Designing a Bipolar PAM Communication System

Nicolas Hernandez
Mckelvey School of Engineering
Electrical and Systems Engineering
St. Louis, USA
h.nicolas@wustl.edu

Caden Wilcox
Mckelvey School of Engineering
Electrical and Systems Engineering
St. Louis, USA
c.b.wilcox@wustl.edu

Ashwin Deodhar
McKelvey School of Engineering
Electrical and Systems Engineering
St. Louis, USA
a.deodhar@wustl.edu

***Abstract*—A bipolar pulse-amplitude modulation (BPAM) was utilized to transmit three messages over three adjacent frequency bands using a sinc and SRRC pulse. The original messages were encoded to be bipolar, oversampled, modulated with the pulse, and then upconverted at a specific adjacent frequencies. The signal was then subjected to varying levels of noise after upconversion, and were then downconverted, demodulated, and unencoded.The performance of the two pulses and bandwidth used were evaluated based on error rate, signal-to-noise ratio(SNR), and Fast Fourier Transform plots. It was found that the SRRC pulse with a bandwidth of 40Hz produced the lowest error rate and the highest SNR.**

I. Introduction

In BPAM, a symbol represents a discrete message, such as a byte, which is modulated onto the carrier signal. When encoding ASCII characters, each symbol corresponds to a byte consisting of 8 bits, with 7 bits utilized to represent an ASCII character. BPAM variation employs orthogonal pulses for modulating these symbols, ensuring minimal interference between adjacent symbols. Orthogonal pulses are crucial in BPAM as they maintain orthogonality between the signal waveforms, which aids in avoiding signal distortion and overlap. This property allows for accurate and efficient demodulation at the receiver end. By using BPAM to encode ASCII characters as symbols, the technique effectively conveys information through distinct amplitude levels, leveraging orthogonal pulses to facilitate effective communication between a transmitter and receiver. The overall methodology is shown in Figure 1.

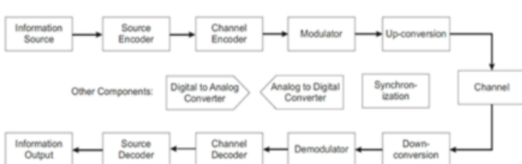

Fig. 1: Complete Method for the Bipolar Pulse Amplitude Modulation System.

An important factor in the modulation process is the symbol rate which describes how many bytes are being sent per second. This is dependant on available bandwidth and how quickly a signal needs to be transmitted. If this rate is too quick it can cause interference between impulses and degrade the quality of the transmitted signal. However if it is too slow it may be useless in transmitting the information so it is necessary to tune it the specifics of a situation. The methodology described in the paper assumes a symbol rate of 10 symbols per second. To avoid signal degradation or inefficiency, it is essential to tune the symbol rate appropriately, adhering to the Nyquist Criterion indicated in Equation 1, which states that the pulse duration should be at least twice the symbol period. Additionally, the oversampling rate, which samples the signal at a higher rate than the Nyquist rate, plays a significant role in avoiding Intersymbol Interference (ISI) and improving signal reconstruction.

$$1/Ts \sum_{n=-\infty}^{\infty} H(f - k/T_s) = 1 \tag{1}$$

for all frequencies f, where T_s is the symbol period.

II. Methods

A. *Source and Channel Encoding:* Three distinct messages first undergo source encoding, where ASCII characters are converted to their binary equivalents. Subsequently channel encoding occurs, where the binary data is transformed into a bipolar vector by replacing 0's with -1's. The messages are now in the appropriate format to be processed further in the BPAM system.
B. *Oversampling*: In order to reduce aliasing and ISI as much as possible, the process of oversampling occurs. This process spaces out the symbols onto the available bandwidth to avoid interference, shown in Figure 3. This can be implemented by the addition of buffers filled with 0s into the matrix in between each sample.The highest frequency component in a signal is related to the symbol rate. In most cases, the highest frequency component is considered to be equal to the symbol rate. With a symbol rate of 10 symbols per second, the Nyquist rate would be 20 samples per second, and oversampling could be used to further improve signal quality and reduce interference. A sample rate of 50 samples per second was used.
C. *Pulse Generation:* The pulses used in the BPAM communication model was a sinc and SRRC pulse. Sinc pulses restrict frequency content to meet bandwidth constraints, which result in an increased pulse spread due to infinite time-domain sequences. Alternatively, SRRC pulses

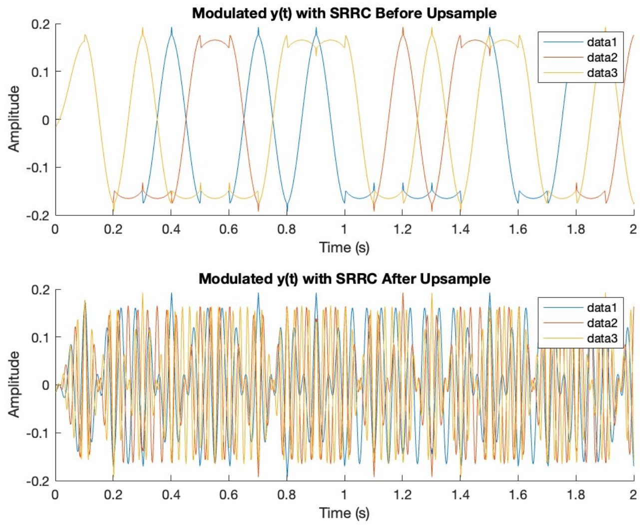

Fig. 2: Three Encoded Messages on Adjacent Frequencies

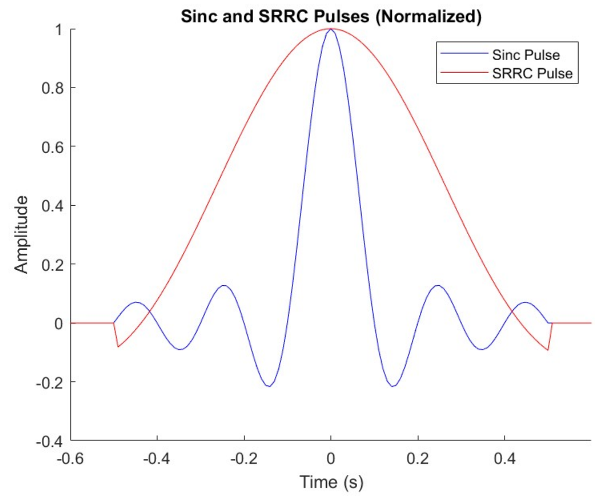

Fig. 3: SINC vs. SRRC

offer a balance between time and frequency domain properties, with their shape determined by the roll-off factor (alpha).The choice of the roll-off factor is a trade-off between bandwidth efficiency and the amount of ISI. A smaller roll-off factor results in better bandwidth efficiency but increases the pulse spread, which can lead to higher ISI if not properly managed. On the other hand, a larger roll-off factor reduces the pulse spread, lowering the risk of ISI, but at the cost of increased bandwidth usage. Ideally if the two were not generally inversely proportionally we would want a high roll and a high bandwidth efficiency.

Bandwidth efficiency, also known as spectral efficiency, is a measure of how effectively a communication system uses the available frequency spectrum to transmit signals. It is usually expressed in bits per second per Hertz and represents the amount of data that can be transmitted in a given bandwidth. Therefore, a higher bandwidth efficiency means that more data can be transmitted using the same amount amount of bandwidth.

The implementation of the SRRC pulse is defined in the time domain by Equation 2.

$$r_p(t) = \frac{sin((\pi * t)/T_s}{(\pi * t)/T_s)} * \frac{cos((\pi * \alpha * t/T_s)}{1 - ((2 * \alpha * t)/T_s)^2} \quad (2)$$

The two pulses are plotted in Figure 3. When comparing the two pulses, the SRRC has a much smoother roll off which makes it practically more applicable, whereas the sinc pulse has larger pulse spread and a less exact roll off.

D. *Modulation:* Modulation is process that maps the message onto the impulse so that it can be carried across the communication line. Modulation was accomplished by taking the convolution of the Bipolar signal vector and either the sinc or SRRC pulse. Due to the modulation scheme the result will have the message encoded into the amplitude of the new pulse.

E. *Up-Conversion and Down-Conversion:* Up conversion is essentially to the communication process as higher frequency signals can propagate over long distances easier. Furthermore the Federal Communications Commission (FCC) requires that certain frequencies be transmitted over different frequencies channels. In order to up convert the signal to a higher frequency the signal is multiplied by a cosine term at the desired transmission frequency. This process also allows for multiple messages to be sent simultaneously over adjacent frequencies by multiplying them by different cosine values of varying frequencies. Then these up sampled messages are added together and transmitted. On the receiver end the signals are once again multiplied by the same respective cosine terms of the same respective frequencies in order to Down-Convert.Two different bandwidths were used to up convert and down convert the signal a bandwidth of 20Hz and 40Hz. At a bandwidth of 20Hz the messages were at up-converted and down-converted 20Hz, 30Hz, and 40Hz. At a bandwidth of 40Hz the messages were at up-converted and down-converted at 20Hz, 40Hz, and 60Hz. In this case the bandwidth is defined as the total frequency width of the band.

F. *Matched Filter and Bit Decisions :* After modulating the pulse and performing up conversion and down conversion, noise was added to the signal. In order to make our simulation more realistic noise was added to our signal. This can be accomplished using a random Gaussian distribution to add in noise that the signal would theoretically encounter in transmission. The noisy data was then passed through a matched filter, which utilizes the properties of orthogonality to enhance the SNR. The matched filter is designed to maximize the output SNR by correlating the received signal with a replica of the transmitted pulse shape. This is achieved by taking the convolution of the noisy data with the original pulse shape, which acts as a low-pass filter. The principle of orthogonality allows the matched filter to distinguish between the desired signal and noise or interference. When the pulse shape is aligned with the desired signal, the correlation is maximized, and the output is at its highest. Conversely, when the pulse shape is not aligned, the correlation is minimized due to orthogonality, thus suppressing noise and interference. After filtering the noisy signal with the matched filter, bit decisions were made by implementing a simple logic statement at every symbol period. If the value was positive, it was assigned a value of one; if the value was negative, it was assigned a value of -1. This process effectively down-converts the signal while maintaining its integrity. The noisy signal, matched filter output and bit decisions are shown in Figure 4.

III. Results and Discussion

The decoding process can be finished by taking the bipolar vector and first converting to binary and then back to ASCII characters in order to retrieve the original message. With a sigma of 0 there were no mistakes in retrieval of the messages. The three messages inputted were equivalent to the three messages outputted. The error rate of the two separate pulses versus noise are analyzed and displayed in Figures 4 and 5. As the sigma value increases from 0 to 2, the error rate of the system increases. A higher sigma value adds more noise into the system, with 0 being no extra noise added. At a higher bandwidth of 40 Hz, shown in Figure 5, the SRRC has a lower error rate than the sinc pulse until a sigma value of 1.4 where it evens out.

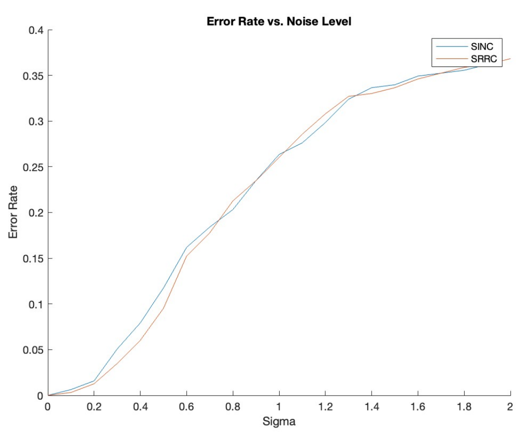

Fig. 4: Comparing the Error Rate of BPAM with Sinc and SRRC Pulse, at Bandwidth 20Hz

The physical effect of the error rate on the Matched filter and Bit decision plot can be seen in the difference between Figure 6, as well as the in the appendix in Figure 13. Each bit decision is taken at a symbol period of 0.1. Figure 4 has a sigma value of 0, so the bit decisions are mainly aligned

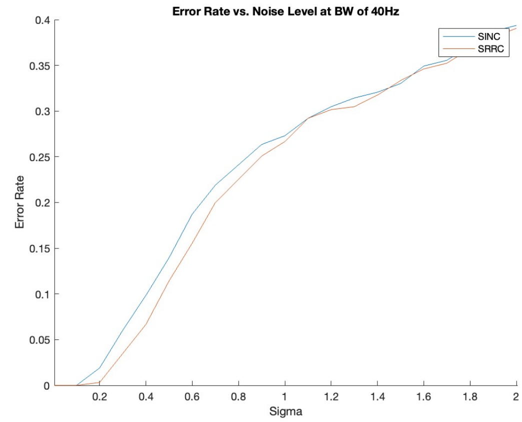

Fig. 5: Comparing the Error Rate of BPAM with Sinc and SRRC Pulse, at Bandwidth 40Hz

with the peaks of the matched filter, which corresponds to a output message that's equivalent to the input message. In Figure 13, the added noise with a sigma value of 0.3 increases the error rate, and does not output the correct message from the input.The added noise makes it harder for the matched filter to determine what the original signal is. This causes the filter to incorrectly determine peaks with high noise levels. With inaccurate determination of what actually is a peak, the bit decisions are incorrectly made.

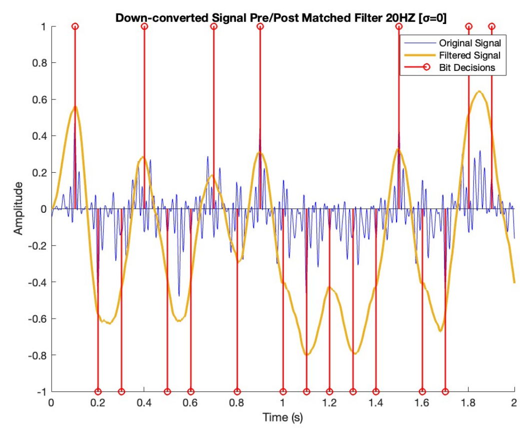

Fig. 6: Down-Converted, Matched Filtered Wave with Bit Decisions

The SNR is shown in Figure 7. This graph describes the signal energy in relation to the error rate. With larger error rates the two pulses perform similarly, but as the error rate drops below 0.15 the SNR pulse outperforms the sinc pulse. In order for the signal energy to overcome the noise, the error rate has to be below 0.03. The x axis is defined by $10 * log(signalenergy/noiseenergy)$. Theoretically, the energy of a signal cannot be negative. Mathematically, when the noise energy is large, the function is the logarithm of a small value which produces a negative number. This is why the x-axis has negative values. In Figure 12 in the appendix, the SRRC at 40 Hz out performed the SRRC at 20 Hz in Figure 7. Below an error rate of 0.25 is when the SRRC outperforms the sinc in Figure 12, whereas it was below 0.15 for 20 Hz.

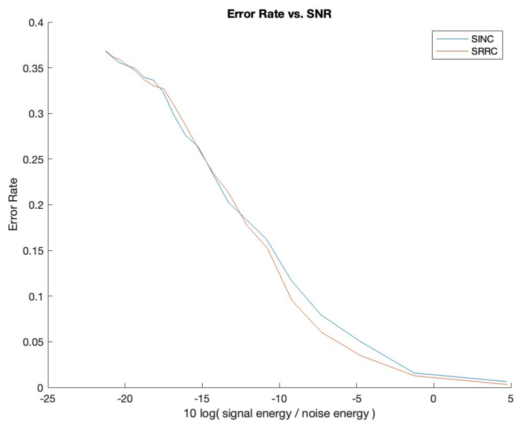

Fig. 7: Error Rate vs. SNR at 20 Hz

The FFT allows a user to visualize the frequencies of a signal and can be used to determine the overlap of the encoded pulses that could lead to aliasing and ISI.

In both pulses there was slight overlap, whereas when the bandwidth was expanded there was no overlap. The SRRC pulse has a greater amount of frequencies in each band at a higher magnitude in contrast to the sinc pulse. In terms of bandwidth efficiency, this means that the SRRC pulse has a broader frequency spectrum compared to the sinc pulse. This indicates to no surprise that larger bandwidth is better for transmitting signals. However, the interference that did exist with a bandwidth of 20 did not effect the error rate until noise was added in. A bandwidth of 40Hz with the SRRC pulse was the most optimal producing the lowest error rates and highest SNR at most error rates.

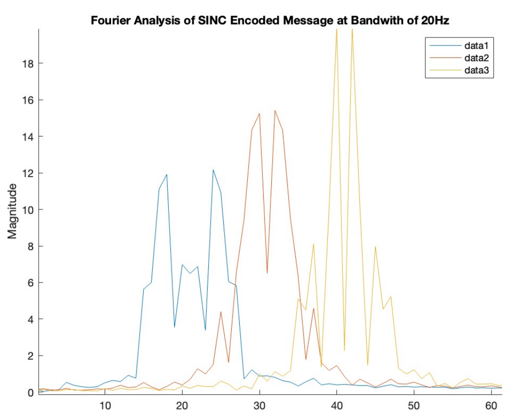

Fig. 8: FFT of Sinc Encoded Message at 20 Hz

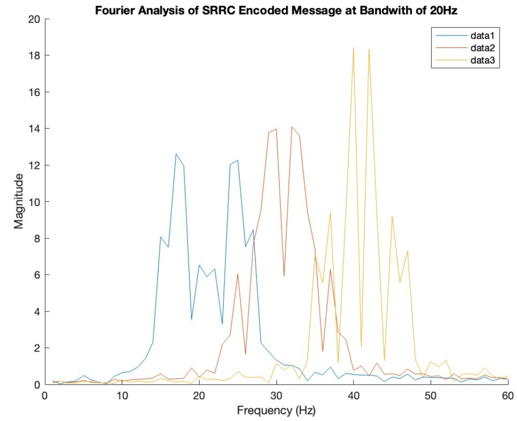

Fig. 9: FFT of SRRC Encoded Message at 20 Hz

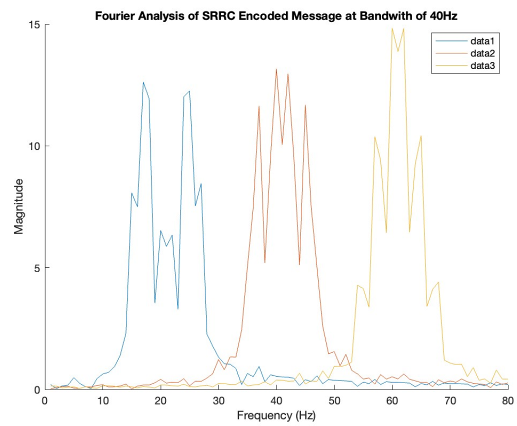

Fig. 10: FFT of SRRC Encoded Message at 40 Hz

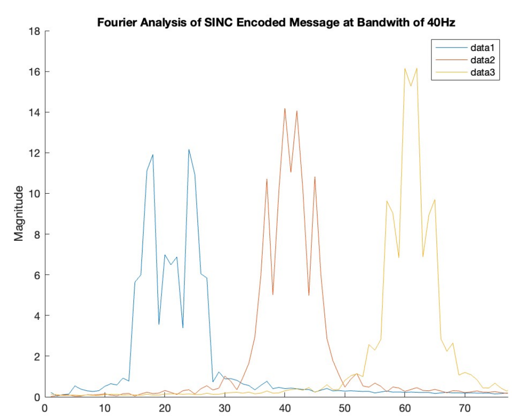

Fig. 11: FFT of SINC Encoded Message at 40 Hz

IV. Conclusion

The study provided valuable insight into the basics for many real life communication systems. Structurally, they function very similarly so the study definitely aided in the understanding of these especially through the process of breaking the larger system into smaller steps like those outlined in the methodology.

While the bipolar nature of the BPAM made the system simpler to implement, it also hindered the efficiency. Only one byte could be encoded per symbol making the system slower than if the amplitude could be encoded with multiple bytes. In a more complex system than a BPAM, which encodes 1's and 0's, more bytes could be encoded on on a multi level BPAM system. This means that the amplitude of each pulse can take on more than two levels. For instance, a 4-PAM system could have -3, -1, 1, 3, which means each pulse can take on four different amplitudes. This proposed system may also require different pulse shapes, and a method of encoding the bits onto the pulse called Quadrature Amplitude Modulation, but would ultimately result in a system that can transmit more bits per unit of time. Furthermore, in order to reduce our bit error rate, an error correcting code block could be added to correct the errors in the transmitted data by adding in redundancies and allowing the receiver to correct errors and improving our bit error rate.

Despite our SRRC pulse system having overlaps in bandwidth which causes ISI, by finding the optimal level of alpha, the filter can optimize the symbols transmitted per unit of time, while decreasing the chance of ISI.Furthermore, our oversampling rate could be adjusted to improve ISI. It was found that a SRRC pulse shape at a bandwidth of 40 Hz with a roll of factor of .5 was fairly effective for a BPAM system and was capable of transmitting messages without bit mistakes at a low to zero noise value.

REFERENCES

[1] Rice, M, Digital Communications: A Discrete-Time Approach, 2008
[2] Patwari, N, ESE 471 Communications Theory and Systems Lecture Notes, 2022
[3] Trobaugh, J, ESE 351 Spring 2023 Case Study 2: PAM Communication, 2023
[4] Overleaf, Documentation, https://www.overleaf.com/learn
[5] Berkeley, ECE 461: Digital Communication, Lecture 8b: Pulse Shaping and Sampling, https://inst.eecs.berkeley.edu/~ee121/sp08/handouts/lecture8b_ece461.pdf#:~:text=The%20advantages%20of%20the%20sinc%20pulse%20are%20similar,that%20we%20can%20tightly%20meet%20any%20bandwidth%20constraint

APPENDIX

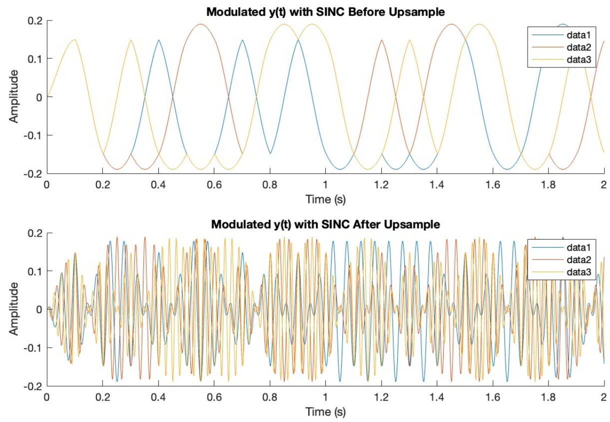

Fig. 12: Three encoded signals

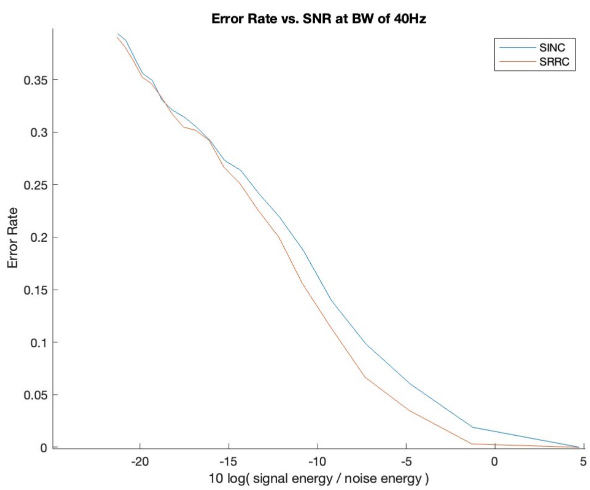

Fig. 13: 40 Hz Error Rate vs. SNR

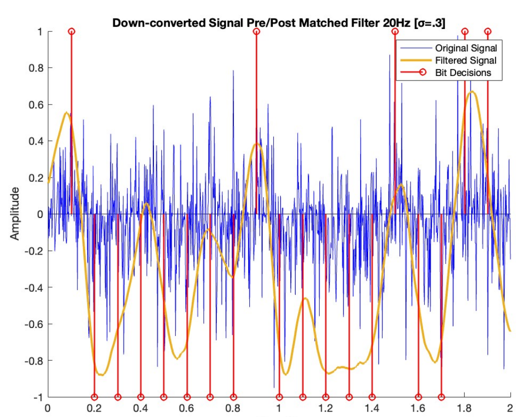

Fig. 14: Sigma .3 Down-converted Signal Bit decisions