Solution Key to Practice Problem Set 1

1. pressure at bottom = 50 + γh
   pressure at bottom = 50 + (12.34)(2) = 74.68 kPa

2. p_A + (8.62)(0.12)=101.5 +(15.57)(0.35)
   p_A =105.9 kPa

3. (9.8)(H-0.15)-[(13.6)(9.8)](0.20)= 0
   H=2.87 m

4. 0 + (9.8)(0.110+0.240)-[(0.83)(9.8)](0.240 +h)= 0
   h= 0.1817m= 181.7 mm

8. The average pressure on the door is the pressure value at the centroid (midpoint) of the door and is determined to be

$$P_{ave} = P_C = \rho g h_C = \rho g(s + b/2)$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(8 + 1.2/2 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= \mathbf{84.4 \text{ kN/m}^2}$$

Then the resultant hydrostatic force on the door becomes

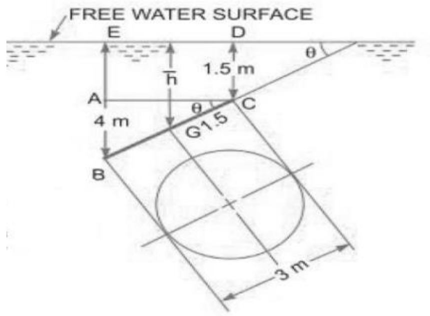
$$F_R = P_{ave}A = (84.4 \text{ kN/m}^2)(1 \text{ m} \times 1.2 \text{ m}) = \mathbf{101.3 \text{ kN}}$$

The pressure center is directly under the midpoint of the door, and its distance from the surface of the lake is determined from Eq. 3–24 by setting P_0 = 0 to be

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.2}{2} + \frac{1.2^2}{12(8 + 1.2/2)} = \mathbf{8.61 \text{ m}}$$

A strong person can lift 100 kg, whose weight is 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN · m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN · m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.

9.



Dia. of plate, $d = 3.0$ m

$\therefore$ Area, $A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3.0)^2 = 7.0685 \text{ m}^2$

Distance $DC = 1.5$ m, $BE = 4$ m

Distance of C.G. from free surface

$$= \bar{h} = CD + GC \sin\theta = 1.5 + 1.5 \sin\theta$$

But $$\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC} = \frac{4.0 - DC}{3.0} = \frac{4.0 - 1.5}{3.0}$$

$$= \frac{2.5}{3.0} = 0.8333$$

$\therefore$ $$\bar{h} = 1.5 + 1.5 \times .8333 = 1.5 + 1.249 = 2.749 \text{ m}$$

(*i*) **Total pressure (F)**

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 7.0685 \times 2.749 = \mathbf{190621\ N.}$$

(*ii*) **Centre of pressure (h*)**

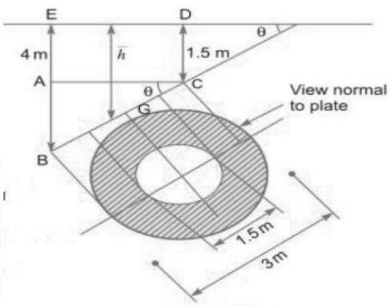
Using equation (3.10), we have $h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

where $I_G = \frac{\pi}{64} d^4 = \frac{\pi}{64} (3)^4 = 3.976 \text{ m}^4$

$$h^* = \frac{3.976 \times (.8333) \times .8333}{7.0685 \times 2.749} + 2.749 = 0.1420 + 2.749$$

$$= \mathbf{2.891\ m.}$$

10.



Dia. of plate, $d = 3.0$ m

$\therefore$ Area of solid plate $= \frac{\pi}{4} d^2 = \frac{\pi}{4} (3)^2 = 7.0685 \text{ m}^2$

Dia. of hole in the plate, $d_0 = 1.5$ m

$\therefore$ Area of hole $= \frac{\pi}{4} {d_0}^2 = \frac{\pi}{4} (1.5)^2 = 1.7671 \text{ m}^2$

$\therefore$ Area of the given plate, A = Area of solid plate – Area of hole

$= 7.0685 - 1.7671 = 5.3014 \text{ m}^2$

Distance $CD = 1.5$, $BE = 4$ m

Distance of C.G. from the free surface,

$$\bar{h} = CD + GC \sin\theta$$
$$= 1.5 + 1.5 \sin\theta$$

But $$\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC} = \frac{4 - 1.5}{3} = \frac{2.5}{3}$$

$\therefore$ $$\bar{h} = 1.5 + 1.5 \times \frac{2.5}{3} = 1.5 + 1.25 = 2.75 \text{ m}$$

(*i*) **Total pressure force (F)**

$$F = \rho g A \bar{h}$$
$$= 1000 \times 9.81 \times 5.3014 \times 2.75$$
$$= 143018 \text{ N} = \mathbf{143.018 \text{ kN}.}$$

(*ii*) **Position of centre of pressure (h*)**

Using equation (3.10), we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where $I_G = \frac{\pi}{64} [d^4 - d_0^4] = \frac{\pi}{64} [3^4 - 1.5^4] \text{ m}^4$

$A = \frac{\pi}{4} [d^2 - d_0^2] = \frac{\pi}{4} [3^2 - 1.5^2] \text{ m}^2$

$\sin\theta = \frac{2.5}{3}$ and $\bar{h} = 2.75$

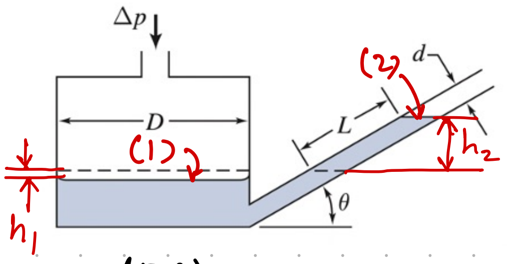
$\therefore$ $$h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right] \times \left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right] \times 2.75} + 2.75$$

$$= \frac{\frac{1}{16}\left[3^2 + 1.5^2\right] \times \left(\frac{2.5}{3}\right)^2}{2.75} + 2.75 = \frac{1 \times 11.25 \times 6.25}{16 \times 2.75 \times 9} + 2.75$$

$$= 0.177 + 2.75 = \mathbf{2.927 \text{ m}.}$$

Problem (5)

a). Apply fundamental equation of Hydrostatics (FHSE) between points (1) and (2)



$$\Delta P = P_1 - P_2 = \rho_l g (h_1 + h_2) \quad \text{— (5A)}$$

- Eliminate h_1

Volume displaced in reservoir = Volume rise in the tube

$$\Rightarrow \frac{\pi D^2}{4} h_1 = \frac{\pi d^2}{4} l \quad \Longrightarrow \quad h_1 = L\left(\frac{d}{D}\right)^2$$

- Also, from the manometer geometry, $h_2 = L\sin\theta$

$$\therefore \text{(5A)} \Rightarrow \Delta P = \rho_l g \left\{ L\sin\theta + L\left(\frac{d}{D}\right)^2 \right\}$$

$$\Delta P = \rho_l g L \left\{ \sin\theta + \left(\frac{d}{D}\right)^2 \right\}$$

$\therefore$ deflection $\boxed{L = \dfrac{\Delta P}{\rho_l g \left\{ \sin\theta + \left(\frac{d}{D}\right)^2 \right\}}}$ — (5B)

b) deflection in a U-tube manometer,

$$h = \frac{\Delta P}{\rho_l g} \quad \text{(refer lecture notes)} \quad \text{— (5C)}$$

$$\therefore \frac{L}{h} = \frac{1}{\sin\theta + \left(\frac{d}{D}\right)^2} \Rightarrow \theta = \sin^{-1}\left\{ \frac{1}{(L/h)} - \left(\frac{d}{D}\right)^2 \right\}$$

— (5D)

$L/h = 6$

$\frac{d}{D} = \frac{8}{85}$

$\Longrightarrow \boxed{\theta = 9.077^\circ}$

manometer sensitivity is the ratio of the manometer deflection to the deflection produced in a straight water manometer

from (5B) and (5C),

$$\boxed{S = \frac{L}{h_w} = \frac{L}{SG \cdot h} = \frac{6}{SG}}$$

S → Specific gravity of fluid.

(5C) eqn. (5B) $\Rightarrow \Delta P = \rho_l g L \left\{ \sin\theta + \left(\frac{d}{D}\right)^2 \right\}$

or $\Delta P = \rho_w g L (SG) \left\{ \sin\theta + \left(\frac{d}{D}\right)^2 \right\}$ —(5E)

Given $\Delta P = 25$ mm of water (gage)

$$\Rightarrow \Delta P = \rho g h_w = (1000)(9.81)(25 \times 10^{-3})$$
$$= 245 \text{ Pa}$$

$h_w = 25\,mm = 0.025\,m$

$\Delta P = 245$ Pa

$\rho_w = 1000\ kg\,m^{-3}$

$g = 9.81\ ms^{-2}$

$L = 15\ cm = 0.15\,m$

$SG = 1.6$

$d/D = 8/85 = 0.0941$

$$\theta = \sin^{-1}\left\{ \frac{\Delta P}{(SG)\rho_w g L} - \left(\frac{d}{D}\right)^2 \right\}$$

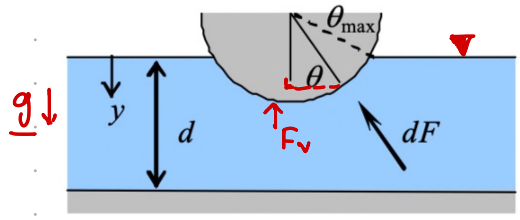
$$\boxed{\theta = 5.463^\circ}$$

Sensitivity, $s = \frac{L}{h_w} = \frac{0.15}{0.025} = 6$

Problem (6)

Governing equations:

FHSE: $\frac{dP}{dy} = \rho g$ — (6A)



vertical hydrostatic force

$$F_v = \int P\,dA_y \quad \text{— (6B)}$$

- For a given depth d,

$$y = d - (R - R\cos\theta) = d - R + R\cos\theta \quad \text{— (6C)}$$

- Force balance on canoe:

$$\Sigma F_y = 0 = Mg - F_v \,, \qquad \text{where } M - \text{total mass of canoe}$$

- $F_v = \int P\,dA_y = \int P\cos\theta\,dA = \int_{-\theta_{max}}^{\theta_{max}} \rho g y L R\cos\theta\,d\theta$

$$= 2\rho g L R \int_0^{\theta_{max}} (d - R + R\cos\theta)\cos\theta\,d\theta$$

$$= 2\rho g L R \int_0^{\theta_{max}} \left[(d-R)\cos\theta + R\cos^2\theta \right] d\theta$$

$$F_v = 2\rho \cancel{g} L R \left\{ (d-R)\sin\theta_{max} + R\cdot\left(\frac{\theta_{max}}{2} + \frac{\sin(2\theta_{max})}{4} \right) \right\} = M\cancel{g}$$

or $$\boxed{M = 2\rho L R \left\{ (d-R)\sin\theta_{max} + R\left(\frac{\theta_{max}}{2} + \frac{\sin(2\theta_{max})}{4} \right) \right\}}$$

Given:

$R = 1\text{ m}$

$L = 6\text{ m}$

$d = 15\text{ m}$

- Maximum possible y. $y_{max} = R$, for $\theta_{max} = 90^\circ$ —(6D)

$$R = d - R + R\,\cancelto{0}{\cos\theta_{max}}$$

$$2R = d \Rightarrow d/R = 2$$

$\therefore$ For a floating canoe, $d/R < 2$.

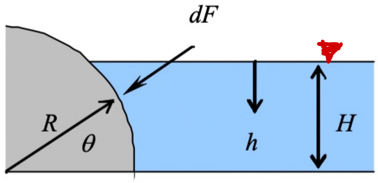
For given conditions, $d/R = 15 > 2 \Rightarrow$ $\boxed{\text{Canoe will Sink.}}$

Results cannot be plotted for the given conditions.

Problem (7)

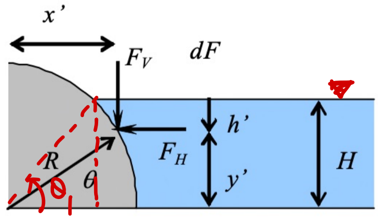
Governing equations

FHSE : $$\frac{dP}{dh} = \rho g \quad \text{— (7A)}$$



vertical HSF, $$F_v = \int P\, dA_y \quad \text{— (7B)}$$

horizontal HSF, $$F_h = P_c \cdot A \quad \text{— (7C)}$$



Moment of vertical force,

$$x' F_v = \int x\, dF_v \quad \text{— (7D)}$$

Line of action of horizontal force,

$$h' = h_c + \frac{I_{xx}}{h_c A} \quad \text{— (7E)}$$

- Assume atmospheric pressure acting on the left of the curved surface.
- integrating FHSE : $P = \rho g h$
- From geometry

$H = 0.7\,m$
$R = 1\,m$
$W = 5\,m$

$$h = H - R\sin\theta$$
$$y = R\sin\theta$$
$$x = R\cos\theta$$
$$dA = WR\,d\theta$$
$$\theta_1 = \sin^{-1}\left(\frac{H}{R}\right) = 44.42^\circ = 0.7753 \text{ rad}$$

$$\therefore F_v = \int P\, dA_y = \int \rho g h \sin\theta\, dA = \int_0^{\theta_1} \rho g (H - R\sin\theta)\sin\theta \cdot WR\,d\theta$$

$$= \rho g W R \int_0^{\theta_1} (H\sin\theta - R\sin^2\theta)\, d\theta$$

$$F_v = \rho g W R \left\{ H(1-\cos\theta_1) - \frac{R\theta_1}{2} + \frac{R\sin(2\theta_1)}{4} \right\}$$

$$= (1000)(9.81)(5)(1) \left\{ 0.7 - 0.7\cos(0.7754) - \frac{(1)(0.7754)}{2} + \frac{(1)\sin(2\times 0.7754)}{4} \right\}$$

$$\boxed{F_v = 3058.33\ N}$$

To calculate line of action of this force,

$$x' F_v = \int R\cos\theta \, \rho g h \sin\theta \, . \, dA$$

$$x' F_v = \rho g W R^2 \int_0^{\theta_1} (H\sin\theta\cos\theta - R\sin^2\theta\cos\theta)\, d\theta$$

$$x' F_v = \rho g W R^2 \left\{ \frac{H}{2}\sin^2\theta_1 - \frac{R}{3}\sin^3\theta_1 \right\}$$

$$\text{or } x' = \frac{\rho g W R^2}{F_v} \left\{ \frac{H}{2}\sin^2\theta_1 - \frac{R}{3}\sin^3\theta_1 \right\} \qquad \Big|\ \sin\theta_1 = 0.7$$

$$= \frac{(1000)(9.81)(5)(1)^2}{3058.33} \left\{ \frac{0.7}{2}(0.7)^2 - \frac{1}{3}(0.7)^3 \right\}$$

$$\boxed{x' = 0.9168\ m}$$

- Horizontal force, $F_H = P_c A = \rho g h_c W H = \rho g \left(\frac{H}{2}\right) W H$

$$\boxed{F_H = 12017.25\ N}$$

- line of action of F_H, $h' = h_c + \frac{I_{xx}}{h_c A} = h_c + \left(\frac{WH^3}{12}\right)\left(\frac{1}{WHh_c}\right)$

$$h' = \frac{H}{2} + \frac{H}{6} = \frac{2}{3}H$$

$$\boxed{h' = 0.4667\ m}$$