2021 Online Physics Olympiad: Invitational Contest

Experimental Exam

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General Instructions

The experimental examination consists of 1 long answer question worth 50 points over 1 full day from August 15, 0:01 am GMT.

- **The team leader should submit their final solution document in this google form. We don't anticipate a tie, but in the rare circumstance that there is one, the time you submit will be used to break it.**
- If you wish to request a clarification, please use this form. To see all clarifications, view this document.
- Participants are given a google form where they are allowed to submit up-to 1 gigabyte of data for their solutions. It is recommended that participants write their solutions in LaTeX. However, handwritten solutions (or a combination of both) are accepted too. If participants have more than one photo of a handwritten solution (`jpg, png`, etc), it is required to organize them in the correct order in a `pdf` before submitting.

Specific Rules

For any part of this paper, you are allowed to use online tools and resources to help you, as long as you are not requesting help from anyone outside of your team. Allowable resources include Wikipedia, research papers, Wolfram Alpha, Python, Excel, etc.

However, you must document every resource that you use and cite them when applicable. As a general rule of thumb, you should derive any results that cannot be found on Wikipedia. Therefore, solutions along the lines of: "By Wolfram Alpha, this is true." will not be accepted. Be reasonable please.

Every time you are asked to run an experiment, you must provide the input parameters and a screenshot of the output.

Accessing the Program

To access the Python notebook, follow this **link**. You will be able to perform all the code online, without downloading anything. If you cannot access the link, we will also provide the source code on our website.

Background Information

In this problem, you will use a computer simulation written in *Python* to complete a series of questions relating to slit interference patterns. While you do not need to understand how exactly the code produces the results, it may be beneficial to understand the algorithm the code takes.

The program uses a simple algorithm to determine the interference pattern.

1. An aperture pattern is created by specifying `slits`.
2. The slits are divided into uniform $\lambda \times \lambda$ segments.
3. For each position x on the screen, a wave is generated (and represented as a phasor) between the point x and the center of *each* segment created in the previous step.
4. These phasors are added up and squared to get the intensity.
5. The final intensity pattern is normalized such that the maximum value is 1.

For this entire problem, assume that the heights of the slits are λ.

Part One

Let us first gain confidence in using the program. To do so, we will derive

$$\frac{\sin(x)}{x} = \cos(x/2)\cos(x/4)\cos(x/8)\cdots \tag{1}$$

via a series of questions.

Problem 1.1 (1 pt)

Suppose monochromatic, coherent light of wavelength λ falls down onto two slits of width w, with midpoints separated by a distance d. Find the amplitude function $A(\theta)$ for the interference pattern produced as a result, where θ is the deviation angle from the center. Assume the screen is far away. A proof is not necessarily necessary, but will help with partial credit in the event that your answer is wrong.

Problem 1.2 (4 pts)

Currently, the code simulates the interference patterns of two slits with width $w = \lambda$ separated by a distance of $d = 10\lambda$. Modify the code to simulate:

- A double slit experiment with wide slits as described in problem 1.

The program will output the intensity function. Make note of the minimum and maximum in the intensity in the experimental results, and compare it to the theoretical results (which you derived in the previous question). Do they agree?

If they do not agree, provide possible reasoning to why they do not agree.

Problem 1.3 (3 pts)

Suppose we have some pattern of slits with overall width w that produces an interference pattern with amplitude $A_0(\theta)$. Suppose we place two of these patterns with midpoints separated a distance $d \geq w$ apart (so that they do not overlap). Find, with proof, as a function of A_0 and other parameters, the amplitude function $A(\theta)$ for the interference pattern produced as a result.

Verify this result experimentally using the code.

Problem 1.4 (5 pts)

Complete the problem by showing

$$\frac{\sin(x)}{x} = \cos(x/2)\cos(x/4)\cos(x/8)\cdots \tag{2}$$

There is no experimental portion associated with this part.

Part Two

Problem 2.1 (8 pts)

Recall that the intensity of the interference pattern from two thin slits behaves like $\cos^2(x)$. Is it possible to have a series of thin slits such that the *amplitude* pattern behaves exactly like the intensity pattern from the two thin slits case? Specifically, the amplitude $A_1(x)$ from one pattern of slits behaves like the intensity $I_2(x)$ from another pattern of slits if:

$$\frac{I_2(x)}{I_{2,\max}} = \frac{A_1(x)}{A_{1,\max}} \tag{3}$$

What sort of aperture would create such an $A(x)$? Verify this experimentally. You will notice that the pattern will deviate towards the edge. At what angle does this deviation become significant? Note that "significant" is subjective, so you will need to provide justification for how you define significant.

Solve this problem using Fourier Optics (There are at least 2 separate ways to do so. As long as you borrow concepts from Fourier Optics, you will receive full points):

- Here are four potentially useful references from Wikipedia. **Any result you use that is not in these references must be derived. This holds for the following two problems as well.**
  - Fraunhofer Diffraction Equation
  - Fourier Optics
  - Convolution
  - Fourier Transform

  Remember that asking for help on public forums or seeking help from other students, other teams, professors, i.e. is strictly prohibited.

Problem 2.2 (4 pts)

In the previous question, you have constructed a series of thin slits such that the amplitude behaves like $\cos^2(x)$. As a result, the intensity behaves like $\cos^4(x)$.

Now, construct a series of slits such that the amplitude function behaves like the intensity pattern from the previous question, i.e. $\cos^4(x)$. You may choose to verify this experimentally, but it is not necessary to get full marks.

Problem 2.3 (4 pts)

Now generalize this to an arbitrary $\cos^n(x)$. If you want the amplitude function to behave like $\cos^n(x)$ where n is a non-negative integer, what should the slit pattern be?

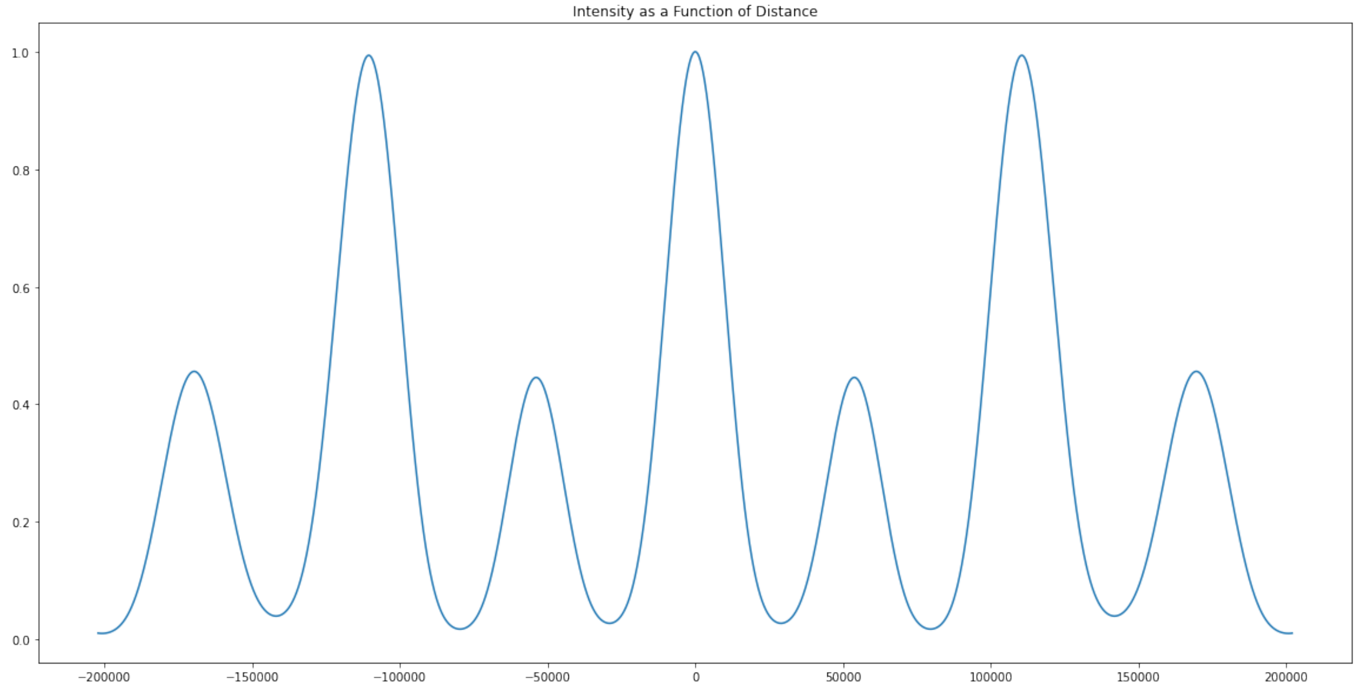
Hint: If you are stuck, try coming up with a conjecture based off of the $\cos^2(x)$ and $\cos^4(x)$ cases, list out how much amplitude they let in, and have everything share a common denominator. If you did the previous parts correctly, the numerators should follow a very familiar pattern.

Part Three

In the previous two parts, you have mostly been asked questions that can be solved analytically, and then used the code to double check your answer. Now, we will ask a few questions that requires data analysis.

Problem 3.1 (11 pts)

The following diagram shows the intensity of the interference pattern produced by a series of slits (where each slit can reduce the wavelength by some factor). The wall is 25 cm away and 500 nm light is used. The x-axis represents locations on the screen, with units of λ.

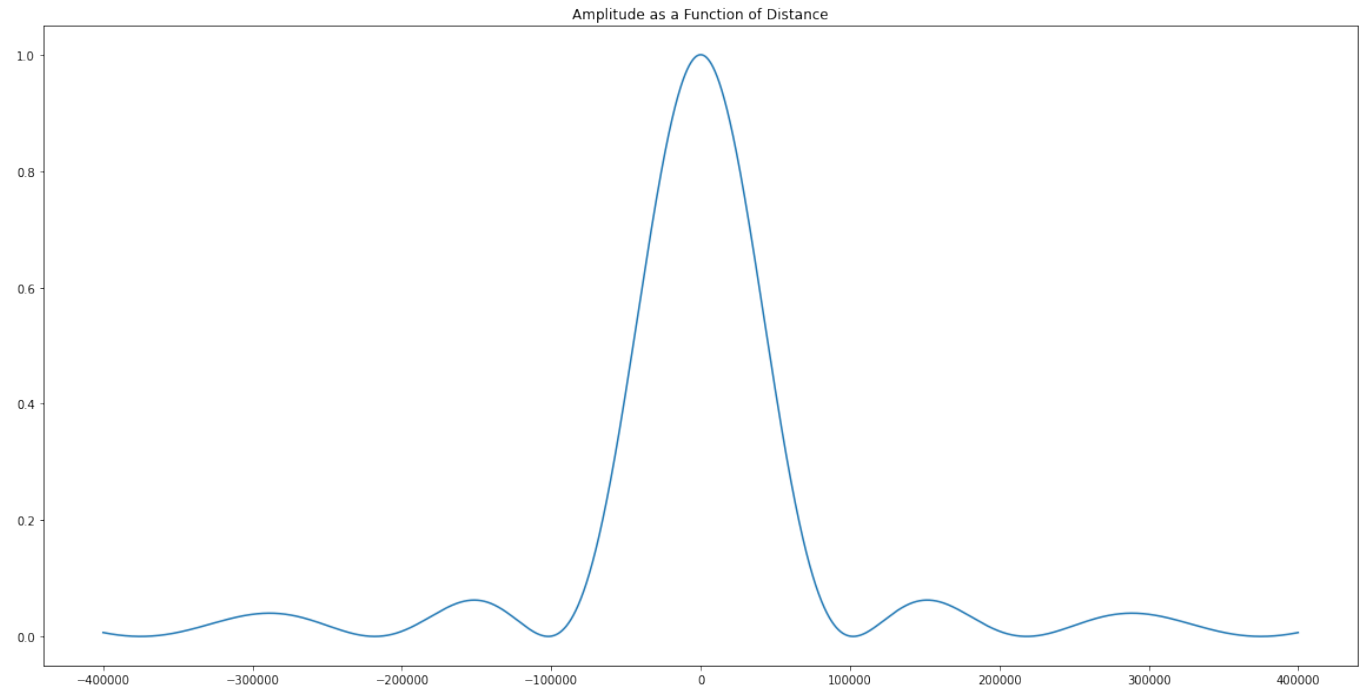


Determine the aperture pattern to the best of your ability. The raw data file is attached on the website.

Note: You may use any online tool and/or resource to do this problem, so long as you are not asking help from people outside your team.

Problem 3.2 (7 pts)

The following diagram shows the **amplitude** of the interference pattern produced by a series of 9 thin slits, centered at $-4\lambda, -3\lambda, \ldots, +4\lambda$. Each slit reduces the wavelength by some factor (this factor isn't necessarily the same for each slit).



Again, the screen is the same distance away as the previous problem and the same scale is used. Determine how much each slit reduces the amplitude of light that passes through.

Let f_x be the reduction factor of the slit centered at x, which has units of λ. For example, the rightmost slit is located at $x = 4$. Make a plot of (x, f_x) for $0 \le x \le 4$ and make note of patterns you see.

Problem 3.3 (3 pts)

The interference pattern from the previous problem can be approximated (at least in the small angle range) by a relatively simple function. Find this function.

Hint: Look at the pattern that was hinted at in the previous question