

Sorting Technique

Bubble Sort :-

1 Type of Sorting

- **Comparison-based:** sorts by comparing adjacent elements.
- **In-place:** does not require extra memory aside from a few variables.
- **Stable:** preserves relative order of equal elements.

2 Algorithm Idea

- Repeatedly compare adjacent elements and swap if they are in the wrong order.
- After each pass, the largest unsorted element “bubbles up” to its correct position.
- Continue passes until no swaps are required.

3 Characteristics

- **Stable** → preserves order of equal elements.
- **In-place** → only constant extra memory needed.
- Simple and easy to implement.
- Can be **adaptive** if optimized: stops early when array is already sorted.

4 Time Complexity

Let n = number of elements

1. **Worst Case (reverse sorted)** → $O(n^2)$
 - Maximum number of comparisons and swaps.
2. **Average Case (random order)** → $O(n^2)$
 - On average, half the comparisons require swaps.
3. **Best Case (already sorted, with optimization)** → $O(n)$
 - Only one pass required; no swaps detected → algorithm stops early.

Reasoning:

- Two nested loops → outer loop for passes, inner loop for comparisons → gives quadratic complexity in worst and average cases.
- Early stopping reduces best-case to linear.

5 Space Complexity

- **Auxiliary Space:** $O(1)$
- Reason: Sorting is done in-place with a temporary variable for swaps.

6 Stability

- Yes
- Equal elements are not swapped, so their relative order remains intact.

7 Pros and Cons

Pros:

- Simple to implement.
- Stable and in-place.
- Adaptive if optimized.

Cons:

- Inefficient for large arrays ($O(n^2)$).
- Not suitable for practical large-scale applications.

✓ Summary Table (Bubble Sort)

Feature	Value
Type	Comparison-based, In-place
Stability	Stable
Best Case TC	$O(n)$
Average Case TC	$O(n^2)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(1)$
Adaptive	Yes (if optimized)
Use	Small arrays, teaching purposes

Selection Sort :-

1 Type of Sorting

- **Comparison-based:** finds the minimum (or maximum) element in the unsorted portion and places it in the correct position.
- **In-place:** only uses a constant amount of extra memory.
- **Not stable** by default: swapping may change the relative order of equal elements.

2 Algorithm Idea

- Divide the array into **sorted** and **unsorted** parts.
- Repeatedly select the **minimum element** from the unsorted part and swap it with the first unsorted element.
- Continue until the whole array is sorted.

3 Characteristics

- **Not stable** → swapping can move equal elements out of order.
- **In-place** → constant extra space.
- Simple and easy to implement.
- **Not adaptive** → always performs the same number of comparisons, even if the array is partially sorted.

4 Time Complexity

Let n = number of elements

1. Worst Case (reverse sorted) $\rightarrow O(n^2)$

- Outer loop runs n times, inner loop scans remaining unsorted elements $\rightarrow n(n-1)/2$ comparisons.

2. Average Case (random order) $\rightarrow O(n^2)$

- Same number of comparisons regardless of element arrangement.

3. Best Case (already sorted) $\rightarrow O(n^2)$

- Even if array is sorted, it still scans for the minimum each pass.

Reasoning:

- **Comparisons:** always $n(n-1)/2 \rightarrow$ quadratic.
- **Swaps:** exactly $n-1 \rightarrow$ fewer than Bubble Sort (advantage for large swaps).

5 Space Complexity

- **Auxiliary Space:** $O(1)$
- Sorting done in-place using a temporary variable for swapping.

6 Stability

- **Not stable** by default.
- Example: if two equal elements are swapped, their relative order changes.

7 Pros and Cons

Pros:

- Simple and easy to implement.
- Performs **minimum number of swaps** (useful if swapping is expensive).
- In-place → low memory usage.

Cons:

- Inefficient for large arrays ($O(n^2)$ comparisons).
- Not stable.
- Not adaptive → does not benefit from partially sorted arrays.

✓ Summary Table (Selection Sort)

Feature	Value
Type	Comparison-based, In-place
Stability	Not stable
Best Case TC	$O(n^2)$
Average Case TC	$O(n^2)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(1)$
Adaptive	No
Swaps	Minimum ($n-1$)
Use	Small arrays or when swaps are costly

Insertion Sort :-

1 Type of Sorting

- **Comparison-based:** elements are compared and shifted to insert them at the correct position.
- **In-place:** only a small amount of extra memory is required.
- **Stable:** preserves the relative order of equal elements.

2 Algorithm Idea

- Divide the array into **sorted** and **unsorted** parts.
- Pick each element from the unsorted part and **insert it into its correct position** in the sorted part by shifting larger elements.
- Continue until the entire array is sorted.

3 Characteristics

- **Stable** → equal elements retain their relative order.
- **In-place** → $O(1)$ extra space.
- **Adaptive** → performs efficiently if the array is nearly sorted.
- Simple to implement and efficient for **small datasets**.

4 Time Complexity

Let n = number of elements

1. **Best Case (already sorted)** → $O(n)$
 - Only comparisons, no shifting required.
2. **Average Case (random array)** → $O(n^2)$
 - Elements need to be shifted for insertion; roughly $n^2/4$ comparisons and shifts.
3. **Worst Case (reverse sorted)** → $O(n^2)$
 - Every element has to be compared with all elements in the sorted part and shifted
→ $n(n-1)/2$ comparisons and shifts.

Reasoning:

- Inner loop performs shifting for each element → dominates time complexity.
- Outer loop runs $n-1$ times → total comparisons and shifts lead to $O(n^2)$ in average/worst cases.

5 Space Complexity

- **Auxiliary Space:** $O(1)$ → Sorting is in-place using a temporary variable to hold the element being inserted.

6 Stability

- Yes
- Equal elements are never swapped out of order; only shifting occurs.

7 Pros and Cons

Pros:

- Stable and in-place.
- Adaptive → very efficient for nearly sorted arrays.
- Simple to implement.
- Good for **small arrays**.

Cons:

- Inefficient for large, randomly ordered arrays ($O(n^2)$).
- Not suitable for large-scale datasets.

✓ Summary Table (Insertion Sort)

Feature	Value
Type	Comparison-based, In-place
Stability	Stable
Best Case TC	$O(n)$
Average Case TC	$O(n^2)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(1)$
Adaptive	Yes
Use	Small arrays, nearly sorted arrays, teaching purposes

Merge Sort :-

1 Type of Sorting

- **Comparison-based:** elements are compared during the merge step.
- **Divide & Conquer:** splits the array into halves, sorts each half recursively, then merges them.
- **Stable:** preserves the relative order of equal elements.
- **Not in-place:** requires extra memory for merging.

2 Algorithm Idea

- Recursively **divide** the array into two halves until each subarray has one element.
- **Merge** the subarrays back together in sorted order.
- Continue until the entire array is merged and sorted.

3 Characteristics

- **Stable** → equal elements maintain relative order.
- **Not in-place** → requires $O(n)$ extra memory for temporary arrays during merging.
- **Divide & Conquer approach** → very efficient for large datasets.
- Consistently fast regardless of input order.

4 Time Complexity

Let n = number of elements

1. **Best Case** → $O(n \log n)$
2. **Average Case** → $O(n \log n)$
3. **Worst Case** → $O(n \log n)$

Reasoning:

- **Division:** array is recursively split in half → $\log_2(n)$ levels.
- **Merging:** each level requires $O(n)$ comparisons and copying.
- Total = $O(n)$ per level × $\log_2(n)$ levels → **$O(n \log n)$** .

5 Space Complexity

- **Auxiliary Space:** $O(n)$
- Reason: Temporary arrays are needed to merge sorted subarrays.

6 Stability

- Yes
- During merging, when two elements are equal, the element from the left subarray is copied first → relative order preserved.

7 Pros and Cons

Pros:

- Stable and reliable.
- Predictable $O(n \log n)$ time for all cases.
- Efficient for large datasets.
- Works well for **linked lists** (can be done in-place for linked lists).

Cons:

- Requires extra memory → not in-place for arrays.
- Slightly slower than Quick Sort in practice due to memory overhead.

✓ Summary Table (Merge Sort)

Feature	Value
Type	Comparison-based, Divide & Conquer
Stability	Stable
Best Case TC	$O(n \log n)$
Average Case TC	$O(n \log n)$
Worst Case TC	$O(n \log n)$
Space Complexity	$O(n)$
Adaptive	No
Use	Large datasets, stable sorting, linked lists

Quick Sort :-

1 Type of Sorting

- **Comparison-based:** elements are compared to a **pivot** during partitioning.
- **Divide & Conquer:** partitions the array into smaller subarrays recursively.
- **Not stable:** swapping may change the relative order of equal elements.
- **In-place:** uses a small amount of extra memory for recursion.

2 Algorithm Idea

- Choose a **pivot element** (first, last, middle, or median).
- **Partition** the array: elements less than the pivot go left, greater go right.
- Recursively sort the left and right subarrays.
- Combine results → the array is sorted.

3 Characteristics

- **Not stable** → equal elements may swap positions.
- **In-place** → low memory usage ($O(\log n)$ recursion stack).
- **Adaptive** depends on pivot choice → performance can vary.
- **Efficient** for large datasets on average.
- Highly popular in practice due to low overhead.

4 Time Complexity

Let n = number of elements

1. **Best Case** → $O(n \log n)$
 - Pivot divides array into two nearly equal halves each time.
2. **Average Case** → $O(n \log n)$
 - Random pivot selection usually gives balanced partitions.
3. **Worst Case** → $O(n^2)$
 - Pivot always the smallest or largest element → unbalanced partitions (e.g., already sorted with first/last pivot).

Reasoning:

- **Partitioning:** $O(n)$ comparisons per level.
- **Recursion levels:** $\log_2(n)$ for balanced splits → $O(n \log n)$.
- **Worst case:** recursion depth = n → $O(n^2)$.

5 Space Complexity

- **Auxiliary Space:** $O(\log n)$ on average (for recursion stack).
- In-place sorting → no extra arrays needed.

6 Stability

- **No** → swapping during partitioning may change relative order of equal elements.

7 Pros and Cons

Pros:

- Fast in practice → low overhead, in-place.
- Average-case $O(n \log n)$ → suitable for large datasets.
- Simple implementation and widely used.

Cons:

- Not stable.
- Worst-case $O(n^2)$ if pivot choice is poor.
- Recursive → recursion stack may grow to $O(n)$ in worst case.

✓ Summary Table (Quick Sort)

Feature	Value
Type	Comparison-based, Divide & Conquer
Stability	Not stable
Best Case TC	$O(n \log n)$
Average Case TC	$O(n \log n)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(\log n)$
Adaptive	Yes, with good pivot selection
Use	Large datasets, fast in practice, in-place sorting

Counting Sort :-

1 Type of Sorting

- **Non-comparison-based**: sorts elements by **counting their occurrences**.
- **Stable**: preserves relative order of equal elements.
- **Not in-place**: requires extra memory for count array.
- Suitable for **integers or small-range discrete values**.

2 Algorithm Idea

- Find the **maximum value (k)** in the array.
- Create a **count array** of size $k+1$ to store the frequency of each element.
- Compute the **cumulative frequency** to determine the correct position of each element.
- Place each element in the output array according to the cumulative frequency.

3 Characteristics

- **Stable** → preserves relative order of equal elements.
- **Not in-place** → requires $O(k + n)$ extra space for count and output arrays.
- Efficient for **integers with small range**.
- **Deterministic** → time complexity depends on n (number of elements) and k (range), not input order.

4 Time Complexity

Let n = number of elements, k = maximum element value

1. **Best Case:** $O(n + k)$
2. **Average Case:** $O(n + k)$
3. **Worst Case:** $O(n + k)$

Reasoning:

- Counting frequencies → $O(n)$
- Cumulative frequency calculation → $O(k)$
- Placing elements in output array → $O(n)$
- Total = $O(n + k)$

Note: Unlike comparison sorts, Counting Sort does **not rely on element comparisons**, so it can achieve linear time for small ranges.

5 Space Complexity

- **Auxiliary Space:** $O(n + k)$
 - Count array $\rightarrow O(k)$
 - Output array $\rightarrow O(n)$

6 Stability

- **Yes**
- When inserting elements into the output array using cumulative frequency, **elements from left to right** maintain their relative order.

7 Pros and Cons

Pros:

- Linear time for small ranges \rightarrow very fast.
- Stable \rightarrow maintains relative order.
- Deterministic \rightarrow predictable performance.

Cons:

- Not in-place \rightarrow extra memory needed.
- Not suitable for large ranges $\rightarrow O(k)$ can be very large.
- Works only for integers (or values that can be mapped to integers).

✓ Summary Table (Counting Sort)

Feature	Value
Type	Non-comparison-based
Stability	Stable
Best Case TC	$O(n + k)$
Average Case TC	$O(n + k)$
Worst Case TC	$O(n + k)$
Space Complexity	$O(n + k)$
Adaptive	No
Use	Small-range integers, counting problems, linear-time sorting

Radix Sort :-

1 Type of Sorting

- **Non-comparison-based**: sorts elements **digit by digit** rather than by comparing values directly.
- **Stable** → preserves relative order of equal elements.
- **Not in-place** → requires extra memory for intermediate arrays (usually uses Counting Sort as a subroutine).
- Suitable for **integers or fixed-length strings**.

2 Algorithm Idea

- Determine the **maximum number of digits** in the array elements.
- Sort the array **digit by digit** from least significant digit (LSD) to most significant digit (MSD) using a **stable sort** (usually Counting Sort).
- After the last digit sort, the array becomes completely sorted.

3 Characteristics

- **Stable** → uses a stable subroutine (like Counting Sort) for each digit.
- **Not in-place** → auxiliary arrays are used for counting or temporary storage.
- Works efficiently when **range of digits is small** compared to the number of elements.
- **Deterministic** → time complexity depends on n (number of elements) and k (number of digits).

4 Time Complexity

Let n = number of elements, d = number of digits, k = base (e.g., 0–9 for decimal digits)

1. **Best Case:** $O(d \times (n + k))$
2. **Average Case:** $O(d \times (n + k))$
3. **Worst Case:** $O(d \times (n + k))$

Reasoning:

- Each digit is sorted using Counting Sort → $O(n + k)$ per digit.
- Number of digits = d → total time = $O(d \times (n + k))$.

Note: For fixed-length integers, d is usually constant → Radix Sort can run in **linear time $O(n)$** .

5 Space Complexity

- **Auxiliary Space:** $O(n + k)$
 - For Counting Sort on each digit.
 - Temporary arrays needed to store intermediate results.

6 Stability

- **Yes**
- Stability of the subroutine (Counting Sort) ensures that sorting by each digit does not change the relative order of equal elements.

7 Pros and Cons

Pros:

- Linear time sorting for small digit numbers → very efficient for large arrays of integers.
- Stable → preserves order of equal elements.
- Deterministic → predictable performance.

Cons:

- Not in-place → uses extra memory.
- Works only for integers or fixed-length strings.
- Efficiency depends on **number of digits**; not suitable for very large numbers with many digits.

✓ Summary Table (Radix Sort)

Feature	Value
Type	Non-comparison-based
Stability	Stable
Best Case TC	$O(d \times (n + k))$
Average Case TC	$O(d \times (n + k))$
Worst Case TC	$O(d \times (n + k))$
Space Complexity	$O(n + k)$
Adaptive	No
Use	Sorting integers or fixed-length strings efficiently

Bucket Sort :-

1 Type of Sorting

- **Non-comparison-based** (distribution sort): elements are distributed into **buckets** based on their value.
- **Stable** if the sorting used within each bucket is stable (like Insertion Sort).
- **Not in-place** → uses extra memory for buckets.
- Efficient for **uniformly distributed data**, especially floating-point numbers in the range (0,1).

2 Algorithm Idea

- Divide the interval of input values into **k buckets**.
- Distribute each element into its corresponding bucket.
- Sort each bucket individually (usually using Insertion Sort).
- Concatenate all buckets to get the sorted array.

3 Characteristics

- **Stable** → maintains relative order if stable sort is used inside buckets.
- **Not in-place** → needs extra memory for buckets.
- Works best when data is **uniformly distributed**.
- Adaptive for data that is **already partially sorted** within buckets.

4 Time Complexity

Let n = number of elements, k = number of buckets

1. **Best Case (even distribution in buckets)** → $O(n + k)$
 - Each bucket has roughly n/k elements → sorting inside each bucket is fast ($O(n/k)$ per bucket).
2. **Average Case (uniform distribution)** → $O(n + k)$
 - Most elements are evenly distributed → sorting inside buckets is efficient.
3. **Worst Case (all elements in one bucket)** → $O(n^2)$
 - All elements fall into a single bucket → sorting that bucket (e.g., with Insertion Sort) takes $O(n^2)$.**Reasoning:**
 - Distribution into buckets → $O(n)$
 - Sorting each bucket → depends on elements per bucket
 - Concatenating buckets → $O(n)$

5 Space Complexity

- **Auxiliary Space:** $O(n + k)$
 - Buckets store subsets of elements temporarily.

6 Stability

- **Yes**, if the sorting algorithm used inside buckets is stable.

7 Pros and Cons

Pros:

- Linear time for uniformly distributed data → very fast.
- Stable if stable sort is used in buckets.
- Adaptive → benefits from partially sorted distributions.

Cons:

- Not in-place → extra memory needed.
- Efficiency depends on **distribution of data**.
- Worst-case $O(n^2)$ if data is skewed.
- Works best for **floating-point numbers in (0,1)** or normalized data.

✓ Summary Table (Bucket Sort)

Feature	Value
Type	Non-comparison-based, distribution sort
Stability	Stable (if stable sort used in buckets)
Best Case TC	$O(n + k)$
Average Case TC	$O(n + k)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(n + k)$
Adaptive	Yes (depends on distribution)
Use	Uniformly distributed numbers, floating-point numbers in (0,1)

Heap Sort :-

Type of Sorting

- **Comparison-based:** sorts elements by using a **binary heap** data structure.
- **In-place:** uses no extra array for sorting aside from a few variables.
- **Not stable:** swapping elements in the heap may change the relative order of equal elements.
- **Divide & Conquer / Selection-based:** repeatedly extracts the maximum (or minimum) from the heap and places it at the correct position.

2 Algorithm Idea

- Build a **max-heap** from the array (largest element at the root).
- Swap the root with the last element → largest element moves to its correct position.
- Reduce heap size by one and **heapify** the root to restore the heap property.
- Repeat until all elements are sorted.

3 Characteristics

- **Not stable** → swapping during heapify can change relative order.
- **In-place** → uses constant extra memory.
- **Deterministic** → worst-case time is always $O(n \log n)$.
- Works well for **large datasets** and when memory efficiency is important.

4 Time Complexity

Let n = number of elements

1. **Best Case** → $O(n \log n)$
 - Building heap and heapifying elements still requires $\log n$ operations per element.
2. **Average Case** → $O(n \log n)$
 - Each heapify operation takes $\log n$, done for n elements.
3. **Worst Case** → $O(n \log n)$
 - Same as average case → consistent performance.

Reasoning:

- **Building max-heap:** $O(n)$
- **Heapify during extraction:** $O(\log n)$ per element $\times n$ elements → $O(n \log n)$
- Total = $O(n) + O(n \log n) = O(n \log n)$

5 Space Complexity

- **Auxiliary Space:** $O(1)$
- Reason: In-place sorting; no extra arrays needed.

6 Stability

- No
- Heapify operations swap elements arbitrarily → equal elements may change order.

7 Pros and Cons

Pros:

- In-place → low memory usage.
- Predictable $O(n \log n)$ time → good for large datasets.
- Simple implementation using arrays.

Cons:

- Not stable.
- Slower than Quick Sort in practice due to less locality of reference.
- Not adaptive → performance does not improve for partially sorted arrays.

✓ Summary Table (Heap Sort)

Feature	Value
Type	Comparison-based, in-place, selection-like
Stability	Not stable
Best Case TC	$O(n \log n)$
Average Case TC	$O(n \log n)$
Worst Case TC	$O(n \log n)$
Space Complexity	$O(1)$
Adaptive	No
Use	Large datasets, in-place sorting, predictable performance

Shell Sort :-

1 Type of Sorting

- **Comparison-based:** generalization of **Insertion Sort**, compares elements that are a gap apart.
- **In-place:** only a small amount of extra memory is needed.
- **Not stable:** swapping distant elements can change the relative order of equal elements.
- Efficient for **medium-sized arrays**.

2 Algorithm Idea

- Choose a **gap sequence** (e.g., $n/2, n/4, \dots, 1$).
- For each gap, perform **gapped insertion sort**: compare elements that are gap apart and swap if needed.
- Reduce the gap until it becomes 1 \rightarrow final pass is a normal insertion sort.

3 Characteristics

- **Not stable** \rightarrow elements far apart may swap.
- **In-place** \rightarrow only a few variables needed for swapping.
- **Adaptive** \rightarrow performs better than Insertion Sort on partially sorted arrays.
- Efficiency depends on the **gap sequence** chosen.

4 Time Complexity

Let n = number of elements

1. **Best Case (nearly sorted array, good gap sequence)** $\rightarrow O(n \log n)$
2. **Average Case** \rightarrow depends on gap sequence, typically $O(n^{(3/2)})$
3. **Worst Case (original Shell sequence)** $\rightarrow O(n^2)$

Reasoning:

- The outer loop reduces gaps, inner loop performs gapped insertion sort.
- Fewer comparisons for larger gaps \rightarrow reduces total shifts compared to normal Insertion Sort.
- Using better sequences (Hibbard, Sedgewick) improves performance to $O(n^{(4/3)}) - O(n^{(3/2)})$.

5 Space Complexity

- **Auxiliary Space:** $O(1)$ → in-place sorting.

6 Stability

- No
- Swapping elements that are far apart can disturb the relative order of equal elements.

7 Pros and Cons

Pros:

- In-place → low memory usage.
- Adaptive → faster than Bubble or Insertion Sort for medium-sized arrays.
- Simple implementation with gap sequence.

Cons:

- Not stable.
- Time complexity depends on gap sequence.
- Worst-case still $O(n^2)$ with original gap sequence.

✓ Summary Table (Shell Sort)

Feature	Value
Type	Comparison-based, in-place
Stability	Not stable
Best Case TC	$O(n \log n)$ (with good gap)
Average Case TC	$O(n^{(3/2)})$
Worst Case TC	$O(n^2)$
Space Complexity	$O(1)$
Adaptive	Yes
Use	Medium-sized arrays, partially sorted arrays

Tim Sort :-

1 Type of Sorting

- **Hybrid sorting algorithm:** combines **Merge Sort** and **Insertion Sort**.
- **Comparison-based.**
- **Stable:** preserves relative order of equal elements.
- Designed to perform well on **real-world data** (partially ordered arrays).

2 Algorithm Idea

- Divide the array into **small segments (runs)**.
- Sort each run using **Insertion Sort** (efficient for small arrays).
- Merge runs using **Merge Sort** strategy.
- Repeat until the entire array is merged into a fully sorted array.

3 Characteristics

- **Stable** → preserves relative order of equal elements.
- **Not in-place** → requires $O(n)$ extra space for merging runs.
- **Adaptive** → extremely efficient for partially sorted arrays.
- Designed for **real-world practical sorting** → used in Python and Java.

4 Time Complexity

Let n = number of elements, run size = minrun

1. **Best Case (already sorted) → $O(n)$**
 - Insertion Sort on small runs is linear.
2. **Average Case → $O(n \log n)$**
3. **Worst Case → $O(n \log n)$**
 - Merging runs dominates time complexity.

Reasoning:

- Runs are sorted with Insertion Sort → $O(n)$ for small or nearly sorted runs.
- Merge process → $O(n \log n)$ total across all levels.
- Adaptivity gives best-case linear time for nearly sorted data.

5 Space Complexity

- **Auxiliary Space:** $O(n)$
- Needed for temporary arrays during merging runs.

6 Stability

- **Yes**
- Merge step preserves the order of equal elements.

7 Pros and Cons

Pros:

- Stable.
- Adaptive → very fast on partially sorted arrays.
- Efficient in practice → widely used in standard libraries.
- Handles large datasets well.

Cons:

- Not in-place → extra memory required.
- Slightly more complex than basic sorts like Quick or Merge Sort.

✓ Summary Table (Tim Sort)

Feature	Value
Type	Hybrid (Insertion + Merge), Comparison-based
Stability	Stable
Best Case TC	$O(n)$
Average Case TC	$O(n \log n)$
Worst Case TC	$O(n \log n)$
Space Complexity	$O(n)$
Adaptive	Yes
Use	Real-world sorting, partially sorted arrays, large datasets

Cocktail (Shaker) Sort :-

1 Type of Sorting

- **Comparison-based, in-place** sorting algorithm.
- **Stable** → preserves relative order of equal elements.
- Variant of **Bubble Sort**, but sorts in **both directions**.

2 Algorithm Idea

- Traverse the array **forward**, bubbling the largest element to the end.
- Then traverse **backward**, bubbling the smallest element to the beginning.
- Repeat forward and backward passes until the array is sorted.

3 Characteristics

- **Stable** → preserves order of equal elements.
- **In-place** → uses only a few extra variables for swapping.
- **Adaptive** → can stop early if no swaps occur during a pass.
- More efficient than Bubble Sort on **partially sorted arrays**.

4 Time Complexity

Let n = number of elements

1. **Best Case (already sorted)** → $O(n)$
 - Only one forward and backward pass needed; no swaps.
2. **Average Case** → $O(n^2)$
 - Comparisons and swaps similar to Bubble Sort.
3. **Worst Case (reverse sorted)** → $O(n^2)$
 - Maximum comparisons and swaps required.

Reasoning:

- Two nested loops: outer loop controls passes, inner loop performs comparisons/swaps.
- Forward and backward traversals slightly reduce passes compared to Bubble Sort.

5 Space Complexity

- **Auxiliary Space:** $O(1)$
- Sorting is done in-place with a temporary variable for swapping.

6 Stability

- Yes
- Equal elements are not swapped out of order.

7 Pros and Cons

Pros:

- Stable and in-place.
- Adaptive → better than Bubble Sort on nearly sorted arrays.
- Simple to implement.

Cons:

- Time complexity still $O(n^2)$ for large arrays → inefficient for large datasets.
- Slightly more complex than Bubble Sort but gains minor improvement.

✓ Summary Table (Cocktail Sort / Shaker Sort)

Feature	Value
Type	Comparison-based, in-place
Stability	Stable
Best Case TC	$O(n)$
Average Case TC	$O(n^2)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(1)$
Adaptive	Yes
Use	Small or partially sorted arrays

Comb Sort :-

1 Type of Sorting

- **Comparison-based, in-place** sorting algorithm.
- **Improvement over Bubble Sort** → eliminates small values near the end (turtles) faster.
- **Not stable** by default.

2 Algorithm Idea

- Use a **gap sequence** instead of comparing only adjacent elements.
- Start with a gap = n (array size) and reduce it using a **shrink factor** (commonly 1.3).
- Compare elements that are **gap apart** and swap if needed.
- Repeat until gap = 1 and no swaps occur → array is sorted.

3 Characteristics

- **Not stable** → swapping distant elements may change relative order of equal elements.
- **In-place** → uses only a few variables.
- Faster than Bubble Sort → handles small values near the end more efficiently.
- Adaptive to some extent → reduces number of swaps for partially sorted arrays.

4 Time Complexity

Let n = number of elements

1. **Best Case (already sorted)** → $O(n \log n)$
 - Fewer comparisons needed; gap sequence quickly converges.
2. **Average Case** → $O(n^2 / 2^p) \sim O(n^2 / \log n)$
 - Depends on shrink factor; better than Bubble Sort in practice.
3. **Worst Case** → $O(n^2)$
 - If the gap sequence is not optimal, may require many comparisons.

Reasoning:

- Gap reduces the number of small-distance swaps → removes turtles faster.
- Final pass with gap = 1 is like Bubble Sort, but fewer elements are left to swap.

5 Space Complexity

- **Auxiliary Space:** $O(1)$
- Sorting done in-place.

6 Stability

- **No**
- Swapping elements with a large gap may change relative order of equal elements.

7 Pros and Cons

Pros:

- Faster than Bubble Sort and Cocktail Sort in practice.
- In-place \rightarrow low memory usage.
- Simple to implement.

Cons:

- Not stable.
- Worst-case $O(n^2) \rightarrow$ inefficient for large datasets.
- Time complexity depends on shrink factor.

✓ Summary Table (Comb Sort)

Feature	Value
Type	Comparison-based, in-place
Stability	Not stable
Best Case TC	$O(n \log n)$
Average Case TC	$\sim O(n^2 / \log n)$
Worst Case TC	$O(n^2)$
Space Complexity	$O(1)$
Adaptive	Somewhat
Use	Small to medium arrays, faster Bubble Sort alternative