Implementation of Least Squares Monte Carlo

Using Longstaff-Schwartz algorithm

Longstaff-Schwartz algorithm

- Proposed in **Longstaff and Schwartz (2001)**.
- Computes price of options that can be exercised before maturity.
- Uses least-squares regression to estimate conditional expectations.
- Useful to price American-style options.

American Options

Allow for **early exercise**:

The value at each time step is the maximum between the payoff of exercising the option at that time step (early exercise value) and the expected value of holding the option to maturity (continuation value).

For an **American call** option:

$$V_t^i = \max\left((S_t^i - K)^+,\ e^{-(T-t)r}(E[S_T|S_t^i] - K)^+\right)$$

Motivation: Why do we need the LSMC algorithm?

- How can we evaluate the **continuation value** $E[S_T|S_t^i]$?

  Nested Monte Carlo simulations for every S_t^i until maturity $\rightarrow$ **unfeasible** for large numbers

- Binomial Tree: high discretization error if used with long time steps.

  Will **underestimate** the number of **early exercise opportunities** as it only provides two outcomes for the value of the underlying.

  On the other hand, time complexity: $O(2^n)$

LSMC algorithm

We take as example an **American call** option with 1 year maturity, exercisable at times 1,2 and 3. Furthermore:

$$S_0 = 1,\ K = 1.1,\ r = 0.1,\ \sigma = 0.2$$

```
GBM_paths = pm.BS_path(1, 0.1, 0.2, 1, nSteps = 3, nPaths = 8)
```