Coursework 1 – Transient Conduction

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1 *Part A: Using lumped capacitance*

1.1 Assumptions

- Internal temperature of the steel ball is uniform at any time t.

- No change in water temperature.

- No heat transfer by radiation.

- Material is standard carbon steel.

- Material properties are constant (taken at average temperature $T = 469^o C$).

1.2 Properties

Table 1: Properties from problem definition

Property	Value	Unit
Characteristic length, L	5	cm
Diameter, D	10	cm
Temperature of the water, T_w	38	$^o C$
Initial temperature of steel ball, $T_{s,1}$	900	$^o C$
Final temperature of steel ball, $T_{s,2}$	200	$^o C$
Heat transfer coefficient, h	600	$W/m^2 K$

Table 2: Properties from literature

Property	Value at $T_{avg}(469 \ ^o C)$	Unit	Source
Specific heat capacity, Cp	552	$J \cdot kg^{-1} K^{-1}$	[3]
Density, ρ	7.8 x 10^3	$kg \cdot m^{-3}$	[1]
Conductivity, k	40	$W \cdot m^{-1} K^{-1}$	[3]

The density of steel is assumed to be constant over the temperature range so the value in table 2, which is given at 300K, is assumed to be accurate. To confirm this assumption is acceptable the elongation was calculated using the ISO 834 standard equations[3]. This showed the overall change

Table 3: Properties error analysis

T [^oC]	C_p [$J \cdot kg^{-1}K^{-1}$]	$\Delta\%$ from T_{469}	k [$W \cdot m^{-1}K^{-1}$]	$\Delta\%$ from T_{469}
900	650	56	27.3	-29
469	416	0	38.4	0
38	454	9	52.7	37

in volume of the sphere was 3% over the full temperature range of the problem. As $V \propto \rho$ this change is low enough to be discounted and for the assumption to be justified.

Similarly the variation in specific heat capacity (C_p) and conductivity (k) were considered over the full temperature range as shown in Table 3. The value of k varies from the T_{mean} value by roughly $\pm 30\%$ but the T_{mean} value is only 4% different from the mean of values of k at the two temperature extremes. This fact coupled with the information given in ISO 835 standard [3] suggest an approximately linear change in h, which makes the assumption useful.

The variation in C_p is less clear cut, with the value at T_{mean} falling much closer to the value at T_{38} than the value at T_{900} as shown in Table 3. This variation is due to the non-linear variation of C_p with temperature [3]. Further investigation would be needed get an accurate estimate of the level of error this introduces to the result but this falls beyond the scope of this problem set so the approximation is used as a best estimate.

1.3 Schematic

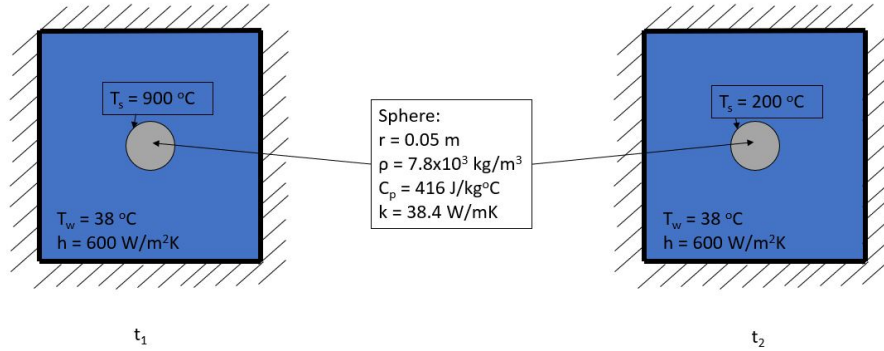
Figure 1: Part A schematic at initial and final state.

1.4 Analysis

Energy balance for closed system gives the following equation.

$$\dot{Q} = hA(T_s - T_f) = C_p \rho V \frac{dT_c}{dt} \tag{1}$$

Where $\dot{Q}$ is heat [W], h is the heat transfer coefficient [$W/m^2 K$], A is the surface area between the ball and water [m^2], T_s is the temperature of the steel ball [^oC], T_f is the temperature of the water [^oC], C_p is the specific heat capacity [J/mK], ρ is the density of the steel ball [kg/m^3], V is

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the volume of the steel ball $[m^3]$ and t is the time $[s]$.

Rearranging (1) to separate the variables gives.

$$\frac{1}{T_s - T_f}dT_c = \frac{hA}{C_p\rho V}dt \tag{2}$$

Which integrates to give.

$$\ln\left(\frac{T_{s1} - T_f}{T_{s2} - T_f}\right) = \frac{hA}{C_p\rho V}(t_2 - t_1) \tag{3}$$

Where t_i and T_{si} are the time $[s]$ and temperature $[^oC]$ at state i receptively.

Rearranging (3) to make t_2 the subject gives.

$$t_2 = \frac{C_p\rho V}{hA}\left(\ln\left(\frac{T_{s1} - T_f}{T_{s2} - T_f}\right)\right) \tag{4}$$

Substituting in the values for the variables given in Tables 1 and 2 gives the final value.

$$\boldsymbol{t_2 = 200s} \tag{5}$$

Where t_2 is the time for the steel ball to reach a temperature of 200^oC under given assumptions.

2 *Part B: Lumped capacitance justification*

The lumped capacitance method is only valid if the ratio of the conductive heat transfer to convective heat transfer is low. This ratio is known as the Biot number, Bi, and is given by.

$$Bi = \frac{h \cdot L_c}{k} \tag{6}$$

Where h is convective coefficient $[W/m^2K]$, L_c is the characteristic length $[m]$ and k is the conductivity $[W/m \cdot K]$.

Applying the values from Tables 1 and 2, choosing to set $L_c = R$ and substituting into equation (6) gives:

$$Bi = 0.7 \tag{7}$$

If $Bi > 0.1$ then the lumped capacitance method is no longer applicable as the assumptions made introduce non-trivial errors [1]. This means that the result in part A is likely inaccurate.

It is worth noting that the choice of L_c is significant. It is common to select L_c to be the maximum distance over which a temperature gradient would occur, as has been done above, but the method from the mathematical derivation uses $L_c = \frac{V}{A_s}$ which for a sphere gives $L_c = \frac{R}{3}$. This means the use of $L_c = R$ will tend to overestimate the value of Bi. In this case however, using $L_c = \frac{R}{3}$ gives $Bi = 0.25$ so the result can still be assumed to be inaccurate using the lumped capacitance method despite the overestimate.

3   Part C: Transient conduction

3.1   Assumptions

- No change in water temperature.

- No heat transfer by radiation.

- Material is standard carbon steel.

- Material properties are constant (taken at average temperature $T = 469^oC$).

3.2   Time interval to cool centre of sphere to 200^oC

As the initial and final temperature of the sphere are specified as well as the temperature of the fluid which is assumed to be constant, the ratio below can be calculated using the values in Table 1.

$$\frac{T_{(0,t)} - T_f}{T_i - T_f} = 0.19 \tag{8}$$

Taking the inverse of (7) gives 1.33. This and (1) allow the Fourier number to be determined from the Heisler chart for a sphere [2], which is determined to be.

$$Fo = 0.67 \tag{9}$$

The Fourier number is defined below.

$$Fo = \frac{tk}{\rho C_p R^2} \tag{10}$$

Using the result from (9) and substituting in the values from Tables 1 and 2 into (10) gives the final result time taken.

$$t = \frac{F_o \rho C_p R^2}{k} = 182s \tag{11}$$

Comparing this result and (5), the result gained from the lumped capacitance method, it can be seen the two are of a similar magnitude but (11) is lower than (5). The similar magnitudes matches with expectations as the assumptions used in section 1 were found to be near the boundary of $Bi = 0.1$ so the error introduced by applying them would be significant but still give an answer of the same magnitude. Where it differs from expectation is that the temperature gradient that was found to be important for this case in section 2 should take additional time to reach the centre point of the steel ball so the transient response would be expected to take longer to reach the target temperature than the lumped capacitance method. The fact that this is not the case here may be attributed to errors introduced by the approximations mentioned in section 1 and errors from reading from Heisler charts.

3.3   Time interval to cool surface of sphere to 200^oC

To find the centre temperature of the sphere when the surface reaches 200^oC a quantification of the temperature gradient is needed. The Heisler charts can again be used to find this using $\frac{1}{Bi} = 1.33$ and that $\frac{r}{r_0} = 1$ for this example, giving the following.

$$\frac{T - T_\infty}{T_c - T_\infty} = 0.65 \tag{12}$$

Where T is the surface temperature $[^oC]$, T_∞ is the water temperature $[^oC]$ and T_c is the centre temperature $[^oC]$.

Rearranging (12) and using the temperatures in Table 1 gives.

$$T = 287^oC \tag{13}$$

Applying this to (8) and again using the $\frac{1}{Bi} = 1.33$ result allows the Fourier number to be found from the Heisler chart [1].

$$Fo = 0.42 \tag{14}$$

Substituting (14) into (10) gives the final result.

$$t = 114s \tag{15}$$

It makes intuitive sense that (15) is lower than (11) as the surface will cool faster then the centre.

4 *Part D: Non-infinite water bath*

If the tub of water is non-infinite it is possible to quantify how the temperature of the water will change during the process to check if the assumption of constant temperature is valid.

From the Heisler charts it is possible to find the proportion of total heat the steel contains that has dissipated at a given time. The temperature rise of the tank of water (Figure 2) when the centre of the steel ball has reached 200^oC is considered here.
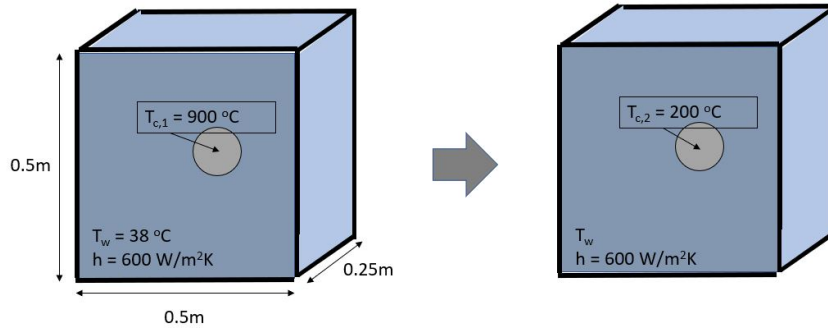

Figure 2: Part D schematic at initial and final state.

Using the results of (7) and (9) and referencing the Heisler chart [1] it is possible to calculate the following.

$$\frac{Q}{Q_o} = 0.7 \tag{16}$$

Where Q is the heat transferred at time t [W] and Q_o is the total heat stored initially in the steel ball [W].

Q_o can be calculated simply.

$$Q_o = \rho_s C_{p,s} V_s T_{c,1} \tag{17}$$

Where ρ_s is the density of steel, $C_{p,s}$ is the heat capacity of steel, V_s is the volume of the steel ball $[m^3]$ and $T_{c,1}$ is inital temperature 900^oC.

Given the continued assumption that no heat is lost via the sides of the water tank all the heat, Q, that leaves the steel ball is absorbed by the water, shown in the following expression.

$$Q = \rho_w C_{p,w} V_w \Delta T_w \tag{18}$$

Where the variables are the same as (17) but for water, denoted by the subscript w on each.

Substituting (18) and (17) into (16), rearranging to make ΔT_w the subject and using the known values shown in Figure 2 gives.

$$\frac{Q}{Q_o} = \frac{\rho_w C_{p,w} V_w \Delta T_w}{\rho_s C_{p,s} V_s T_{c,1}} \tag{19}$$

$$\Delta T_w = 5.4^oC \tag{20}$$

Doing the same analysis using the Fo number for the surface reaching 200^oC and using the assumptions detailed in the lumped capacitance method give values of ΔT_w of $\pm 1^oC$, so the temperature change is similar for all examples.

This is a 14% change in the temperature and a more in depth analysis would look at the change in water properties over a 5^oC temperature rise to quantify this further.

5   *Part E: Equilibrium temperature*

Over a long time period the steel ball and water will reach a uniform equilibrium temperature. As part D has shown this temperature cannot be assumed to be the initial water temperature of $T_w = 38^oC$. The method for finding the true equilibrium does not need to account for the transient effects as it happens over a long enough time-frame that they are no longer relevant.

$$Q = \rho_w C_{p,w} V_w (T_\infty - T_w) = \rho_s C_{p,s} V_s (T_s - T_\infty) \tag{21}$$

Where T_∞ is the equilibrium temperature $[^oC]$.

Rearranging (21) to make T_∞ the subject and solving given properties used in Part D gives.

$$\boldsymbol{T_\infty = 30.5^oC} \tag{22}$$

The final temperature that this system would reach under the given assumptions and boundary conditions.

References

[1] T. L. Bergman and Frank P. Incropera, editors. *Fundamentals of heat and mass transfer*. Wiley, Hoboken, NJ, 7th ed edition, 2011.

[2] P.S. Ghoshdastidar. 4.5 multidimensional transient problems: Application of heisler charts, 2012.

[3] Jean-Marc Franssen and Paulo Vila Real. *Fire Design of Steel Structures : EC1: Actions on Structures; Part 1-2: Actions on Structure Exposed to Fire; EC3: Design of Steel Structures; Part 1-2: Structural Fire Design*, volume Second revised edition of *ECCS-SCI Eurocode Design Manuals*. Ernst & Sohn, [Place of publication not identified], 2015.