Lecture 9

$$\begin{tikzcd} z \arrow[d] \\ x \end{tikzcd}$$

Bayesian network

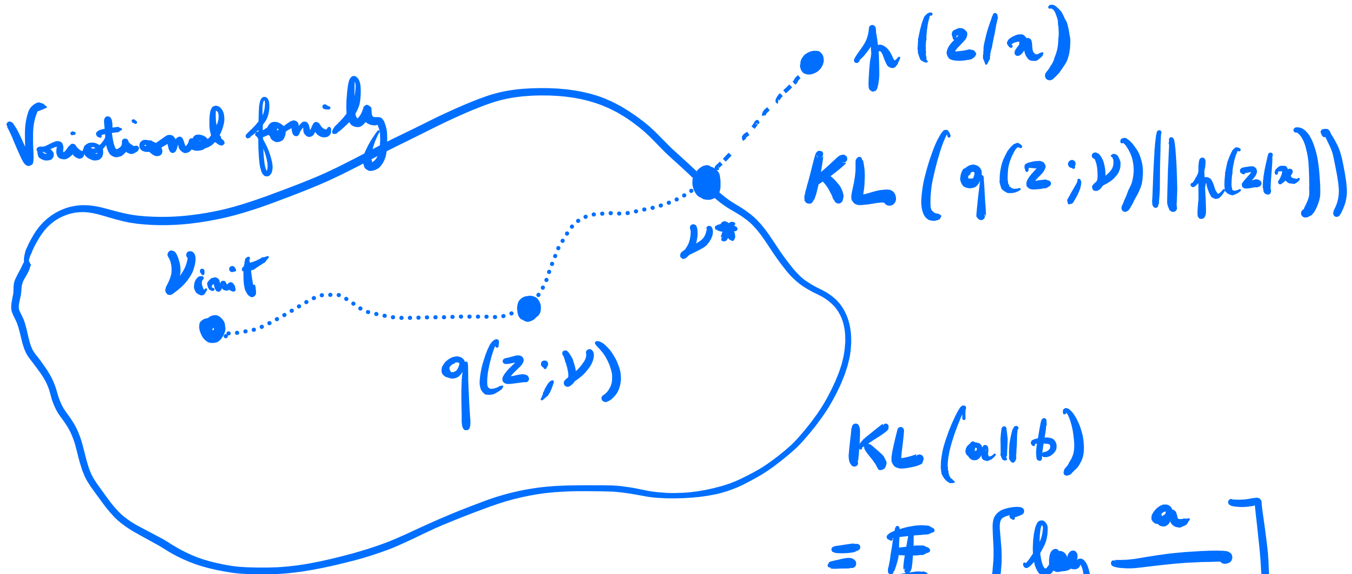
$p(x, z)$

Inference :

$$p(z|x) = \frac{p(x|z)\,p(z)}{p(x)}$$

$$= \frac{p(x|z)\,p(z)}{\underbrace{\int}_{\text{intractable}} p(x,z)\,dz}$$

Variational inference :



$$KL(a\|b) = \mathbb{E}_a\left[\log \frac{a}{b}\right]$$

$$\nu^* = \arg\min_{\nu} \underbrace{KL\left(q(z;\nu) \,\|\, p(z|x)\right)}$$

$$= \mathbb{E}_{q(z;\nu)}\left[\log \frac{q(z;\nu)}{p(z|x)}\right]$$

$$= \mathbb{E}_{q(z;\nu)}\left[\log q(z;\nu) - \log \underbrace{p(z|x)}_{\frac{p(x,z)}{p(x)}}\right]$$

$$= \mathbb{E}_{q(z;\nu)}\left[\log q(z;\nu) - \log p(x,z)\right] + \log p(x)$$

$$= \arg\min_{\nu} \mathbb{E}_{q(z;\nu)}\left[\log q(z;\nu) - \log p(x,z)\right]$$

$$= \arg\max_{\nu} \underbrace{\mathbb{E}_{q(z;\nu)}\left[\log p(x,z) - \log q(z;\nu)\right]}_{ELBO(x;\nu)}$$

$$ELBO(x;\nu) = \mathbb{E}_{q(z;\nu)}\left[\log p(x|z)\,p(z) - \log q(z;\nu)\right]$$

$$= \mathbb{E}_{q(z;\nu)}\left[\log p(x|z)\right] - KL\left(q(z;\nu) \,\|\, p(z)\right)$$