Question of the Day

Fill in the blank:

“All models are _____, but some are ________.”

“If you have a hammer, then everything looks like _________”

Your alien friend sees that people use umbrellas in the rain, and infer that umbrellas are a rain-blessing artifact. He is committing the fallacy of ____________

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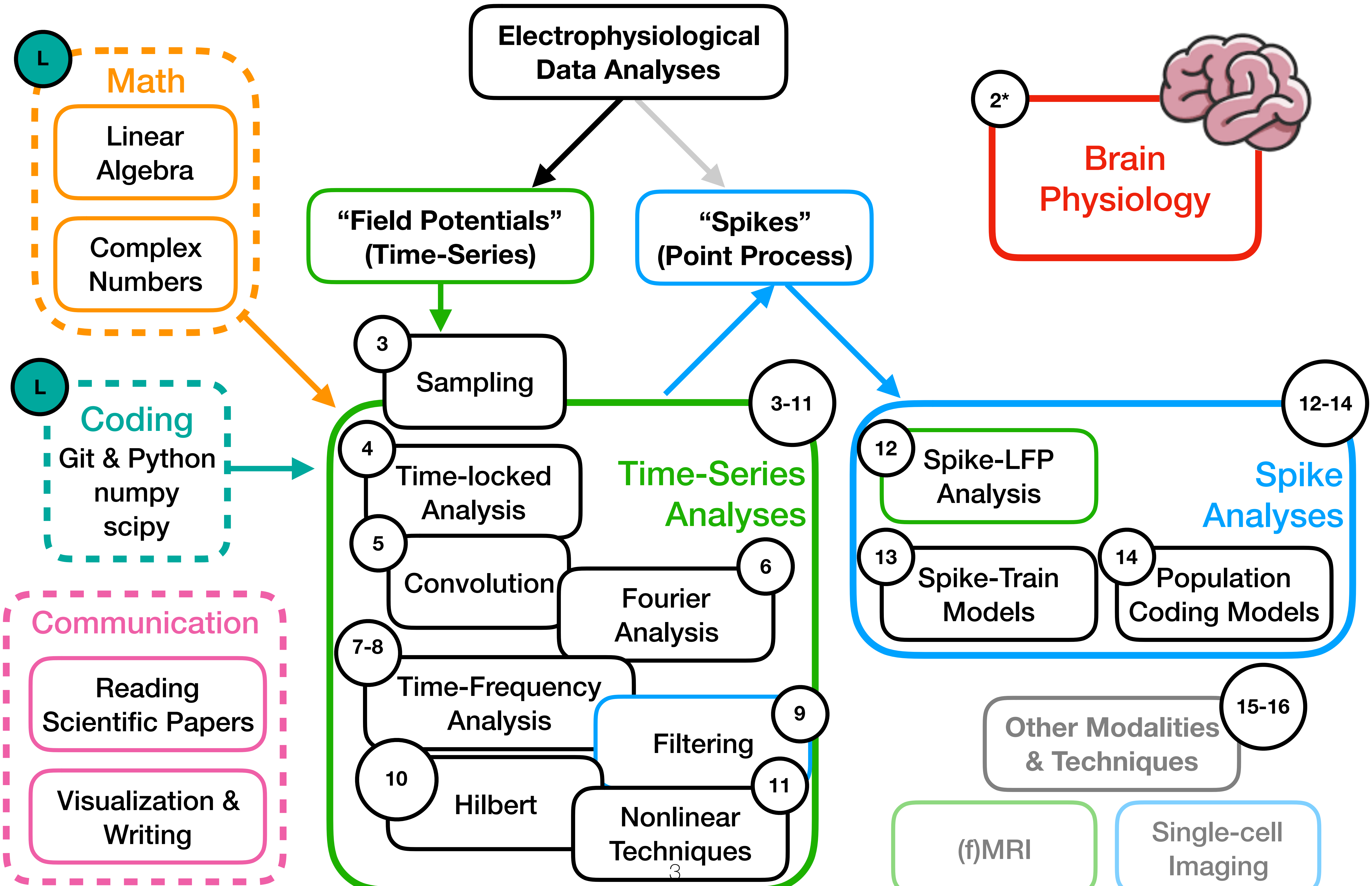
Spikes: Rate Models & Rate-LFP Analyses

Lecture 13
July 24, 2019

Course Outline: Road Map



Goals for Today

1. Conceptualize neuron as computational device

2. Compute spike counts, smoothing & firing rate

3. Understand correlation and rate-LFP analyses

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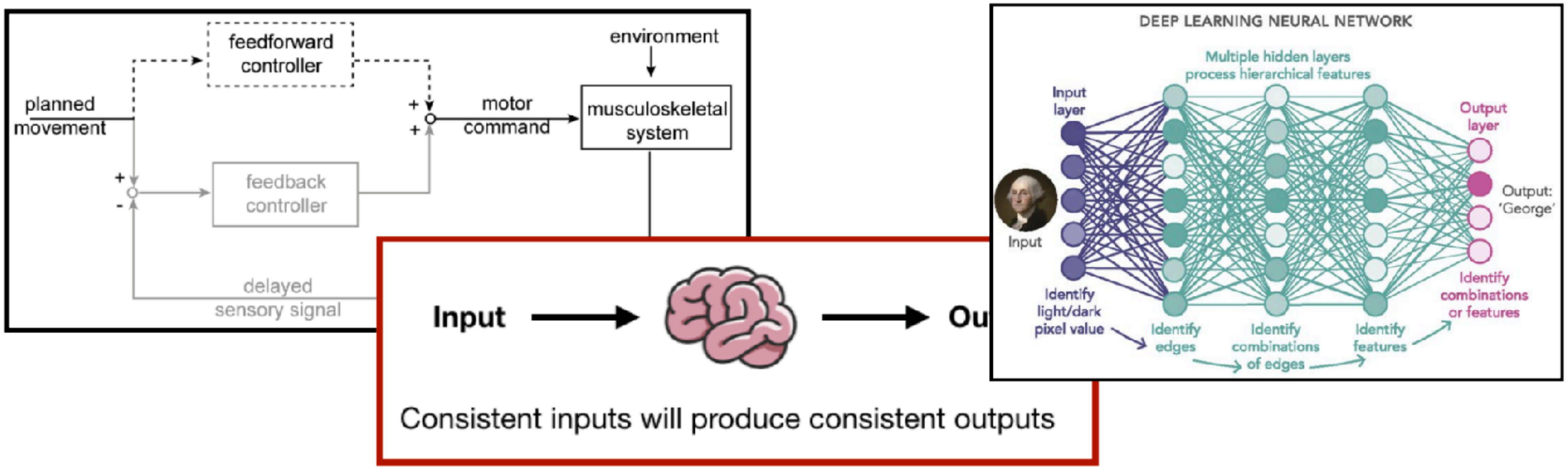
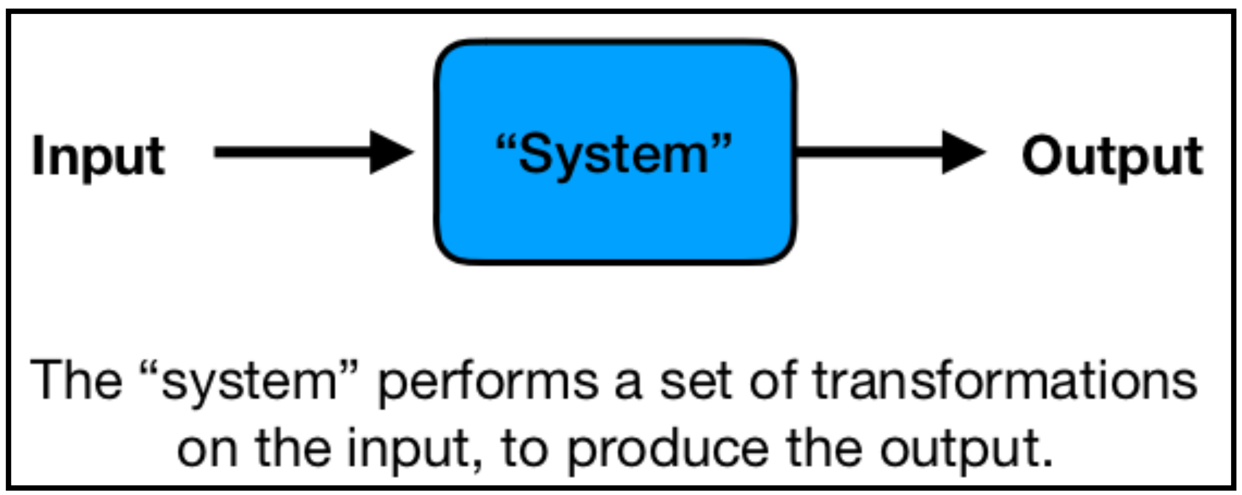
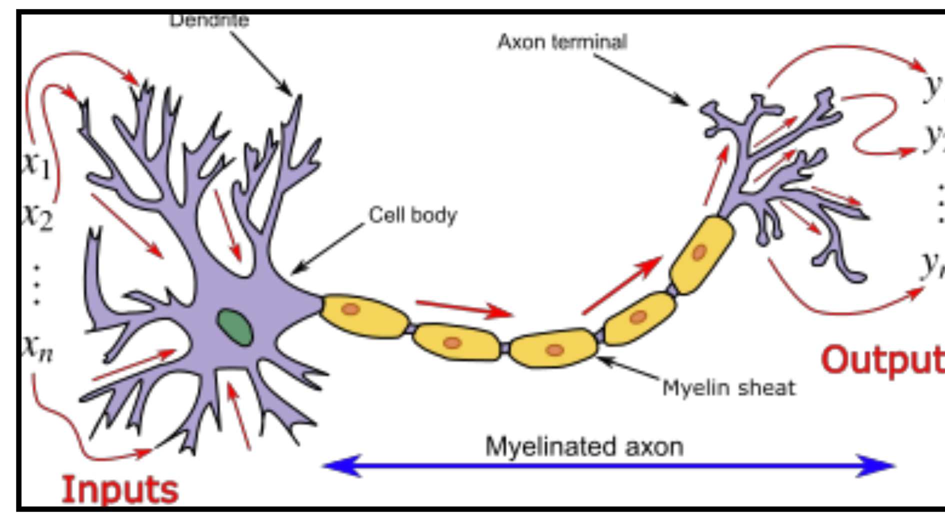
Single Neuron as a Computational Device



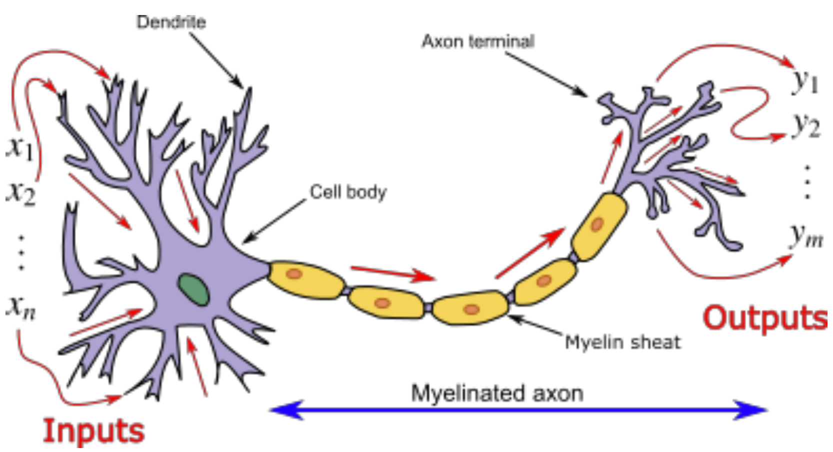


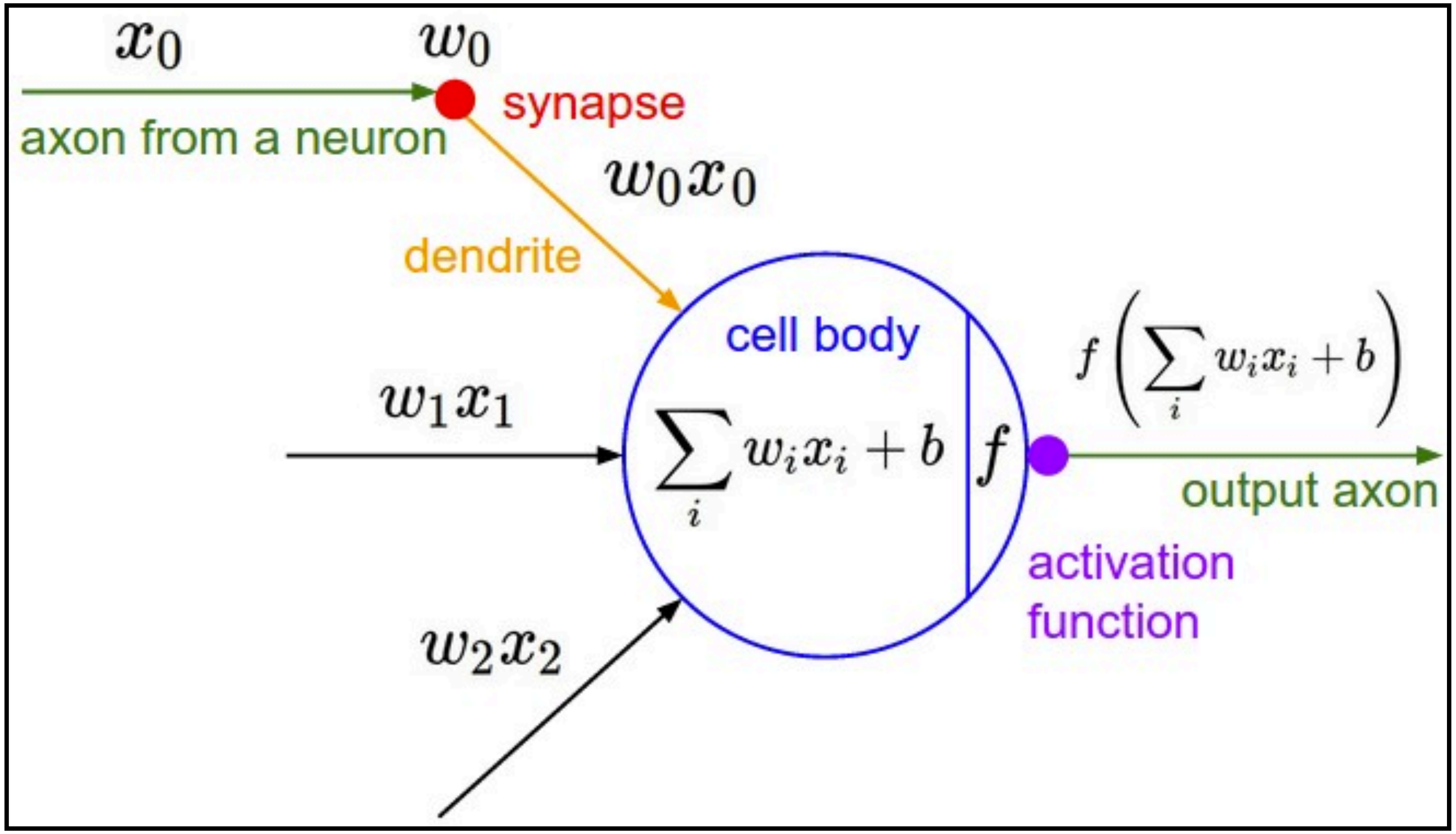


Systems engineering view: neuron receives (sensory or synaptic) input, performs a "computation", and sends the result as an output.

This is **a model**, or abstraction, of the biological cell!

Single Neuron as a Computational Device



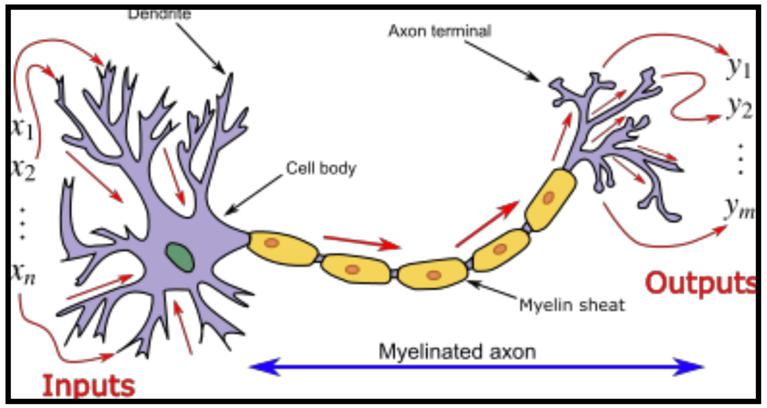


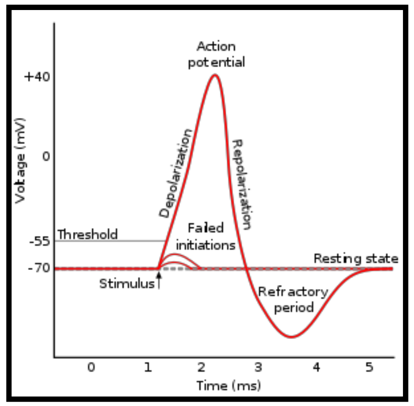
$$\boxed{f\left(\sum_i w_i x_i + b\right)}$$

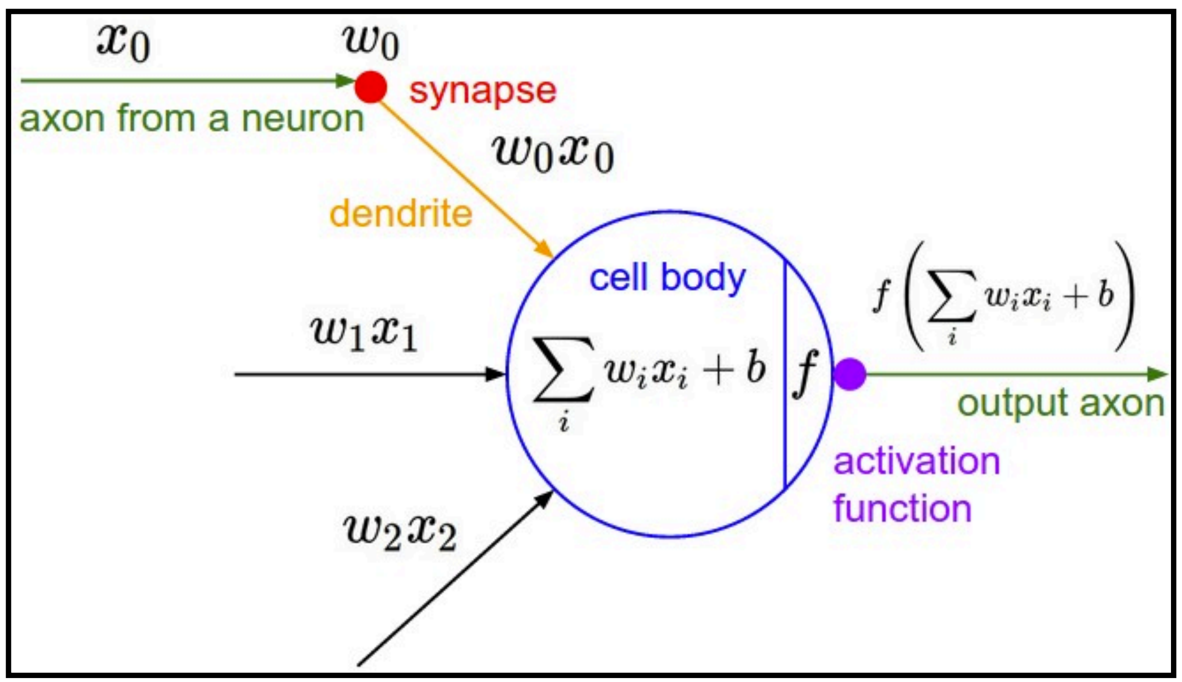
dot product
w: “synaptic” weights
x: inputs
b: bias
f( ): nonlinearity

But what is x, physically?

Information Encoding







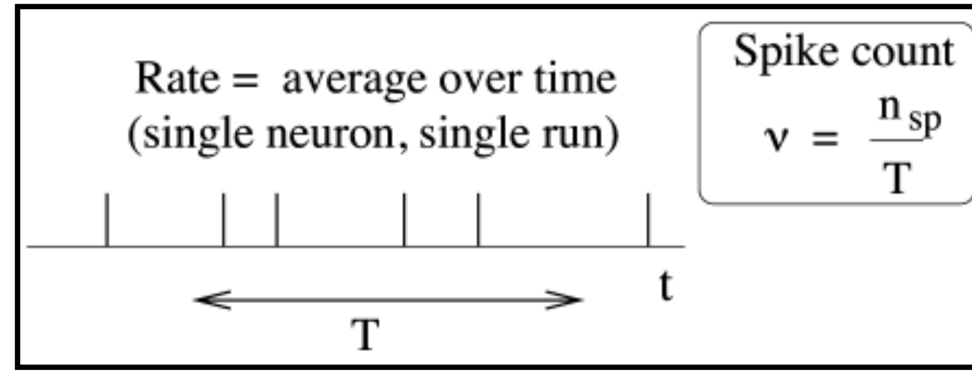
The biological neuron only receives and emits discrete **action potentials.**

What aspect of the action potential “encodes information”, that we can manipulate computationally?

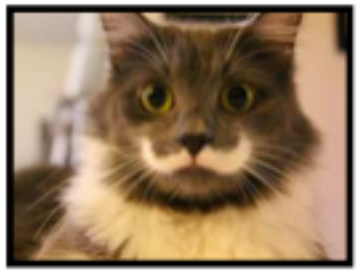
Action potentials (more or less) have constant amplitudes and widths…

One view (rate coding):
Number of spikes within a time window.
Spike Count: over a longer window
Firing Rate: “instantaneous” quantity

Spikes, Spike Counts, & Firing Rate



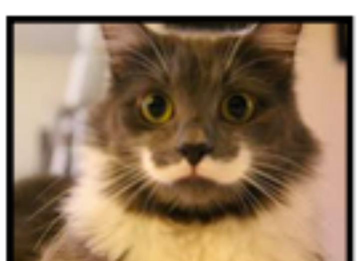
Stimulus **Response**

Trial 1   10 spikes -> 100Hz

Trial 2   2 spikes -> 20Hz

Trial 3   4 spikes -> 40Hz

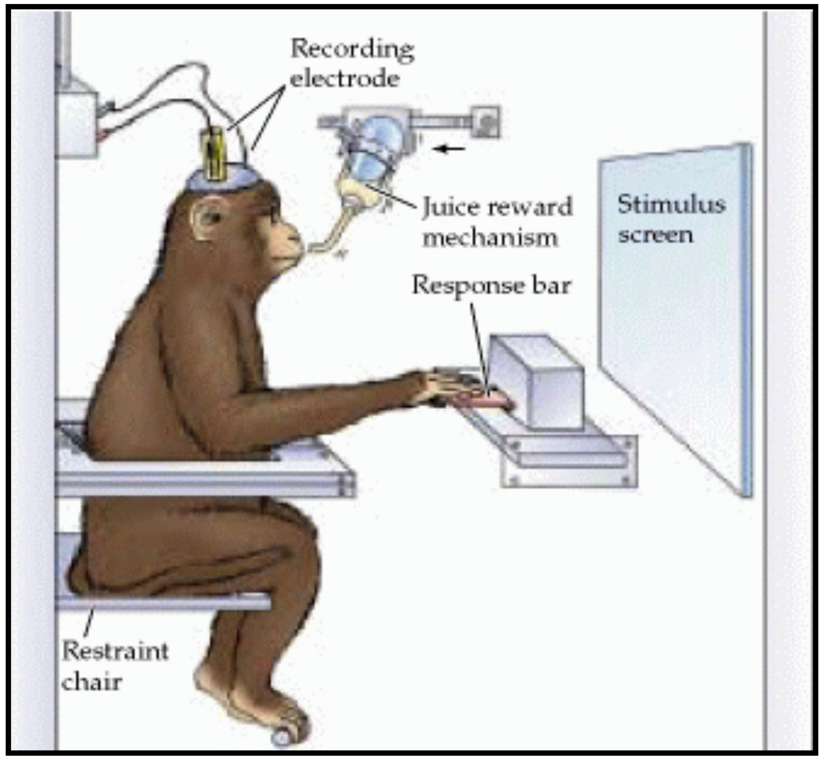
...

Trial N   12 spikes -> 120Hz

100 ms

This neuron is a “cat” neuron.

Receptive Fields & Stimulus-Tuned Neurons



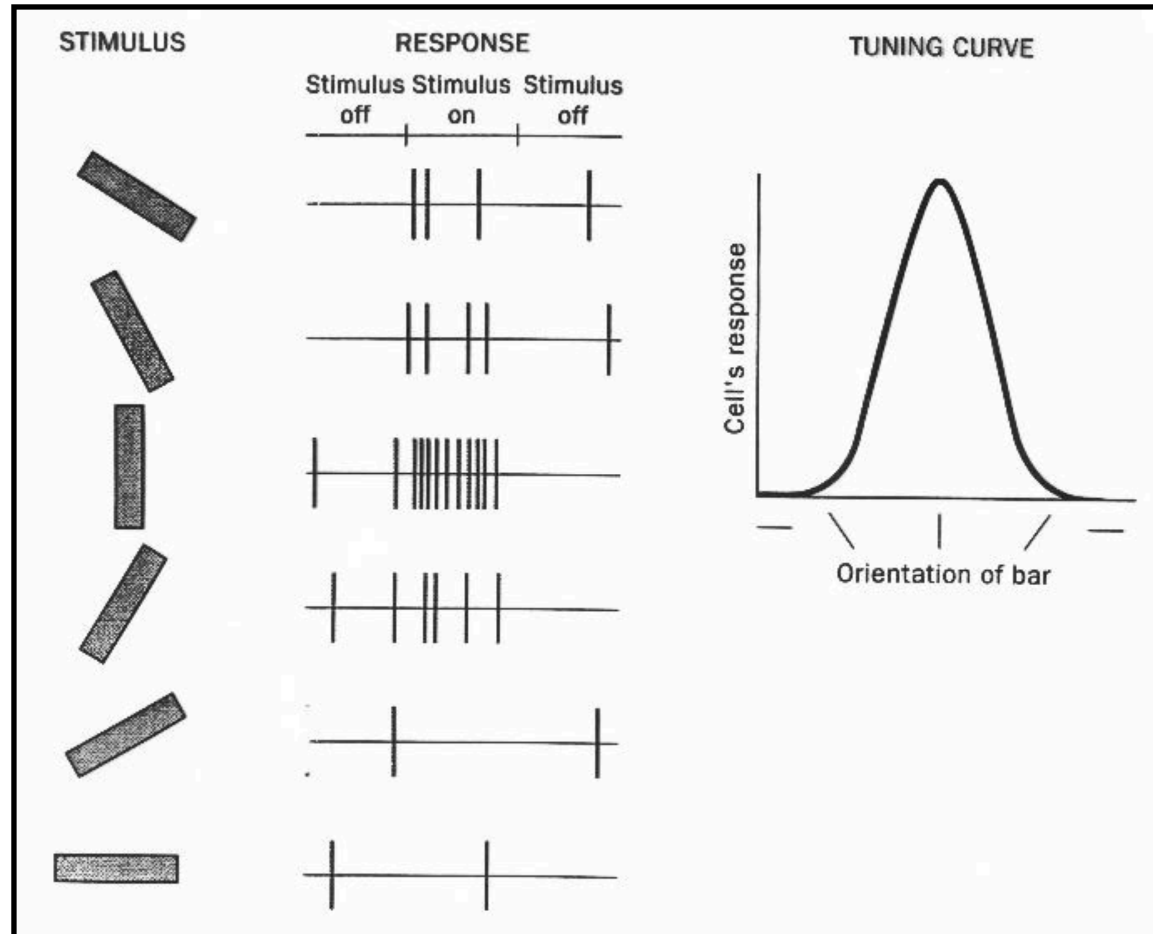


FIGURE 4.8 Response of a single cortical cell to bars presented at various orientations.

Stimulus with **varying aspects** are presented, e.g.:

- location
- orientation (angle)
- color
- sound frequency
- etc…

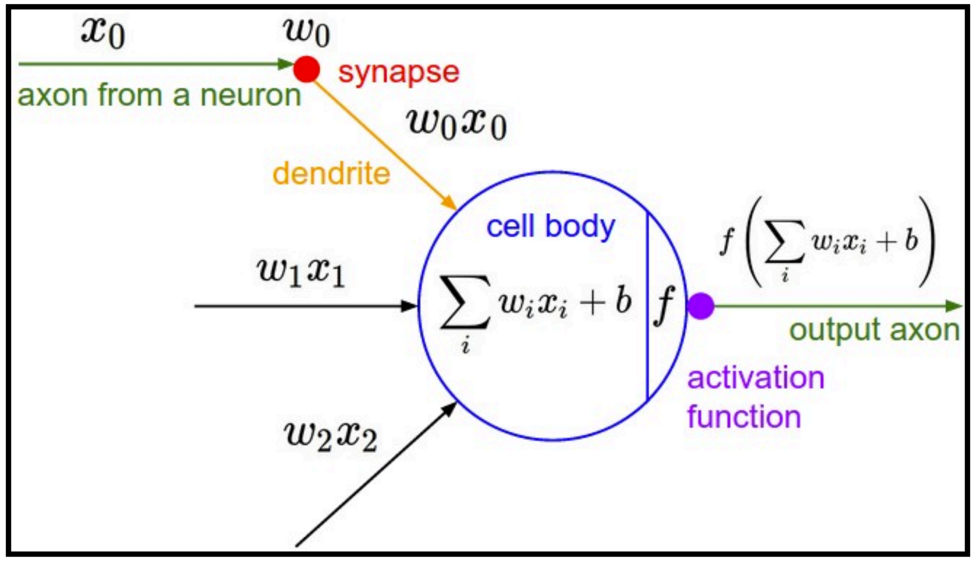
"This neuron has an orientation preference."

Receptive Fields & Stimulus-Tuned Neurons

Tuning	Location in the Brain	Special Name?
place	hippocampus	place cells & grid cells
motion	retina, or V5/MT	direction cells/motion cells
grandmother	IT/hippocampus	gnostic/grandmother/Jennifer aniston cells
numerosity	PFC	numerons
biological motions	premotor/SMA	mirror neuron
phallic images		

Google: "__ tuned neuron" or "neuron responsive to ___" or "__ cell neuroscience"

Single Neuron as a Computational Device



$$f\left(\sum_i w_i x_i + b\right)$$

dot product

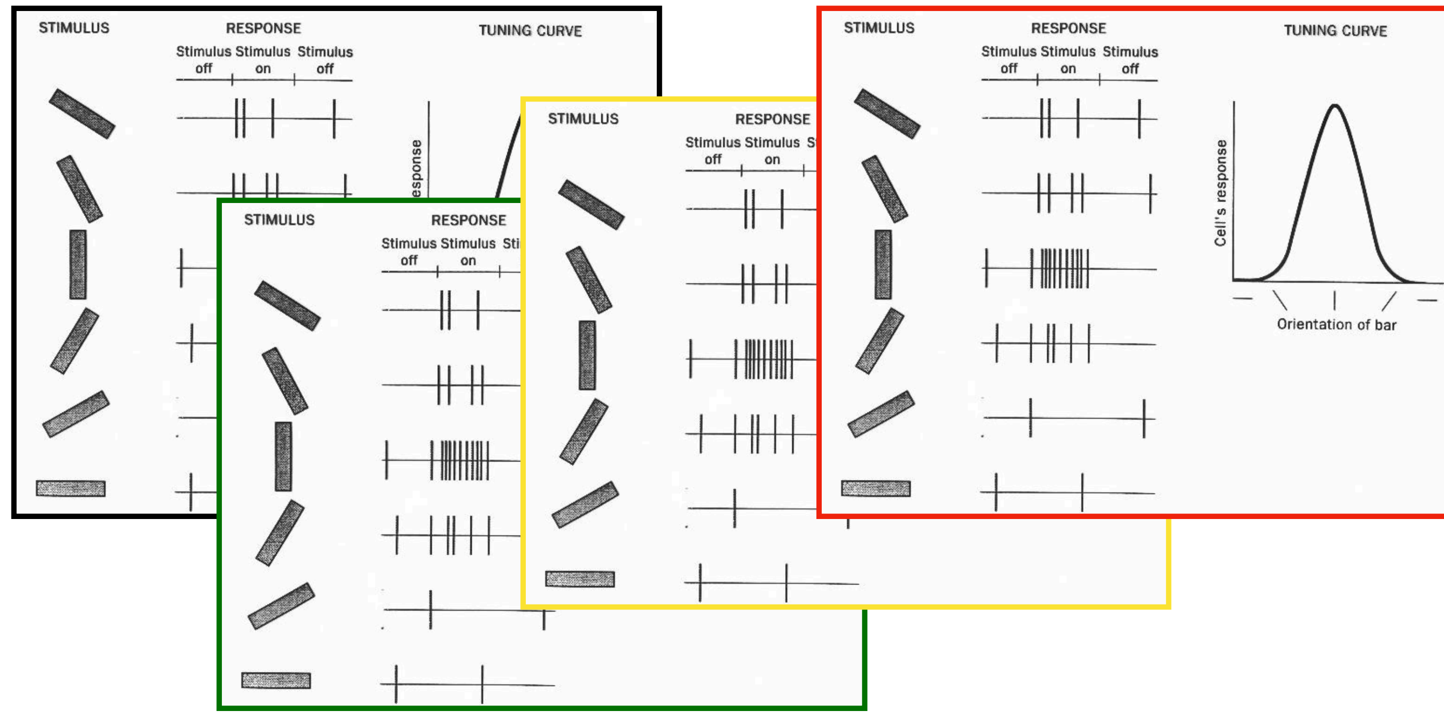
w: “synaptic” weights

x: inputs

b: bias

f( ): nonlinearity

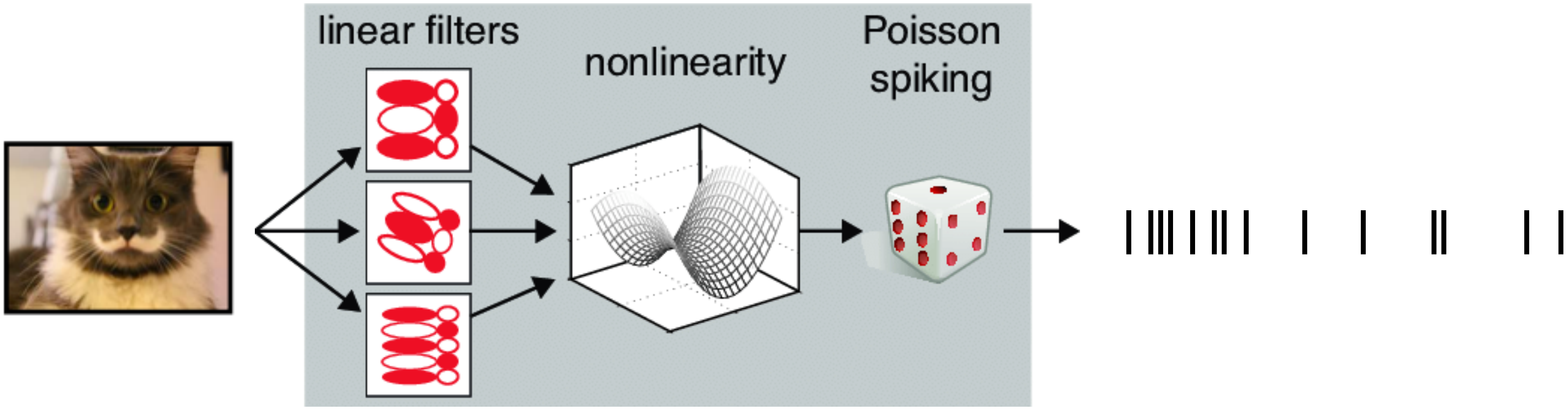
Different neurons have different “tuning curves”, i.e., sensitive to different values of the variable.

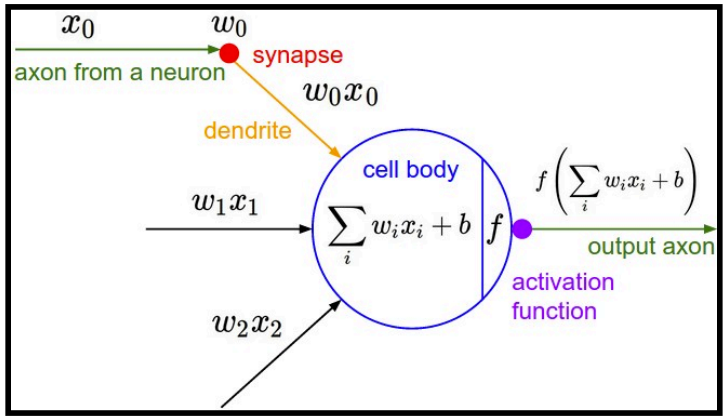


downstream neuron combines them linearly via dot product (linear filter)

Naive model of a neuron’s function!

Linear-Nonlinear Poisson Model





linear: dot product / filter of stimulus tuning

nonlinear: activation function, e.g., sigmoid, ReLu

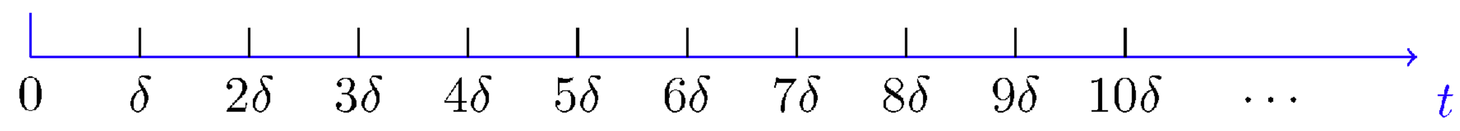
Poisson: spikes are randomly emitted as a Poisson process, given the **average rate parameter**

$$P\{N(t) = n\} = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$

Brief Intro to Poisson Spikes

Poisson: spikes are randomly emitted as a Poisson process, given the **average rate parameter**

$$P\{N(t) = n\} = \frac{(\lambda t)^n}{n!} e^{-\lambda t}.$$



Generate a random number between 0-1 at every time step.

If that number is greater than **(rate * interval length)** -> no spike.

Otherwise -> spike

Exercise: rate = 5Hz, interval length = 0.1s, total time = 2 seconds

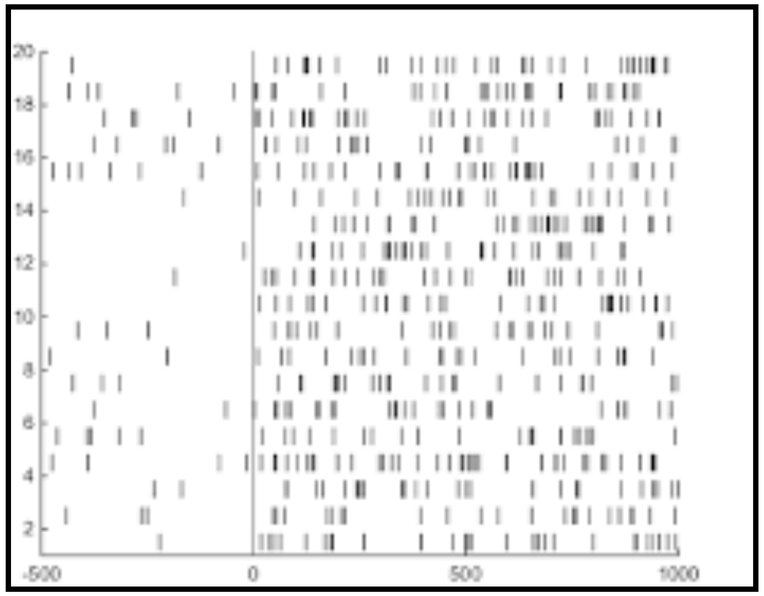
nice property:
for some duration T, average count = rate * T; variance = rate

Rate Encoding Model

Model of how a neuron encodes information via spike trains.

Assumption: information (e.g., stimulus intensity) is encoded via a neuron's firing rate.

Assumes precise spike timing on the millisecond scale does not matter (in contrast to **spike timing encoding models**).



Given some spike timestamps, we want to estimate firing rate.

1. Conceptualize neuron as computational device

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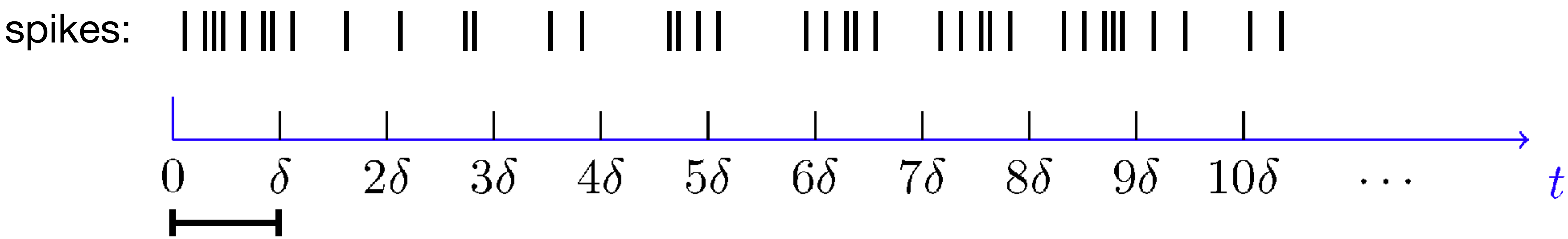
Spike Times to Spike Counts



dt = 5ms

spike count vector

6	2	3	2	3	2	4	5	5	2	...

or

8	5	5	9	7	...

or

34

Time bin width is a parameter choice!

Smaller bins = better temporal resolution, but noisier estimate
Bigger bins = worse temporal resolution, more accurate estimate

Spike Counts to Firing Rate

Spike count vector is usually very noisy, we want a more continuous & smooth firing rate estimate.

Solution: take “moving average”.

6	2	3	2	3	2	4	5	5	2	…

4	2.5	2.5	2.5	2.5	3	4.5	5	3.5		

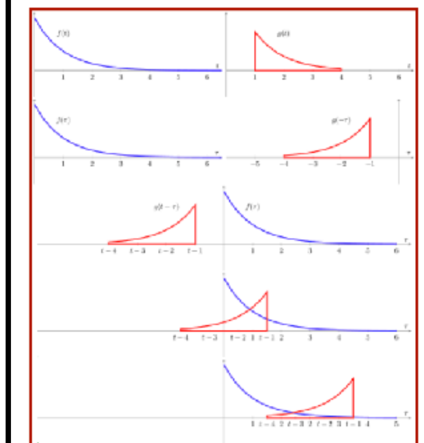
Moving Average Smoothing as Filter

Spike count vector is usually very noisy, we want a more continuous & smooth firing rate estimate.

Solution: take "moving average".

6	2	3	2	3	2	4	5	5	2	…

$$(f * g)[n] = \sum_{m=-\infty}^{\infty} f[m]g[n-m]$$



Input: x(n) = [1,0,3,7,2,5]

IRF: [2,-1,-1,0]

IRF[-n]: [0, -1, -1, 2]

[0, -1, -1, 2]
[1,0,3,7,2,5] 0

[0, -1, -1, 2]
[1,0,3,7,2,5] 2

[0, -1, -1, 2]
[1,0,3,7,2,5] -1

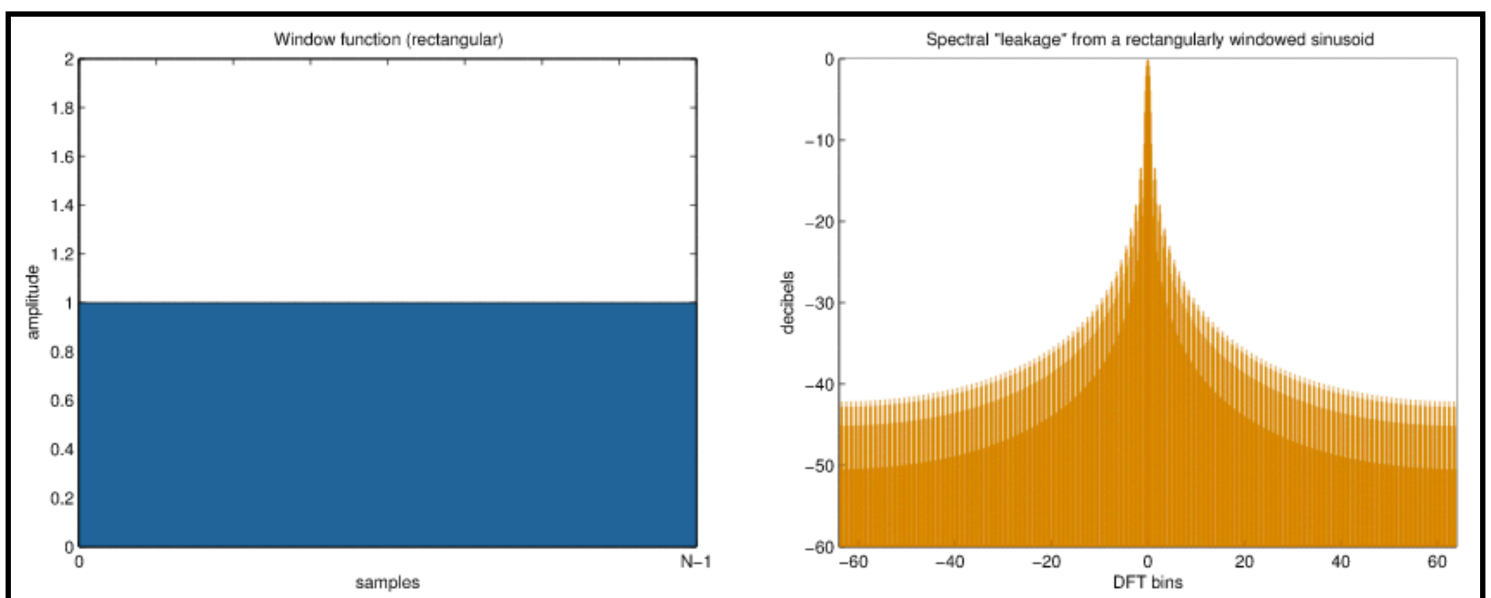
[0,-1,-1,2]
[1,0, 3, 7,2,5] 5

[0,-1,-1,2]
[1,0, 3, 7,2,5] -5

Equivalent to **convolution** with:

0.5	0.5

2-point boxcar or rectangular window

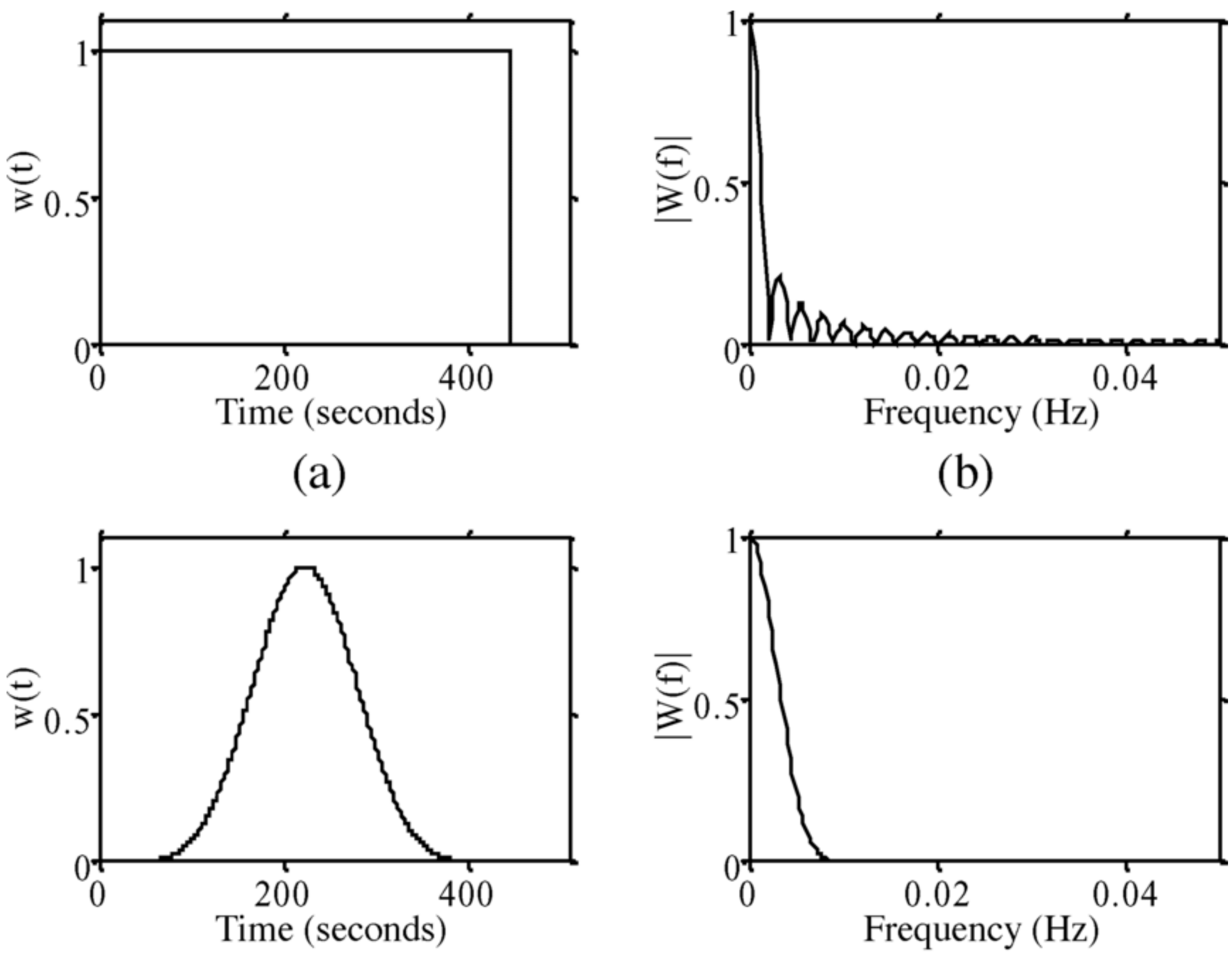


Firing Rate: Smoothed Spike Count

Time Domain | Frequency Domain

Rectangular Window

Gaussian Window



(a) (b) (c) (d)

Smoothing is applying a low-pass filter!

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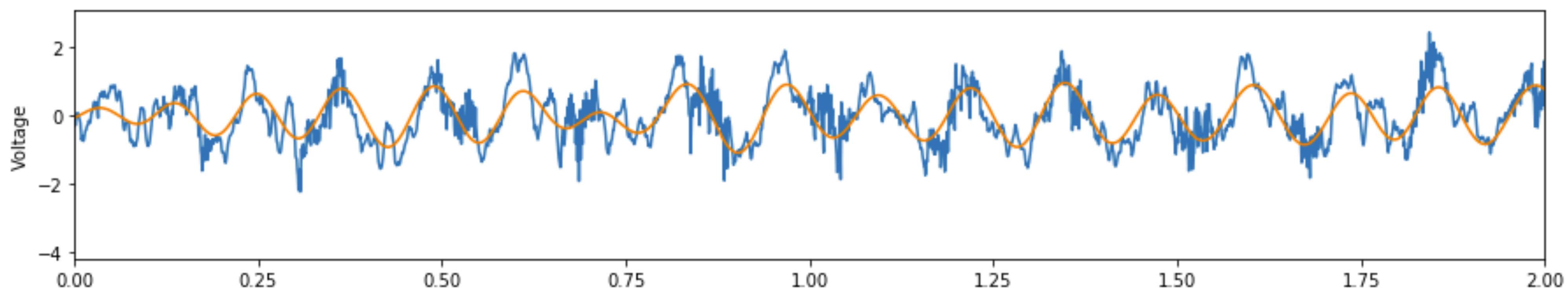
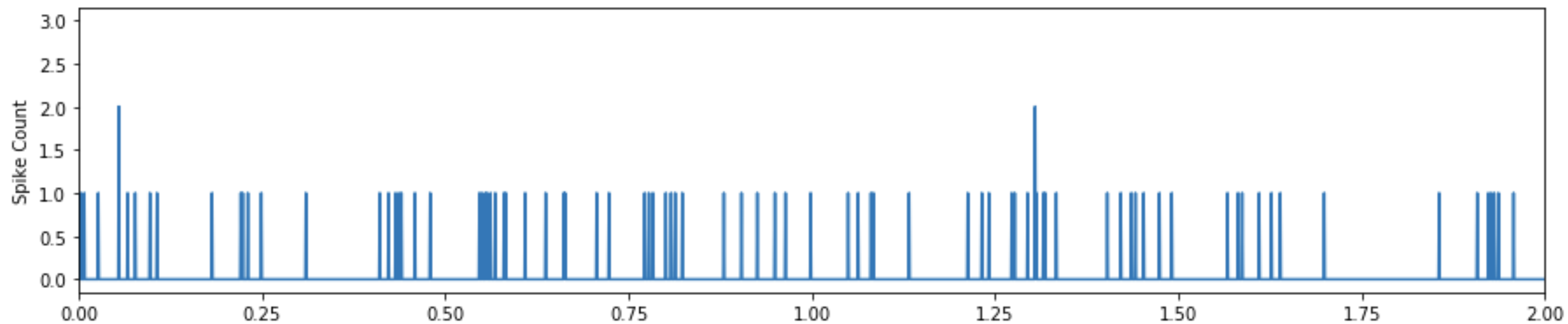
Firing Rate & LFP Analysis

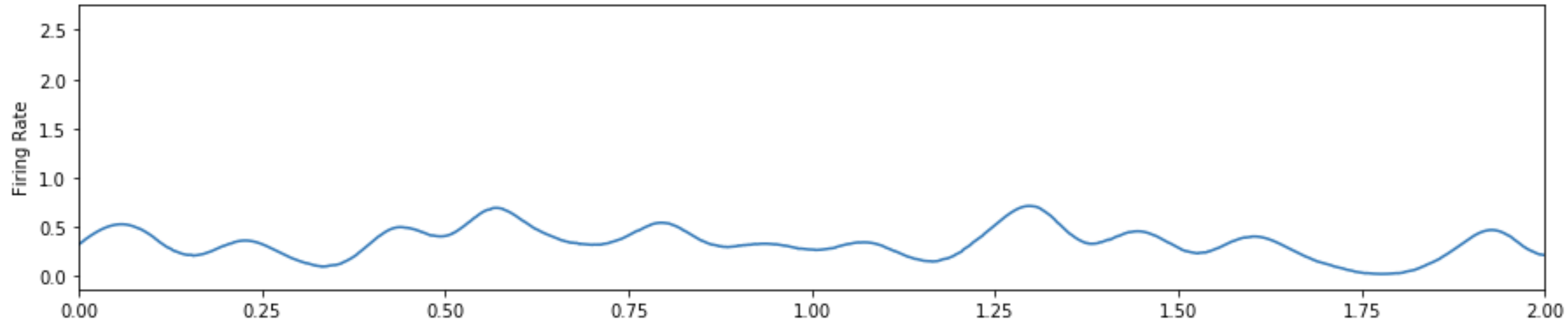
LFP



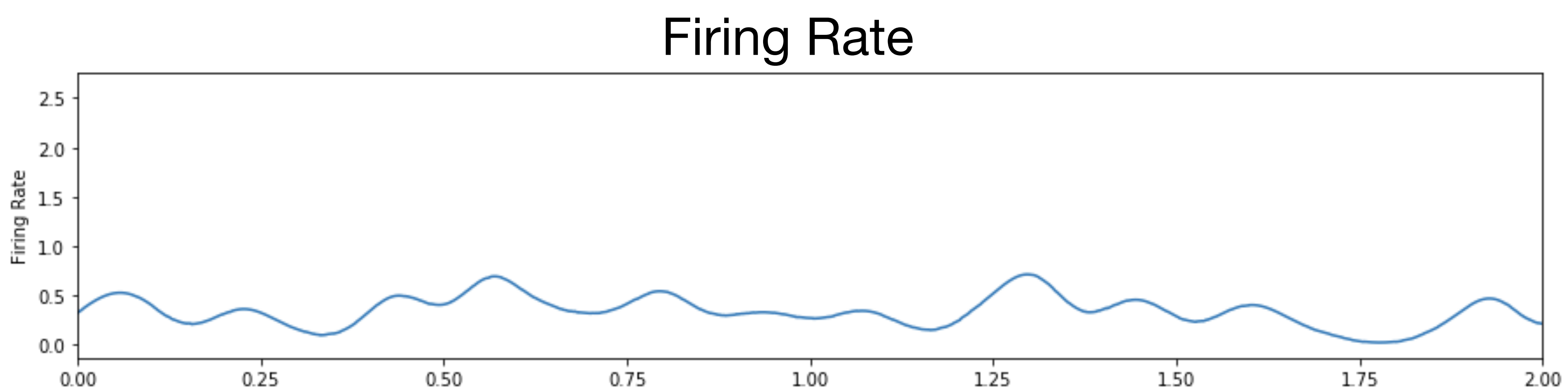
spikes: 

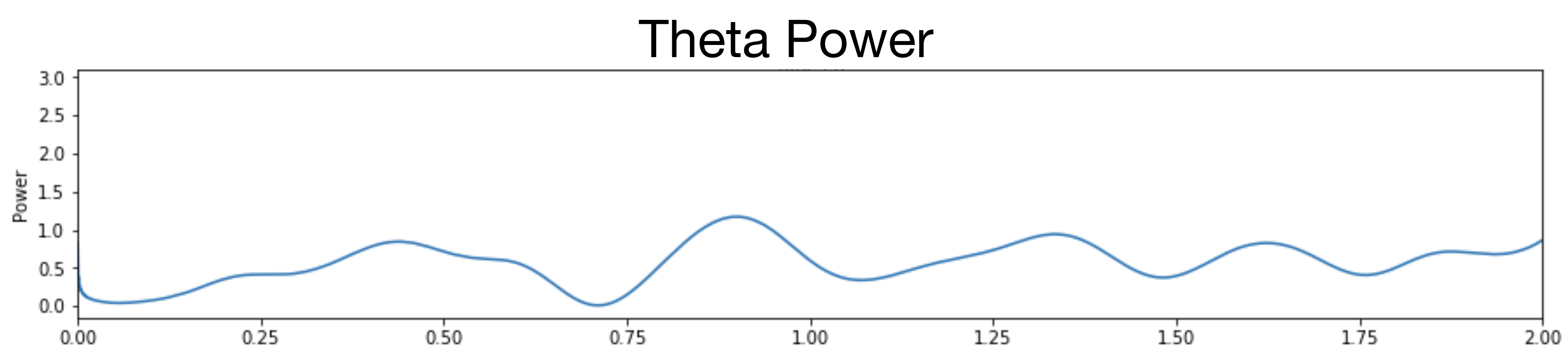
Use same time bin width (dt) to bin spike counts





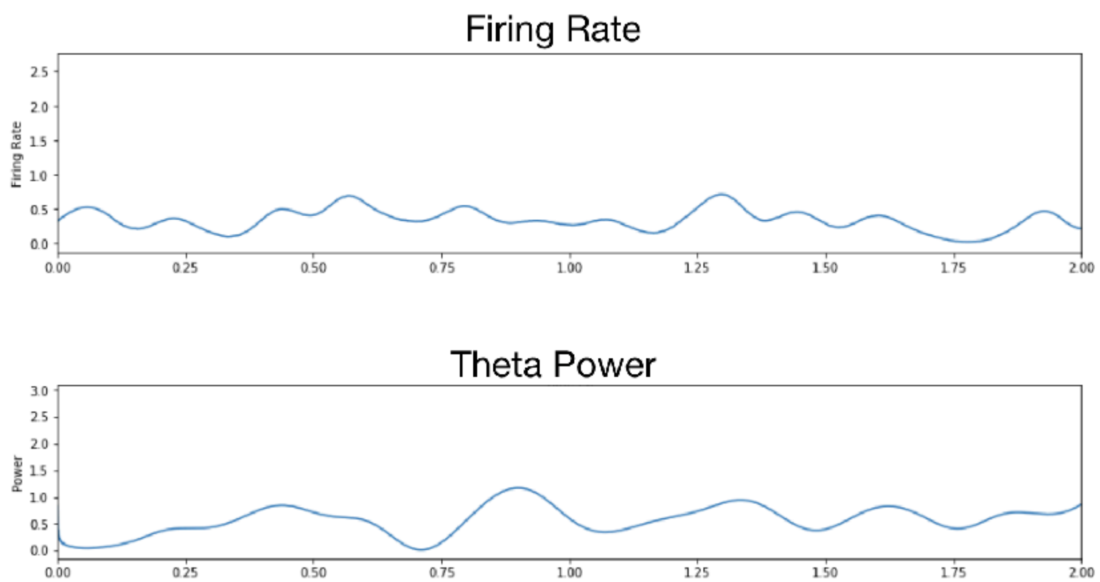
Firing Rate & LFP Analysis





How to quantify their relationship?

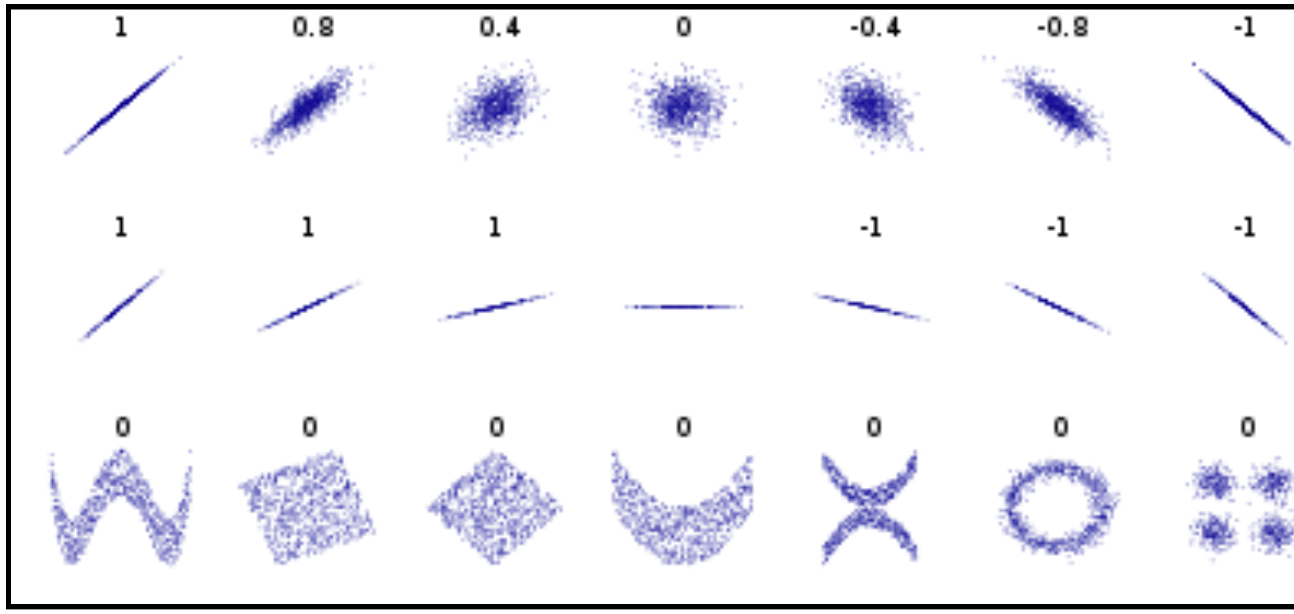
Correlation



How to quantify their relationship?

Pearson Correlation Coefficient

$$\rho_{X,Y} = \mathrm{corr}(X,Y) = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{\mathrm{E}[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$



For two discrete signals x and y:

$$r_{xy} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i-\bar{x})^2 \sum_{i=1}^{n}(y_i-\bar{y})^2}},$$

Correlation

For two discrete signals x and y:

$$\boxed{r_{xy} \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{(n-1)s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2 \sum_{i=1}^{n}(y_i - \bar{y})^2}},}$$

standard deviation
of x and y

covariance

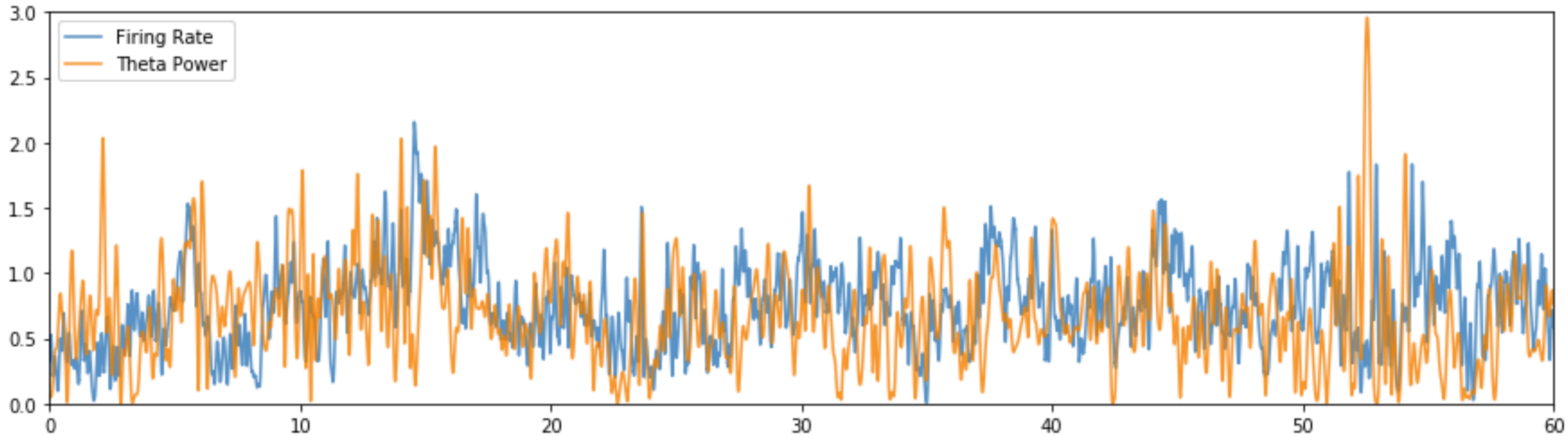
$$\boxed{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}$$

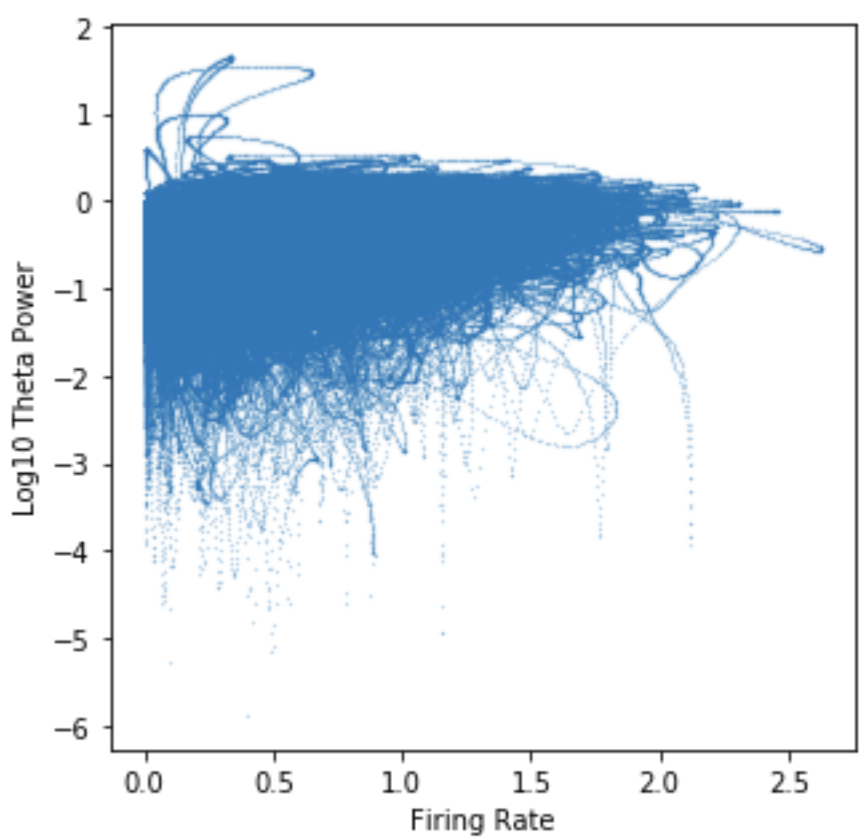
what is this operation if $\bar{x} = 0,\ \bar{y} = 0$

dot product!

Pearson correlation coefficient is the dot product between 2 mean-subtracted signals, normalized by their standard deviations

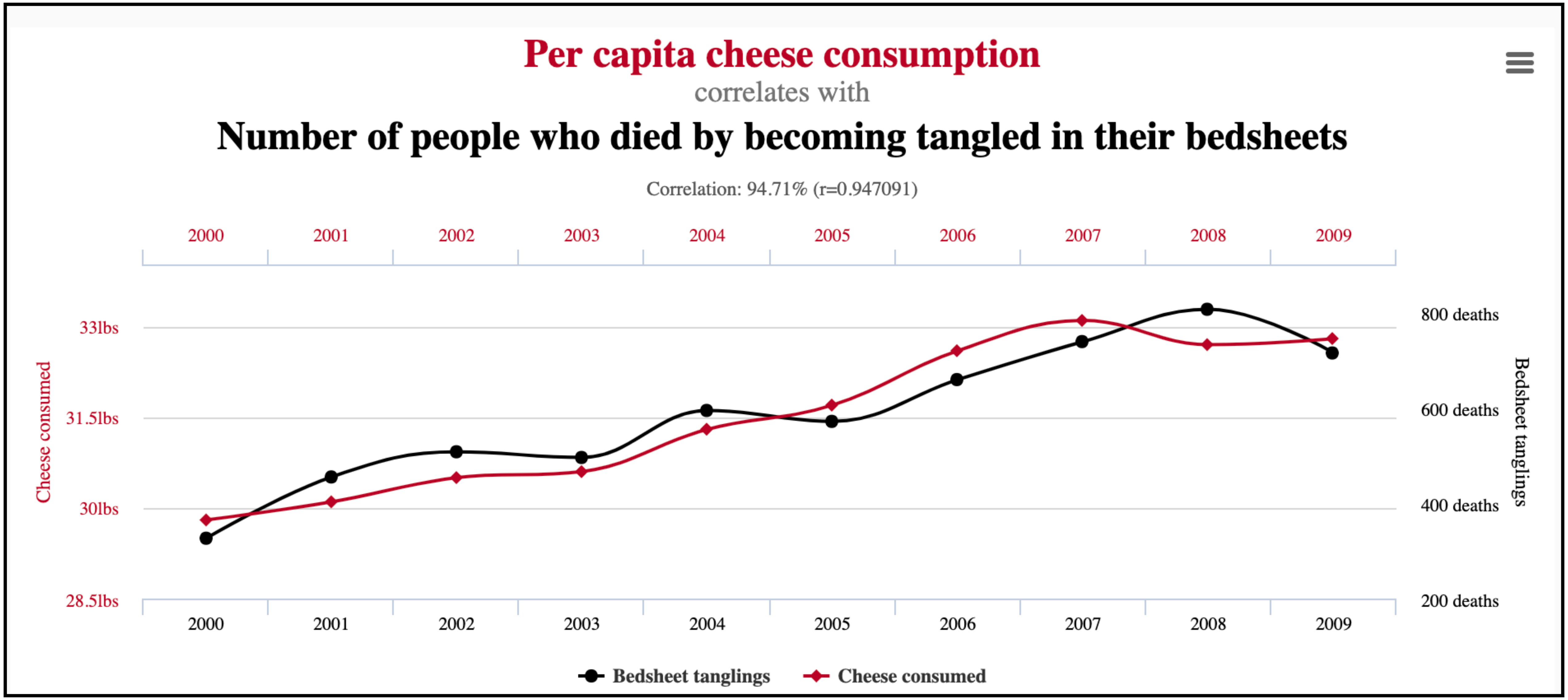
Rate-Power Correlation





np.corrcoef( )

Correlation Does Not Imply Causation



Summary

1. Conceptualize neuron as computational device

2. Compute spike counts, smoothing & firing rate

3. Understand correlation and rate-LFP analyses

https://tinyurl.com/cogs118c-att