Coursework 1 – Transient Conduction

Adam Duncan

October 22, 2021

1 *Part A: Using lumped capacitance*

1.1 Assumptions

- Internal temperature of the steel ball is uniform at any time t.
- No change in water temperature
- No heat transfer by radiation
- Material is standard carbon steel
- Material properties constant (taken at average temperature $T = 469^oC$)

1.2 Schematic

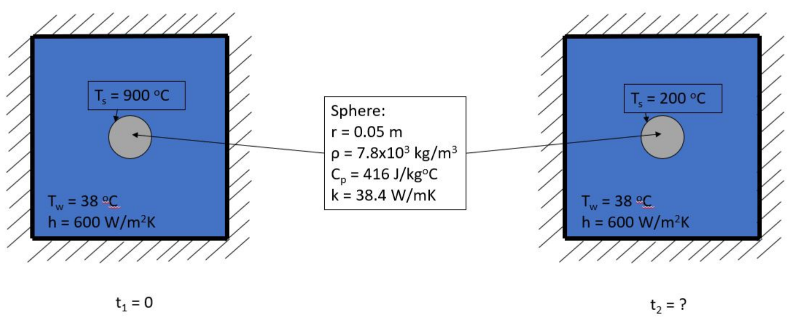

Figure 1: Part A schematic at initial and final state.

1.3 Analysis

Energy balance for closed system gives the following equation.

$$\dot{Q} = hA(T_c - T_f) = C_p \rho V \frac{dT_c}{dt} \tag{1}$$

Where $\dot{Q}$ is energy flow $[W]$, h is the heat transfer coefficient $[W/m^2K]$, A is the surface area between the ball and water $[m^2]$, T_c is the temperature of the steel ball $[^oC]$, T_f is the temperature of the water $[^oC]$, C_p is the specific heat capacity $[J/mK]$, ρ is the density of the steel ball$[kg/m^3]$, V is the volume of the steel ball $[m^3]$ and t is the time $[s]$.

Rearranging to separate the variables gives.

$$\frac{1}{T_c - T_f} dT_c = \frac{hA}{C_p \rho V} dt \tag{2}$$

Which integrates to give.

$$\ln\left(\frac{T_{c1} - T_f}{T_{c2} - T_f}\right) = \frac{hA}{C_p \rho V}(t_2 - t_1) \tag{3}$$

Where t_i is the time [s] at state i.

$$Bi = \frac{hL_c}{k} \tag{4}$$

Where h is conductivity [W/mK]

$$t = \frac{f_0 \rho C_p R^2}{k} \tag{5}$$

2 *Part B: Lumped capacitance justification*

3 *Part C: Transient conduction*

4 *Part D: Non-infinite water bath*

5 *Part E: Equilibrium temperature*