Coursework 1 – Transient Conduction

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1 *Part A: Using lumped capacitance*

1.1 Assumptions

- Internal temperature of the steel ball is uniform at any time t.
- No change in water temperature
- No heat transfer by radiation
- Material is standard carbon steel
- Material properties constant (taken at average temperature $T = 469^oC$)

1.2 Properties

Table 1: Properties from problem

Property	Value	Unit
Characteristic length, L	5	cm
Diameter, D	10	cm
Temperature of the water, T_w	38	^oC
Initial temperature of steel ball, $T_{s,1}$	900	^oC
Final temperature of steel ball, $T_{s,2}$	200	^oC
Heat transfer coefficient, h	600	W/m^2K

Table 2: Properties from literature

Property	Value at $T_{avg}(469\ ^oC)$	Unit	Source
Specific heat capacity, Cp	552	J/kgK	
Density	7.8 x 10^3	kg/m^3	
Conductivity	40	W/mK	

1.3 Schematic

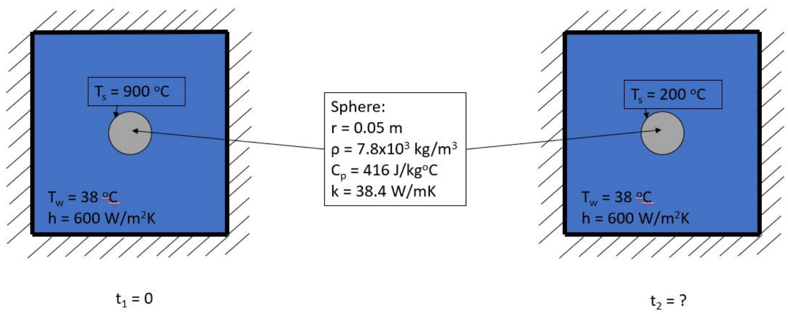

Figure 1: Part A schematic at initial and final state.

1.4 Analysis

Energy balance for closed system gives the following equation.

$$\dot{Q} = hA(T_s - T_f) = C_p \rho V \frac{dT_c}{dt} \tag{1}$$

Where $\dot{Q}$ is energy flow $[W]$, h is the heat transfer coefficient $[W/m^2K]$, A is the surface area between the ball and water $[m^2]$, T_s is the temperature of the steel ball $[^oC]$, T_f is the temperature of the water $[^oC]$, C_p is the specific heat capacity $[J/mK]$, ρ is the density of the steel ball$[kg/m^3]$, V is the volume of the steel ball $[m^3]$ and t is the time $[s]$.

Rearranging (1) to separate the variables gives.

$$\frac{1}{T_s - T_f} dT_c = \frac{hA}{C_p \rho V} dt \tag{2}$$

Which integrates to give.

$$\ln\left(\frac{T_{s1} - T_f}{T_{s2} - T_f}\right) = \frac{hA}{C_p \rho V}(t_2 - t_1) \tag{3}$$

Where t_i and T_{si} are the time [s] and temperature [^oC] receptively at state i.

Rearranging (3) to make t_2 the subject gives.

$$t_2 = \frac{C_p \rho V}{hA}\left(\ln\left(\frac{T_{s1} - T_f}{T_{s2} - T_f}\right)\right) \tag{4}$$

Substituting in the values for the variables given in Figure 1 gives the final value.

$$\boldsymbol{t_2 = 205s} \tag{5}$$

Where t_2 is the time for the steel ball to reach a temperature of 200^oC under given assumptions.

2 *Part B: Lumped capacitance justification*

$$Bi = \frac{hL_c}{k} \tag{6}$$

Where h is conductivity [W/mK]

$$t = \frac{f_0 \rho C_p R^2}{k} \tag{7}$$

3 *Part C: Transient conduction*

4 *Part D: Non-infinite water bath*

5 *Part E: Equilibrium temperature*