Mechanics Vector Pair Trick

Problem 1 A book of mass M is positioned against a vertical wall. The coefficient of friction between the book and the wall is μ . To keep the book from falling, push on it with a force F applied at an angle θ with respect to the horizontal. What is the minimum force F needed? At what angle θ does it becomes impossible for the book to be kept up?

Problem 2 A block is placed on an inclined plane with angle of inclination θ . At what angle α with respect to the ground should you pull the block such that the block starts to move upwards at a constant velocity and that the tension force F_T is minimized? What is this minimum force? The coefficient of friction is μ .

Problem 3 A block is placed on an inclined plane with angle of inclination θ and coefficient of static friction μ . The condition $\mu < \tan \theta$ is met. The plane accelerates horizontally such that the block stays still with respect to the ramp. What is the *range* of accelerations that will make this possible?

Problem 4 A ladder is leaning on a smooth wall and makes an angle θ with the ground. The coefficient of friction between the ladder and the ground is μ . What is the minimum possible μ such that the ladder is in static equilibrium?

Hint: Take the intersection of two force vectors, and balance torques around that point. (The general result you'll find is that in a system with three forces, all force vectors must be concurrent).