ELE466 - Quarter Car model

Car simulation

Erick Stanzah, Nandana Lathesh, Yixuan Huo, Simon Martin

Supervisor(s): Prof. Dr Anthony Rossiter and Prof. Dr Viktor Fedun

07/11/25

Abstract

This report investigates the modelling and simulation of a simplified car suspension system comprising of three masses representing the wheel, chassis, and car seat. The aim was to develop a complete mathematical model describing the system's dynamic behaviour through mechanical analysis. A free-body diagram was constructed, and corresponding ordinary differential equations (ODEs) were derived and converted into matrix, Laplace, and state-space forms. These formulations were implemented in MATLAB and Simulink to simulate the system's response to various inputs and parameter changes. The simulation results demonstrate how parameter variations influence the dynamic performance of the suspension.

Table of Contents

1 The Plan

During the initial planning stage, a structured approach was established to ensure clear task allocation, consistent progress, and effective collaboration throughout the project. Regular meetings were scheduled to review progress, address challenges, and coordinate the development of the suspension model and simulations. Each meeting defined specific objectives and assigned responsibilities to ensure that the modelling and analysis tasks were completed efficiently.

The technical workflow followed a staged progression:

- **Problem Analysis** – A detailed free-body diagram of the mechanical system was constructed to identify all relevant forces, displacements, and interactions.
- **Mathematical Derivation** – The governing ordinary differential equations (ODEs) describing the motion were derived, converted into Laplace domain, and expressed in the matrix form for analytical examination.
- **State-Space Representation** – The equations were reformulated into a state-space model suitable for numerical simulation and control analysis.
- **Simulation and Analysis** – The derived models were implemented in MATLAB and Simulink, allowing the system's response to various input conditions and parameter variations to be evaluated.

This structured methodology ensured a clear connection between theoretical derivation and practical implementation. A shared online workspace was used for organising MATLAB scripts, simulation files, and documentation, supporting efficient collaboration. Regular communication channels were maintained to coordinate ongoing work, and contingency measures were planned to address potential disruptions such as delays or technical issues.

2 Modelling Assumptions

2.1 Linearity of Springs and Dampers

Both springs and dampers are assumed to behave linearly. The restoring spring force is proportional to displacement, following **Hooke's Law:**

$$F_s = -k\,x \tag{1}$$

where F_s is the spring force, k is stiffness (N/m), and x is displacement from equilibrium. The negative sign shows the force acts opposite to the displacement.

Damping forces are **viscous**, being proportional to relative velocity:

$$F_d = -c\,\dot{x} \tag{2}$$

where F_d is the damping force, c is the damping coefficient (N·s/m), and $\dot{x}$ is velocity. These linear relationships keep k_i and c_i constant, simplifying the system to linear differential equations.

2.2 Small Displacements

It is assumed that all displacements and velocities are sufficiently small such that:

- Geometric non-linearities and large-angle effects can be neglected.
- The system behaves linearly, with no large deformations or rotations.
- Linear ordinary differential equations (ODEs) and the principle of superposition are valid.

2.3 Lumped Parameter Model

Each body (wheel, chassis, and seat) is represented by a single lumped mass point:

- M_1: Wheel/axle assembly.
- M_2: Chassis.
- M_3: Seat and occupant.
- Each mass moves only in the vertical direction (with upwards forces considered as positive); rotational and lateral dynamics are neglected.
- Springs and dampers connect these masses directly, and their effects are not spatially distributed.

2.4 No External Disturbances or Friction

- Apart from the road excitation $z(t)$, no other external forces (such as aerodynamic drag or rolling friction) are considered.
- Damping arises solely from the defined dampers C_1, C_2, and C_3.

2.5 Lack of Gravity in Dynamic Equations

- The model uses displacements from equilibrium, so the constant gravitational term Mass * gravity cancels out.

- The ODEs represent motion about the static equilibrium position.

3 System Modelling

This section develops the mathematical model of the three-mass car suspension system, representing the wheel/axle (M_1), chassis (M_2), and seat/occupant (M_3). The model describes vertical dynamics using linear springs, dampers, and masses connected in series. The objective is to derive the governing equations of motion, express them in Laplace form, and obtain a state-space representation suitable for MATLAB and Simulink analysis.

3.1 Parameters and Inputs

Table 1 lists the system parameters and their physical meaning. The road disturbance $z(t)$ is defined as the vertical input displacement acting on the wheel, representing a bump or uneven surface.

Table 1: System parameters and descriptions.

Symbol	Description	Units
M_1, M_2, M_3	Mass of wheel, chassis, and seat	kg
k_1, k_2, k_3	Spring stiffnesses	N/m
c_1, c_2, c_3	Damping coefficients	N·s/m
$z(t)$	Road input displacement	m

3.2 System Description

The wheel (M_1) is connected to the road through the tire spring k_3 and to the chassis (M_2) through k_2 and c_2. The chassis is connected to the seat (M_3) via k_1, c_1, and c_3. All motion is assumed vertical, with y_1, y_2, y_3 representing absolute displacements of each mass.

3.3 Equations of Motion

Applying Newton's Second Law to each mass gives:

$$M_1 \ddot{y}_1 = k_2(y_2 - y_1) + c_2(\dot{y}_2 - \dot{y}_1) + k_3(z - y_1) \tag{3}$$

$$M_2 \ddot{y}_2 = (c_1 + c_3)(\dot{y}_3 - \dot{y}_2) + c_2(\dot{y}_1 - \dot{y}_2) + k_1(y_3 - y_2) + k_2(y_1 - y_2) \tag{4}$$

$$M_3 \ddot{y}_3 = (c_1 + c_3)(\dot{y}_2 - \dot{y}_3) + k_1(y_2 - y_3) \tag{5}$$

Here, $k_i(y_j - y_i)$ represents spring forces, and $c_i(\dot{y}_j - \dot{y}_i)$ represents viscous damping opposing relative motion.

3.4 Laplace-Domain Representation

Assuming zero initial conditions, the Laplace transforms become:

$$M_1 s^2 Y_1 = (C_2 s + K_2)(Y_2 - Y_1) + K_3(Z - Y_1), \tag{6}$$

$$M_2 s^2 Y_2 = (C_1 + C_3)s(Y_3 - Y_2) + (C_2 s + K_2)(Y_1 - Y_2) + K_1(Y_3 - Y_2), \tag{7}$$

$$M_3 s^2 Y_3 = (C_1 + C_3)s(Y_2 - Y_3) + K_1(Y_2 - Y_3). \tag{8}$$

$$\begin{bmatrix} M_1 s^2 + C_2 s + k_2 + k_3 & -(C_2 s + k_2) & 0 \\ -(C_2 s + k_2) & M_2 s^2 + (C_1 + C_2 + C_3)s + k_1 + k_2 & -\big((C_1 + C_3)s + k_1\big) \\ 0 & -\big((C_1 + C_3)s + k_1\big) & M_3 s^2 + (C_1 + C_3)s + k_1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} k_3 \\ 0 \\ 0 \end{bmatrix} Z \tag{9}$$

3.5 State-Space Representation

Defining the state vector:

$$x = \begin{bmatrix} \dot{y}_1 & \dot{y}_2 & \dot{y}_3 & y_1 & y_2 & y_3 \end{bmatrix}^{\mathrm{T}}, \quad \dot{x} = Ax + Bz, \quad y = Cx + Dz$$

where:

$$
A = \begin{bmatrix}
-\frac{C_2}{M_1} & \frac{C_2}{M_1} & 0 & \frac{-K_2-K_3}{M_1} & \frac{K_2}{M_1} & 0 \\[4pt]
\frac{C_2}{M_2} & \frac{-C_1-C_2-C_3}{M_2} & \frac{C_1+C_3}{M_2} & \frac{K_2}{M_2} & \frac{-K_1-K_2}{M_2} & \frac{K_1}{M_2} \\[4pt]
0 & \frac{C_1+C_3}{M_3} & \frac{-C_1-C_3}{M_3} & 0 & \frac{K_1}{M_3} & -\frac{K_1}{M_3} \\[4pt]
1 & 0 & 0 & 0 & 0 & 0 \\[4pt]
0 & 1 & 0 & 0 & 0 & 0 \\[4pt]
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}, \quad
B = \begin{bmatrix}
\frac{K_3}{M_1} \\[4pt]
0 \\[4pt]
0 \\[4pt]
0 \\[4pt]
0 \\[4pt]
0
\end{bmatrix}
$$

Typical output matrices are:

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix} \qquad
D = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

This representation enables direct simulation using MATLAB's `ss()` and `step()` functions or Simulink's State-Space block.

3.6 Static Equilibrium

At rest, gravitational forces $M_i g$ are balanced by static spring extensions. As the model describes small oscillations about equilibrium, these constant terms cancel, allowing $M_i g$ to be omitted from the dynamic equations.

4 Simulation

4.1 Model Implementation

The suspension system was implemented in SIMULINK using a mass–spring–damper structure. Each mass (wheel, chassis, and seat) was modelled as an individual subsystem, with springs and dampers defining the force relationships between them. This block-based arrangement reflects the physical structure of the suspension and allows the vertical dynamics to be visualised clearly during simulation.

A separate MATLAB parameter script **ELE466_group_1.mlx** was used to define all

mass, stiffness, and damping values. Using a single parameter file allows fast modification of suspension characteristics without altering the SIMULINK MODEL, enabling efficient tuning and sensitivity analysis.

To verify that the mathematical model had been implemented correctly, three equivalent Simulink models were created:

- **Equations-of-Motion Model:** constructed directly from the force-balance expressions for each mass. This model was used as the primary simulation framework, as its structure made it straightforward to modify suspension parameters. For example, the additional damper C_4 could be introduced and later removed during tuning without requiring major changes to the model.

- **Transfer-Function Model:** formed by applying the Laplace Transform to the differential equations and implementing the resulting transfer functions in Simulink.

- **State-Space Model:** implemented using the A, B, C, and D matrices derived from the system equations.

To view Simulink in a larger view click the following link: (9.2)

(a) Main System

(b) Wheel Sub-System

(c) Chassis Sub-System

(d) Seat Sub-System
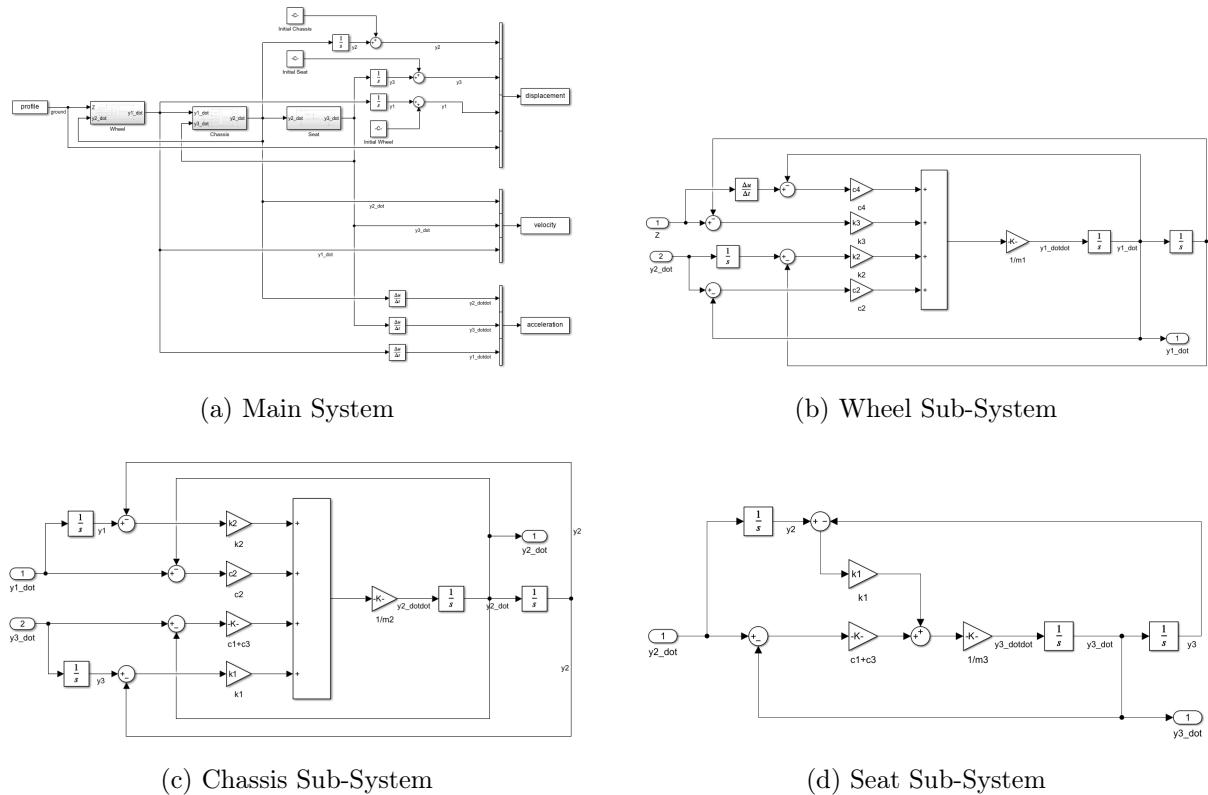
Figure 1: System and Sub-Systems based on the Equations of Motion

All models were subjected to the same road input $z(t)$, and the resulting displacements

and accelerations (y_1, y_2, y_3) were compared. The responses matched across all three implementations, confirming that the equations of motion, transfer-function form, and state-space representation were consistent and correctly implemented.

# 5   Results and Analysis: Comparison of control and Tuned Suspension

The model was evaluated under three types of road excitation: a bump input, a wavy road input and a upwards slope. For each case, the responses of the Control suspension and the Tuned suspension were compared in terms of displacement, velocity and acceleration of the wheel, chassis and seat masses.

5.1   Upwards Slope Input

The upwards slope input (9.1.2) represents a gradual increase in the height of the road and therefore acts as a **low-frequency excitation**. In the **Control** suspension, the chassis and seat responses exhibit a **lag followed by overshoot**, meaning the vehicle body rises later than the wheel and then rises too far. This behaviour is visible in both the displacement and velocity responses, where the control system shows a **sharper rise and longer settling period**. Small but noticeable acceleration peaks are also present, indicating that some of the road elevation change is still transmitted to the seat.

The **Tuned** suspension responds more smoothly to the same input. The chassis and seat displacement follow the slope **more gradually**, with **reduced overshoot** and **lower peak velocity**. The acceleration response is also significantly smoother. This indicates that the increased damping provides **better control of low-frequency body motion**, allowing the vehicle to adjust to gradual changes in road height without unnecessary oscillation.

Overall, the tuned system offers **improved stability and ride comfort** when travelling over slopes, speed platforms, or gentle surface transitions.

(a) Displacement Tuned (Upwards Slope Input)

(b) Velocity Tuned (Upwards Slope Input)

(c) Acceleration Tuned (Upwards Slope Input)
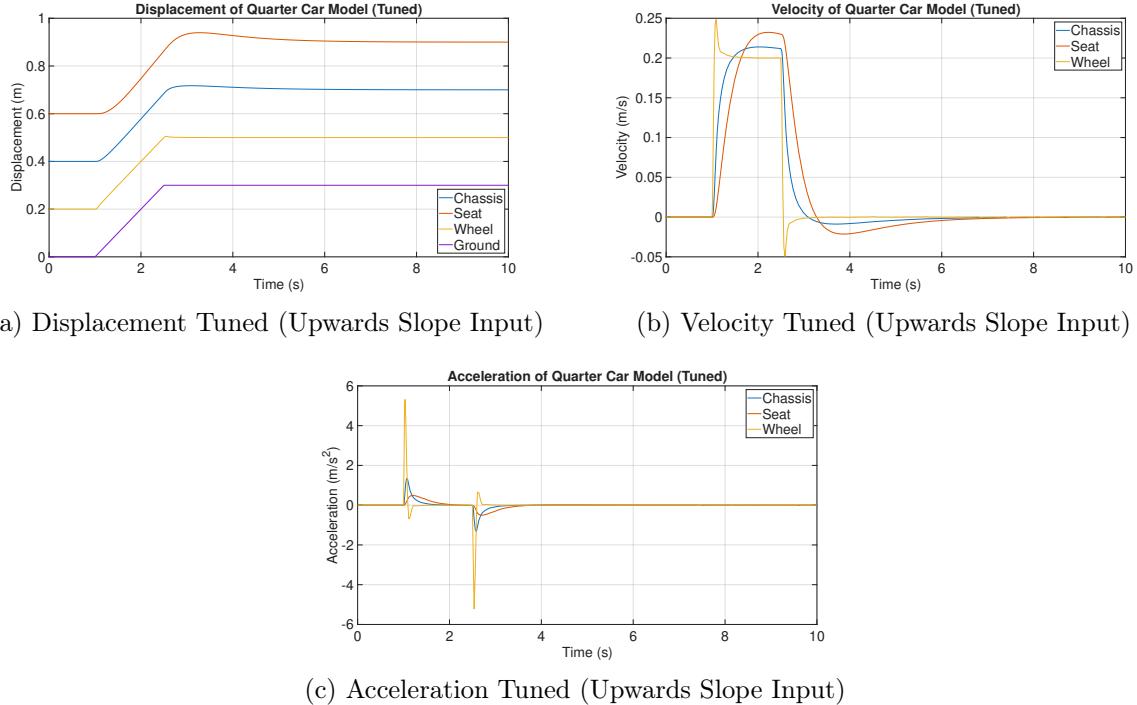
Figure 2: Response of Tuned Suspension to Upwards Slope Input

5.2 Bump Input

The bump input (9.1.5) produces a **sudden and high-frequency disturbance** to the wheel. In the **Control** suspension, this results in a **sharp increase in seat and chassis acceleration**, indicating that a significant portion of the impact is transmitted into the vehicle body. The response also shows **overshoot and sustained oscillation** before settling.

In contrast, the **Tuned** suspension shows a **reduced peak acceleration** and a **faster decay**, demonstrating improved **shock absorption**. The lower oscillation amplitude indicates that the tuned damping **dissipates energy more effectively**, resulting in a smoother and more controlled transient response.

5.3 Wavy Road Input:

The wavy road input (9.1.4) introduces a **continuous, periodic variation** in road height. For the **Control** suspension, both velocity and acceleration responses show **sustained oscillations**, meaning vibrations are transmitted through the chassis and seat with limited attenuation. This reflects **insufficient damping** for continuous, mid-frequency excitation.

The **Tuned** suspension exhibits **lower oscillation amplitude** and **faster decay** of periodic disturbances. This leads to improved **vibration isolation**, particularly at the

8

seat mass, which is closely associated with the occupant's perceived ride comfort. When the wavy road amplitude was increased to approximately $2\,\text{m}$ trough-to-peak (9.1.3), the suspension travel was exceeded, causing the wheel, chassis, and seat masses to move into contact during the return phase of each wave. After consideration, it was determined that this input does not represent realistic road conditions, as such large surface variations are not typically encountered in normal driving. Therefore, the performance evaluation focused on more representative inputs (bump, slope, and moderate wavy profiles) to provide meaningful and practical conclusions regarding suspension behaviour.

5.4 Overall Performance Evaluation

Across all road input conditions, the **Tuned suspension** consistently demonstrated improved dynamic response. In particular, it:

- **Reduces seat displacement and acceleration** (by lowering the values of k1 and increasing c1 + c3), leading to **improved ride comfort**.
- **Lowers oscillation amplitude**, indicating **more effective damping performance**.
- **Improves settling behaviour**, allowing a **faster return to equilibrium** after disturbances.
- **Maintains chassis stability** (smaller k2 produces smoother chassis oscillations) while still **reducing vibration transmission** to the upper masses.
- **Reduces the wheel's acceleration** (affected by lowering k3 and increasing c2), causing less oscillations on the wheel.

In contrast, the **Control suspension** exhibits **greater vibration transmission** and **slower damping**, particularly under continuous or periodic road excitations.

These results confirm that **tuning the damping and stiffness parameters** leads to a **clear improvement in ride comfort** without **compromising overall system stability**.

Note on Damper C_4

An additional damper C_4 was initially placed in parallel with the tire spring k_3. However, simulations showed that C_4 did not improve ride comfort and sometimes increased high-frequency vibration transmission at the wheel. Since tuning the existing parameters already achieved the desired performance, C_4 was removed and the tire was modelled as a spring only.

6   Model validation

To ensure that the mathematical model and its implementation were correct, several Simulink models of the suspension system were constructed and compared.

The main model (from the equations of motion), contains the full suspension structure with the "subsystems" representing each mass (Wheel, chassis, and seat). This configuration reflects the physical layout of the system and was used for tuning and performance analysis.

Two validate the equations of motion, two simplified models were also built.

- State-Space Matrix (Figure 35). This model uses the A,B,C, and D matrices from the system equations. It's only purpose is to verify that the state-space representations matches the physical block model.

- Laplace/ Transfer Function Model (Figure 34). This model directly uses the Laplace-domain expressions derived from the differential equations. The purpose of this model is to confirm that the equations of motion were correctly formed.

7   Improvements

Although the model provides a useful representation of the suspension behaviour, several limitations exist suggesting potential areas for future improvement.

- Introduce **non-linear damper and spring models** to more accurately represent real suspension components.

- Extend the model to include **pitch and roll motion** of the chassis (full-car or half-car model).

- Replace the tyre spring with a **tyre model including damping and contact patch dynamics**.

- Incorporate **variable mass or load-dependent stiffness** to represent different seating conditions.

- Implement a **semi-active or active suspension controller** to further reduce seat acceleration and improve ride comfort.

8   Conclusion

This report presented the modelling, simulation and analysis of a quarter-car suspension system consisting of three masses connected by linear springs and dampers. The equations

of motion were derived and implemented in SIMULINK, with additional state-space and transfer-function models used to validate mathematical correctness. All models were shown to produce consistent responses, confirming the validity of the derived system representation.

The system was evaluated under upwards slope, bump, and wavy road inputs. The tuned suspension demonstrated **reduced seat acceleration**, **lower oscillation amplitudes**, and **faster settling** compared to the control configuration, indicating improved ride comfort while maintaining chassis stability. During the modelling process, an additional damper C_4 was initially included at the wheel–axle interface. However, simulation results showed that this provided no improvement in vibration isolation and in some cases increased high-frequency transmission. Therefore, the tire was modelled as a **spring only**, consistent with standard quarter-car models.

Overall, the results show that appropriate tuning of stiffness and damping parameters can significantly improve passenger comfort without compromising dynamic performance, and that simplifying the tire to a pure spring element provides an accurate and efficient representation for this class of suspension analysis.

9   Appendix

Group Meeting Log

Erick Stanzah (ES), Nandana Lathesh (NL), Yixuan Huo (YH), Simon Martin (SM)

Date	In Attendance	Summary of Discussion	Plan for Next Meeting
09/10/25	ES, NL, SM, YH	First meeting. Introduced group members and discussed key strengths and roles.	Check lecture notes and reference sources as initial preparation.
13/10/25 (Morning)	ES, NL, SM, YH	Constructed free-body diagram and derived Equations of Motion. Verified results collaboratively.	Continue with Laplace Transformation and State-Space representation.
13/10/25 (Noon)	ES, NL, SM, YH	Performed Laplace Transform and derived State-Space Matrix.	Begin MATLAB scripting and Simulink model construction.
16/10/25	ES, NL, SM, YH	Constructed Simulink model from Equations of Motion and parameter file.	Determine suitable validation approach.
20/10/25	ES, NL, SM, YH	Created Transfer-Function and State-Space Simulink models for comparison.	Generate different road input functions.
23/10/25 (Morning)	ES, NL, SM, YH	Implemented upwards slope, bump, and wavy road inputs in MATLAB.	Run simulations with varying parameters.
23/10/25 (Noon)	ES, NL, SM, YH	Performed simulations and analysed impact of parameter changes.	Begin tuning suspension parameters.
27/10/25	ES, NL, SM, YH	Tuned model for improved response across all road conditions.	Begin writing report and planning presentation.
30/10/25	ES, NL, SM, YH	Drafted report and discussed presentation structure.	Prepare slides.
03/11/25	ES, NL, SM, YH	Completed presentation slides and rehearsed.	Finalise report and tidy MATLAB/Simulink code.
06/11/25	ES, NL, SM, YH	Final checks: report, code and Simulink cleaned and verified.	—

During the first meeting, we created a WhatsApp group chat as our method of communication and also did tasks allocation. However, we decided that everyone should do the derivation of the Equations of Motion as it's the first question and also to keep everyone on the same page. Afterwards, everyone did their parts equally and we kept track of each

other's progress during the lab or via the group chat. For long discussions, we had several group meetings outside of the lab hours. The only thing we would want to add for next time is to use a shared folder as we mostly used emails or sent the files on the group chat.

9.1 Evidence of Graphs

9.1.1 (c_4) Upwards slope Excitation
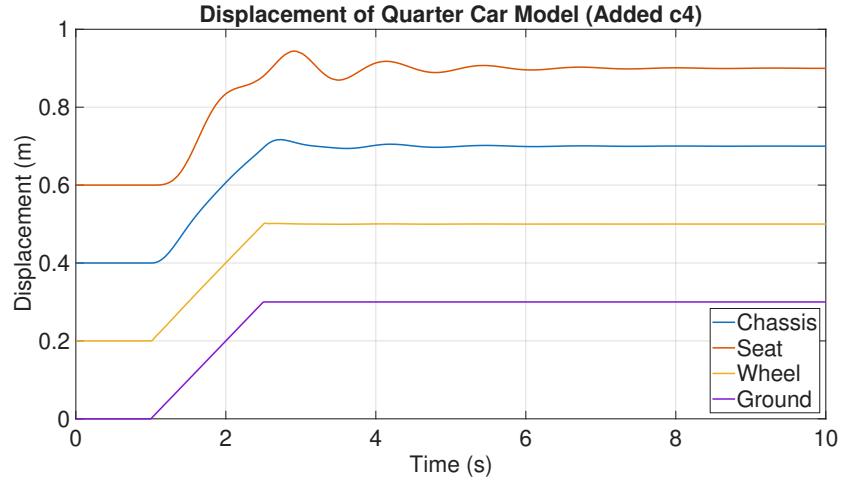
Figure 3: Displacement (c_4)
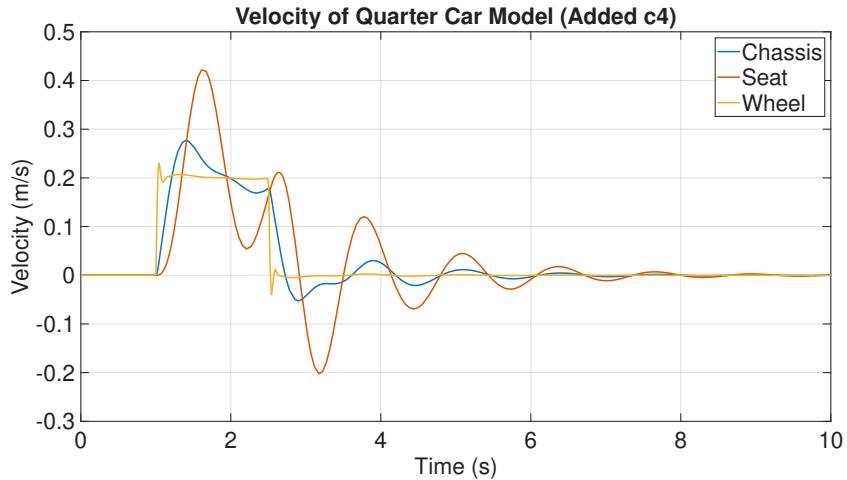
Figure 4: Velocity of (c_4)
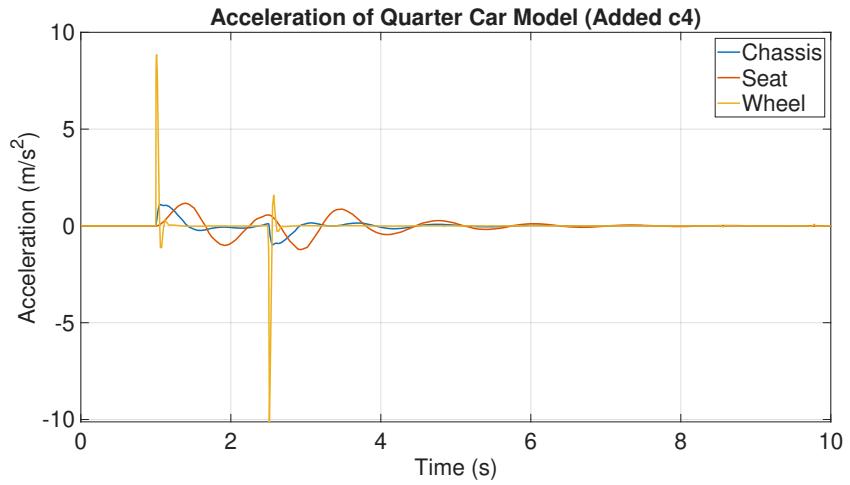
Figure 5: Acceleration (c_4)

9.1.2 (Upwards Slope Excitation)
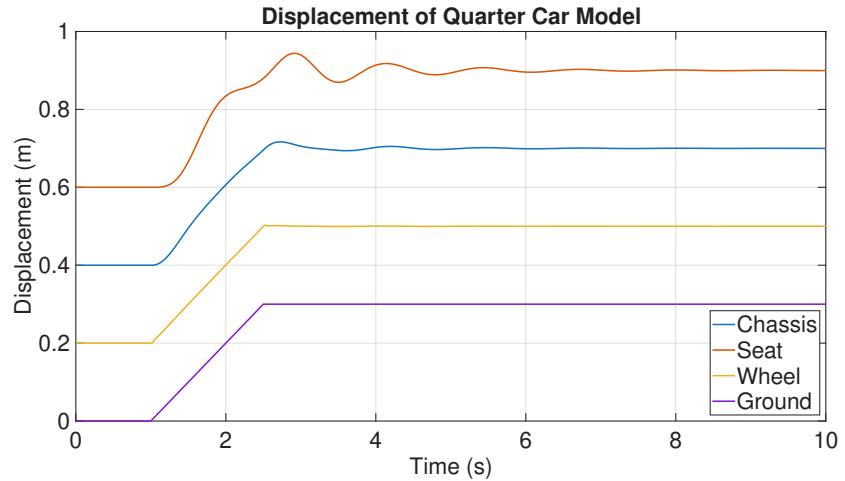
Figure 6: Upwards Slope Displacement Control
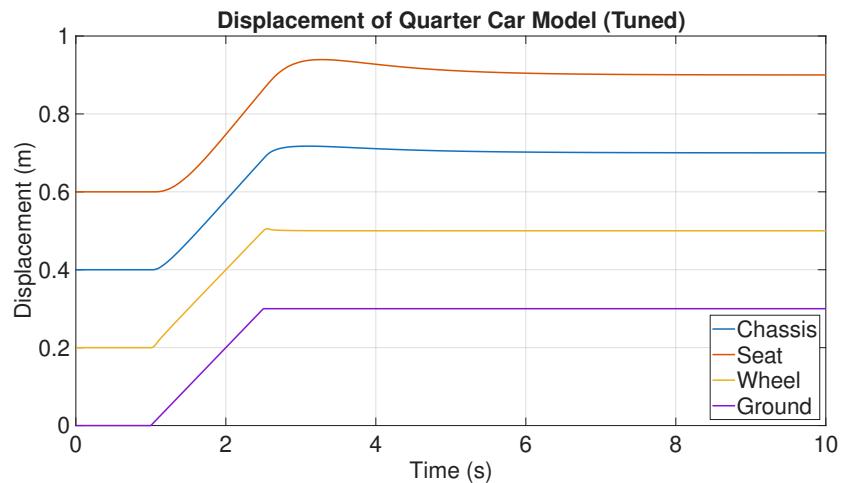
Figure 7: Upwards Slope Displacement Tuned
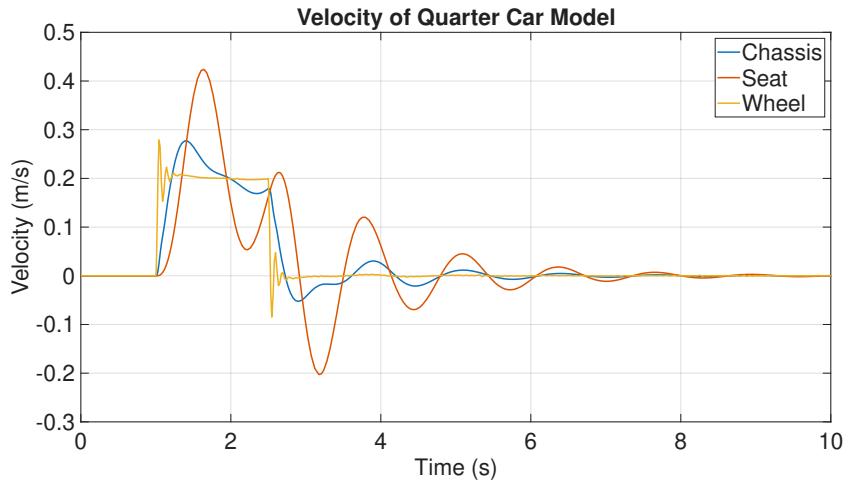
Figure 8: Upwards Slope Control Velocity
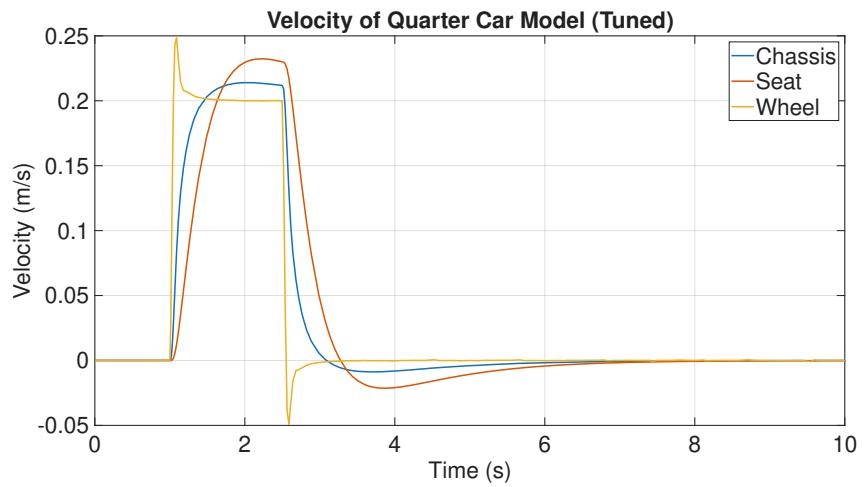
Figure 9: Upwards Slope Velocity Tuned
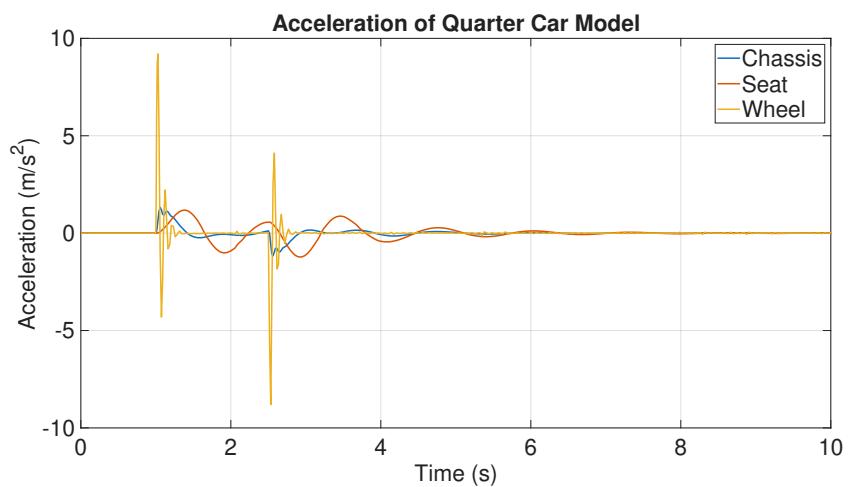
Figure 10: Upwards Slope Acceleration Control

15
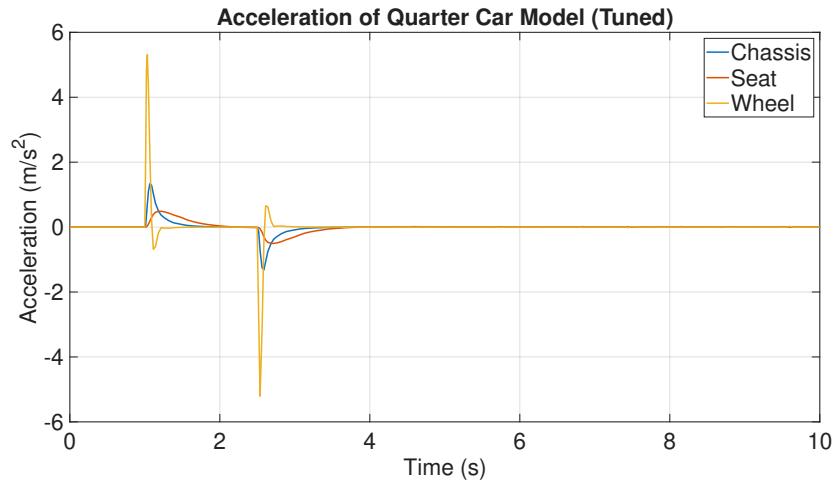
Figure 11: Upwards Slope Acceleration Tuned

9.1.3 Wavy Excitation 1 Meter Amplitude
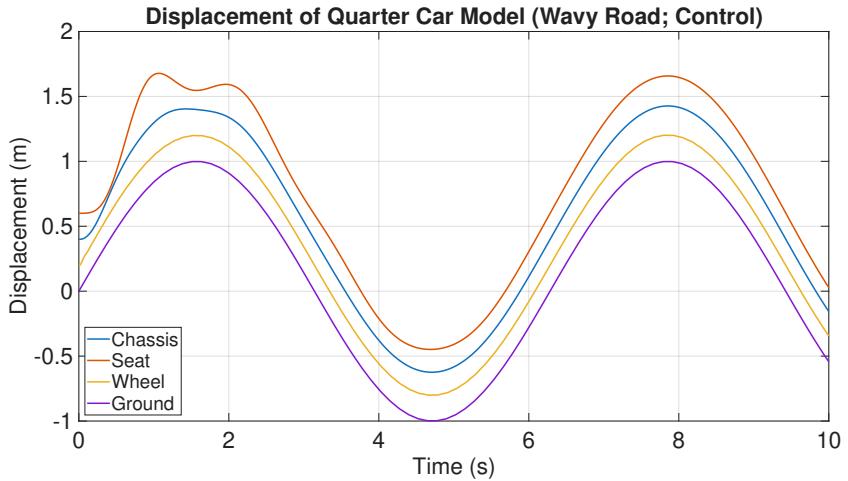
Figure 12: Wavy Displacement Control - 1 Meter Amplitude
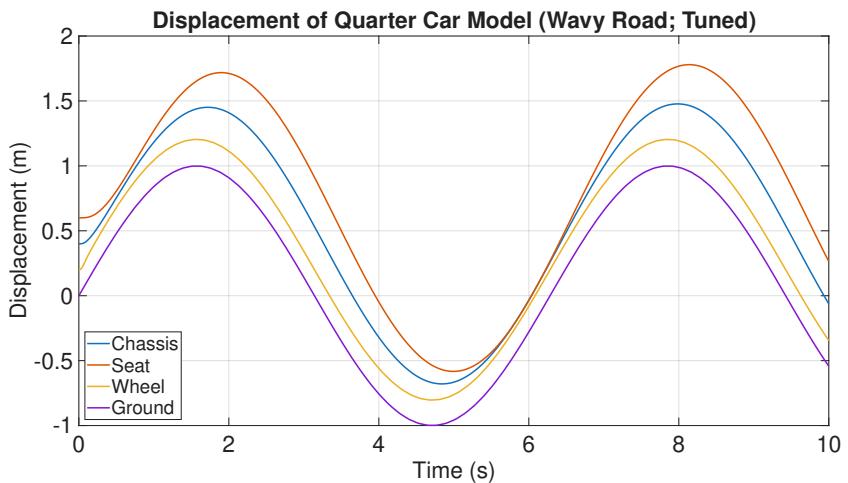
Figure 13: Wavy Displacement Tuned - 1 Meter Amplitude

16
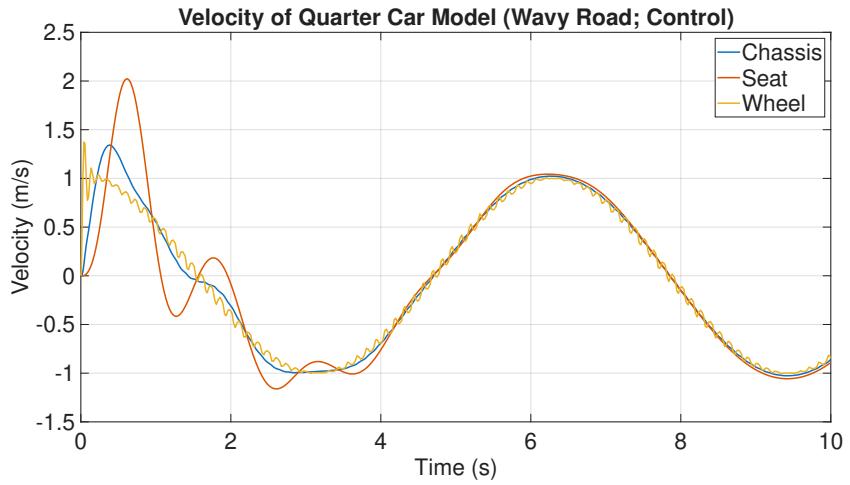
Figure 14: Wavy Velocity Control - 1 Meter Amplitude
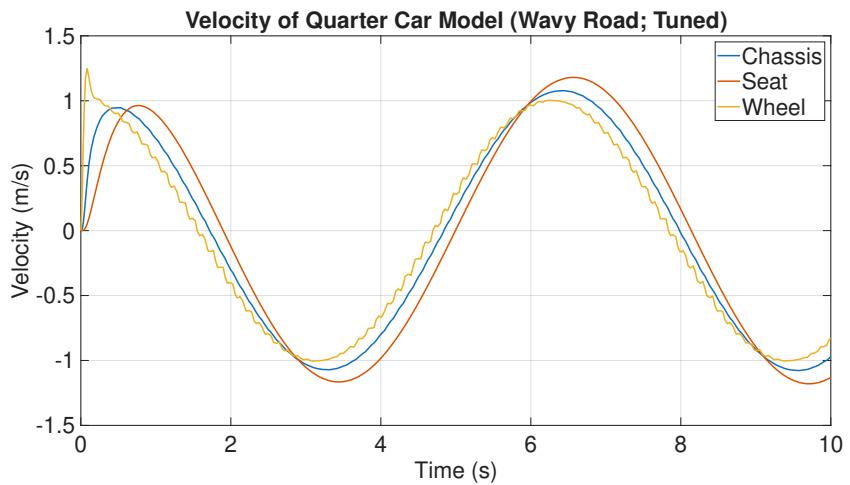
Figure 15: Wavy Velocity Tuned - 1 Meter Amplitude
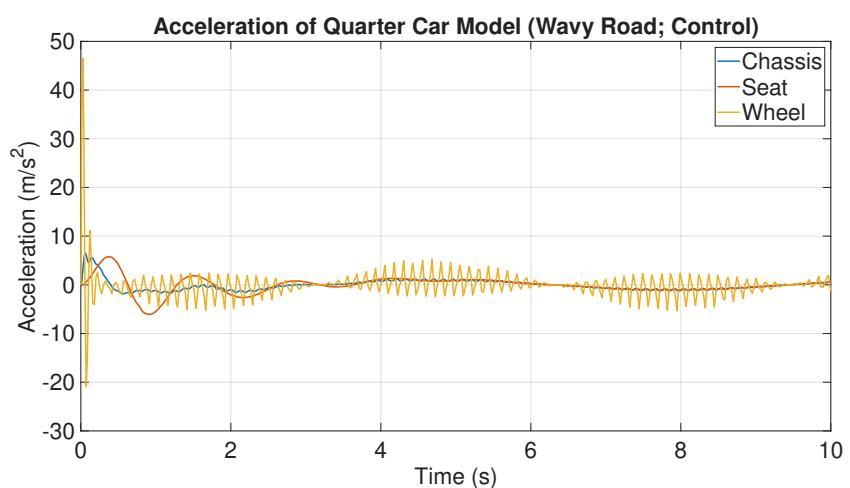
Figure 16: Wavy Acceleration Control - 1 Meter Amplitude

17
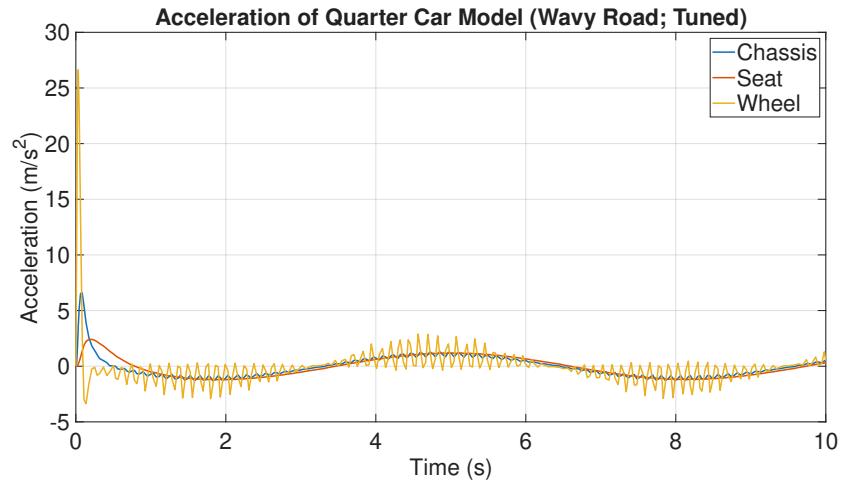
Figure 17: Wavy Acceleration Tuned - 1 Meter Amplitude

9.1.4 Wavy Road Excitation - 0.2 Meter Amplitude
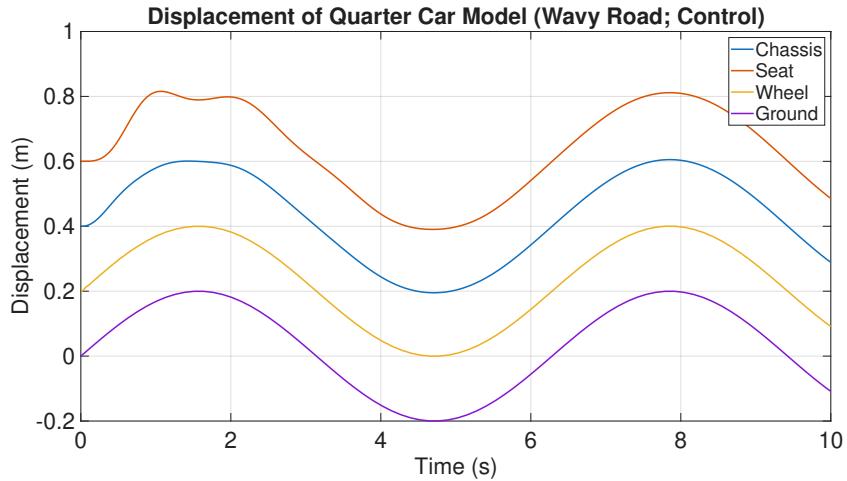
Figure 18: Wavy Road Displacement Control - 0.2 Meter Amplitude
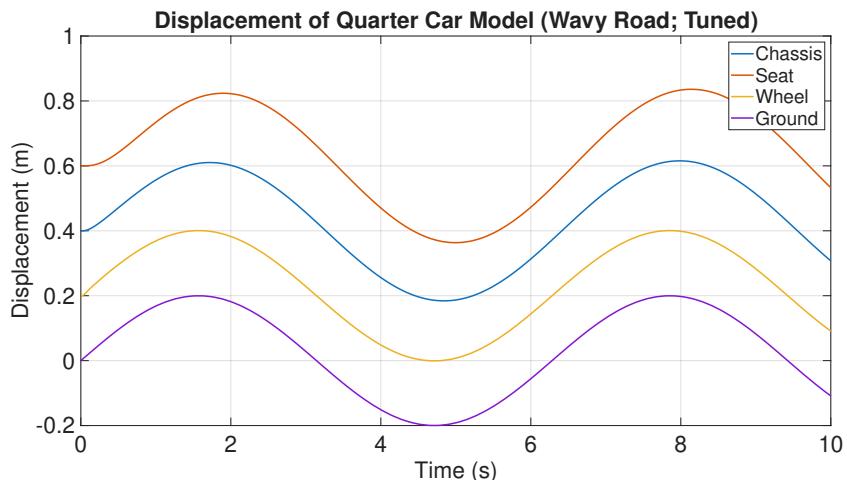
Figure 19: Wavy Road Displacement Tuned - 0.2 Meter Amplitude
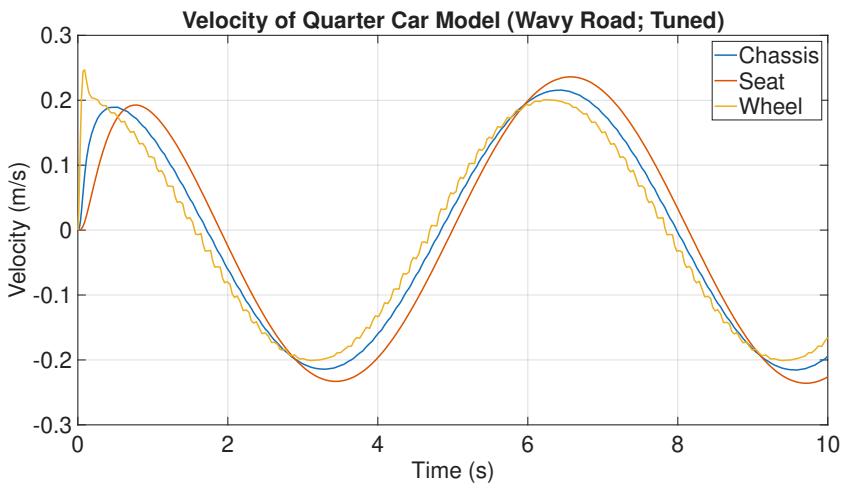
Figure 21: Wavy Road Velocity Tuned - 0.2 Meter Amplitude
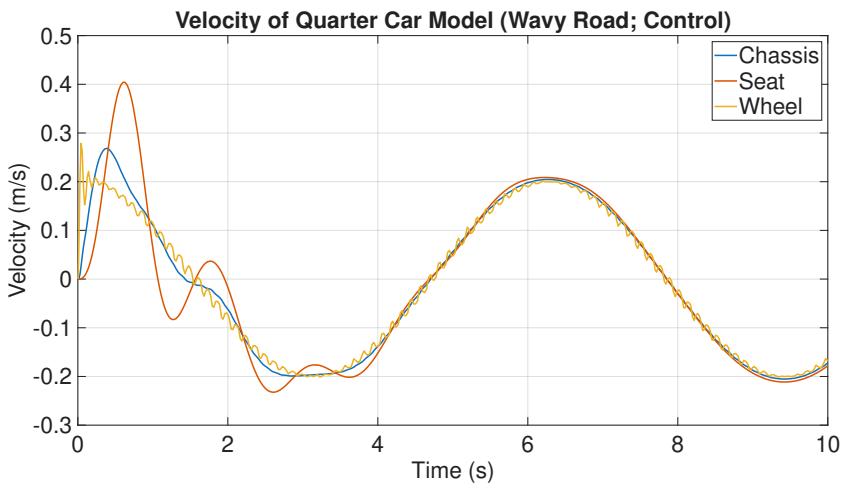
Figure 20: Wavy road velocity control - 0.2 Meter Amplitude
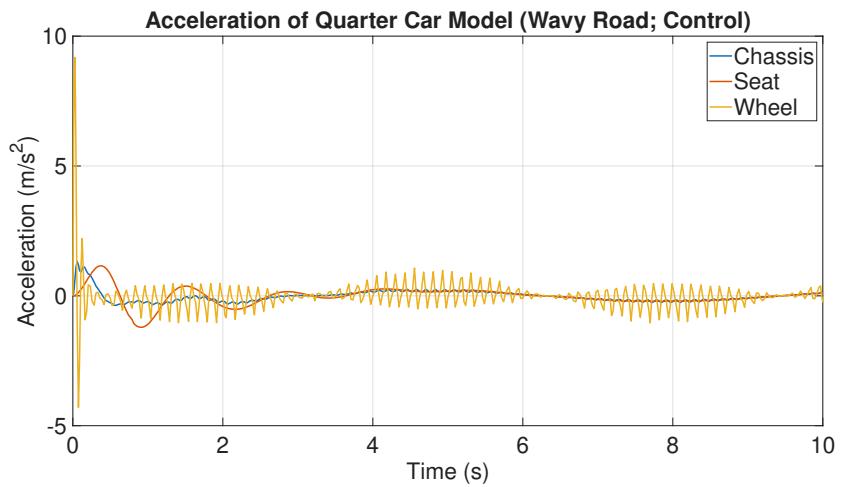
Figure 22: Wavy Road Acceleration control - 0.2 Meter Amplitude
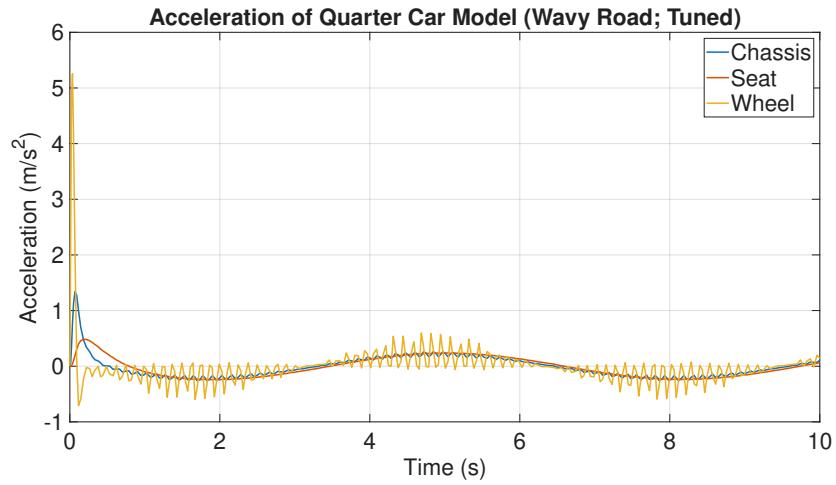
Figure 23: Wavy road velocity Control - 0.2 Meter Amplitude

9.1.5 Bump Excitation
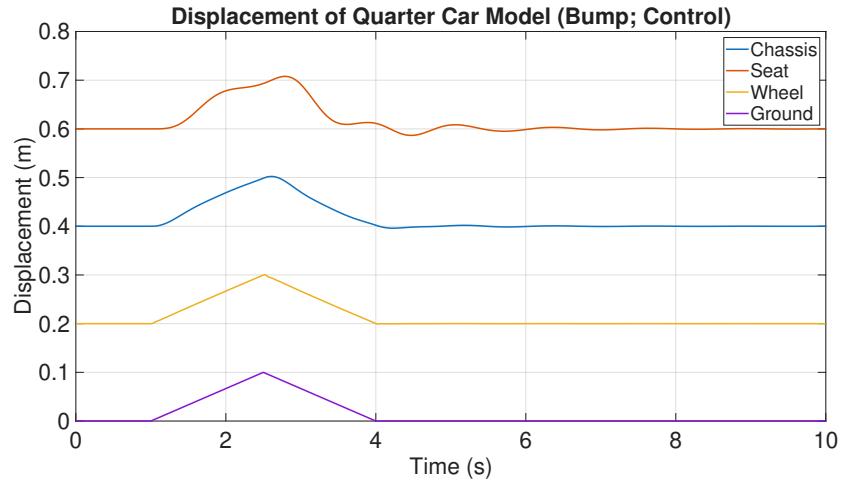
Figure 24: Displacement Bump Control
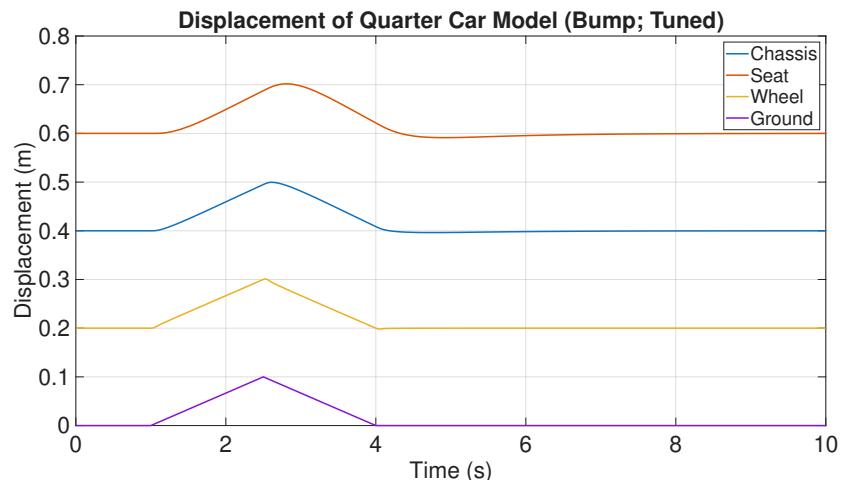
Figure 25: Displacement Bump Tuned
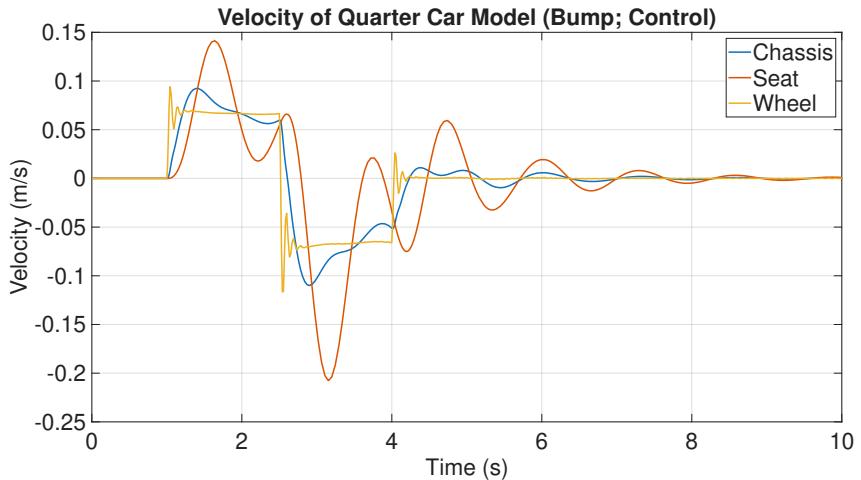
Figure 26: Velocity Bump Control
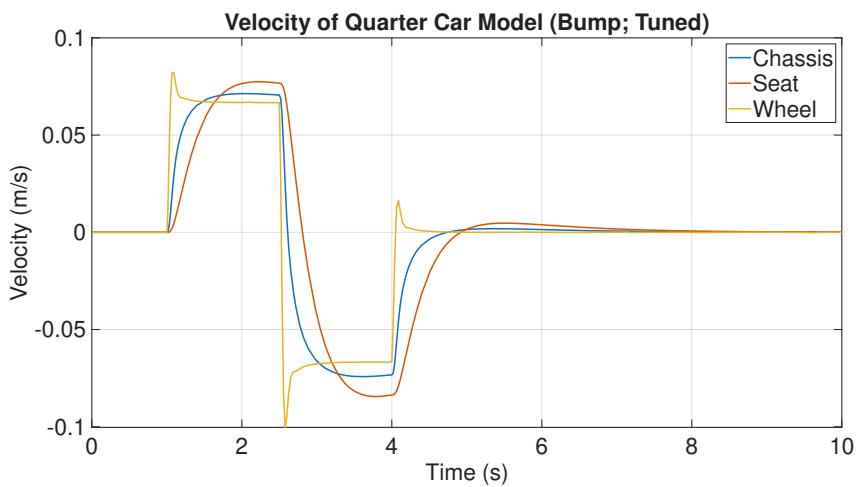

Figure 27: Velocity Bump Tuned
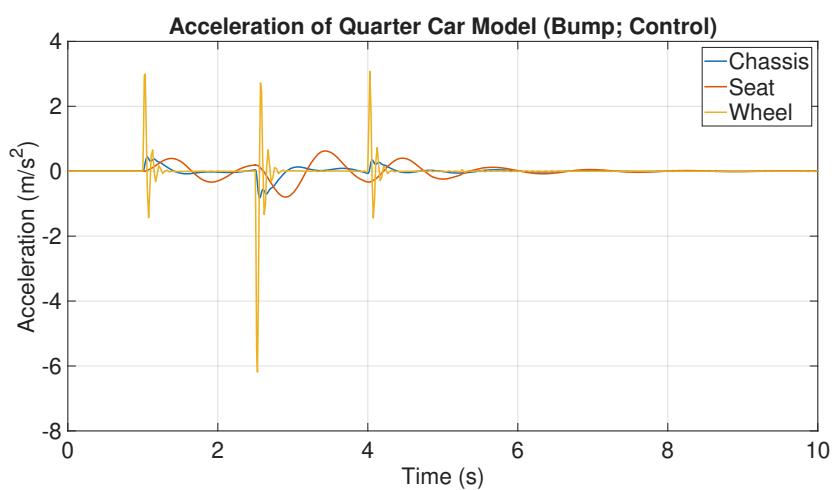

Figure 28: Acceleration Bump Count
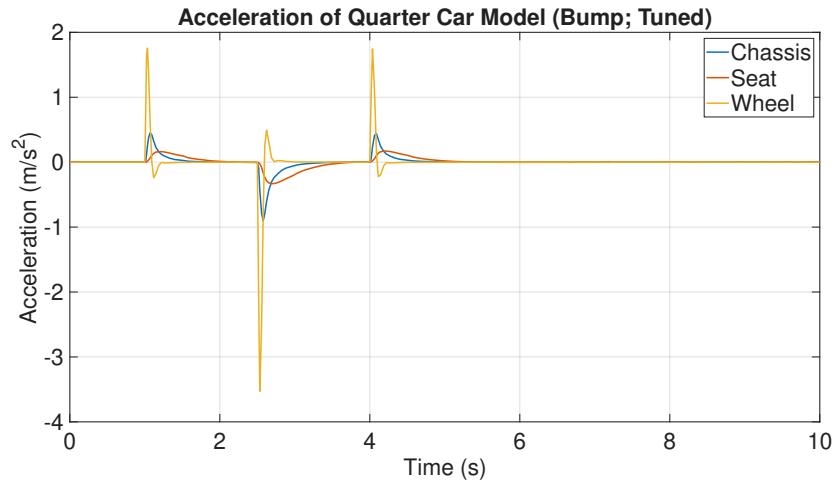
Figure 29: Acceleration Bump Tuned

9.2   Evidence of Simulink Models
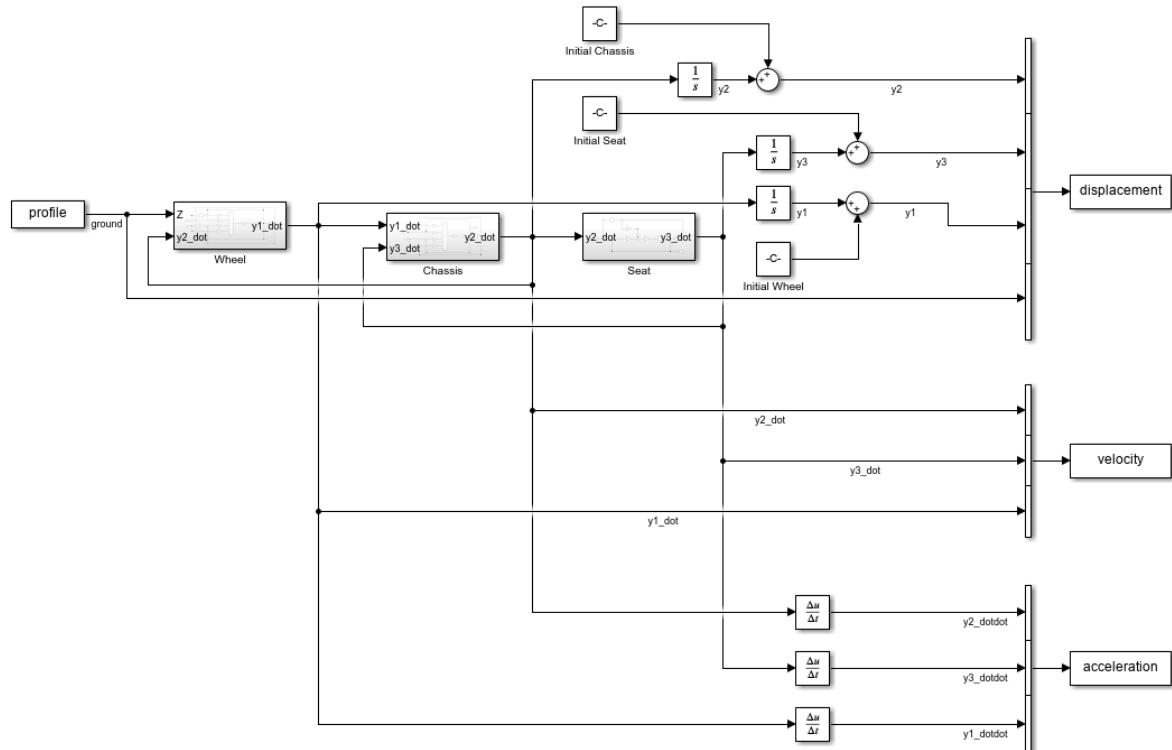
Figure 30: Main system EOM
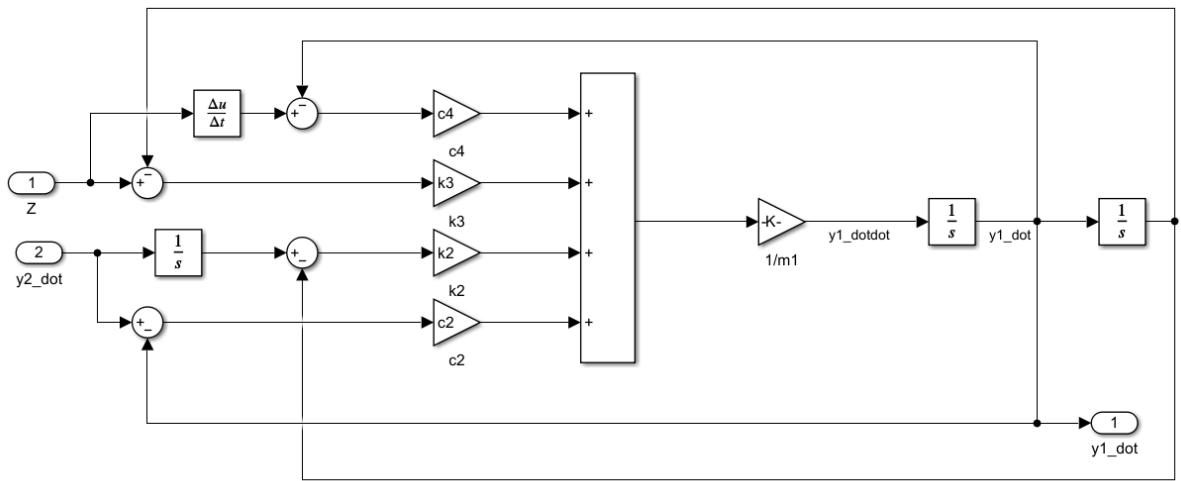
Figure 31: Wheel Sub-System
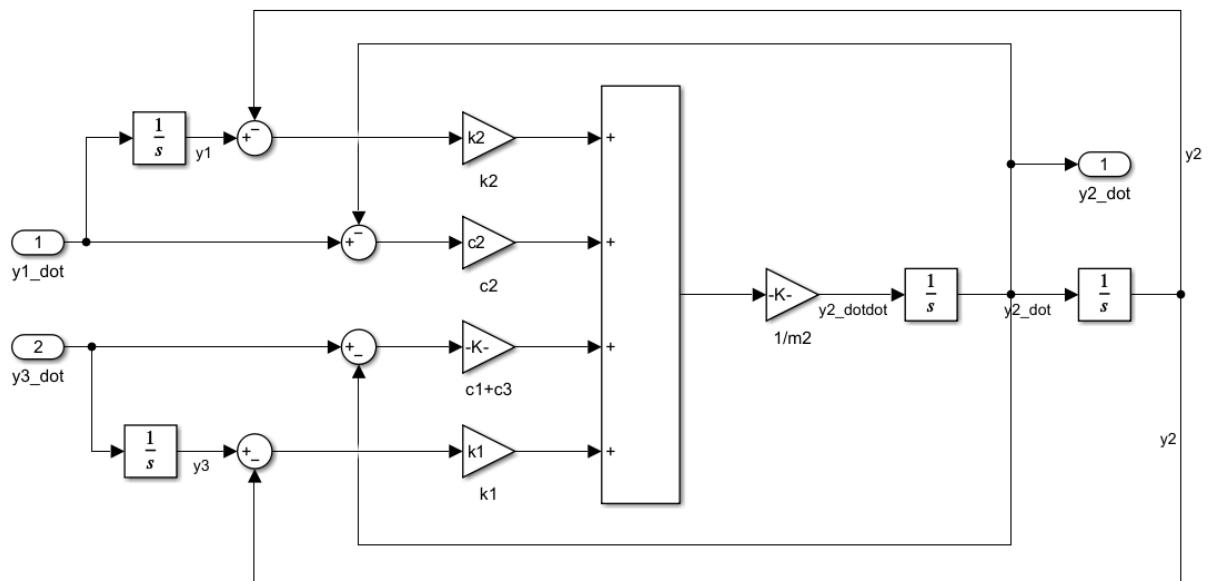
Figure 32: Chassis Sub-System Model
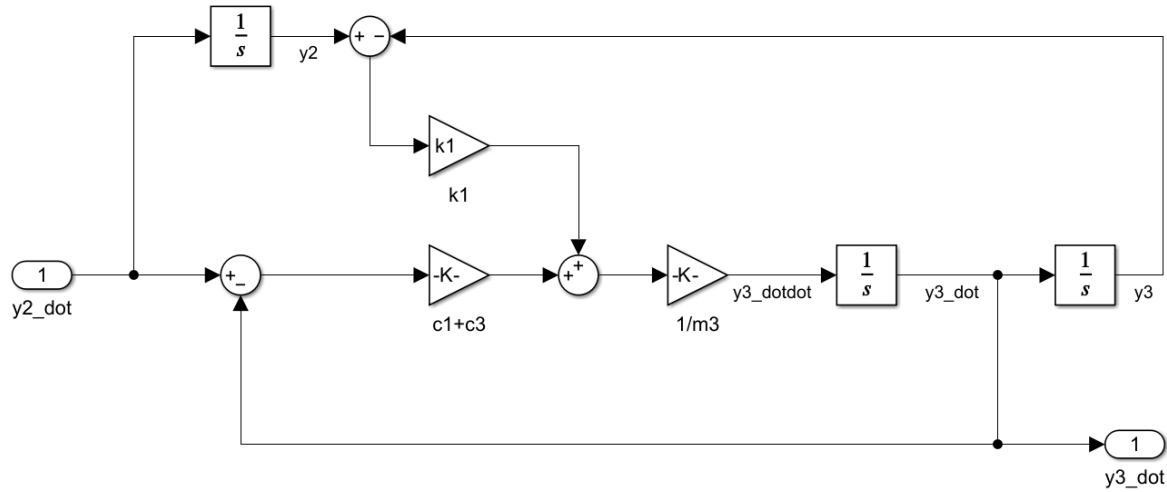
Figure 33: Seat Sub-System simulink
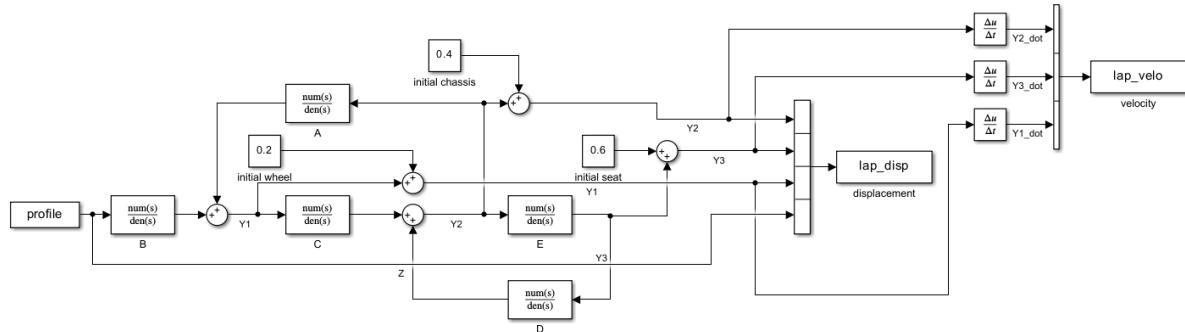
Figure 34: Laplace Transformations simulink
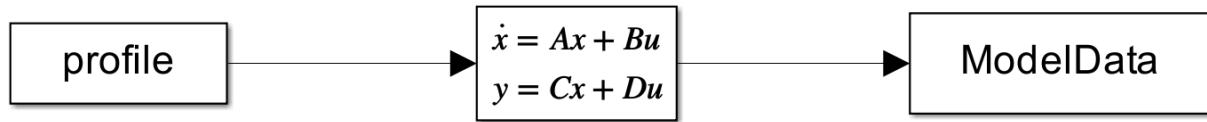
Figure 35: State Space Matrix Model

References

[1] Control Education Group (no date) *Control Education.* The University of Sheffield. Available at: https://controleducation.sites.sheffield.ac.uk/ (Accessed: 10 November 2025).

[2] Nouri, K., Loussifi, H. and Braiek, N. (2011) 'Modelling and wavelet-based identification of 3-DOF vehicle suspension system', *Journal of Software Engineering and Applications*, 4, pp. 672–681. doi: 10.4236/jsea.2011.412079.