Lecture 10

Value function

$$V(\phi,\theta) = \mathbb{E}_{x\sim p(x)}\left[\log d(x;\phi)\right] + \mathbb{E}_{z\sim p(z)}\left[\log\left(1-d(g(z;\theta);\phi)\right)\right]$$

$$\boxed{\theta^* = \arg\min_\theta \max_\phi V(\phi,\theta)}$$

① θ is fixed

$$d(x;\phi^*_\theta) = \frac{p(x)}{q(x;\theta)+p(x)}, \quad \forall x$$

② $$\min_\theta \max_\phi V(\phi,\theta) = \min_\theta V(\phi^*_\theta,\theta)$$

$$= \min_\theta \mathbb{E}_{x\sim p(x)}\left[\log \frac{p(x)}{q(x;\theta)+p(x)}\right] + \mathbb{E}_{x\sim q(x;\theta)}\left[\log \frac{q(x;\theta)}{q(x;\theta)+p(x)}\right]$$

$$= \min_\theta KL\left(p(x)\,\middle\|\,\tfrac{1}{2}(q(x;\theta)+p(x))\right) - \log 2$$

$KL(p\|q) = \mathbb{E}_p\left[\log \frac{p}{q}\right]$

$$+ KL\left(q(x;\theta)\,\middle\|\,\tfrac{1}{2}(q(x;\theta)+p(x))\right) - \log 2$$

$$= \min_\theta 2JSD\left(p(x)\,\|\,q(x;\theta)\right) - \log 4$$

$$\theta^* = \arg\min_{\theta} \max_{\phi} V(\phi, \theta)$$

$$= \arg\min_{\theta} JSD(p(x) \,||\, q(x;\theta))$$