^*$ that moves the robot from $\boldsymbol{r}^{start}_{CoM,\,j}$ to $\boldsymbol{r}^{goal}_{CoM,\,j}$ solving the following NLP problem:

$$\min_{\boldsymbol{x}}\; w_{CoM} \sum_{k=1}^{n_{step}} \sum_{j=1}^{2} \|\boldsymbol{r}^{[k]}_{CoM,\,j} - \boldsymbol{r}^{goal}_{CoM,\,j}\|^2 + w_F \sum_{k=1}^{n_{step}} \|\boldsymbol{F}^{[k]}_C\|^2 \tag{1}$$

$$m\boldsymbol{g} + \boldsymbol{G}_{CD}\boldsymbol{F}^{[k]}_c = \boldsymbol{0} \quad \text{centroidal dynamics} \tag{2}$$

$$\boldsymbol{f}_{env}(\boldsymbol{r}_{C,\,i}) = \boldsymbol{0} \quad \text{feet touch the floor} \tag{3}$$

$$\begin{cases} \boldsymbol{F}_{C,\,i} \cdot \boldsymbol{n}_{C,\,i} > F_{thr} \\ \|\boldsymbol{F}^t_{C,\,i}\| \le \mu_i(\boldsymbol{F}_{C,\,i} \cdot \boldsymbol{n}_{C,\,i}) \end{cases} \quad \text{friction cones} \tag{4}$$

$$\|\boldsymbol{r}^{[k]}_{C,\,i} - \boldsymbol{r}^{[k+1]}_{C,\,i}\|^2 \le 0.09 \quad \text{the step is inside a circle of radius 0.3, centered in } \boldsymbol{r}^{[k]}_{C,\,i} \tag{5}$$

$$1 \le \|\boldsymbol{r}^{[k]}_{C,\,BR} - \boldsymbol{r}^{[k]}_{C,\,FR}\|^2 \le 9 \quad \text{distance between feet is bounded} \tag{6}$$

$$1 \le \|\boldsymbol{r}^{[k]}_{C,\,BL} - \boldsymbol{r}^{[k]}_{C,\,FL}\|^2 \le 9 \quad \text{''} \tag{7}$$

$$0.25 \le \|\boldsymbol{r}^{[k]}_{C,\,BR} - \boldsymbol{r}^{[k]}_{C,\,BL}\|^2 \le 2.25 \quad \text{''} \tag{8}$$

$$0.25 \le \|\boldsymbol{r}^{[k]}_{C,\,FR} - \boldsymbol{r}^{[k]}_{C,\,FL}\|^2 \le 2.25 \quad \text{''} \tag{9}$$

$$\|\boldsymbol{r}^{[k]}_{C,\,i} - \boldsymbol{r}^{[k+1]}_{C,\,i}\|^2\; \|\boldsymbol{r}^{[k]}_{C,\,j} - \boldsymbol{r}^{[k+1]}_{C,\,j}\|^2 = 0,\; i \ne j \quad \text{only one foot can be moved} \tag{10}$$

$$\|\boldsymbol{r}^{[k]}_{CoM,\,j} - \boldsymbol{r}^{[k]}_{C,\,i}\|^2 = 0.5,\; \text{i = 1, 2 if j = 1 or i = 3, 4 if j = 2} \tag{11}$$

the distance between CoMs and feet is bounded

where:

$$\boldsymbol{F}^n_{C,\,i} = (\boldsymbol{F}_{C,\,i} \cdot \boldsymbol{n}_{C,\,i})\boldsymbol{n}_{C,\,i}$$
$$\boldsymbol{F}^t_{C,\,i} = \boldsymbol{F}_{C,\,i} - \boldsymbol{F}^n_{C,\,i}$$
$$\boldsymbol{n}_{C,\,i} = -\frac{\nabla \boldsymbol{f}_{env}(\boldsymbol{r}_{C,\,i})}{\|\nabla \boldsymbol{f}_{env}(\boldsymbol{r}_{C,\,i})\|}.$$