Coursework 1 – Transient Conduction

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1 *Part A: Using lumped capacitance*

1.1 Assumptions

- Internal temperature of the steel ball is uniform at any time t.
- No change in water temperature
- No heat transfer by radiation
- Material is standard carbon steel
- Material properties constant (taken at average temperature $T = 469^oC$)

1.2 Properties

Table 1: Properties from problem

Property	Value	Unit
Characteristic length, L	5	cm
Diameter, D	10	cm
Temperature of the water, T_w	38	^oC
Initial temperature of steel ball, $T_{s,1}$	900	^oC
Final temperature of steel ball, $T_{s,2}$	200	^oC
Heat transfer coefficient, h	600	W/m^2K

Table 2: Properties from literature

Property	Value at $T_{avg}(469\ ^{o}C)$	Unit	Source
Specific heat capacity, Cp	552	$J \cdot kg^{-1}K^{-1}$	[3]
Density	7.8 x 10^3	$kg \cdot m^{-3}$	[1]
Conductivity	40	$W \cdot m^{-1}K^{-1}$	[3]

The density of steel is assumed to be constant over the temperature range so the value in table 2, which is given at 300K, is assumed to be accurate. To confirm this assumption is acceptable the elongation was calculated using the ISO 834 standard equations[3]. This showed the overall change in volume of the sphere was 3% over the full temperature range of the problem. As $V \propto \rho$ this change is low enough to be discounted and for the assumption to be justified.

Table 3: Properties error analysis

T $[^{o}C]$	C_p $[J \cdot kg^{-1}K^{-1}]$	$\Delta\%$ from T_{469}	k $[W \cdot m^{-1}K^{-1}]$	$\Delta\%$ from T_{469}
900	650	56	27.3	-29
469	416	0	38.4	0
38	454	9	52.7	37

Similarly the variation in specific heat capacity (C_p) and conductivity (k) were considered over the full temperature range as shown in Table 3. The value of k varies from the T_{mean} value by roughly 30% either way but the T_{mean} value is only 4% different from the mean of the two temperature extremes. This fact coupled with the information given in ISO 835 standard [3] suggest a linear change in h, which makes the approximation useful.

The variation in C_p is less clear cut, with the value at T_{mean} falling much closer to the value at T_{38} than the value at T_{900} as showing in table 3. This variation is due to the non-linear variation of C_p with temperature [3]. Further investigation would be needed get an accurate estimate of the level of error this approximation introduces to the result but this falls beyond the scope of this problem set so the approximation is used as a best estimate.

1.3 Schematic

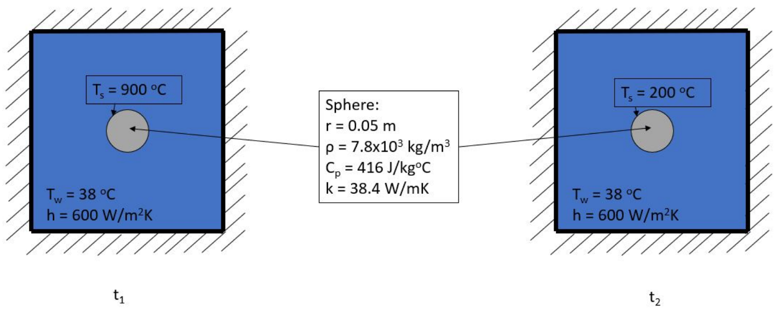

Figure 1: Part A schematic at initial and final state.

1.4 Analysis

Energy balance for closed system gives the following equation.

$$\dot{Q} = hA(T_s - T_f) = C_p \rho V \frac{dT_c}{dt} \tag{1}$$

Where $\dot{Q}$ is heat $[W]$, h is the heat transfer coefficient $[W/m^2K]$, A is the surface area between the ball and water $[m^2]$, T_s is the temperature of the steel ball $[^oC]$, T_f is the temperature of the water $[^oC]$, C_p is the specific heat capacity $[J/mK]$, ρ is the density of the steel ball$[kg/m^3]$, V is the volume of the steel ball $[m^3]$ and t is the time $[s]$.

Rearranging (1) to separate the variables gives.

$$\frac{1}{T_s - T_f} dT_c = \frac{hA}{C_p \rho V} dt \tag{2}$$

Which integrates to give.

$$\ln\left(\frac{T_{s1} - T_f}{T_{s2} - T_f}\right) = \frac{hA}{C_p \rho V}(t_2 - t_1) \tag{3}$$

Where t_i and T_{si} are the time [s] and temperature $[^oC]$ receptively at state i.

Rearranging (3) to make t_2 the subject gives.

$$t_2 = \frac{C_p \rho V}{hA}\left(\ln\left(\frac{T_{s1} - T_f}{T_{s2} - T_f}\right)\right) \tag{4}$$

Substituting in the values for the variables given in Figure 1 gives the final value.

$$\boldsymbol{t_2 = 205s} \tag{5}$$

Where t_2 is the time for the steel ball to reach a temperature of 200^oC under given assumptions.

2 *Part B: Lumped capacitance justification*

The lumped capacitance method is only valid if the ratio of the conductive heat transfer to convective heat transfer is low. This ratio is known as the Biot number, Bi, and is given by.

$$Bi = \frac{h \cdot L_c}{k} \tag{6}$$

Where h is convective coefficient $[W/m^2K]$, L_c is the characteristic length [m] and k is the conductivity $[W/m \cdot K]$.

Applying the values from Tables 1 and 2, choosing to set $L_c = R$ and substituting into equation 6 gives:

$$Bi = 0.7 \tag{7}$$

If $Bi > 0.1$ then the lumped capacitance method is no longer applicable as the assumptions made introduce non-trivial errors [1]. This means that the result in part A likely inaccurate.

It is worth noting that the choice of L_c is significant. it is common to select L_c to be the maximum distance over which a temperature gradient would occur, as has been done above, but the method from the mathematical derivation is to use $L_c = \frac{V}{A_s}$ which for a sphere gives $L_c = \frac{R}{3}$. This means the use of $L_c = R$ will tend to overestimate the value of Bi. In this case however, using $L_c = \frac{R}{3}$ gives $Bi = 0.25$ so the result can still be assumed to be inaccurate using the lumped capacitance method despite the overestimate.

3 *Part C: Transient conduction*

3.1 Time interval to cool centre of sphere to 200^oC

As the initial and final temperature of the sphere are specified and as well as the temperature of the fluid which is assumed to be constant the ratio below can be calculated.

$$\frac{T_{(0,t)} - T_f}{T_i - T_f} \tag{8}$$

Using the values in table 1 the answer to (8) is 0.19. Taking the inverse of (7) gives 1.7. These two values allows the Fourier number to be determined from the Heisler chart [2], which is determined to be.

$$Fr = 1.2 \tag{9}$$

The Fourier number is defined below.

$$Fr = \frac{tk}{\rho C_p R^2} \tag{10}$$

Using the result from (9) and the values in tables 1 and 2 into (10) gives the final result time taken.

$$\boldsymbol{t = \frac{F_o \rho C_p R^2}{k} = 257s} \tag{11}$$

Comparing the results of (5) and (11) the result gained from the lumped capacitance method is of a similar maginitude but lower than the transient solution. This matches with expectations as the temperature gradient that was found to be important for this case in section 2 would take additional time to reach the centre point.

3.2 title

4 *Part D: Non-infinite water bath*

5 *Part E: Equilibrium temperature*

References

[1] T. L. Bergman and Frank P. Incropera, editors. *Fundamentals of heat and mass transfer*. Wiley, Hoboken, NJ, 7th ed edition, 2011.

[2] P.S. Ghoshdastidar. 4.5 multidimensional transient problems: Application of heisler charts, 2012.

[3] Jean-Marc Franssen and Paulo Vila Real. *Fire Design of Steel Structures : EC1: Actions on Structures; Part 1-2: Actions on Structure Exposed to Fire; EC3: Design of Steel Structures; Part 1-2: Structural Fire Design*, volume Second revised edition of *ECCS-SCI Eurocode Design Manuals*. Ernst & Sohn, [Place of publication not identified], 2015.