Experiments in Location Games

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(ABSTRACT)

Location theory, also known as spatial economics, is a specialization of microeconomic theory that considers markets where economic activity does not occur in a single point in space. The study of firm and consumer behavior in various theoretical spaces, transportation cost, and information assumptions have led to various outcomes, as far back as Hotelling (1929) and earlier. This dissertation introduces classroom experiments to examine some of the basic models of location theory. The first chapter is an introduction to the subfield of location theory, and the second chapter is a literature review with a focus on the development of location games that are relevant to the models investigated in this dissertation. para The third chapter presents experiments on basic Hotelling location models. Results are compared to pure- and mixed-strategy Nash equilibria in models with two to six sellers. For groups of two firms, there is clear evidence of communication for the purpose of influencing the other seller as part of a multiperiod strategy. However, it is unclear whether the motivation for this signaling is to coordinate on a fair outcome, to maximize one's own payoff, or for deception. Whatever the motivation, a clear pattern emerges, which is not explained by theory. For groups of three firms, there is evidence of risk aversion as players avoid riskier central locations. For groups of four and five firms, results do not readily attain the unique pure-strategy Nash equilibrium due to a previously unexplored coordination problem. In the four-player groups, players choose a combination of pure-strategy focal points and the mixed strategy support; however, players also choose the center of the market, which is not explained by theory. In the six-player case, there are two equilibria in pure strategies, providing an even

more complex coordination problem. The distributions of strategies vary systematically as the number of seller increases in a manner consistent with Nash predictions. para In the fourth chapter, I introduce one-way communication in order to facilitate the solution of the coordination problem. This design has the potential to yield a cleaner test of the theory by removing the confounding coordination difficulty. I run groups of two to four firms in a fixed-matching repeated game. In these experiments, one subject in each group is designated to be the communicator. Prior to each round of play, the communicator sends a suggested location set to the other players. If the communicator understands the strategic situation, they can specify locations to the other players that result in an equilibrium outcome. Previous experiments suggest that this kind of "cheap talk" can facilitate equilibrium selection in coordination games. The results show some improvement in the ability of subjects to coordinate on Nash equilibrium play. However, there is considerable heterogeneity across groups. I conclude that one-way communication enhances the ability of subjects to coordinate, but bounded rationality and poor leadership preclude the convergence to equilibrium.

Experiments in Location Games

Anthony E. Dziepak

(GENERAL AUDIENCE ABSTRACT)

Location theory is the study of how sellers and buyers interact in geographical space. Models consider various dimensions and shapes of spaces and assumptions about transportation costs, how sellers should position their products, and how buyers shop around. In the classic example from Hotelling (1929), there is a boardwalk and two ice cream vendors have to decide where they will position their ice cream carts. Under certain assumptions, the two vendors will want to locate next to each other in the middle of the boardwalk in order to maximize their market share. However, it is observed that this may differ from the best locations from a social welfare perspective. If the carts were more spread out, buyers would not have to walk as far to get their ice cream. If there are instead three or more vendors, the outcome is different. para This dissertation introduces classroom experiments to see if what is observed aligns with the theory. I conducted several experiments in a computer lab with students playing the role as ice cream vendors. First, with groups of two sellers, there are some dynamic movements that are not addressed on theory. In groups of three, theory does not predict a stable outcome, but a pattern emerges in the classroom experiments. For groups of four and more, additional patterns emerge as a result of a previously unexplored coordination problem—players don't know where the other players will locate. para In a follow-up set of experiments, I introduce one-way communication in order to address the coordination problem. In these experiments, one subject in each group is designated to be the communicator. Prior to each round of play, the communicator sends a suggested location set to the other players. I conclude that one-way communication enhances the

ability of subjects to coordinate, but sometimes the communicator does not put out an agreeable proposal. Overall, the research reveals the importance of having a communication environment and game structure that will support trust and understanding.

Dedication

This dissertation is dedicated to the memory of my grandmother, Helen DeSocio, and of my father, Anthony J. Dziepak.

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Quote: "A hypothesis or theory is clear, decisive, and positive, but it is believed by no one but the man who created it. Experimental findings, on the other hand, are messy, inexact things which are believed by everyone except the man who did that work." para – Harlow Shapley

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Chapter 1

Introduction

1.1 Background and Context

Spatial economics is a subfield of microeconomics that considers economic models in which all economic activity are not assumed to occur in a single point in space. Spatial economics is rooted in the introduction of transportation costs in a geographically dispersed market. If all economic activity took place at a point, or if transportation was costless in time and money, there are no location problems. Instead, suppose that consumers are geographically dispersed, while the firm produces at one point. If this is the case, interesting problems of pricing, location, and solicitation strategies may arise. This is the basis of location market models, which give rise to location games.

Location games can also be interpreted not just as geographical space, but as quality space, a political spectrum, a color scale, or some other characteristic. Location games can thus be generalized as a study of product differentiation across some continuous product characteristic. Therefore, the study of the mathematical properties, and the human navigation behaviors across physical geography or across figurative product space can be helpful in understanding markets with product differentiation more generally.

1.2 Taxonomy of Location Games

Spatial markets have been modeled with various parameters and assumptions which affect the properties and results. These include the transportation cost function, the market space and distribution of potential buyers, the number and location of firms, the pricing policy, the equilibrium (solution) concept (Nash, minimax, conjectural variations, and others), and the existence of barriers to entry. More recent extensions of the models consider the information set, the process of obtaining information, the presence of collusion, and the application of location games to international trade. This section will describe the various parameters that will determine the shopping behavior by the buyers and location and pricing strategies by the sellers in the models.

1.2.1 Market Space Dimensions

The earliest spatial models used a line segment market space, which had an inherent property of asymmetry of location due to its endpoints. The unit line is the most studied space, and it arguably has the most intuitive interpretation for both a geographical space to non-geographical application, including a color or electromagnetic spectrum, the political spectrum, quality, and others.

Perhaps the second most common model is the unit circle, in which the endpoints of the unit line are connected to form a closed loop. Any given location on the unit circle is symmetric with respect to the market space. Interpretation of the unit circle as an abstract space has been interpreted as horizontal product differentiation, such as color preference. As a consequence of "closing the loop" from a unit circle vs. a unit line, equilibrium characteristics may differ greatly between otherwise similar examples on the two market spaces. Most of the literature has been written using these two basic market spaces.

Other spaces that have been studied: The plane segment is a two-dimensional extension of the unit line. The sphere (ball) surface is a two-dimensional extension of the unit circle. A cylindrical market space may represent one horizontal characteristic and one vertical characteristic. Furthermore, networks are connected line segments, which may model the situation of transportation routes; thus different models of space not only have different equilibrium characteristics, but different interpretations.

1.2.2 Interpretation of Space

In the original spatial economic models, the space has initially been interpreted as a market that operates within a non-point geographical space across which buyers are distributed. With nonzero transportation costs added to the purchase price of the good, buyers seek to minimize the full cost of purchase, thus defining market areas for the firms. Later, the same structural models have been interpreted as variety space, where potential buyers' location in space indicates their ideal variety of the product. Likewise, the firm's location in variety space indicates the variety of a good that the firm chooses to produce, and the distance between the firm and the buyer represents the deviation from the buyer's ideal variety.

Another interpretation of this variety space is the positioning of political candidates for an election on the political spectrum ("left" to "right"). Voters, distributed on the issue space, would vote for the candidate closest to their own position on the issue.

Variety space has been further defined as either horizontal or vertical, where vertical differentiation indicates quality differences, and horizontal differentiation is some other characteristic. In terms of utility, all potential buyers rank products on the vertical scale in the same order whereas this ranking differs in horizontal space. In other words, everyone prefers a higher-quality product, but color preferences, an example of horizontal variation, may not be consistent from one buyer to another.

1.2.3 Information Structure

Full information is another assumption in the modeling of markets. Full information means the buyers know the locations and prices of all of the sellers, so there is no shopping cost to discover the prices. Incomplete information introduces shopping behaviors. For example, a basic shopping behavior can be introduced in which buyers know the average price across two firms, but they cannot match firm with price. Each potential buyer knows a distribution function that gives the probability that the price observed, after visiting a number of firms, is below some minimum price. Searching is costly in terms of time spent or distance traveled, and thus the buyer searches until the additional search is more costly than the expected gain. One could, instead, consider a sequential search, where the buyer takes into account the new information gathered at each search.

1.2.4 Method of Solicitation

An important distinction between the solicitation of information by phone and the solicitation of information in person is that costs can only be shared (combined) in solicitation in person. That is, visiting two shops in one mall is about as costly as visiting one shop in a mall; however, phoning two shops in a mall is about twice as costly as phoning one shop at the mall. This key distinction plays a role in the clustering of firms, and the characteristics of the equilibria.

1.2.5 Clustering of Firms

Location models have been used to examine the observed practice of clustering, which is the tendency of firms to locate close to one another. Notwithstanding urban planning and zoning effects, clustering results in two opposing considerations for the seller. .has a cost to the sellers of increased competition. However, firms may be drawn to each other in order to capture market share from competitors. In an environment of costly information, clustering may facilitate consumer search. Consumers may be attracted to clusters for this reason (because it facilitates search), or because consumers have ”competitive expectations” about the cluster prices. Furthermore, firms might cluster to take advantage of these competitive expectations.

1.2.6 Demand Function

The demand function of the individual potential buyers is a parameter that affects the equilibrium characteristics of the model. Demand can be perfectly inelastic, or downward-sloping. There may or may not be a reservation price. Demand for a good may be unitary or multiunitary. If demand is perfectly inelastic, and there is no reservation price, this is equivalent to modeling just the location problem without regard to price.

Closely related to the demand is the distribution of the consumers. The most common is uniform distribution, but several nonuniform distributions, such as a triangular distribution, a normal distribution, or discontinuities in a rectangular distribution, have been used.

1.2.7 Transportation Costs

The transportation cost function can dramatically alter the results of the model. For instance, in the original model with a unit line and linear transportation costs (i.e. the cost is a linear function of distance), firms have an incentive to locate closer to each other. However, if the transportation costs are quadratic, the firms want to locate as far as possible from each other. Aside from linear and quadratic costs, the two most often considered, one might also consider a general transportation cost, or examining the effects of general properties of the transportation costs on the results of a particular model. Such properties may have an impact on the results are the degree of convexity, the continuity, and the differentiability (which effects the continuity of the payoff function).

An interesting cost structure to examine might be square-root transportation costs, or some similar costs, where costs increase with distance at a decreasing rate. The interpretation of this type of cost is that in a world of multimodal transportation, the diminishing transportation costs to approximate real-world tradeoffs between transportation modes. The firm may choose among various modes of transportation to deliver the good to a consumer at any given distance. The modes of transportation differ with respect to fixed and variable costs.

For instance, carrying the box in person has very little fixed cost, but great variable costs; whereas a truck may have greater fixed, and less variable costs. The lower envelope of all modes for all distances represents the most economical for any given distance. As the number of modes increases, the lower envelope becomes a smooth, diminishing curve.

Also, how about 2 firms, with different transportation costs? One with high fixed, the other with high variable. These two models probably would have similar outcomes: one firm cross-hauling is a possibility here.

1.2.8 Tiebreaker Rule and Other Rules of the Game

One of the concerns of location models is how to treat the cases where two or more firms wish to locate at the same point in the market space. This sometimes results in discontinuous demand functions. The usual treatment is that firms are allowed to locate at the same point, and they will divide profits evenly.

Another treatment is that they cannot locate at the same space, that one firm is to the left and one firm is to the right. Their profits will thus be divided at the point of their location, with one firm serving the buyers to the left, and the other serving the buyers to the right. These two treatments can result in different solutions.

1.2.9 Other Factors

The number of firms is a parameter of location models. Most common is the analysis of two firms. Models with three or more firms have been done, and each number of firms can greatly change the equilibrium.

As with other economic competition models, the possibility of entry and barriers to entry can affect the market structure.

Finally, the equilibrium concept is a parameter in spatial models. Nash is the predominant equilibrium concept used in these models. However, other concepts have been postulated in attempt to reach a reasonable solution. Such concepts typically include various conjectural variations and minimax reasoning.

1.3 Experimental Considerations

The application of experimental analysis to location games had started in the 2000s and there has not been much research conducted to date. While there are few papers that are direct application of experimental analysis of location games, there are several papers addressing issues specific to experimental analysis in general that do not come up in theoretical analysis.

The first issue is coordination. In theory, in some models, there are certain location combinations that are a Nash equilibrium. However, in experimental practice, even if subjects wish to choose this equilibrium, there is a problem in determining who goes where. For example, in a simultaneous-play game, where two firms want to locate at the endpoints, there needs to be a mechanism to assign firms to one or the other endpoint.

Fairness is a quality that has been explored in experimental economics. The desire to be treated fairly and to treat others fairly is a motivator for player behavior. In selfish fairness, players may desire an outcome in which no other player receives higher payoffs. Players may be concerned about the fairness to others in the group. This either may be altruistic, or it may be rational expectations—a player may reason that an outcome that is not minimally fair to others in its group may be unstable, or proposals that are not reasonably fair may not be plausible.

Reciprocity is another form of fairness. In a repeated game, a player that is ahead of others in cumulative earnings may get different group treatment than one who is behind. A leader who treats subjects fairly may develop esteem.

Whereas experiments have been designed to test specific human social behavior, the application of experimental analysis of location games may include evidence of specific human behaviors in an applied context. For example, there have been experiments designed specifically to test for reciprocity—subjects punishing other subjects who behave noncooperatively in the context of social exchange[1]. While these experiments are concerned primarily in the characterization of the behavior of subjects in a location game context, part of that characterization may be that reciprocal behavior is a plausible explanation of some observed actions.

1.4 Objective

The objective of this research was to observe and present results of basic location models in an experimental setting and to compare these results with what is predicted by theory. In some instances, patterns are observed that are not addressed in theory.

1.5 Navigation

The second chapter is a literature review. The third chapter presents results of experiments on simple Hotelling location models. Results are compared to pure- and mixed-strategy Nash equilibria for the simple location problem with two to six sellers. For groups of two firms, there is clear evidence of communication for the purpose of influencing the other seller as part of a multiperiod strategy. However, whether the motivation for this signaling is to coordinate on a fair outcome or to deceive is unclear. For groups of three firms, there is evidence of risk aversion as players avoid riskier central locations. For groups of four and five firms, results do not readily attain the unique pure-strategy Nash equilibrium due to a previously unexplored coordination problem. In the four-player groups, players choose a combination of pure-strategy focal points and the mixed strategy support; however, players also choose the center of the market, which cannot be explained by theory. In the six-player

[1] See Hoffman, McCabe, and Smith (1998).

case, there are two equilibria in pure strategies, providing an even more complex coordination problem. The distributions of strategies vary systematically as the number of seller increases in a manner consistent with Nash predictions.

In the fourth chapter I introduce one-way communication in order to facilitate the solution of the coordination problem. This design has the potential to yield a cleaner test of the theory by removing the confounding coordination difficulty. I run groups of two to four firms in a fixed-matching repeated game. In these experiments, one subject in each group is designated to be the communicator. Prior to each round of play, the communicator sends a suggested location set to the other players. A leader who understands the strategic situation may specify locations to the other players that result in an equilibrium outcome. Previous experiments suggest that this kind of "cheap talk" can facilitate equilibrium selection in coordination games. The results show some improvement in the ability of subjects to coordinate on Nash equilibrium play. However, there is considerable heterogeneity across groups. I conclude that one-way communication enhances the ability of subjects to coordinate, but that bounded rationality and poor leadership preclude the convergence to equilibrium.

Finally, in the backmatter, there is a master bibliography, which includes all references cited in this dissertation.

Chapter 2

Review of Literature

The following is a description of the papers that lead through the major developments of location theory and the experimental analysis of location games. Thisse and Norman (1994) is a nice reprint-volume introduction to the basic models and variations of product differentiation.

2.1 Seminal Location Theory Papers

The origin of the analysis of spatial competition, as far as the author can tell, begins with comments in Sraffa (1926). Previous to that article, all oligopolistic analysis has assumed that the product is homogeneous in the respect that all market activities occur at one point; i.e. there is no space dimension of the market. Sraffa discusses how firms have some degree of local monopoly due to established business circles or geographical differentiation in the presence of transportation costs.

The unit line model of spatial differentiation was introduced in Hotelling (1929). The model presented was a line of fixed length, which may be a transcontinental railroad or Main Street in a long town. Other interpretations might be two pushcart ice cream vendors along a linear beach or boardwalk. In this model, transportation costs are embodied in buyers' walking efforts, which are linear with distance, and the quality of the costless product is assumed to be identical among both vendors. Where should a vendor set up shop, and how much should it charge?

Assuming prices are fixed and equal among all firms, and with location as the decision variable[1], one firm can gain market share by moving toward the other firm. As one firm moves toward the right toward the other firm at a rate of epsilon, the halfway point between the two firms (the point that divides the customers that buy purchase at Firm 1 from the customers that purchase from Firm 2) also moves right at a rate of epsilon/2. Consequently, the two firms will want to move toward each other, until they are both located at the same point, in the middle of the unit line space. This is a demonstration of "clustering (referred to at the time as agglomeration)," where vendors play strategically to maximize their market share. Hotelling then commented on how the location model might explain the observation that some notable commodities of the day were similar in quality and too homogenous, including shoes, protestant sects, and cider. The other key observation of this model is that the equilibrium outcome is not the socially maximizing outcome. In terms of minimizing walking distance, it would be preferred if the vendors were located at ¼ and ¾.

Boulding (1966) first used the term "principle of minimum differentiation," referring to the tendency of firms to locate within close proximity, referred to in this paper as "clustering." He suggested this principle as a possible explanation for a wide range of occurrences, including the reason why all automobiles are so much alike, why nickel-and-dime stores tend to cluster together, and why industries tend to concentrate in one quarter of the city.

The pure-strategy equilibria for three or more firms were characterized by Eaton and Lipsey (1975). For three firms, with price fixed, no locational pure-strategy equilibria exist. However, locational equilibria exist for four or more firms. For four firms, a unique equilibrium exists with two firms paired at the first and third quartiles. For five firms, there are two

[1]The Hotelling model also modeled both price and location as a decision variable, leading to additional results.

pairs of firms located at 1/6 and 5/6, and the fifth firm is unpaired, and located at 1/2. The unpaired firm earns twice the profit as the other paired firms. For six firms, there are two equilibria: three pairs at (1/6, 1/2, 5/6), or two sets of pairs at 1/8 and 7/8, and single firms at 3/8 and 5/8.

In general, for $n > 3$, there are two pairs of peripheral firms, with the interior firms either single or paired. General properties are that the unpaired interior firms earn payoffs at least as great as the paired peripheral firms. This paper also considered the unit circle and two-dimensional market spaces, such as the unit plane and a disc.

2.2 Mixed Strategies

While a pure-strategies equilibrium does not exist for the three-firm case, Shaked (1982) showed the existence and computation of the mixed strategy equilibrium. If three firms play in the support [¼, ¾] with uniform distribution, that is a mixed-strategy Nash equilibrium.

Dasgupta and Maskin (1986) is the general paper that determines the existence of Nash equilibria (pure and mixed strategies) in games with discontinuous payoff functions. Simon (1987) extends Dasgupta and Maskin in providing a general, formal treatment of discontinuities in the payoff function. Osborne and Pitchik (1986) show that mixed-strategy equilibria exist for larger groups because the model satisfies the conditions required in Dasgupta and Maskin (1986). However, these equilibria are hard to solve.

Denzau, Kats, and Slutsky (1985) discusses the possible motivations of the sellers in a pure location game. Rank minimization (by market share, as opposed to absolute market share maximization) is considered as an alternative motivation. This has experimental implications. If all sellers are share maximizers, then results are identical to Eaton and Lipsey

(1975). It is thus a more complete characterization of location games taking into account various motivations of the players.

2.3 Experimental Analysis Applied to Location Games

There has not been a lot of location experimental literature to date.

Benson and Feinberg (1988) is an example of an early paper, which addresses producer's information about rivals' prices. This is an experimental paper in a location model but with prices as the decision variable.

Brown-Kruse, Cronshaw, and Schenk (1993) is closer to the experimental design that I had considered. However, their model contains a demand function that orchestrates a cooperative and a competitive equilibrium, reducing it essentially to a prisoner's dilemma game in a location game context.

Two papers that are similar to my topic are Collins and Sherstyuk (2000) and Huck, Müller, and Vriend (2002). The first paper examines a location game with three players in a group in a multiperiod game, in which players are randomly rematched after each period. The second paper explores a four-player location game in a fixed-matching, repeated game.

2.4 Research Gap

There has been very little experimental study of the behavior of subjects in a location setting. Most experimental papers design experiments with a design that optimizes the specific behavior that experimenters want to test. However, this research is seeking to use experimental methods to observe and explain behavior in spatial economics. The theory literature also does not address the coordination problem that would arise in many simultaneous play models.

I was interested first in doing the basic location models to observe the similarities to and differences from theory. Then I added a treatment of cheap talk to address the coordination problem.

Chapter 3

Location Experiments

Abstract: *This chapter presents results of experiments on basic Hotelling location models. Results are compared to pure- and mixed-strategy Nash equilibria in models with two to six sellers. For groups of two firms, there is clear evidence of communication for the purpose of influencing the other seller as part of a multiperiod strategy. However, it is unclear whether the motivation for this signaling is to coordinate on a fair outcome, to maximize one's own payoff, or for deception. Whatever the motivation, a clear pattern emerges, which is not explained by theory. For groups of three firms, there is evidence of risk aversion as players avoid riskier central locations. For groups of four and five firms, results do not readily attain the unique pure-strategy Nash equilibrium due to a previously unexplored coordination problem. In the four-player groups, players choose a combination of pure-strategy focal points and the mixed strategy support; however, players also choose the center of the market, which is not explained by theory. In the six-player case, there are two equilibria in pure strategies, providing an even more complex coordination problem. The distributions of strategies vary systematically as the number of seller increases in a manner consistent with Nash predictions.*

3.1 Introduction

This paper extends our understanding of spatial economic theory by investigating human economic behavior using experimental analysis in the location game framework. Experiments are conducted using groups of two through six firms. This experimental analysis reveals economic behavior that cannot be discovered using traditional theoretical techniques. The experimental analysis also provides a test of how well theory predicts actual behavior.

3.1.1 Location Theory Fundamentals

Location theory analyzes the efficient and strategic placement of resources, production, and consumption in a geographical or otherwise differentiated space. For example, an ice cream vendor may choose the most profitable location for his cart on a boardwalk, taking into account the locations of any competitors and the distribution of potential customers along that boardwalk. This is the scenario that is associated with Hotelling (1929). Other than the geographical interpretation of location theory, it can be interpreted as a location game among political candidates, who may strategically choose a position along a political spectrum in order to maximize votes[1].

Results derived from the study of location theory have been used to describe observed market phenomena such as firm clustering–competition causing retailers to locate next to each other in some cases in order to capture maximum market share[2]. With the reverse effect, it has also been used to describe geographical differentiation–firms locating away from each other in order to limit direct competition[3].

3.1.2 Application of Experimental Analysis to Location Games

Game theory has been the traditional means used to study the behavior of agents in a market with differentiated products. More recently, experimental analysis of games has been used

[1] First known comment on this is Downs (1957).
[2] See Eaton and Lipsey (1979).
[3] See d'Aspremont, Gabszewicz, and Thisse (1979).

to evaluate and extend the game theoretical analysis.

Many of the experimental tests of location theory have been done in the context of voting design and analysis. Enelow and Hinich (1990), Palfrey (1991), and Williams (1997) address specific observed phenomena in voting and elections. These papers are all extensions of the basic location models, with one or two modifications in the assumptions that customize the model to explain a specific finding in voting theory. In that way, it is not really a test of the pure location model.

Brown-Kruse, Cronshaw, and Schenk (1993), provide one of the first experimental tests of spatial competition. A simple location-only (fixed-price) game is used to test both competitive and cooperative outcomes with two firms. This paper uses a demand function such that a competitive Nash Equilibrium exists in which firms locate at (.5, .5), and a cooperative equilibrium in which firms locate at (.25, .75). Experimental results closely resemble both theoretical outcomes depending on the treatment. First, with anonymous pairings negating any possibility of coordination, the competitive outcome resulted. In another treatment, with the possibility of nonbinding communication, the cooperative outcome was observed. However, this paper uses the demand function to orchestrate a clear cooperative and competitive equilibrium, reducing it essentially to a prisoner's dilemma game in location game packaging. In that regard, it is not really an analysis of location games with its continuous or near continuous strategy space.

In contrast, Collins and Sherstyuk (2000) use an inelastic demand, which truly represents the location model. In this paper, each consumer is uniformly distributed across the linear market space and purchases exactly one unit of good with no reservation price. This paper examines only the three-player case, in which no equilibrium exists in pure strategies. My chapter corroborates their results and extends it to 4-6 firms, and to treatments with fixed-matching interaction.

This chapter examines the behavior of subjects in a location game environment and compares the results with the theoretical predictions. It also deals with additional factors that come into play that are not of issue in theory. Those factors include the subject's motivation: whether it is solely to achieve Nash equilibrium, or some other outcome; in the presence of multiple equilibrium, which one; the process of discovering the equilibrium by learning; the process of information gathering about the behavior of other players in the group; whether the players are risk neutral; the process of arriving at a particular outcome; and other goals, such as focal points (which may or may not correspond to a Nash equilibrium), fairness, and coordination problems in a multiperiod simultaneous-play game without communication.

The first issue to address in the experimental setting that is not a factor in theory is the comprehension of equilibrium by the players. There may be a certain amount of learning taking place in the first few periods.

Another issue is the motivation of the players. The players might not necessarily want to achieve the Nash equilibrium. First, the reader should recognize that, in a pure location model, without reservation prices, it is a constant-sum game. Each customer purchases one unit from the nearest seller, so the relative positions of the sellers determine the pie splitting, or market share, of each seller. So the players are dealing with location choice that may affect expected payoffs and variance. If players are not risk neutral, then variance is a factor as well as expected payoffs.

The motivation of the player may be fairness or winning. If players have a strong sense of group fairness, they might want to reach an outcome in which all players' payoffs are equal. However, if a players' goals are to "win the game" by earning more than their "rivals," then they might purposely try avoid an equilibrium or stable outcome in order to Create opportunities.

Even if it is indeed the goal of each player to achieve an equilibrium, the process of arriving at that equilibrium is not clear cut. In a simultaneous-play game, even if players recognize the equilibrium as a set of locations, there is no assignment of individual players to individual locations. That is, in the simultaneous-play game, the present-period location decisions of other group members are not common knowledge. Consequently, a player, in a group of players, all desiring an equilibrium, cannot be assured of an equilibrium outcome because there may be a deficiency of information regarding the assignment of players to positions.

This chapter also looks at the behavior of individual players. It looks at how an individual's location choice is affected by their information set, with particular emphasis on the previous period sales and previous period relative position. It also looks at how the individual adjusts. Do players move centrally or toward the endpoints and when? Do the players seek to be a middle firm or an end firm? Do the players seek locations that would increase fairness?

Finally, focal points play an important issue. Endpoints, midpoints, equilibrium locations, fair locations, or round numbers may serve as focal points. Focal points may or may not correspond to equilibrium positions, so focal points might reinforce equilibrium choice, or they might distract from it.

The coordination problem comes into play when the player group size is greater or equal to three. Coordination is not a problem in the two-player group. Learning is a factor in models of all size. Risk aversion is very likely to be a major issue in three-player groups, and possibly also in groups of four through six. There is no risk-aversion issue in groups of two.

The paper finds clear evidence that players are willing to invest a small price to communicate for the purpose of influencing other group members. However, it is unclear as to whether the motivation for that communication is self-interest, fairness, or an improvement of social welfare. Furthermore, there is evidence of risk minimization by risk-averse players.

The rest of this chapter is organized as follows: Section 3.2 presents the theoretical background and some new material in preparation for the experimental analysis. Section 3.3 presents some experimental background and explains the design and procedure of the experiment. Section 3.4 presents the results and analysis of the data along with a comparison to the theoretical predictions, and draws some hypotheses to explain the subject behavior. Finally, Section 3.5 presents conclusions and extensions.

3.2 Theoretical Considerations

3.2.1 Theoretical Background

The basic location model consists of one or more firms selling a product to consumers distributed across a market space. Hotelling (1929)[4] described the first and simplest interesting model, where two firms choose a location in a linear market space. Transportation costs, representing buyers' walking efforts, are linear with distance, and the quality of the costless product is assumed to be identical for both vendors. Buyers are uniformly distributed on this linear space, and they purchase from the closest seller. Thus a seller could gain market share by moving closer to the other seller. The result is a tendency for the two firms to gravitate towards each other at the midpoint of the market. The unique, pure-strategy equilibrium for two firms is (.5, .5).

Eaton and Lipsey (1975) extend this model to more than two firms. In the case of three firms, there is no pure-strategy equilibrium; for any given combination of locations, at least one firm can increase payoffs by changing location. Pure-strategy Nash equilibria exist for all numbers of firms except three. Four firms form two pairs at 1/4 and 3/4. For five firms,

[4] Hotelling also modeled both location and price as decision variables of the players.

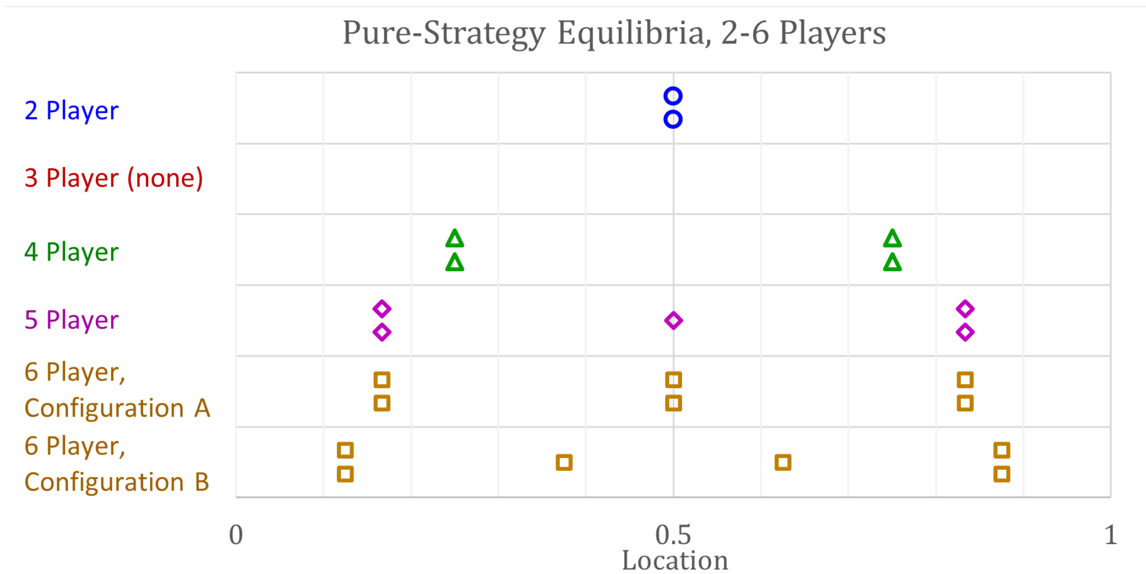

Figure 3.1: Pure-strategy equilibria for 2-6 players on a unit line market space.

there are two pairs at 1/6 and 5/6, with a single firm in the middle at 1/2. In this unique pure-strategy equilibrium, the middle firm earns twice as much as the paired end firms.

With a group of six firms, there are two pure-strategy equilibria configurations. One is with three pairs: at 1/6, 1/2, and 5/6, and all firms get equal payoff. The other is with two pairs at 1/8 and 7/8, and single firms located at 3/8 and 5/8. The single, interior firms get payoff double that of the paired end firms. There are also multiple pure-strategy equilibria for $n > 6$. Figure ?? shows the pure strategies solutions for $n = 2$ to 6 players.

There does exist a mixed-strategy equilibrium for $n = 3$, as characterized by Shaked (1982). There is a unique, symmetric mixed-strategy equilibrium in which all players select the support [1/4, 3/4] with uniform distribution, and each player gets equal payoff of 1/3.

More generally, Dasgupta and Maskin (1986) investigate the existence of mixed-strategy equilibria in games with discontinuous payoff functions. They show that a class of economic games, including the three-firm location game, possess a mixed-strategy Nash equilibrium. In fact, any n-player location game of this sort will have a mixed-strategy equilibrium.

3.2.2 Additional Examination

Osborne and Pitchik (1987) specifically show that a symmetric mixed-strategy equilibrium exists for finite $n > 3$, and that the density function is continuous and differentiable along the support. However, this equilibrium is difficult to solve. The following exercise will illustrate the difficulty in characterizing these equilibria.

First, if one supposes that the distribution of a symmetric mixed-strategy equilibrium is uniform, as it is for $n = 3$, one can solve for the supports (see Appendix A). However, the distribution on that support is not necessarily uniform. For a uniform distribution on such a support to be a Nash equilibrium, it must be that when $n - 1$ firms are playing the equilibrium, there is no other single location with a higher expected payoff. All single locations within the support have to yield an expected payoff of $1/n$. Therefore, all linear combinations or distributions within this support will also yield an expected payoff equal to $1/n$.

Lemma 3.1. *The distribution of a symmetric mixed-strategy equilibrium is not uniform for $n = 4$.*

Proof. Here is a counterexample. For $n = 4$ if $n-1$ firms are playing a uniform distribution and the nth firm plays a location of .5, then the expected payoff is .2396, which is less than .25 (calculations in Appendix B). The difference cannot be attributed to rounding, and there are no discontinuities in the payoff function. Therefore, player n can improve their payoff by removing some mass from the vicinity of .5 and distribute it nearer the endpoints of the support and improve their expected payoff. Therefore the equilibrium distribution is not uniform. $\square$

Furthermore, since the distribution is not uniform, in particular, since the distribution is

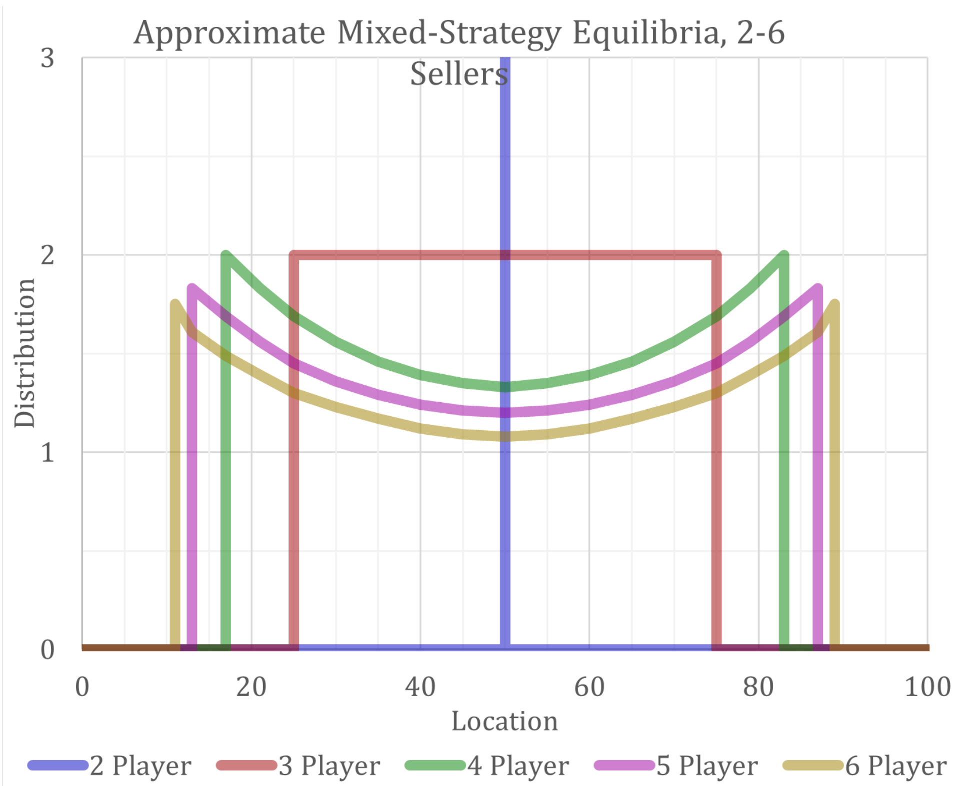

Figure 3.2: Pure-strategy equilibria for 2-6 players on a unit line market space.

higher at the endpoints, the expected location of the leftmost adjacent firm playing the equilibrium distribution is further left. This, in turn, moves the support endpoints in slightly.

In conclusion, what can be said about the support and distribution of the mixed strategies equilibrium is that the support is somewhat shorter than indicated in Figure ??, and that the distribution is not uniform. For $n = 4$, and probably also for $n = 5$ and 6, the distribution is less in the middle and greater in the endpoints.

Now let's take this to the other extreme and consider a case where a strategy is a mix between two discrete locations. One can redistribute mass from the middle and redistribute it on the endpoints until one is left with two spikes. The obvious place to start is ¼ and ¾.

Lemma 3.2. *For $n = 4$, the mixed-strategy play of [.5(25), .5(75)] is not an equilibrium.*

Proof. Assuming the other three firms play this, the fourth firm can improve their expected payoff by playing interior to the interval [25, 75]. Their payoff would be 25 in the case that they are an internal firm, but their payoff would be much greater in the event that all three other firms are to one side of them. Thus if such an equilibrium exists, the distance between the two peaks must be reduced in order to reduce the payoff of an internal firm. □

Lemma 3.3. *Lemma 3: For* $n = 4$*, the mixed-strategy play of* $[.5(x), .5(100 - x)]$ *is not an equilibrium.*

Proof. If all of the distribution is on the two spikes, then those two spikes cannot be moved closer because then a firm can deviate toward the endpoints (just outside the spikes) and receive a guaranteed payoff strictly greater than 25. Therefore, there are no mixed-strategies equilibrium with two spikes. □

Therefore, a mixed-strategy equilibrium exists, the distribution is bimodal, the distribution is intermediate between a uniform distribution and two spikes. It would be hard to solve for the actual distribution. However, from the above analysis, I can place some bounds on the distribution. First, I can bound the support to that of the support if the distribution were uniform. If any rival firm plays with any support outside these bounds, they can make a unilateral improvement in expected payoff by moving that support anywhere inside the bounded region. Second, the support must extend to at least the outermost spikes of the pure-strategy equilibrium. If not, a player can always play pure strategy outside this region and guarantee a payoff greater than $1/n$.

To go further and characterize the shape of the density function, I can say that this is bi-modal, with a U-shaped distribution. Osborne and Pitchik (1986) show that it is continuous and differentiable across the support. Suppose the function is quadratic. One would have to find a quadratic density function such that the playing all points on the support with pure strategies yield a payoff of $1/n$. Geometrically, I have fitted a quadratic function to the endpoints and the midpoint, but I have not found the formal solution (See Appendix C).

Another method of characterizing this distribution is with a finite approximation as used in Huck, Müller, and Vriend (2002). The solution for 100 discreet locations has a U-shaped density function although it has a peculiar zigzag shape. If you look at the shape of the distribution, you can imagine the average density as the number of discrete positions gets very large: the average density is a quadratic with a minimum at .5. Whatever the technique used to estimate the density function, my analysis indicates that it is U-shaped with a support somewhat less than [1/6, 5/6]. I also suspect that the shapes are similar for $n = 5$ and 6. An approximation of the mixed strategies supports and density functions are shown in Figure 3.2. The main focus of this dissertation is to report experimental results, and I will leave the solution of the distribution for future work.

The next section will describe the experimental design and procedure.

3.3 Experimental Design and Procedure

3.3.1 Design

Subjects participated in a simple location game similar to the one described in the theoretical section. In this model, price is fixed and equal across firms; thus location is the only decision variable for the subjects. The market space is described as a road of 100 blocks, with one customer per block. The sellers may choose locations at the intersection of each block; that is, the location must be a whole number from 0 to 100. Automated customers purchase one unit per period from the nearest seller. In the case of two or more firms locating at the same point, customers are divided evenly across such sellers. Complete instructions are given in

Appendix B.

The game is repeated, with the number of periods varying from 16-30, depending upon the treatment. Players are not informed of how many total periods are to be played.

The experiment is a 5x2 factorial design, as shown in Table 3.1. The first factor is the number of sellers in the market, with market groups of two, three, four, five, and six. The second factor is the matching protocol. Firms are either matched in a market of fixed size for the duration of the experiment and play a fixed-matching game, or they are randomly rematched in each period of the game.

Table 3.1: List of treatments and sessions conducted.

# players per group	# sessions	# groups	# players	# periods
Fixed groups				
2	1	4	8	30
3	2	5	15	30
4	2	5	20	30
5	3	6	30	30/30/15
6	2	3	18	30/17
Randomly rematched groups				
2	1	5	10	30
3	1	4	12	30
5	1	2	10	30

The matching protocol was varied for two reasons. First, fixed matching might more readily converge, as a long-term, multiperiod interaction may be more conducive to coordination. The second reason is to enhance comparability with other studies. Brown-Kruse, et al. (1993) used fixed matching with repeated interaction, but Collins and Sherstyuk (2000) rematched subjects between each round. A fixed-matching game gives the players the opportunity to communicate and coordinate whereas a rematched game might be more likely to match theoretical mixed-strategy equilibrium. In most real-world applications, the situation more closely resembles a repeated game.

The experiment was implemented using a program written in Java programming language. The program ran on a web server. Subjects at individual computers accessed the program through a Java applet in a web browser.

3.3.2 Procedure

The experiments were conducted using a computerized decision environment, which allowed automation of the calculation and tabulation of results, randomization of player groupings, and the maintenance of anonymity.

127 subjects were recruited from undergraduate classes at Virginia Tech. Each sat at an individual computer screen in a computer lab. A session consisted of 10-12 subjects, matched into groups of the appropriate size. Subjects were not told with whom they were matched. The subjects interacted with each other through the program, which simulated a hypothetical market environment as determined by the experimental design. The subjects were motivated to play well by cash payment that depended upon their choices in the market, and which they received at the conclusion of the session.

Students were paid a $3 show-up fee, plus their earnings in the experiment, which averaged $8, for a total of $11 average earnings. As shown in Table 3.2, the pay rate per unit sold was adjusted according to number of players in each group in order to equalize earnings across treatments. (If the price were the same across sessions, as the larger groups split the pie among more players, their expected payoff per period would be lower.)

Students were informed that the actual cash payout would be the total of their show-up fee plus the earnings, rounded up to the nearest quarter. The average earning rate of each subject was about 27 cents per period across all treatments.

Table 3.2: Earnings rate by number of sellers per group.

# players per group	Earnings rate (cents per unit sold)	Average earnings (per subject per period)
2	0.53	$0.269
3	0.80	$0.269
4	1.07	$0.273
5	1.33	$0.271
6	1.60	$0.273

3.3.3 Timeline

The timing of events in a typical session occurs as follows: After logging in, the students are presented a line figure depicting the market space. Students enter their location in a box, which is indicated on the line by a dot. A sample "location choice" screen appears as a figure in Appendix C. After all players enter their choices, the program calculates all sales. The program also records all locations, sales, and accumulated earnings. Each subject observes locations, sales, and accumulated earnings of all players in his group in a table. On the computer screen, the locations of players and market areas are indicated by dots and colored bars, respectively. A sample "results" screen appears as a figure in Appendix C. The subject is also required to record his decisions and earnings on a log sheet. This was done to encourage the subjects to view all the information and to maintain a uniform pace to the session.

3.4 Results

The results section is divided into subsections according to group size. Each group size has a different set of analyses. At the end, I summarize and compare the results across group sizes.

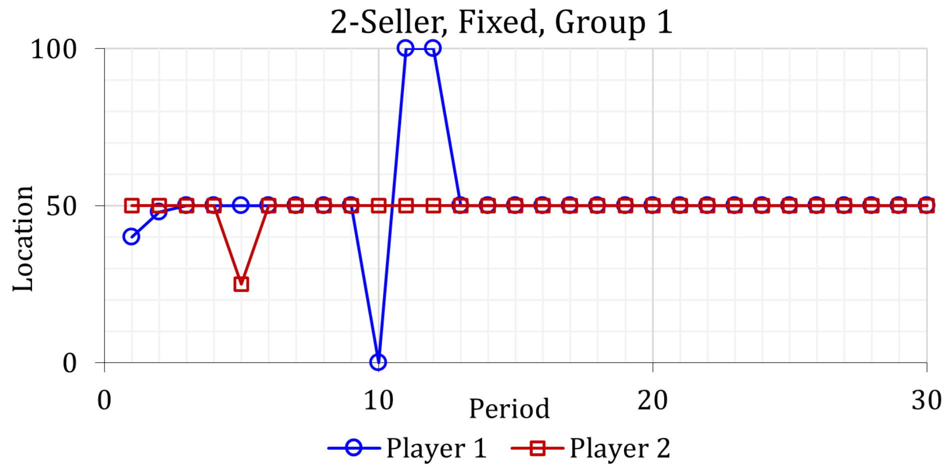

Figure 3.3: Locations by period, 2-player, fixed matching, Group 1.

3.4.1 The Two-Player Case

In the two-player case, the theoretical revenue-maximizing location, assuming that the other player chooses 50, is to also choose 50. Because this is a pure-strategy equilibrium and a mixed-strategy equilibrium does not exist, there is no expected variance issue. Consistent with pure-strategy equilibrium, most players converged or trended to a location of 50, but there was some deviation—especially in earlier periods.

The individual fixed groups show some striking diversity in play dynamics, and I have inserted them here as Figure 3.3 through Figure 3.7. Group 1 shows some early experimentation and a quick convergence to 50-50 play for the final 17 periods. Group 2 shows more early deviant location selection leading to a convergence to near 50-50 in the last 15 periods. Group 4 displays both players converging to 50 from the same side rather than jumping back and forth on both sides of the market midpoint. Group 3 displays the most deviant play with both players apparently attempting to catch their rival on the short side of the market and gain some market share. The players in the randomly rematched treatment tended to deviate less often overall.

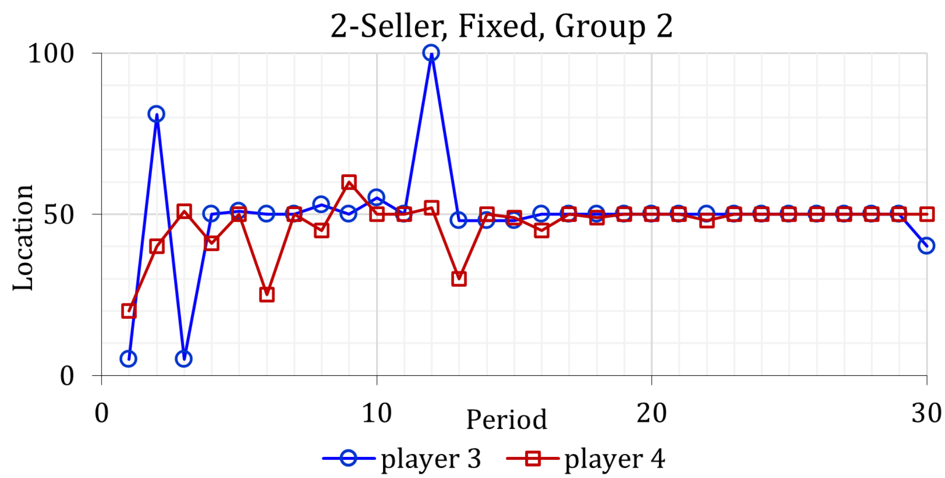

Figure 3.4: Locations by period, 2-player, fixed matching, Group 2.

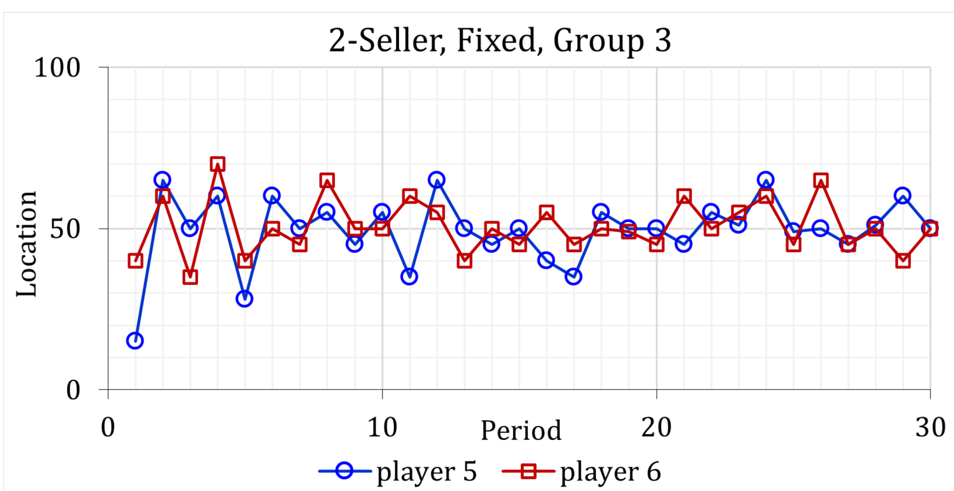

Figure 3.5: Locations by period, 2-player, fixed matching, Group 3.

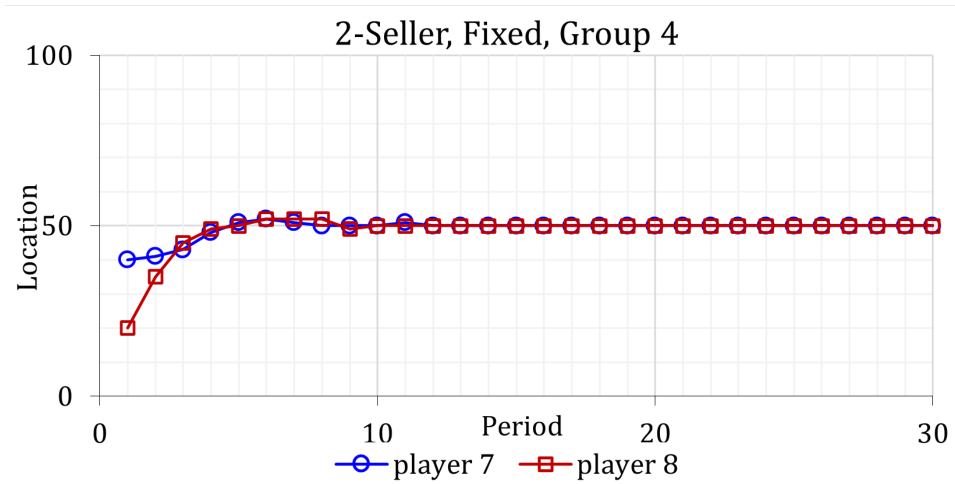

Figure 3.6: Locations by period, 2-player, fixed matching, Group 4.

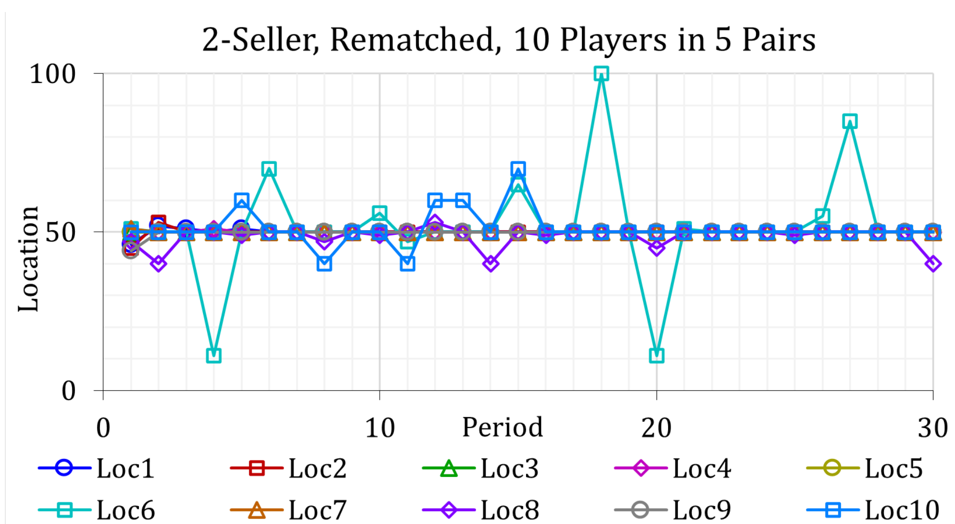

Figure 3.7: Locations by period, 2-player, all randomly rematched players.

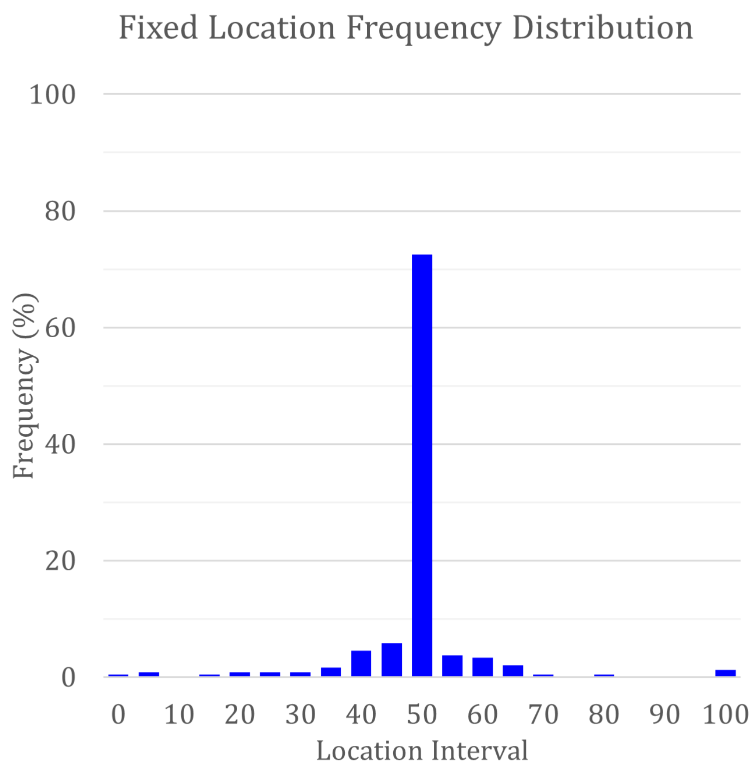

Figure 3.8: Location frequency distribution for fixed, 2-player groups, all periods.

In casual observation of the locations chosen by players in two-player groups, most location choices were at or near 50. The variation from 50 seemed to narrow in later periods. Similarly, the random-rematched two-player groups showed a frequent play of 50 by subjects with some deviation, which diminished with time. It appears that the random-rematched groups had less variation than the fixed-matching groups. The concentration of locations in the center of the market is evident in a distribution function of all of the location choices by five-unit intervals centered on locations divisible by five (0-2, 3-7, 8-12, 13-17, 18-22, .., 93-97, 98-100). The frequency distribution of the fixed-matching data and the random-rematched data is shown in Figure 3.8 and Figure 3.9, respectively.

Both the random-rematched and fixed-matching data show strong spikes at 50 and some variance around this spike. The random-rematched two-player groups show a stronger spike

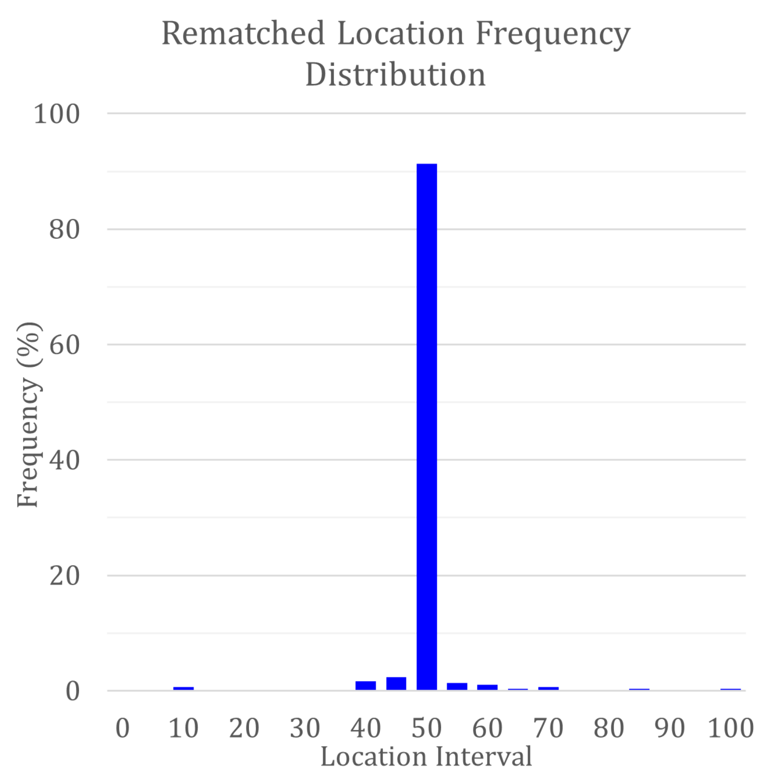

Figure 3.9: Two-firm location frequency distribution for players who were randomly rematched after each period.

and a lower variance. The distribution of the random-rematched treatment was over 90 percent playing in the interval [47, 52]. The fixed-matching treatment distribution is still a strong peak of over 65 percent, but with more spread. The rematched sellers seem to go straight to 50 whereas the fixed-matching groups tried to do something more creative with signaling in the early rounds. To an extent, the non-50 play seems to have an effect on the other player in some groups.

Examining the fixed-matching data, locations within the interval of [48, 52] were played 73

I will now provide more detail on the feedback players experienced depending on the locations they chose. First, in the theoretically optimal play, assuming the rival plays 50, the best response is to also play 50. More comprehensively, the theoretical payoff curve by a player's chosen location is illustrated in Figure 3.10. If the rival plays 50 (black line), the payoff is 50 if the player chooses 50, and the payoff is reduced linearly as one deviates from 50, with payoffs equal to 25 at the endpoints, 0 or 100. If a player chooses 50, the player is guaranteed a payoff of at least 50 regardless of the rival's chosen location. However, there is a chance that the player will receive less than 50 if the player chooses a location other than 50. The colored lines show the payoffs function by location for rivals' locations other than 50. Specifically, this figure shows the payoff functions when the rival's location is 0, 10, 20, 30, and 40; on red, orange, green, blue, and purple-colored lines, respectively.

Notice there are discontinuities in these payoff functions, when both players are co-located, and when the players switch order. For example, when the rival's location is 20, the player faces the green payoff function. If the player chooses 0, the market is split at location = 10, and the player receives 10 and the rival receives 90. Moving to the right from this point, the payoff increases for the player at a rate of ½ for every unit increase in location. When both players co-locate at 20, the two players split the market 50-50. The experimental design allows for co-location, and the rule that two co-located players split their market is

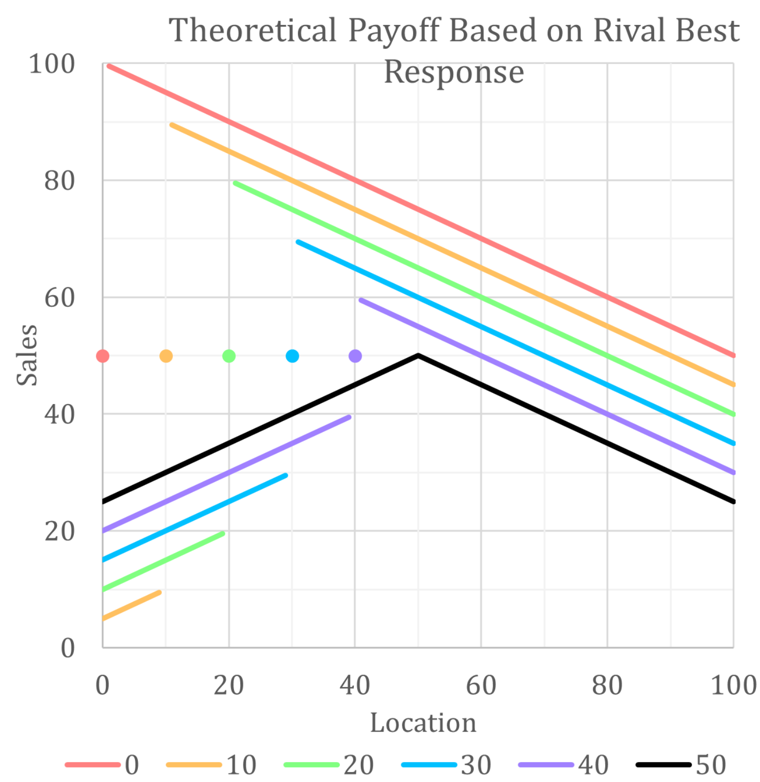

Figure 3.10: Payoff functions conditional on rival's location.

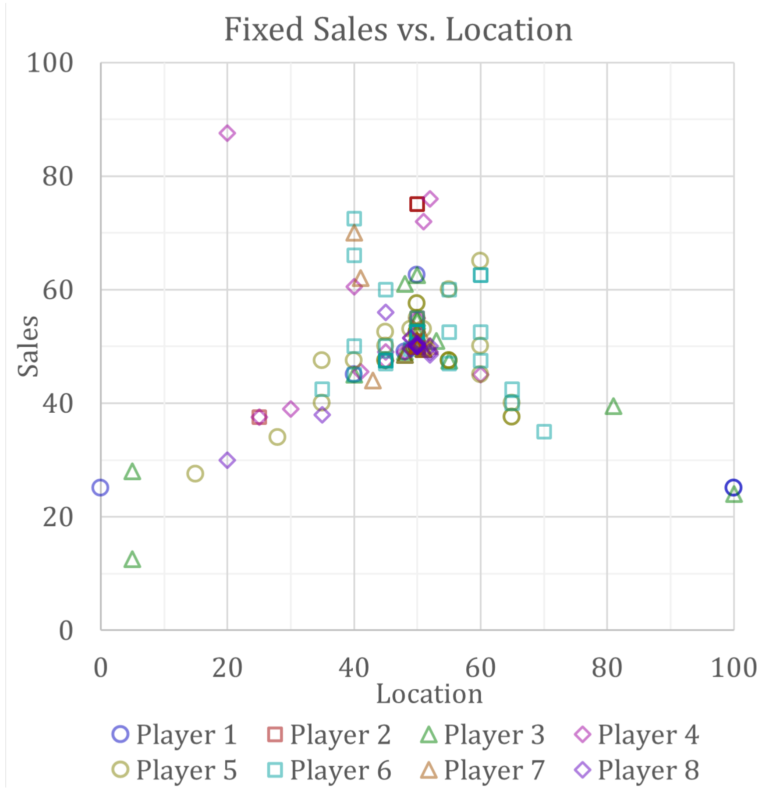

Figure 3.11: Sales by location, all fixed data, 2 players per group.

a common assumption in location models. When the player moves one more unit to the right, that player now captures the right side of the market, and sales jump to 79.5. The experimental design only allows for whole-number location choices. Moving further to the right will decrease the player's market size at a rate of ½ for every unit moved, until a location at the right endpoint of the market results in a payoff of 40.

With these payoff functions in mind—especially the black line—Figure 3.11 and Figure 3.12 show the actual payoffs by location for the fixed groups and the randomly rematched groups, respectively. The observation is that the fixed groups deviate from the black line more than the rematched groups.

Also observe the distribution of points along a vertical line above the point [50,50]. If the player locates at 50 and the rival chooses any location other than 50, the player will be on

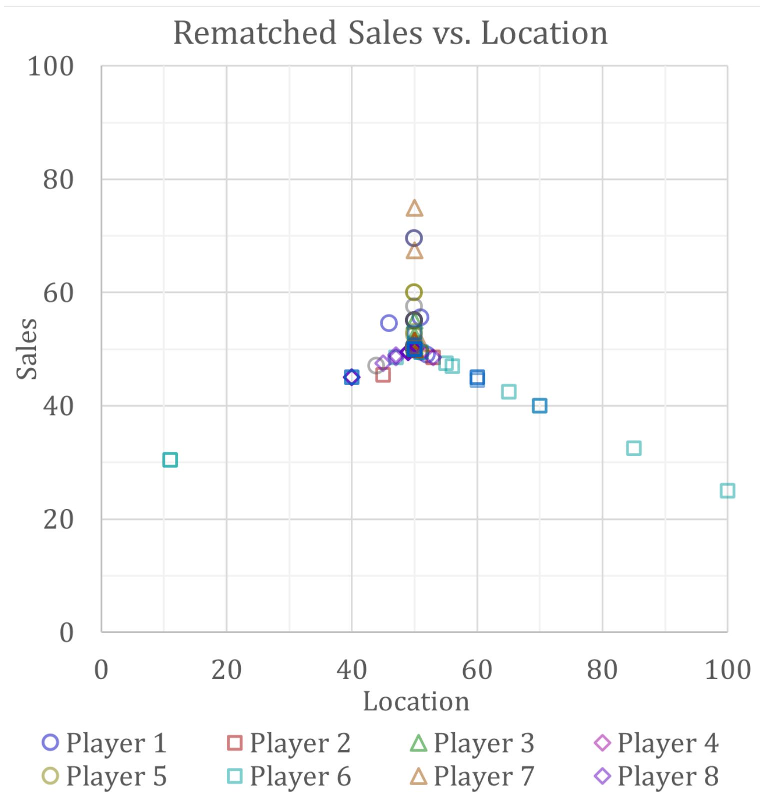

Figure 3.12: Sales by location, all rematched data.

this line and enjoy a payoff greater than 50.

A player might choose another location for one of three reasons. The player may be trying to influence the other player for future rounds–either to coordinate on some outcome other than the Nash equilibrium. Players may be trying to influence their rivals in order to get them to play a nonequilibrium location in the future in order to take advantage of them, or they may believe that the other player will play some other location, and they are trying to earn a profit strictly greater than 50. Especially in the early rounds, the players may be learning the game and figuring out the strategy.

An overall view of the earnings by location reveals that a location choice of 50 always earns at least 50, and earns more when the other player chooses another location. As one chooses a location further away from the center, there is a chance that one could earn more if one still lies between the center and the other player. The further out one moves, the more likely that the other player is less central, so the player stands to lose more.

Next, I graphed the locations over time in a scatterplot to see if there was a visual trend; see Figure 3.13. A casual observation of the graph revealed a trend of reduced variance over time. I then graphed the location distributions of just the last 15 and the last 10 periods. These graphs revealed stronger spikes than the full-period data. This suggests that there was learning and a subsequent elimination of noise, or that there was a change of strategy from an attempt to communicate to Nash equilibrium play.

I then used statistical tests to confirm that our observations are significant. First, in the two-player case, Kolmogorov-Smirnov (K-S) one-sample tests compare the distribution of play with the theoretical equilibrium of playing 50 with probability one. The K-S test uses the largest difference between the two cumulative distributions. The numbers are given in Table 3. First, there was a significant difference between the full fixed-matching data and

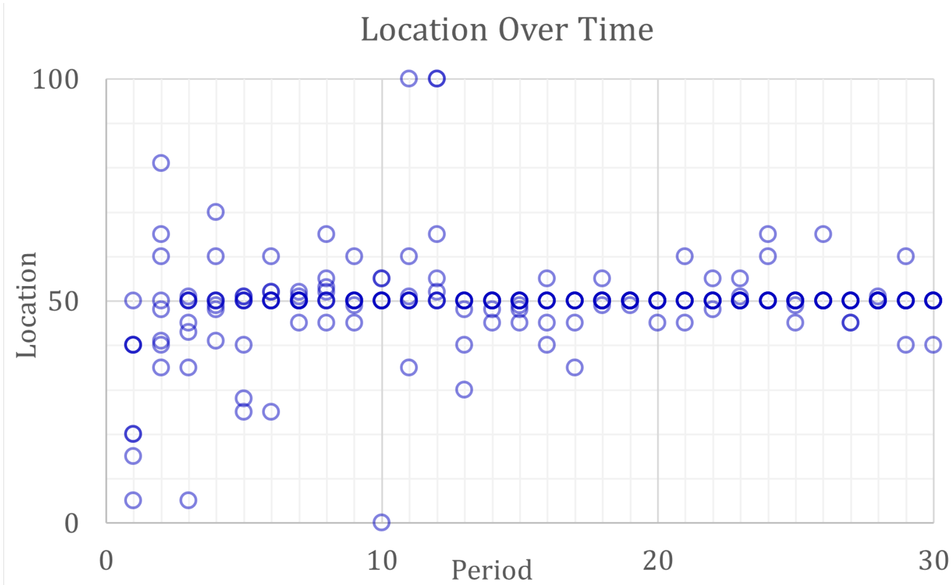

Figure 3.13: Location over time, all fixed group data.

the theory. However, there was no significant difference between the last 15 periods of fixed-matching data with theory. Furthermore, there was no significant difference between the full-period rematched data and the theoretical prediction. No significant difference means I cannot reject the possibility that the two samples come from the same pool.

Table 3.3: Significance of differences in observed distributions from theory, $N = 2$.

Matching treatment	Number of observations	D, largest difference	P-value
Fixed, all periods	240	0.2125	0.00001
Fixed, last 15 periods	120	0.1250	0.15335
Rematched	300	0.0633	0.30057

3.4.2 The Three-Player Case

The three-firm case is significantly different from the two-player case because there is no pure-strategy Nash equilibrium. A casual observation of the three-player groups' location choices by period, group by group, demonstrate a lot of jumping from one side of the market

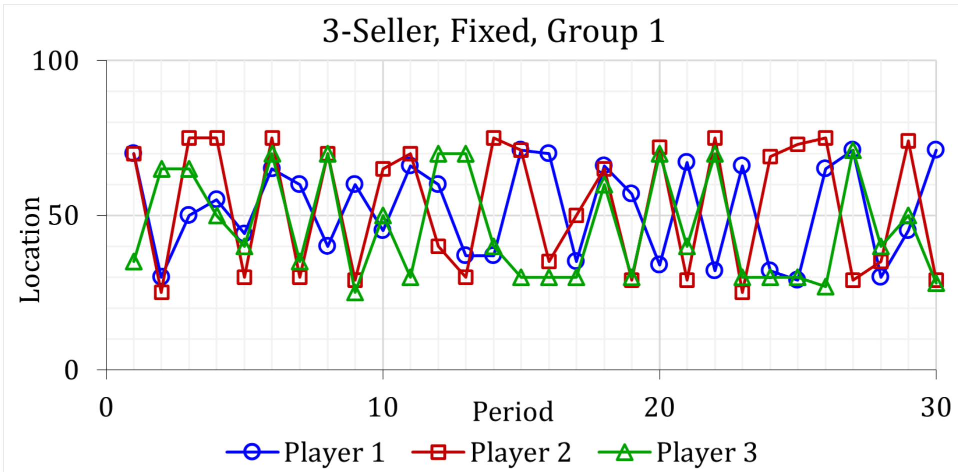

Figure 3.14: Locations by period, 3-player, fixed matching, Group 1.

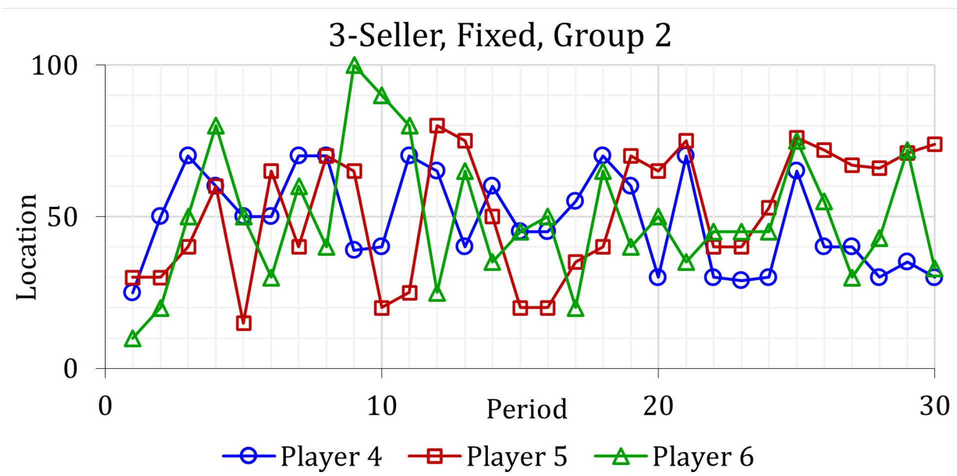

Figure 3.15: Locations by period, 3-player, fixed matching, Group 2.

to the other; see Figure 3.14 through Figure 3.18. Some sellers are more alternating, from one side to the other, while others change their location in smaller increments. No single player ever settled into a single location that was unchanging across periods.

Frequency distributions are shown for the three-player, fixed-matching groups in Figure 3.19. There is an obvious bimodal shape to the graphs with peaks at 30 and 70. A frequency distribution of the last 15 periods and the last 10 periods (Figure 3.20 and Figure 3.21, respectively) show stronger peaks than that of the full-period data. Comparing the last

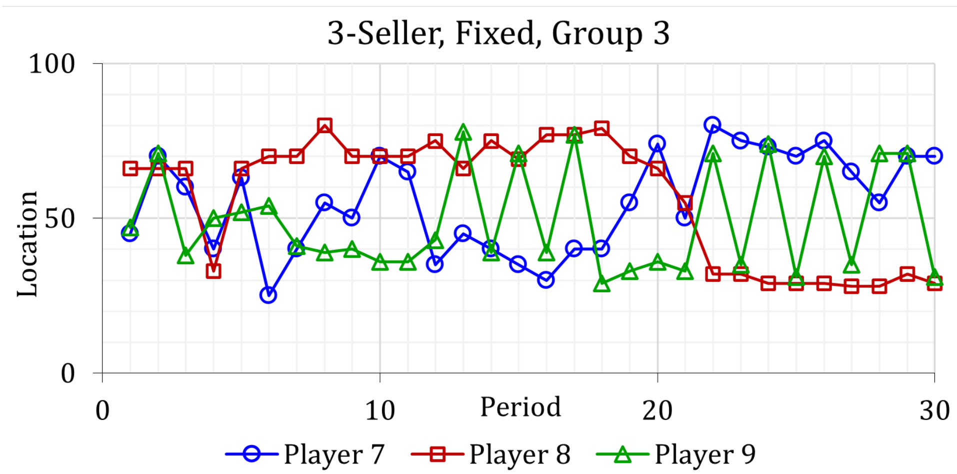

Figure 3.16: Locations by period, 3-player, fixed matching, Group 3.

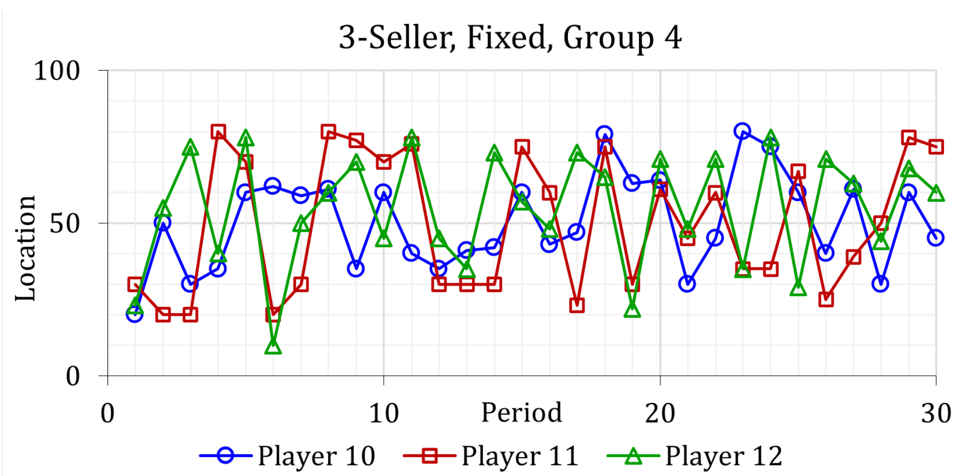

Figure 3.17: Locations by period, 3-player, fixed matching, Group 4.

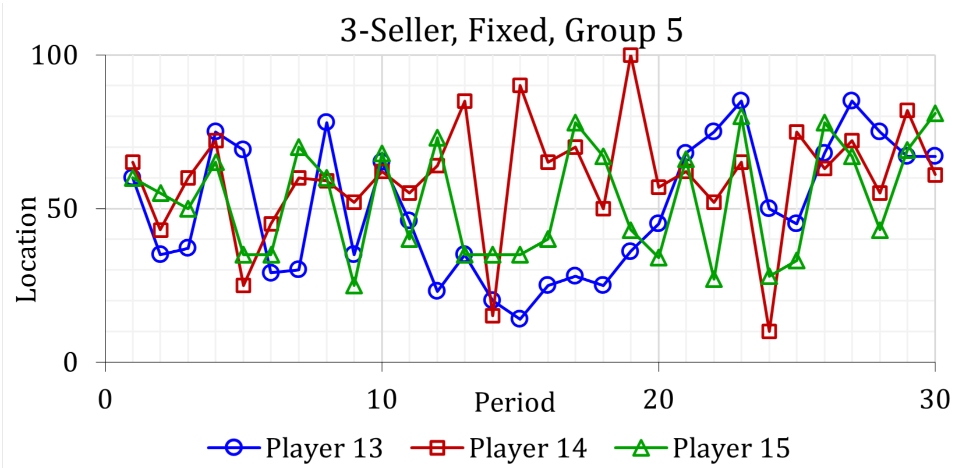

Figure 3.18: Locations by period, 3-player, fixed matching, Group 5.

periods only to the full-period data, it appears that sales have a higher variance toward the middle (especially between 35 and 65) and lower variance toward the ends.

The randomly-rematched, three-player data was less peaked than the fixed-matching data. It was more rectangular, more like the theoretical distribution. First, Figure 3.22 and Figure 3.23 show the locations of two rematched sessions by period. Figure 3.24 shows the rematched location frequency distribution, which has more distribution across the middle of the market and diminished peaks at 30 and 70.

The three-firm case has no pure-strategy equilibrium, but it does have a mixed-strategy equilibrium. The results show a significant difference in distribution from the theoretical mixed-strategy distribution. However, the support is a good predictor of the location of the player. That is, there were few location choices outside the mixed-strategy support.

For three firms, the theoretical expected payoff for a third firm, as a function of location choice, assuming the other two firms are playing the mixed-strategy equilibrium, is flat and maximized for the whole mixed-strategy support. Figure 3.25 shows the theoretical expected payoff of a third firm, as a function of location, assuming the other firms play the

Figure 3.19: 3-player, fixed location frequency distribution, all data.

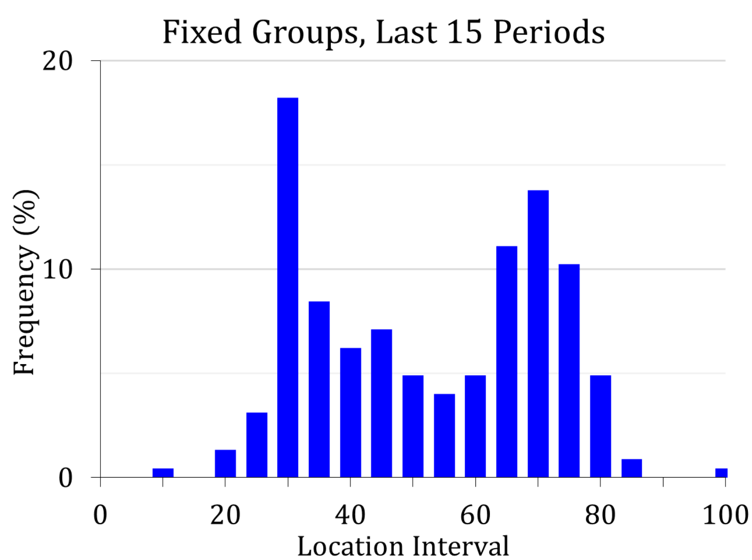

Figure 3.20: 3-player, fixed location frequency distribution, last 15 periods.

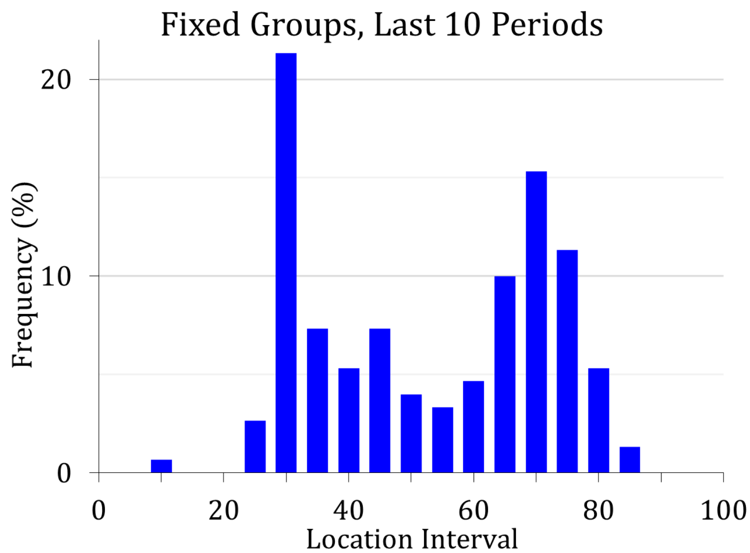

Figure 3.21: 3-player, fixed location frequency distribution, last 10 periods.

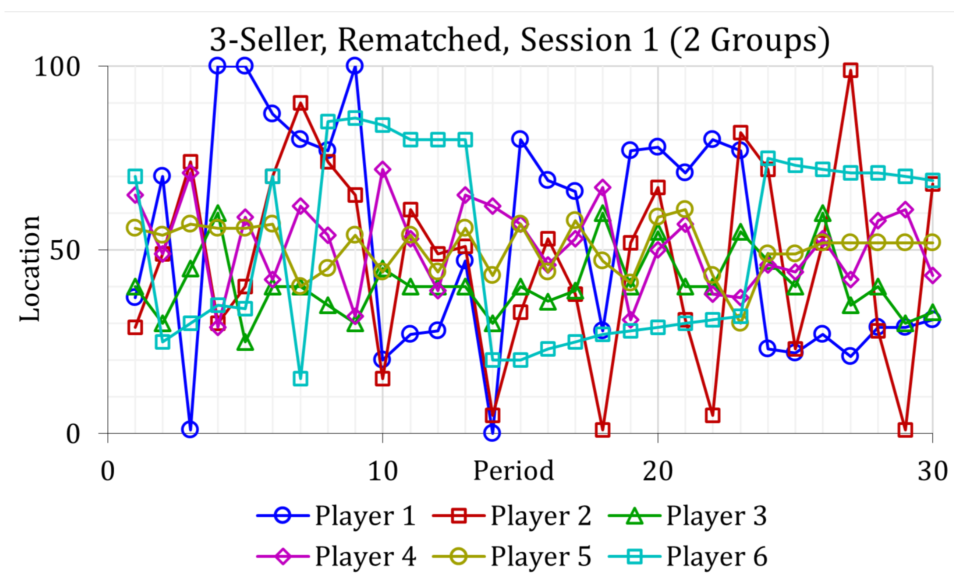

Figure 3.22: Locations by period, 3-player, randomly rematched, Session 1 (2 groups).

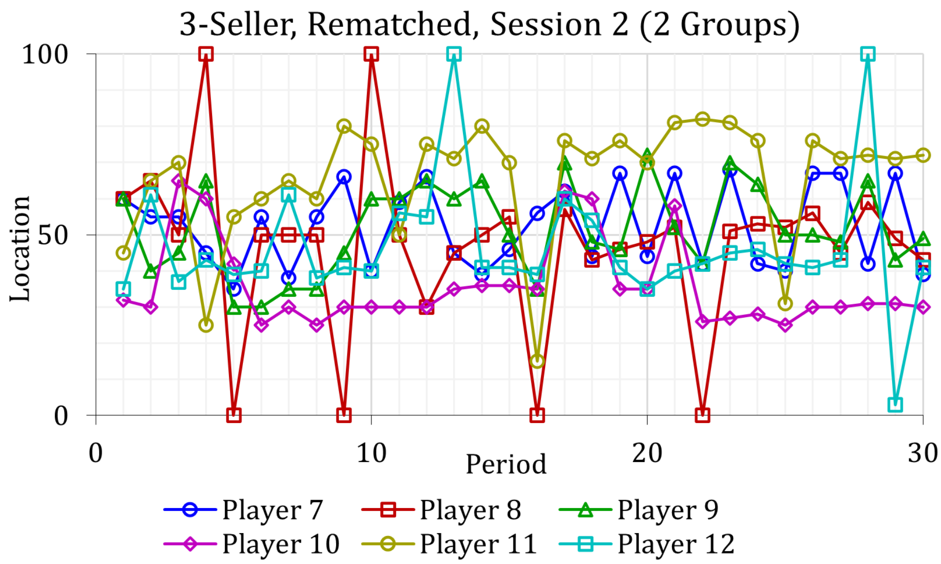

Figure 3.23: Locations by period, 3-player, randomly rematched, Session 2 (2 groups).

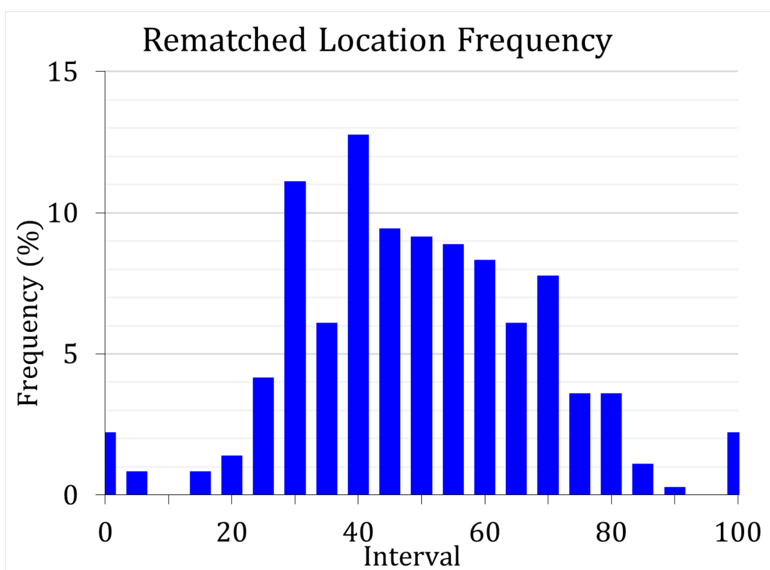

Figure 3.24: 3-player, randomly rematched location frequency distribution, all data.

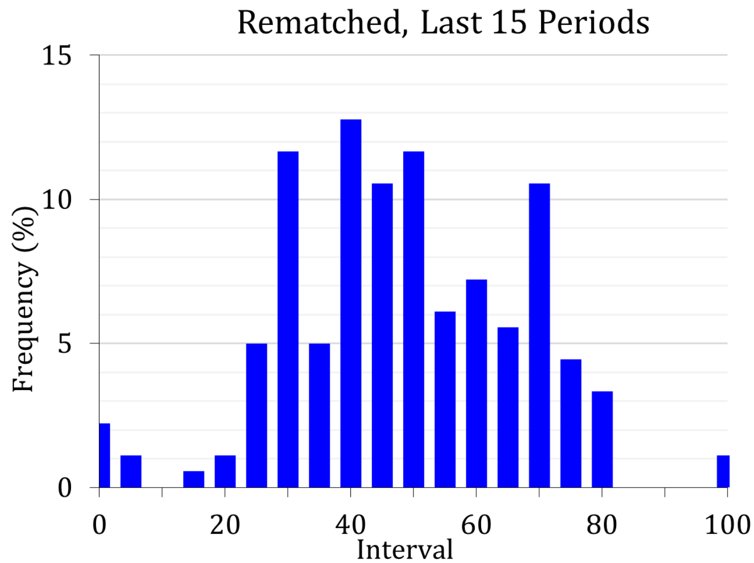

Figure 3.25: 3-player, randomly rematched location frequency distribution, last 15 periods.

mixed-strategy equilibrium. The payoff is flat and equal to 1/3 for the entire mixed-strategy support of [25, 75]. So a risk-neutral mixed-strategy player would randomize among equally appealing locations within this region, and one would observe a rectangular distribution of locations. However, Figure 3.25 also illustrates the variance of payoffs as a function of location. Players who locate more centrally will experience more variance. So a risk-averse player might avoid the center.

Figure 9 shows the actual average payoffs from the data, excluding the first 5 periods. When compared to the theoretical expected value, the payoffs are slightly peaked at 40 and 60. In the late data, there are peaks at 35 and 65, with a minipeak at 50. So the players who are willing to locate slightly more towards the center than others, and take on some more variance, are rewarded with slightly higher payoff.

The three-player data also show a clear trend to move out of the center over time. To measure this, I define centrality as the distance from the endpoints. Specifically, centrality equals location for $0 <$ location 50; 100-location otherwise. In a market space of length

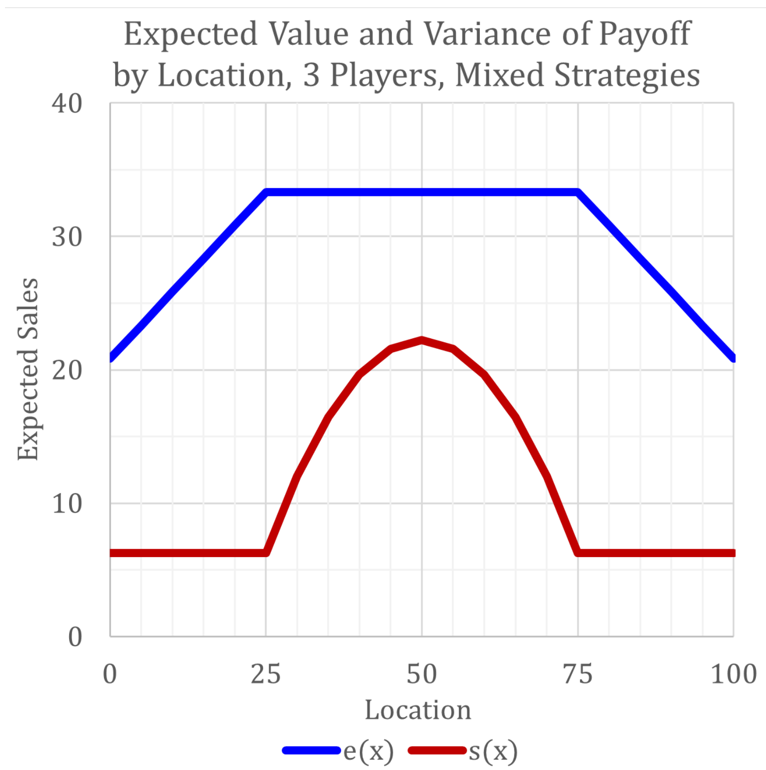

Figure 3.26: Expected value and variance of payoff by location, 3 players.

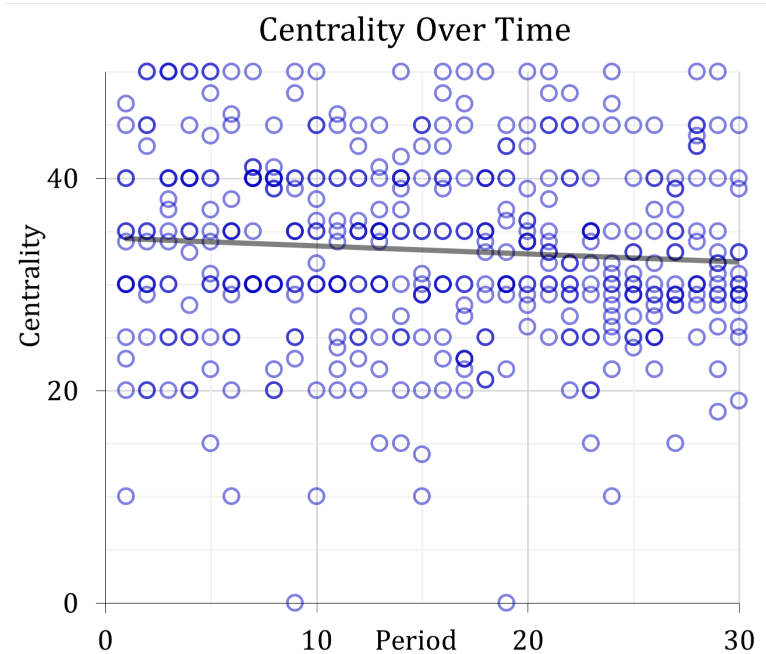

Figure 3.27: Centrality by time period, all 3-player fixed data.

100, the maximum value of centrality would be 50. For three firms, the average centrality was 33.23, the median centrality was 33 and the mode was 30, for all fixed-matching groups, all periods.

To observe a change in peak strength over time, I have graphed average distance from the median centrality by period in Figure 3.26. There is a downward trend in the distance from the mean centrality of 33. This supports the observation that the peaks are becoming more pronounced over time.

I used K-S one-sample tests to compare distributions to the theoretical prediction in Table 6, for $n = 3$. In all cases the full-period fixed-matching data were significantly different from theory. The late-period fixed-matching data were still significant, but not as strong, probably due to the smaller sample size. The rematched data were borderline significant.

Furthermore, the interquartile range and the standard deviations of the experimental results are higher than the theoretical values. For instance, in the three-firm fixed-matching data,

Table 3.4: Significance of differences in observed distributions from theory, 3 players.

Matching treatment	Number of observations	D, largest difference	P-value
Fixed, all periods	450	0.1511	0.00003
Fixed, last 10 periods	150	0.2000	0.00247
Rematched	300	0.0917	0.08024

the mixed-strategy equilibrium has quartiles at 37.5 and 62.5, yielding an interquartile range of 25, whereas the experimental results had quartiles at 30 and 65, yielding an interquartile range of 35. The standard deviation of the three-firm fixed-matching data is 18.81, versus 14.43 for the theoretical equilibrium. The values for these stats in a bimodal distribution would be larger than a uniform distribution. The interquartile range and standard deviations for all treatments are given in Table 3.5.

Table 3.5: Interquartile ranges and standard deviations for all treatments.

Matching treatment	Number of players	Interquartile range	Standard deviation
Fixed, all periods	2	0	10.22
Fixed, last 15 periods	2	0	3.62
Rematched	2	0	5.25
Theory (single peak)	2	0	0
Fixed, all periods	3	35	18.81
Fixed, last 10 periods	3	38	18.96
Rematched	3	26.25	19.59
Theory (uniform)	3	25	14.87
Fixed, all periods	4	42	23.02
Fixed, last 15 periods	4	44	23.03
Fixed, all periods	5	51	25.78
Fixed, last 15 periods	5	50	24.87
Rematched	5	50	25.36
Fixed	6	52.25	26.29

I also observed that players avoid locations that lie outside the mixed-strategy support. For three firms, more than 85 percent of observations were in the support segment, including over 91 percent of the data from the last 10 periods. The full-period fixed-matching data was significantly different, but the late-period data was not significantly different. Table 3.6 shows the significance of data outside the mixed-strategy support.

Table 3.6: Testing use of theoretical support segment.

Matching treatment	Number of players	Number of observations	% distribution outside support	P-value
Fixed, all periods	2	240	0.2425	0.00000
Fixed, last 15 periods	2	120	0.2000	0.00822
Rematched	2	300	0.1033	0.04071
Fixed, all periods	3	450	0.1422	0.00011
Fixed, last 10 periods	3	150	0.0867	0.32383
Rematched	3	300	0.1611	0.00041

3.4.3 The Four-Player Case

The four-firm groups showed a slightly wider dispersion and slightly more stable play across periods than the three-player groups. Individual players stayed on one side of the market for more consecutive periods, and individual players repeated previous period locations. The next five figures illustrate the play of the individual fixed group players.

The four-firm location distribution is shown for all periods in Figure 3.33. There is a more even distribution (less peaks) and more distribution at the center than in the three-firm groups, including a minor peak at 50, and more play around 20-30 and 70-80, and the main peaks themselves are slightly further out. The same graph with only the data from the last 15 periods shows greater peaks at 25, 50, and 75, with less distribution outside the quartiles, and slightly less distribution within quartiles; see Figure 3.34.

With four firms, there is a pure-strategy equilibrium and a mixed-strategy equilibrium. In the pure-strategy equilibrium, if the other three firms are playing 25 or 75, with two at one quartile and a third at the other, the fourth firm would expect a payoff of 25 for any location

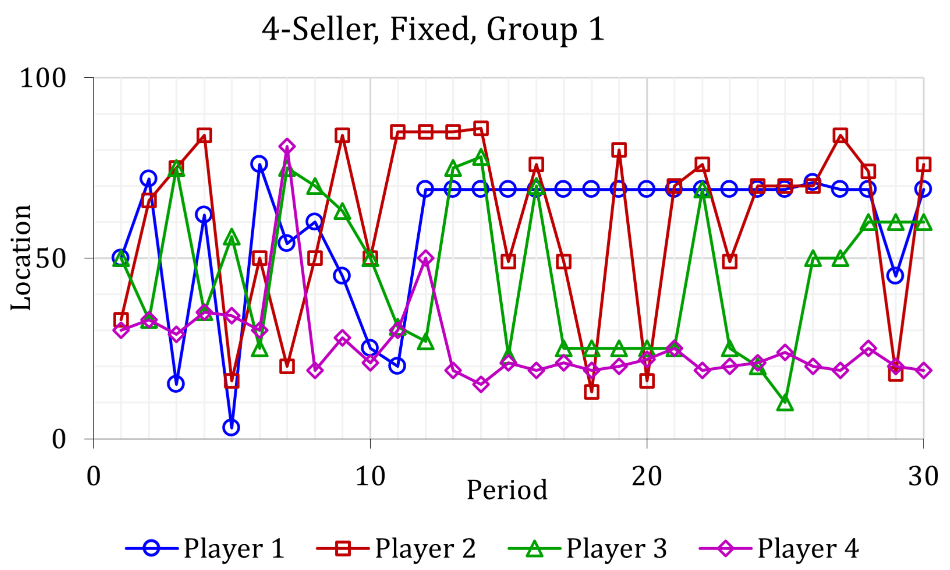

Figure 3.28: Locations by period, 4-player, fixed matching, Group 1.

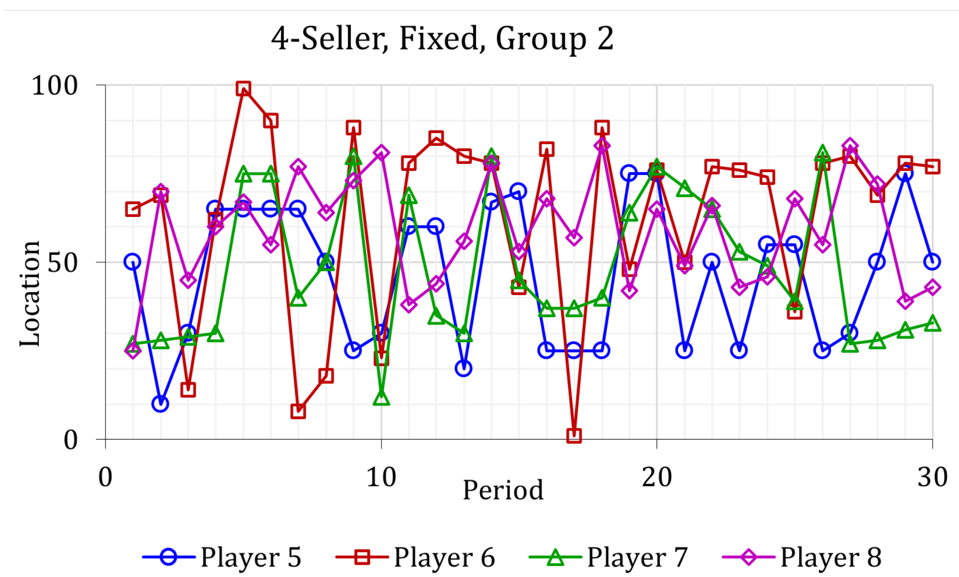

Figure 3.29: Locations by period, 4-player, fixed matching, Group 2.

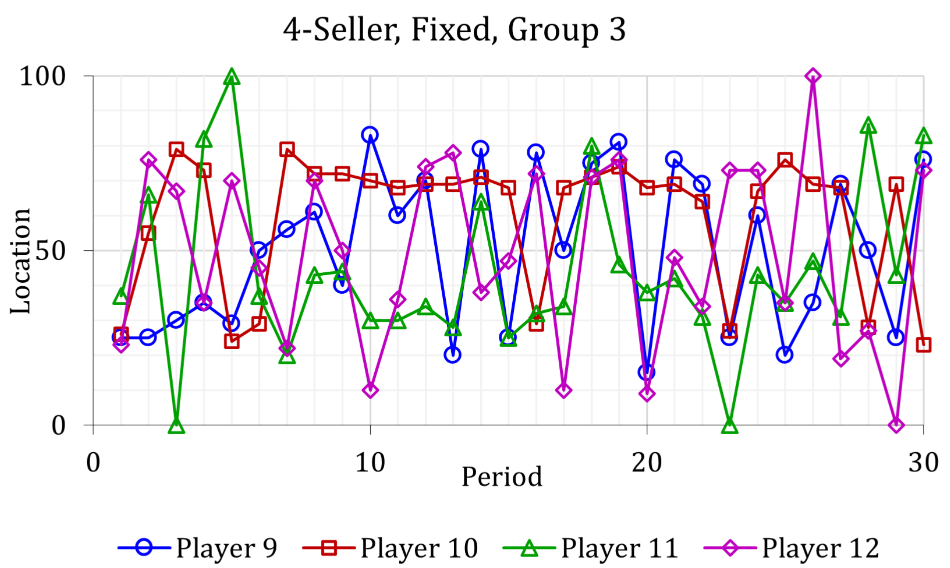

Figure 3.30: Locations by period, 4-player, fixed matching, Group 3.

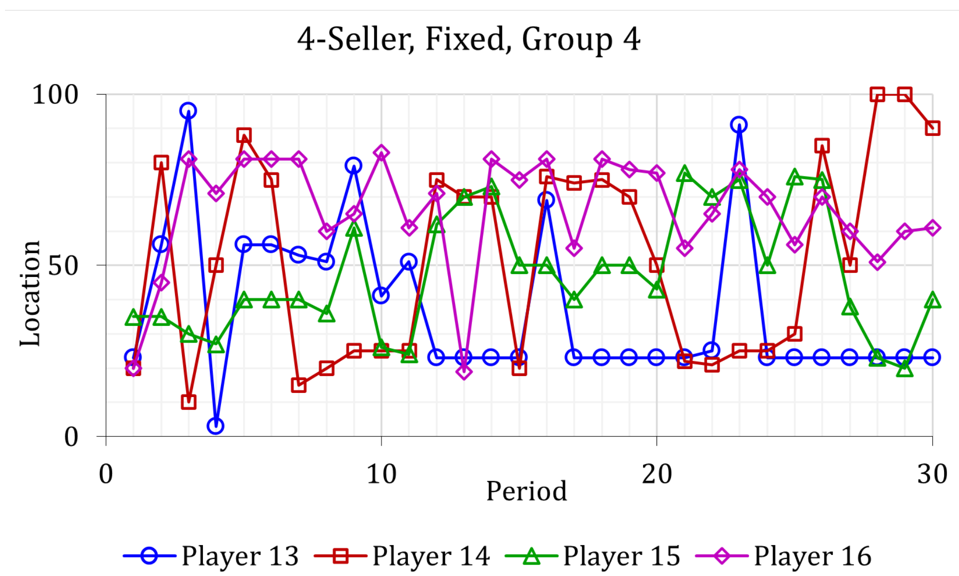

Figure 3.31: Locations by period, 4-player, fixed matching, Group 4.

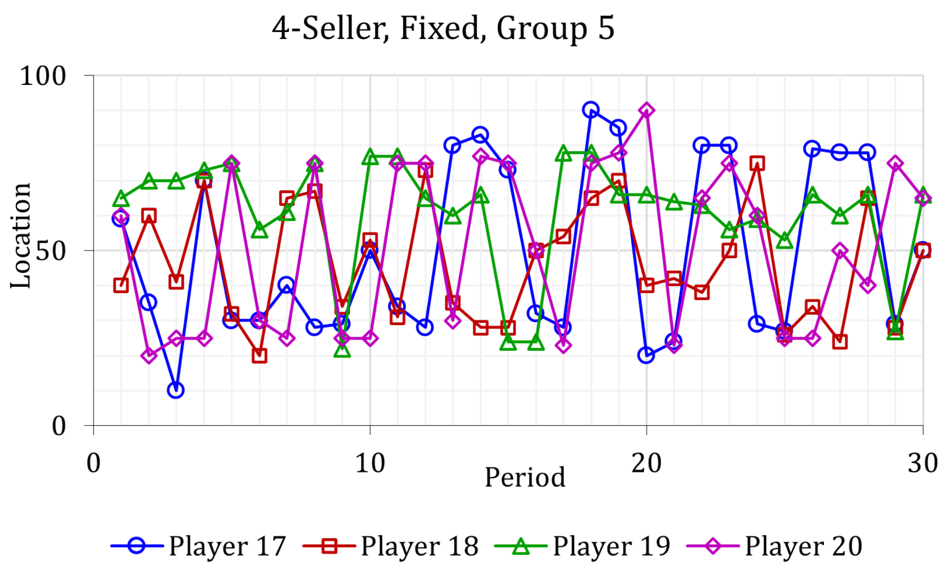

Figure 3.32: Locations by period, 4-player, fixed matching, Group 5.

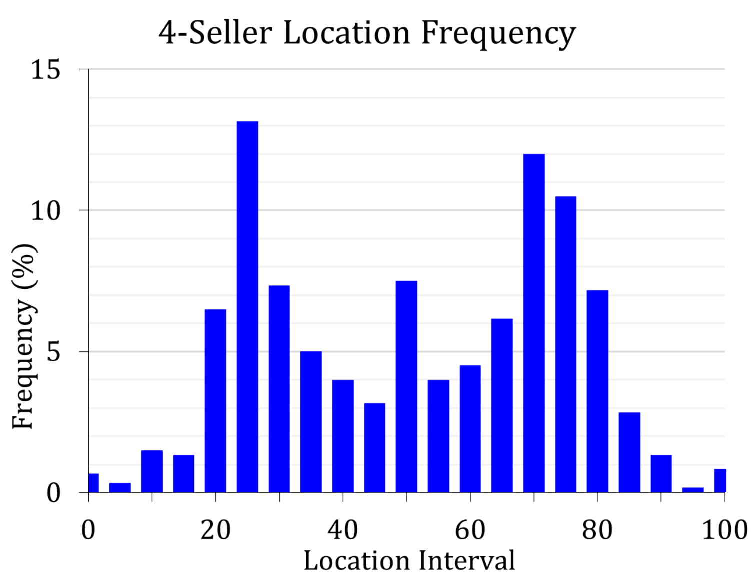

Figure 3.33: 4-player location frequency distribution, all fixed data.

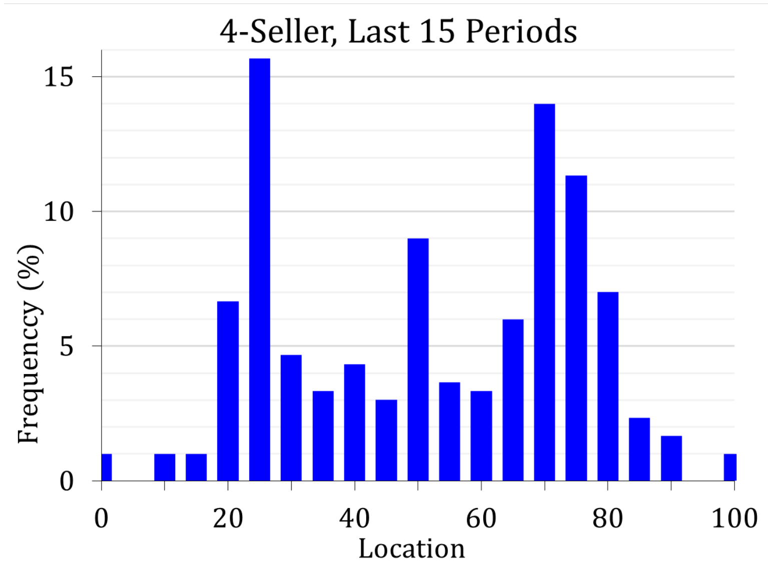

Figure 3.34: 4-player location frequency distribution, fixed, last 15 periods.

inside the interval [25, 75]. Payoffs would be strictly less outside this interval. Therefore, the expected payoff by location would be similar to the three-player case; however, there is a discontinuity in the case that the player chooses the quartile in which two other firms are located. Actual sales for all fixed data is shown in Figure 3.35.

Another important difference is that because there is no means of communication, the three other firms might all choose the same side of the market. In that case, the fourth firm should locate just to the inside of the three other firms. However, if the fourth firm does not know which side of the market the other three are stacked, a risk-neutral, myopic, best-responding player should randomize within the interval [25, 75].

Now considering mixed strategies, I know that the support is somewhat wider than [25, 75], but not wider than [1/6, 5/6]. If the other three firms are playing the mixed strategy, the expected payoff to the fourth firm is equal to 25 for all locations in that support.

The average payoff from the experimental data, excluding the first five periods, is illustrated in Figure 3.36. The locations with the highest payoffs were 30-45 and 55-70.

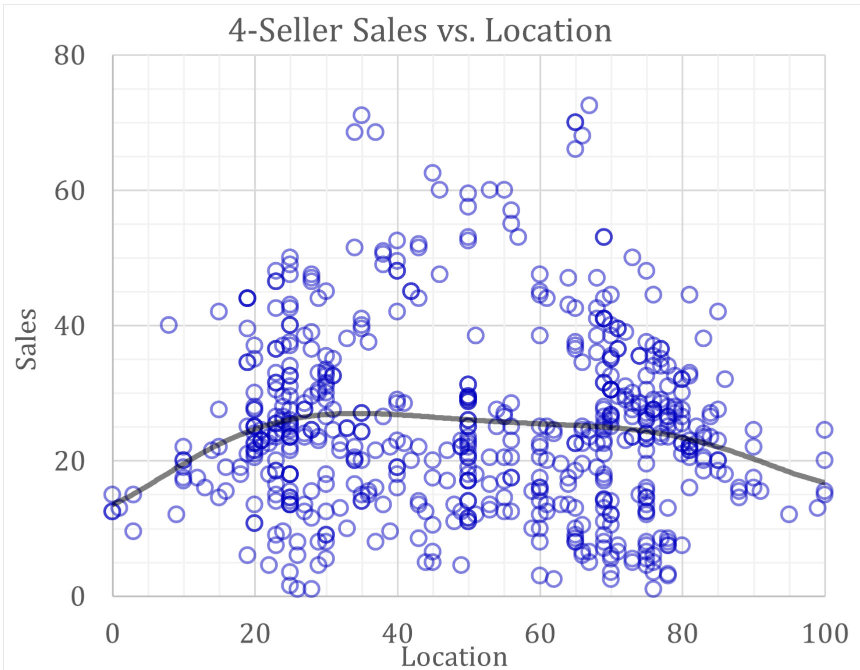

Figure 3.35: Sales by location, four firms, all fixed data.

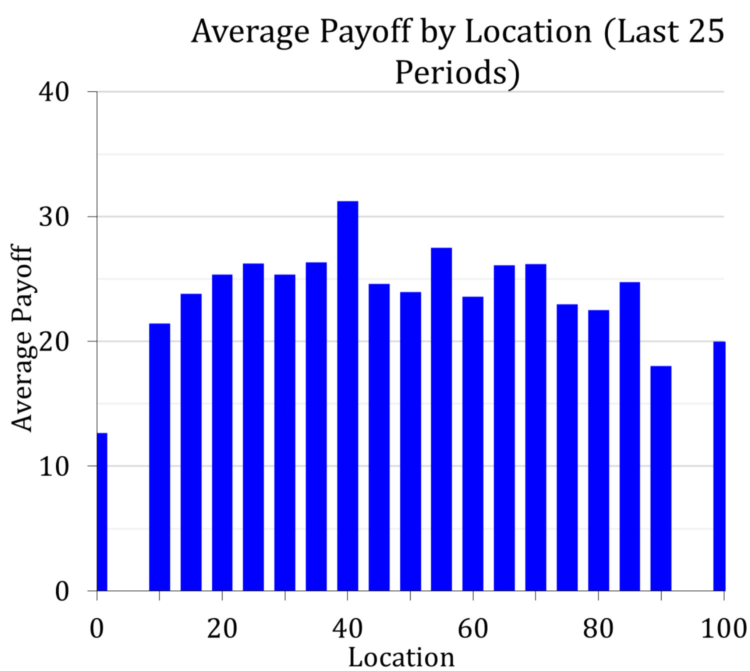

Figure 3.36: Average payoff by location, fixed groups, excluding first five periods.

For four firms, an increase in overall centrality would indicate a strengthening of the middle peak. Overall, there is no significant decrease in centrality over time. However, the incidence of players choosing location on the interval [47, 52] almost doubles from the first to last period. This is not picked up in overall centrality because the middle peak growth is counteracted by a moving of distribution from the near center toward the quartile peaks.

As for the quartile peaks, the average distance from these peaks is flat for the complete data; however, it decreases significantly when locations in the interval [47, 52] are excluded from the data.

For four firms, there are three observed peaks: 25, 50, and 75. The modal centrality for all periods was 25. The 25 and 75 can be explained as being a component of a pure-strategy Nash equilibrium. However, the smaller 50 peak cannot be explained by either a pure-strategy or mixed strategy Nash equilibrium.

A more thorough examination is the closeness to the pure-strategy equilibrium for the group as a whole. To measure group Nash equilibrium closeness, I measure the total distance of the two leftmost firms from 25 and the two rightmost firms from 75. This value decreases insignificantly over periods.

3.4.4 The Five-Player Case

The individual player location choices by period are shown by their group in Figure 3.37 through Figure 3.42.

The frequency distributions for five firms are given in Figure 3.43. The full-data graph includes all six fixed-matching groups, while the late-data graph includes the last 15 periods from just the four groups that were run for 30 periods (Figure 3.44). There is slightly less tendency to locate outside the mixed strategies support in the late data. Also, late play is

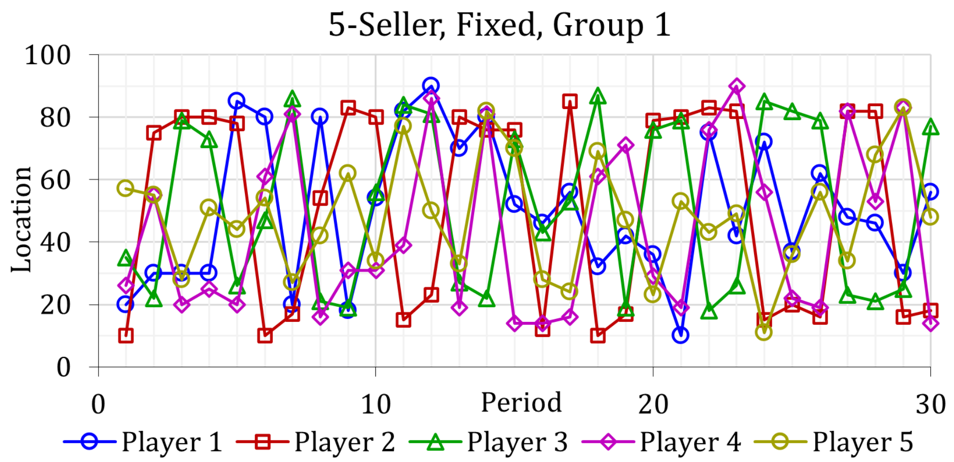

Figure 3.37: Locations by period, 5-player, fixed matching, Group 1.

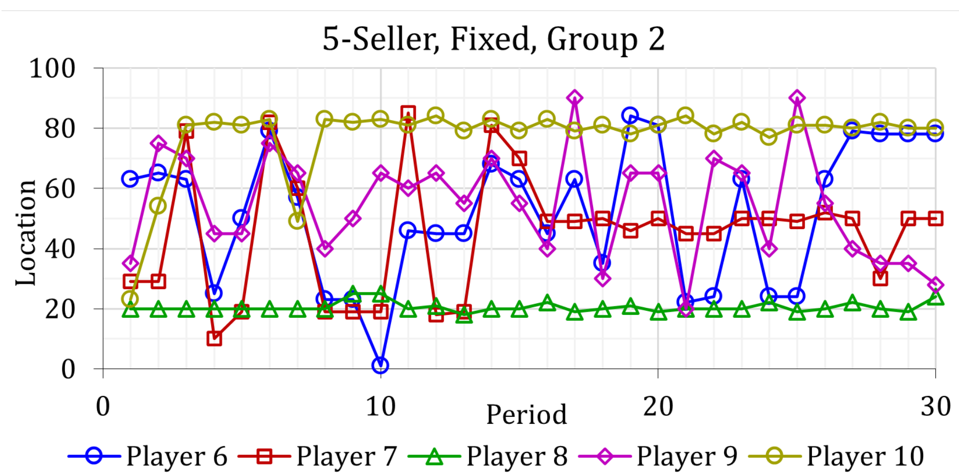

Figure 3.38: Locations by period, 5-player, fixed matching, Group 2.

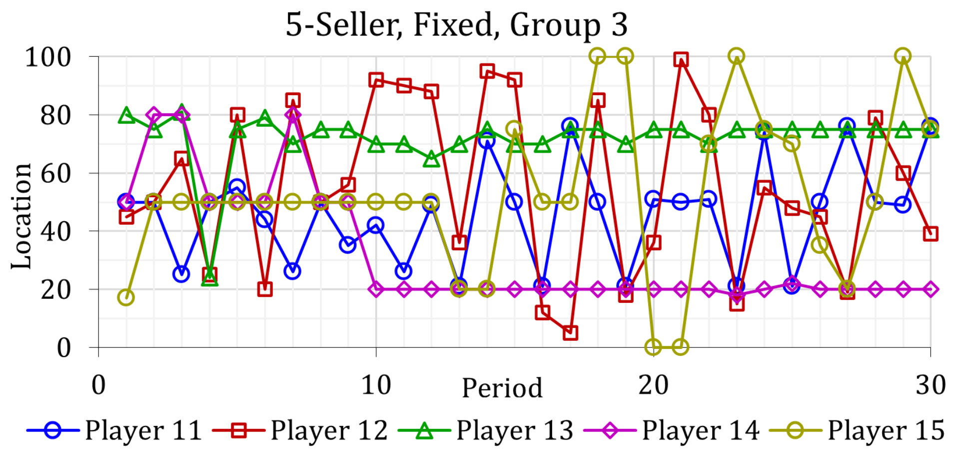

Figure 3.39: Locations by period, 5-player, fixed matching, Group 3.

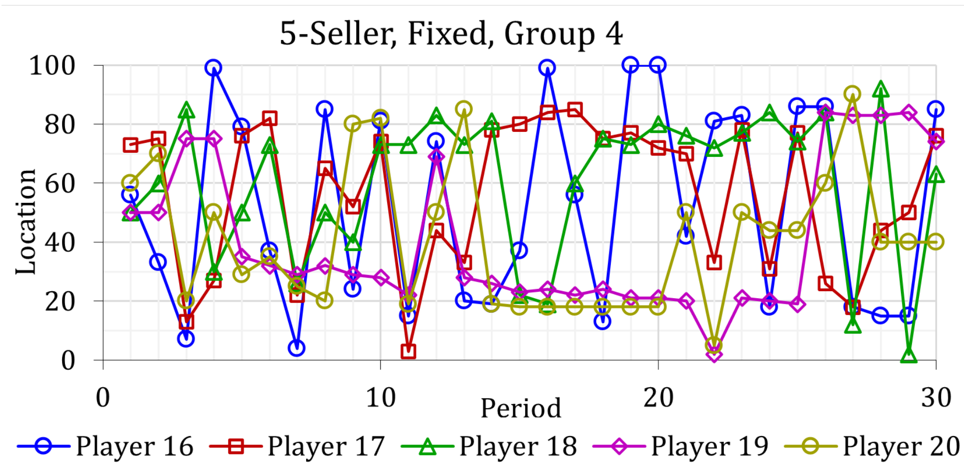

Figure 3.40: Locations by period, 5-player, fixed matching, Group 4.

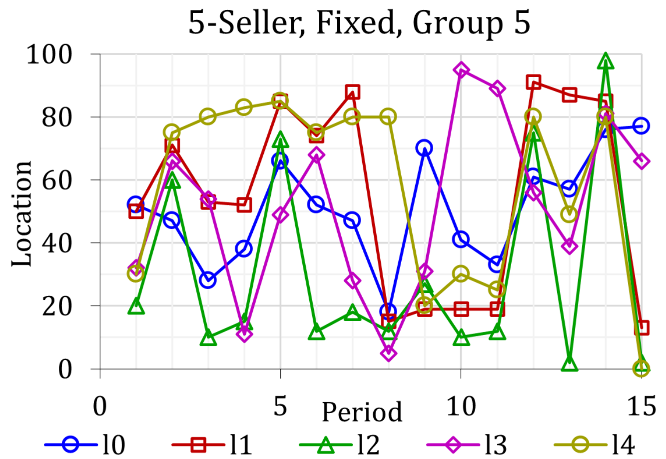

Figure 3.41: Locations by period, 5-player, fixed matching, Group 5.

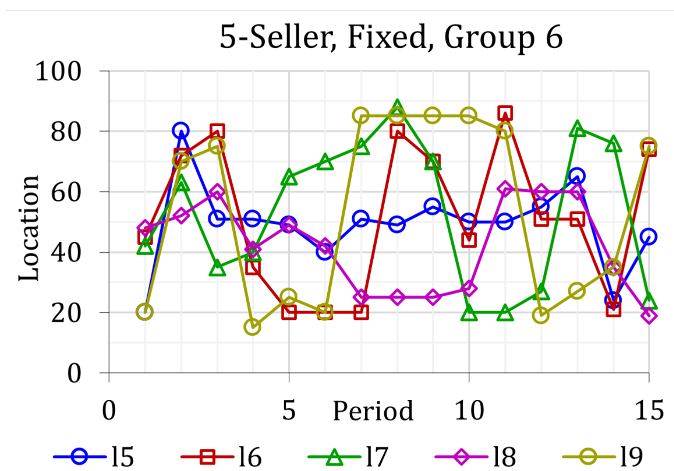

Figure 3.42: Locations by period, 5-player, fixed matching, Group 6.

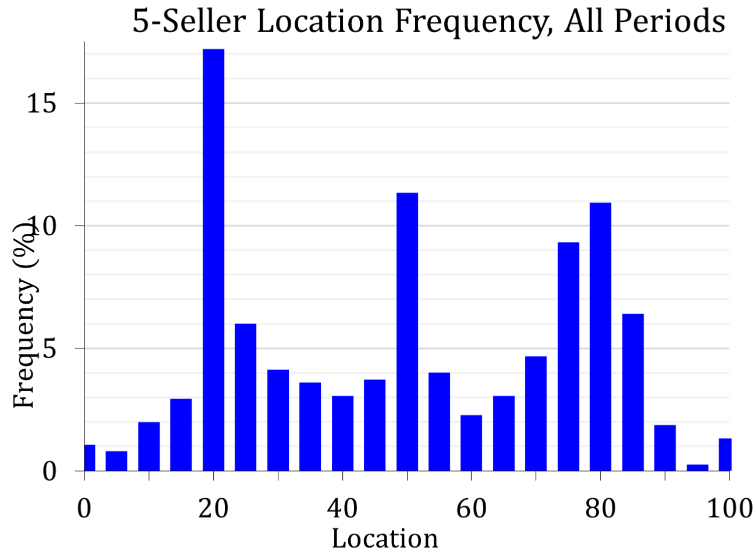

Figure 3.43: 5-player location frequency distribution, all fixed data.

slightly more rectangular. The randomly-rematched distribution, shown in Figure 3.45, is slightly more rectangular, with less pronounced, but still prevalent, peaks. Furthermore, the peaks are further out (15 and 85 as opposed to 20 and 80).

For the five-player case, there are three modes: 20, 50, and 80. The middle mode is almost as prominent as the other two nodes. In this case, the play of 50 is a component of the pure-strategy equilibrium. In fact, in this equilibrium, the single middle player gets double the payoff of the four paired end players at the two outside peaks. However, in out-of-equilibrium play, actual experimental payoffs are not significantly higher in the middle peak.

With five firms, in the theoretical pure-strategy equilibrium, the middle player gets twice the sales as the four end players. One might then expect location frequency to be more concentrated in the middle. If, however, the middle position is already occupied by two other firms and each end position is also occupied, the best response is to locate just outside of one of the endpoints. There are nine unique possible combinations using just the three equilibrium locations.

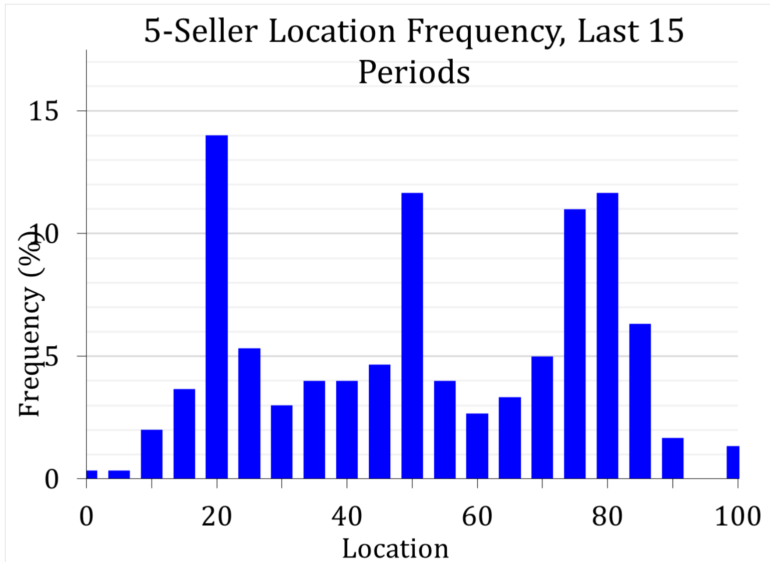

Figure 3.44: 5-player location frequency distribution, fixed, last 15 periods.

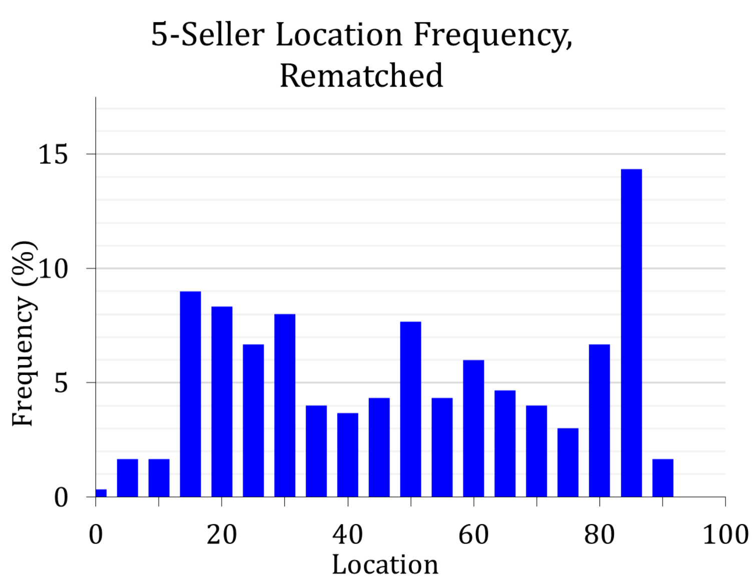

Figure 3.45: 5-player location frequency distribution, rematched data.

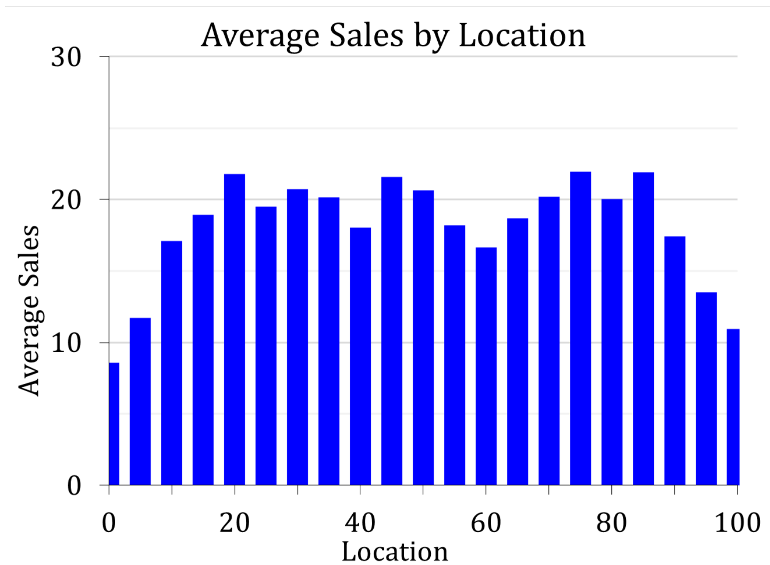

Figure 3.46: 5-player average payoff, fixed data, excluding first 5 periods.

Figure 3.46 shows the average sales by location from the experimental data. The location intervals of 20- 25 and 75-80, as well as 50, showed slightly greater than average payoffs.

Five-player centrality shows no trend over time. The mean centrality over all periods is 27.7. Center peak incidence decreases slightly over time (see Figure 3.47).

In terms of best-response dynamics, assuming the other four players are successfully coordinating and playing a pure strategy, there are two possibilities: either the middle position is open, or one of the end positions is open. In either case, the best response is the interval [1/6, 5/6].

If the other players are choosing components of the pure-strategy equilibrium, but not successfully coordinating, there are chances that three or more players will end up on one end location, or that two or more players will choose the middle position. In this case, it may be advised to avoid these locations. For the endpoints, it is preferred to cheat slightly inward rather than outward to the endpoints, for this yields higher average payoffs.

A summary of the overall observed location distribution and the average earnings by location

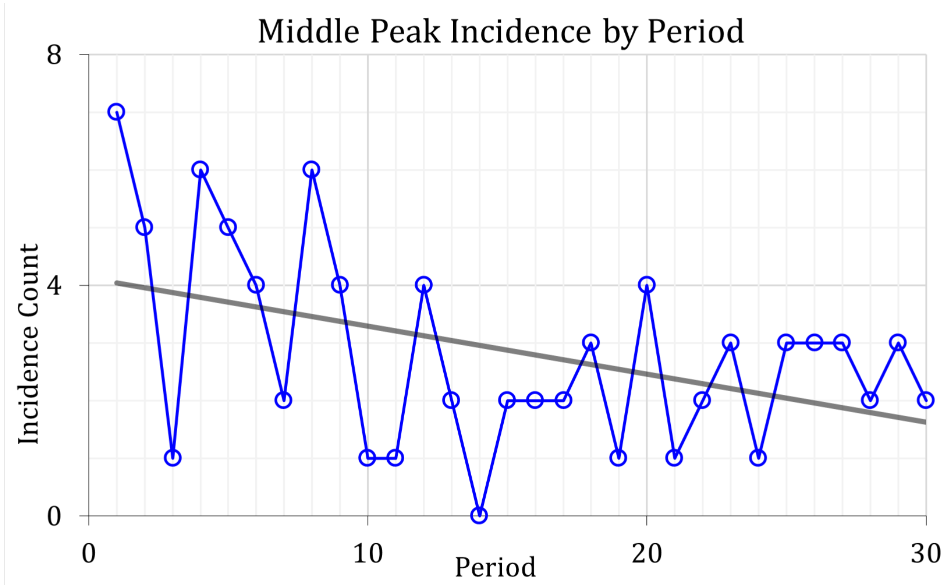

Figure 3.47: Middle peak incidence over time, 5-player, fixed data.

is given in Table 3.7.

3.4.5 The Six-Player Case

The six-player groups are shown in Figure 3.48 through Figure 3.50. There were three fixed groups; there were no rematched groups.

The six-player location frequency distribution is graphed in Figure 3.51. Peaks are further out, and the central peak is more pronounced–almost as strong as the bimodal peaks.

A notable result with six sellers is that locations of 45 and 55 have expected payoffs much higher than the average. Intervals 20-25 and 75-80 also have higher than average payoffs, but not as high as 45 and 55, which average over 20. Payoffs at 50 were below average. The graph of the average payoff by location, excluding the first five periods, is shown in Figure 3.52.

The individual sales by location, for all data, are shown in Figure 3.50.

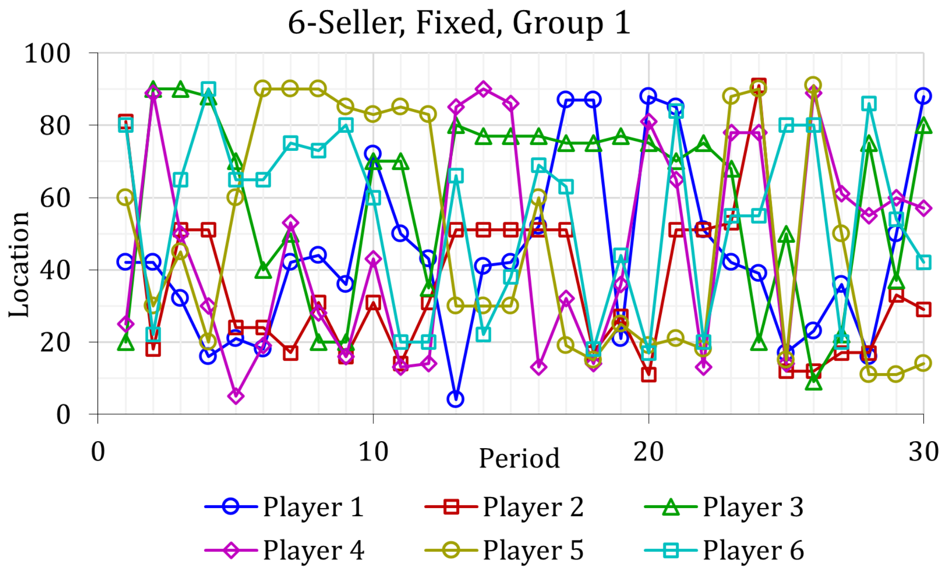

Figure 3.48: Locations by period, 6-player, fixed matching, Group 1.

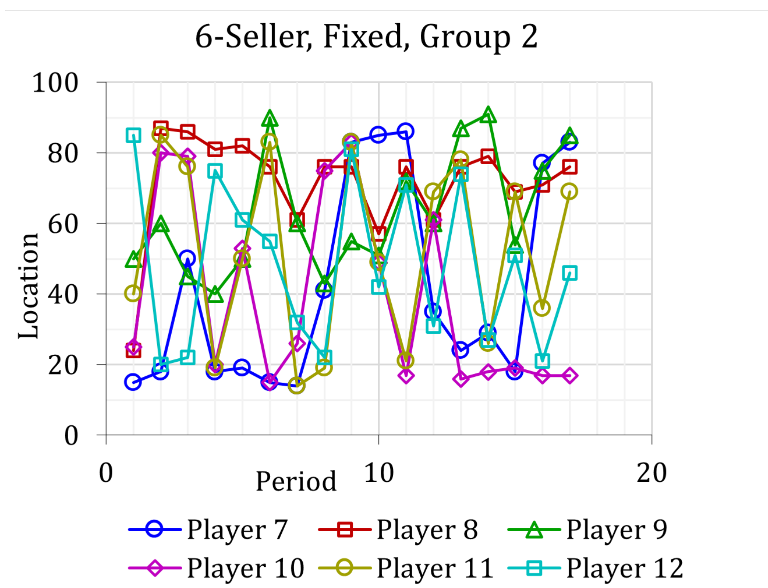

Figure 3.49: Locations by period, 6-player, fixed matching, Group 2.

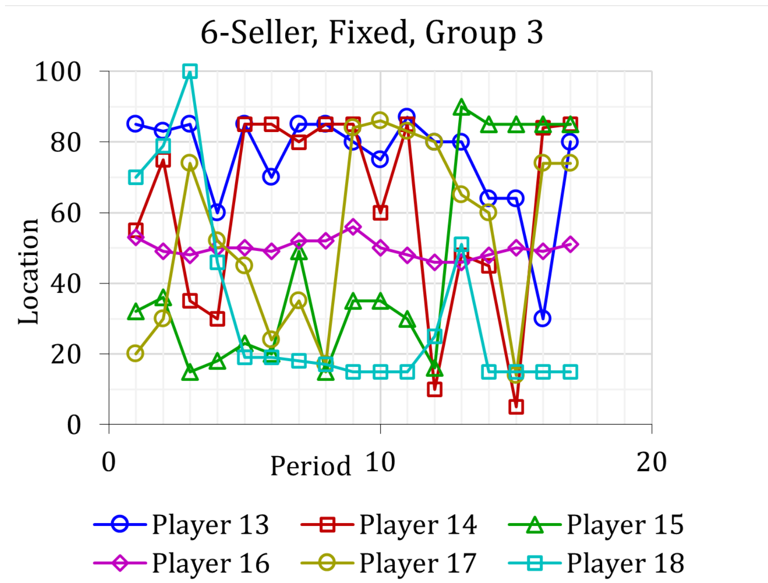

Figure 3.50: Locations by period, 6-player, fixed matching, Group 3.

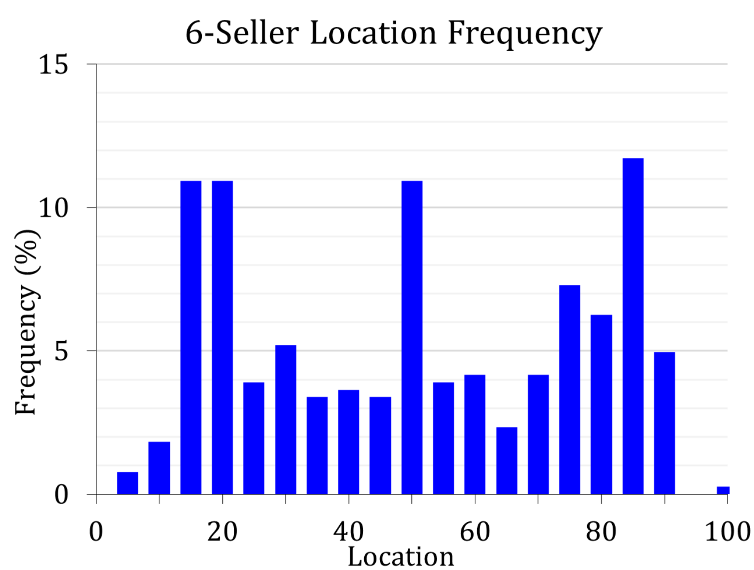

Figure 3.51: 6-player location frequency distribution, all fixed data.

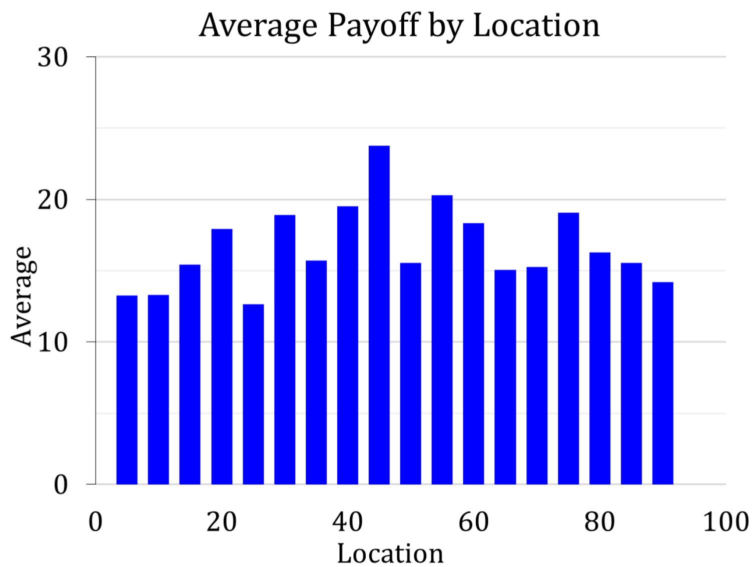

Figure 3.52: 6-player sales by location, all fixed data.

Centrality appears to be lower than the 5-firm case, and centrality decreases over time. A graph of average centrality is shown in Figure 3.51.

There was not sufficient long-period data to detect trends in centrality over time for six firms.

3.5 Overall Comparisons

In this section, I compare and contrast the results of the different group sizes.

The following characteristics support the hypothesis that the theoretical outcomes do not predict the experimental outcome. First, the shape of the distribution for $n = 3$ is a rectangular (uniform) within a support that is a subset of the total market interval. For $n = 4$, the distribution is U-shaped, and the distribution is also probably U-shaped for $n = 5$ and 6.

Table 3.7: Summary of observations regarding the distribution and average earnings by location.

Number of players	Overall observations	Average earnings by location
2	Single peak at 50.	Maximized at 50.
3	Bimodal at 30 and 70.	Above average in [25, 75], but lower than average at 50.
4	Two main peaks at 25 and 75; smaller peak at 50.	Above average in [30, 70], but lower than average at 50.
5	Three peaks: 20, 50, and 80; middle peak almost as strong as end peaks.	Above average in interval [15, 85], but lower than average for locations of 40 and 60.
6	Three even peaks: 15, 50, and 85.	Above average for locations 45 and 55; below average for locations of 50, <15, >85.

Within this support, there are definite observed peaks in the distribution of play in the experimental results. First, there are strong peaks at the endpoints of these intervals for three, four, five, and six firms. Second, there is a smaller peak in the center (50), which is strong for $n = 6$, moderate for $n = 5$, weak for $n = 4$, and nonexistent for $n = 3$.

In comparing these features in the full fixed-matching game data (all periods) with just the late fixed-matching game data (last 10 or 15 periods, 2-5 sellers only), in all cases, these features are stronger in the late-period data.

I also noted that a "stable" outcome was observed over time for $n = 2$, but not for $n > 2$. In this context, stability means that the players settle on a permanent location, and do not change their location for most periods. When this occurs, I conclude that the subjects are playing some kind of pure strategy. In the case of two firms, this coincides with the predicted pure-strategies Nash equilibrium. This could also be explained by focal points, but I believe that the profit-maximizing motive is the predominant force here.

In the three-firm case, no stable outcome was observed. That is, players are rarely choosing the same location for two consecutive periods. There is a little more consistency in four, five, and six firms, but I conclude from observing the data, that the players never settle into a stable location.

Finally, the observed distribution spread increase with n for $n \geq 2$, and the central peak increases with n for $n \geq 2$. Kolmogorov-Smirnov two-sample tests show differences in distributions between treatments in matrix form in Table 3.8. In the fixed-matching data, all groups are significantly different except 5 from 6. In the late-period data, all groups are different except 4 from 5. All of the rematched data are significant.

Table 3.8: Differences in distributions between groups of different sizes, full fixed data.

P-value	3	4	5	6
2	0.00000	0.00000	0.00000	0.00000
3		0.00000	0.00000	0.00000
4			0.00000	0.00008
5				0.07952

Table 3.9: Differences in distributions between groups of different sizes, late fixed data.

P-value	3	4	5
2	0.00000	0.00000	0.00000
3		0.00006	0.00001
4			0.03286

Table 3.10: Differences in distributions between groups of different sizes, randomly rematched data.

P-value	3	5
2	0.00000	0.00000
3		0.00004

Table 3.11 summarizes the time trends in the data.

Table 3.11: Summary of time trends by number of players.

Number of players	Peak strengthening	Pure-strategy NE trend
2	Begins strong and strengthens in the first few periods.	Maximized at 50.
3	Distance from mean peak centrality decreases over time.	No pure-strategy NE.
4	Middle peak incidence doubles over time, distance from quartile peak centrality decreases over time.	Insignificant decrease in distance from NE.
5	Three peaks: 20, 50, 80; middle peak almost as strong as end peaks.	Above average in interval [15, 85], but lower than average for locations of 40 and 60.
6	Three even peaks: 15, 50, and 85.	Above average for locations 45 and 55, below average for locations of 50, <15, >85.

3.5.1 Characterization of Individual Behavior

This section will be merged with the previous section. This section examines various individual behavior models. First, here are some regressions which I have already done:

First, I observe three characteristics in the data with regard to individual behavior. First, players tended to change their locations more when their previous period sales were low. For the three-player case, a simple linear regression of the absolute change in location as a function of previous period sales shows a coefficient of $-.2239$ and an R^2 of .0495. The change in location was especially high when a player received sales of 9 and under. Change in location averaged 29 in this range. Change in sales were lowest in the 60-69 range, averaging 13.

Interestingly, change in location was high for a few extremely high sales (70 and greater, only 5 instances). Perhaps they anticipate that their high sales will provoke movement of their rivals toward them.

Second, players tended to move out of the center when their previous period sales were much lower than average, move toward the center when sales were average, and not change their centrality when sales were very high.

Finally, I observed that players tended to move out of the center when their previous period position was the middle firm; players tended to move centrally when their position was an end firm. This observed tendency is consistent with a myopic best-response strategy. For $n = 3$, average increase in centrality was 3 for end firms and average decrease in centrality was 4 for central firms.

In the two-player case, I observe a convergence to the Nash equilibrium after the first few periods. For n greater than two, I observe a tendency for individual players to play a location in the equilibrium strategy sets, but also a tendency toward a strengthening of peaks at non-equilibrium focal points.

Another thing I want to explore is which multiperiod, dynamic individual strategies paid off, and which failed. To learn this, I have to look at the cumulative sales for all periods.

3.5.2 Interpretation of Observations

Intuitively, there is no barrier for two-player groups to achieve the pure-strategy equilibrium location of [50, 50]. They are not impaired by coordination problems because coordination is unnecessary.

The range of locations chosen increased with group size. The two-player groups are narrowly centered on or about 50 with a few outliers. There is an observable trend of increasing spread of locations from $n = 3$ to $n = 6$. Finally, from these graphs, there is no obvious difference between random-rematching and fixed-rematching treatments.

The lower variance of the randomly-rematched two-player groups suggests that the early variance by the fixed-matching groups may not be learning, but perhaps making some attempt to signal or communicate. If it were learning, one should have observed the same early-period variance in the rematched groups. The rematched groups have lower, less changing variance throughout all 30 periods, and a quicker and stronger convergence to 50. Running both random rematching and fixed-matching groups in the two-player case was useful to distinguish between learning and signaling.

There is convincing evidence that players are attempting a long-term coordination or communication strategy. The main evidence is that the early fixed-matching treatment is significantly more disperse in locations than the random-rematching treatment. The plausible explanation for this is an attempt at communication and coordination, not learning. If it were learning, one would expect to see identical early dispersion, regardless of rematching treatment.

Because this is a zero-sum game, any attempt to communicate and coordinate on a combination of equitable locations other than [50, 50] is purely for the public's welfare. For instance, the socially optimal location combination is [25, 75]—the outcome that minimizes the total consumer walking distance. These costs are not borne by the sellers, nor are there any reservation prices to worry about; every consumer always buys exactly one unit each period.

Another possibility is that the players are trying to lure the other player out of the equilibrium to take advantage of him. This is difficult—the costs are often in excess of the gains. If the rival does not follow the lead of a player that is deviating, the deviator will continue to lose market share.

Whether the motivation is for good of society or for self-interest, locating at a point other than 50 usually incurs a cost. Since this decision to deviate from the equilibrium has to be made unilaterally, it amounts to a soft money gift to the other player. It is then up to the other player to decide how to react to this gift.

From this model, coordination and communication is difficult due to the restrictions of the experimental treatment. However, long-term influence is at least feasible in a fixed matching treatment, and not possible in a random-rematching treatment.

With regard to a group of at least three sellers, there are two relative positions: a middle seller, or one of two end sellers. Moving to the center increases your profits given that both competitors are to one side of you; however, it also increases that chance that you will fall between two adjacent sellers, in which case you will be squeezed from either side. The expected payoff of an end player is high, but the payoff increases as one moves toward the center (assuming the player remains an end player). The expected payoff of a middle player is very low in the three-firm case. The probability of being a middle player decreases as one moves away from the middle.

Therefore, a risk-averse player might optimize between risk and return. In this regard, a location just inside the endpoints of the mixed-strategy support is indicated.

With regard to dynamics, if a player settles on a particular location for multiple periods, a second player can spot this and locate just outside him. However, this is more difficult to do with four or more firms because a third firm can also locate just outside of the crowder.

Multistage best-response: It may be enlightening to explore best-response dynamics beyond a single stage. In the four-player case, if a player assumes that the other three players choose one of the two pure-strategy choices (25 or 75), there are possible outcomes. There is 75 percent chance that two firms will be at one quarter, and the other firm will be at the other quarter. In this case, the best response is to locate anywhere in the interval (25, 75). All such locations are indifferent and give a payoff of 25. There is also a 25 percent chance that all three firms are stacked on one of the two quartiles. In this case, the best response is to locate to the inside of this stack. If the player could not identify the location of the stack, the player would be indifferent to any location in the interval (25, 75). So in conclusion, the interquartile interval seems to be attractive to players considering best-response dynamics.

However, to extend the analysis to multistage best-response dynamics, suppose a player considers the best response to three players randomizing in this interval. This is similar to the best response to the mixed-strategy equilibrium, but with a narrower interval. If a player assumes that the three other players are playing first-stage, pure-strategy-equilibrium best response, the second-stage best response is to play 25 or 75. The payoffs of playing the endpoints are 5/16. All locations inside the interval cannot beat that expected payoff.

3.6 Conclusions and Extensions

3.6.1 Conclusions

In this paper I characterize the results of simple location Hotelling models with location-only implications with regard to communication, risk aversion, and coordination.

Only for two firms do results match the theoretical equilibrium. There is clear evidence of an attempt to communicate for purposes of influencing the other seller. The decision not to choose the equilibrium is a unilateral decision and can be interpreted as a small cost of communication. It is like a soft money gift to the other player. To be rational, it must be part of a multiperiod strategy, and that is why you see it only in the fixed-matching treatment and not a random-rematching treatment.

The purpose of the communication attempt could be motivated in one of two ways: an appeal to fairness or social welfare improvement; or as self-interest, involving an attempt to lure the other player out of equilibrium and then take advantage. Both are very difficult to do in the two-player model with simultaneous play. If the motivation is self-interest, the cost of communication is often greater than the gains.

For three firms, results are different than the symmetric mixed-strategy equilibrium proposed by theory. Players tend to avoid riskier locations, resulting in a bimodal distribution.

Groups of four through six firms resemble outcomes similar in characteristic to the three-firm outcome, but again different than theoretical prediction. Play is weighted more heavily on the endpoints of the mixed-strategy support, indicating a possible risk aversion in the face of a low ability to coordinate.

The locations outside of the mixed-strategy support are avoided, consistent with theory. The distribution of locations increases with the number of sellers, which is consistent with a widening of the mixed strategy support.

However, unlike the rectangular distribution predicted by theory, the experimental results show different distributions. All group sizes show bimodal distributions with peaks near the endpoints of the mixed strategy supports. This indicates risk aversion. In addition, groups of four through six display a minipeak at 50, which is stronger as group size increases.

A challenge of this paper has been to interpret subjects' observable location decisions as unobservable strategies. Players choose individual locations in periods, but this could be a component of a mixed strategy. This issue has been explored in an experimental paper by Shachat and Walker (2004). In this paper, games were constructed to allow players to explicitly choose a mix, or combination of moves. However, I did not choose to allow this in this paper. I might consider this for future research.

The theoretical result of a pure-strategy equilibrium for $n > 3$ is unattainable in this experimental setting, with simultaneous play and discrete periods of payoffs. Assuming all firms wish to arrive at the Nash equilibrium, there is a coordination problem and uncertainty as to which particular firm should occupy which pure strategy slot.

3.6.2 Extensions

One extension that could solve the coordination problem is to introduce communication to the model. In the next chapter, I introduce one-way communication into this model.

The quantal response equilibria literature has been shown to be useful for data analysis in the presence of noise. For a future paper, I may reexplore the dataset using these techniques.

There are other models that could possibly approximate the pure-strategy Nash equilibrium for $n > 3$, without resorting to direct communication. For instance, a game that has dynamic, continuous payoffs, where players move around and are constantly earning payoffs according to their instantaneous locations could provide the coordination feedback necessary. I leave that for future work.

Chapter 4

Location Experiments with Communication

Abstract: *Previous experimental research designed to test location games yield results that differ from the predictions of theory. In a simple setup with four to six firms choosing locations on a unit line, subjects fail to reach the pure-strategy Nash equilibrium. One possible reason for the failure of theory is that subjects are unable to solve the attendant coordination problem. In the simultaneous-play environment there is no mechanism for them to decide which player will take each location specified in the equilibrium strategy set. In this paper I introduce one-way communication in order to facilitate the solution of the coordination problem. This design has the potential to yield a cleaner test of the theory by removing the confounding coordination difficulty. I run groups of two to four firms in a fixed-matching repeated game. In these experiments, one subject in each group is designated to be the communicator or leader. Prior to each round of play, the communicators send proposals to the other players, which include a complete location set for themselves and all other players in their groups. A leader who understands the strategic situation may specify locations to the other players that result in an equilibrium outcome. Alternatively, they may recommend coordinating on another equitable combination that promotes social welfare, such as one that minimizes transport costs for the buyers, although this is not a consideration that is explicitly mentioned in the instructions. Leaders may also submit an inequitable proposal if they think*

they enjoy a favorable position that they can exploit. They may also offer an inequitable proposal that is unfavorable to themselves in order to gain trust or to deceive.

Previous experiments suggest that this kind of "cheap talk" can facilitate equilibrium selection in coordination games. The results show some improvement in the ability of subjects to coordinate on Nash equilibrium play. However, there is considerable heterogeneity across groups. I conclude that one-way communication enhances the ability of subjects to coordinate, but that bounded rationality and poor leadership preclude the convergence to equilibrium.

4.1 Introduction

Experimental methods bring a new dimension to the study of location games. It also brings new issues of human behavior that are assumed away in the theoretical analysis. In the previous chapter, I showed the inability to coordinate in a simultaneous-play environment with no direct means of communication was a major factor in attaining equilibrium in a four-person location game. The motivation of this next chapter is to determine if communication may change the results. I explore a slightly more involved environment in which limited communication is allowed that might solve the coordination problem. Specifically, I introduce "cheap talk" (costless nonbinding communication) to the previous model.

In the basic location game, players are sellers choosing a location in a market space. Price is fixed, and the decision variable is location. In the most fundamental of location games, one could examine a unit line with a uniform distribution of customers who wish to purchase one unit of product from the nearest seller. Knowing this, the sellers will locate strategically to maximize market share.

In this basic model, theory indicates a unique, pure-strategy Nash equilibrium for the number of sellers, $n = 2$, 4, and 5; no pure-strategy equilibrium for n=3, and multiple pure-strategy Nash equilibria for $n > 5$. Furthermore, mixed strategy equilibria exist for $n > 2$.

However, the experimental results do not match theory. In the previous chapter, while two-player groups converged to the pure-strategy Nash equilibrium, the four-player groups did not. One factor that might have played a major role in preventing the firms from achieving the equilibrium is the inability to coordinate. Assuming that the sellers desired to reach the equilibrium, how do you determine which two should locate at .25 and which two go at .75? This problem is not addressed in theory. The coordination problem in simultaneous-play games is well known. Cooper, DeJong, Forsythe, & Ross (1989) illustrate the coordination problem in a simple battle-of-the-sexes experiment. Without communication, coordination is very difficult, if not impossible.

In our example, if all four firms randomize between .25 and .75, the chances of achieving two pairs are 3/8, while the chance of a 3-1 split are 50%. Furthermore, there is a 1/8 chance of all four winding up at the same point. If two pairs are achieved, there is no incentive to deviate, as this is the pure-strategy equilibrium. However, in a 3-1 situation, there is incentive for the lone firm to move toward the triplet. There is also incentive for the triplet members to deviate epsilon to the right or left of the quartile. When all four firms are at the same quartile, each one wants to move epsilon to the open end of the market.

Furthermore, while the pure-strategy play combination of (25, 25, 75, 75) is an equilibrium, the mixed-strategy play of all four firms playing [.5(25), .5(75)] is not. For the above reasons, if the other three are playing this mixed strategy, the fourth firm can earn a higher expected payoff by moving toward the center, at 26 or 74.

The goal of this chapter is to determine if the addition of cheap talk changes the outcome. The model in the previous chapter did not allow sellers to communicate between group members. The participants were isolated in cubicles, and many groups were playing anonymously. The game was simultaneous-move and multiperiod. Players could only take into account the prior period actions of other sellers. They may recognize that their location choices will be viewed by others and used to predict their future location decisions. So they may choose their location for purposes of signaling their future intentions.

However, when a formal communication structure is introduced, 1 allow some players to signal outside the context of their actual location decision. In this model, I allow one person per group to transmit suggested locations to all group members. I compare the results of the experiments with that of the noncommunication version, in terms of the ability to improve coordination on the equilibria and other focal points. I also analyze the individual subject behavior according to his information set. Among other variables, I consider how his location decisions are affected by the communicators' proposals and the previous period behavior of his group members.

Communication can significantly change the results of an experiment. For example, Brown-Kruse, Cronshaw, and Schenk (1993) conduct a simple prisoner's dilemma game in a location game context. Without communication, players choose the competitive equilibrium; with communication, they are able to observe the cooperative outcome.

The introduction of this communication brings a whole set of additional human behavioral issues into the game, such as fairness, credibility, cooperation, and the effect of "cheap talk." Experimental investigation has been designed to study these specific issues. For example, Duffy & Feltovich (1998) and Farrell and Rabin (1996) have explored the relative reliance of players on cheap talk vs. observation of previous-period play in various 2x2 simultaneous, multiperiod games, with varying results depending upon the game.[1]

In my model, the leaders' proposals themselves are judged by credibility by the group mem-

[1] For a survey of cheap talk, see Crawford (1998).

bers. The credibility of these proposals determines whether those proposals or prior period behavior is a better predictor of a subject's location decision. Wilson and Rhodes (1997) explore the effects of leadership behavior and the credibility of communication in coordination games involving one-way communication.

The introduction of one person per group as communicator places this person in a leadership role. There is now the opportunity for the leader to facilitate coordination, assuming this is what the group members want. The groups can suggest which two should be on the left side, and which two can be on the right. However, it also introduces the possibility of leaders taking advantage of their position and proposing inequitable locations.

Do the group members want to achieve equilibrium? In equilibrium, each player receives equal payoff. This is a constant-sum game, where a fixed market is divided among sellers. If a stable outcome is achieved, there is no variance in payoffs from period to period. A risk-averse seller might prefer this. However, a player might wish to avoid an equilibrium in order to "win," or to "beat the market."

What are characteristics that might describe a "good" leader? One characteristic is fairness, embodied in proposals that would result in equitable or near equitable sales for each seller. Another characteristic is credibility, where leaders actually choose the location they suggest for themselves. For example, Wang (1997) shows that a credibility test is a strong criterion to determine if cheap talk is taken seriously. I will call the third characteristic plausibility. Leaders have to appear as if they know what they are doing by making some proposals that make some sense.

A simple way to measure fairness is to take the absolute difference in the payoffs associated with the proposals. It might be acceptable for the leader to take a little more. It might actually be more important for the nonleaders to have equal payoffs among each other rather than for the nonleaders' pay to match the leader's pay.

Leaders who do not follow their own proposals lose credibility. This can be very damaging to their ability to control their subjects. The group may never recover.

In addition to being somewhat fair and credible, there must be some plausibility or apparent rationality to the proposals. Perhaps the proposals correspond to the Nash equilibrium. Or maybe they are at least locations where no one is tempted by the ability to greatly increase his payoffs with a small deviation from the proposals. Or perhaps there is one person who has such a temptation, but the leader has established his credibility, and this player is trustworthy.

In the two-player model, although it has not been explicitly mentioned in the instructions, there is a socially optimal location set of (25, 75) that minimizes the distance consumers have to travel to purchase. Leaders could consider this in their proposals. Otherwise, there is not much purpose to their communication. The other thing that a leader could try to do is leverage their leadership position by proposing a little more for themselves. However, unlike in an ultimatum game, defection is less costly.

When the communication environment is limited, sometimes it is hard for the nonleaders to have confidence in the leaders' proposals. If the leader can only express the what and not the why, the nonleaders may not have confidence that their designated group leader is making good choices, if they are seeking to benefit themselves, promoting equity, or social welfare.

In the three-player model, there is no equilibrium in pure strategies, but there are some proposal sets that are less tempting to deviate from than others. For example, compare the proposals (17, 50, 83) with (0, 67, 67) In the first proposal, it is tempting for the two end players to deviate in, but there is not much temptation for the middle firm to deviate.

”Good” leaders might assign themselves locations in which the temptation is high to deviate, but then not deviate, building credibility[2] and displaying proposals with high plausibility. However, in the first proposal, the leader cannot simultaneously be both end players. Thus proposal 1 has low plausibility. The nonleader end player(s) will be tempted to crowd the middle player.

However, the second proposal has high plausibility. If the leader suggests 0 and actually chooses 0, they builds high credibility. Furthermore, there is no incentive for the players assigned 67 to deviate.

There also might be an element of respect. Nonleaders might respect the leader’s actual or perceived strategic advantage by conceding to him proposals that are slightly favorable to the leader. For example, the leader might suggest (10, 70, 70).

An alternative would be that the leader suggests (0, 67, 67), but then actually chooses 5 or 10. Nonleaders may concede small deviations from proposals, but more credibility is maintained if leaders state their intentions to take a little extra initially at the proposal stage.

To characterize the behavior of the nonleaders, one can characterize the absolute location, or one can talk about the decision whether or not to deviate from the proposals, at what magnitude, and from what motivation. One might classify the nonleaders into several types. They might be totally confused and choosing randomly. They might be myopic best responders to the proposals, or they might be best-responding to the previous period locations.

Deviation is defined as selectin a location other than the proposal. For a leader, deviation is a deception or attempted deception. For a nonleader, deviation is a noncompliance or a rejection. A leapfrog is a deviation that would change the suggested ordering of sellers.

[2]Leaders with credibility may also place nonleaders in vulnerable positions, which may be exploited in future periods.

Crowding is deviation that respects the suggested ordering. Best-response crowding is a particular crowding solution which is also the undercutting solution For example, if the proposals are (20, 30, 50), the leftmost player choosing 22 is crowding. The leftmost player choosing 29 is BR-crowding. The leftmost player choosing 55 is leapfrogging, and choosing 51 is BR-leapfrogging.

The rest of the chapter is organized as follows: In the Section 2, I describe the experimental design and procedure used. In Section 3, I describe the results and compare them to the noncommunication case. In Section 4, I analyze the behavior of individual players based on location strategy and actions as a function of play history and leader proposals. Finally, Section 5 is a conclusion.

4.2 Experimental Design and Procedure

4.2.1 Design

Subjects participated in a modified location game similar to the one described in the theoretical section in the previous chapter. Location is the decision variable for the subjects (price is fixed). The market space is described as a road of 100 blocks, with one customer per block. The sellers may choose locations at the intersection of each block; that is, the location must be a whole number from 0 to 100. Automated customers purchase one unit per period from the nearest seller. In the case of two or more firms locating at the same point, customers are divided evenly across such sellers. This part of the design is identical to the one in the previous chapter.

However, now each group has a "leader," who transmits suggested locations to each member of his/her group. These proposals are nonbinding, one-way communication. The leaders and the nonleaders are all free to choose any location. That is, they can completely ignore the proposals if they choose. Each member can see all of the proposals the leader made by ID number. The nonleaders do not know which ID number the leader is. Complete instructions are given in Appendix A.

The game is repeated, with the number of periods varying from 10-24, depending upon the session. Players are not informed of how many total periods are to be played. In fixed-matching sessions, players are matched with the same individuals for the entire session (not rematched). A limited number of randomly-rematched sessions were also conducted, for 2-player groups only.

The experiment is done with three group sizes: two, three, and four players. A list of treatments and sessions conducted is shown in Table 4.1.

Table 4.1: List of treatments and number of sessions and observations.

Number of players per group	Matching treatment	Number of sessions	Number of groups	Number of periods	Total subjects	Total observations
2	fixed	2	9	20/23	18	384
2	rematched	1	5	20	10	200
3	fixed	4	14	10/16/20	42	672
4	fixed	4	9	16/20/24	36	704

4.2.2 Procedure

The experiments were conducted using a computerized decision environment, which allowed automation of the calculation and tabulation of results, randomization of player groupings, and the maintenance of anonymity. The program was written in Java programming language, and runs through a web server in a Java applet. Subjects at individual computers access the program through a web browser.

121 subjects were recruited from undergraduate classes at Virginia Tech. Each sat at an individual computer screen in a computer lab. A session consisted of 8-12 subjects matched into groups of the appropriate size. Subjects were not told with whom they were matched. The subjects interacted with each other through the program, which simulated a hypothetical market environment as determined by the experimental design. The subjects were motivated to play well with cash payment that depended upon their choices in the market, and which they received at the conclusion of the session.

Students were paid a $3 show-up fee, plus their earnings in the experiment, which averaged $8, for a total of $11 average earnings. As shown in Table 4.2, the pay rate per unit sold was adjusted according to number of players in each group in order to equalize earnings across treatments. (If the price were the same across sessions, as the larger groups split the pie among more players, their expected payoff per period would be lower.)

Table 4.2: List of treatments and number of sessions and observations.

Number of sellers per group	Earnings rate (cents per unit sold)
2	0.67
3	1.00
4	1.33

Players were paid 20 percent more per period for the communication design than in the previous chapter because each period is slower due to the additional step of communication, and there were fewer periods of play per session in the communication treatments.

4.2.3 Timeline

The computers were initialized before the subjects were seated. Instructions were read aloud in an adjacent classroom before the subjects were seated in the computer lab. The assignment of leaders was random. First, the leaders determine their proposals; they can use scrap paper to make calculations and/or diagrams. The leaders first write down those proposals on their log sheets, then types them into the program and sends them to the other group members, at which point the other group members observe the proposals and write them down on their log sheets. The students are then presented a line figure depicting the market space. Students enter their location in a box, which is indicated on the line by a dot. A sample location choice screen appears as a figure in Appendix B.

After all players enter their choices, the program calculates all sales. The program also records all locations, sales, and accumulated earnings. Each subject observes locations, sales, and accumulated earnings of all players in his group in a table. On the computer screen, the locations of players and market areas are indicated by dots and colored bars, respectively. The subjects are also required to record their decisions and earnings on a log sheet. This was done to encourage the subjects to view all the information and to maintain a uniform pace to the session. After all of the desired periods are run, the subjects are dismissed and paid individually.

4.3 Results

4.3.1 The Two-Player Case

Nine groups of two sellers were conducted in two sessions: a five-group session and a four-group session. The five-group session was 20 periods, and the four-group session was 23 periods, for a total of 384 individual observations. In the following several paragraphs, I will show the individual proposal and locations of each group with some period-by-period narration and commentary of what I observe.

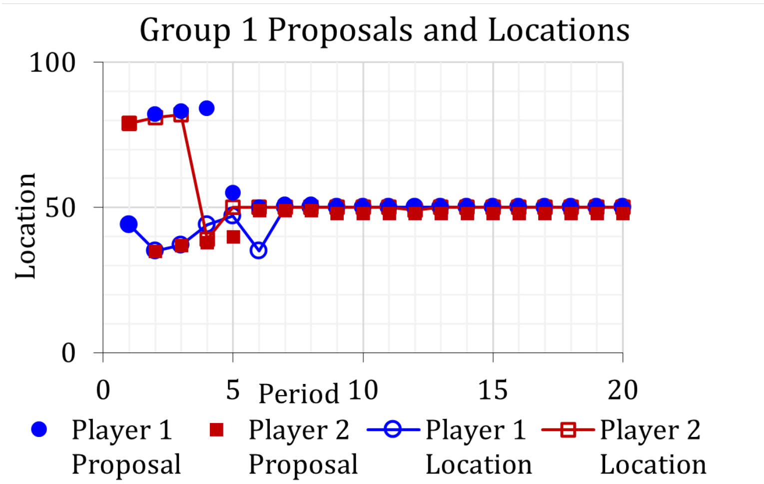

Figure 4.1: Proposals and locations, 2-player, fixed matching, Group 1.

The first group's proposals and locations by period are shown in Figure 4.1. In the first period, the leader proposed a slightly inequitable location pair of (44, 79), and there was complete compliance—both the leader and the nonleader chose the proposed locations. Both players observed their payoffs, including 61.5 for the leader and 38.5 for the nonleader.

In the second period, the leader switches sides and proposes (82, 35). Then both leader and nonleader chose the opposite side of the market from their proposal. Furthermore, the nonleader made an undercut attempt by choosing 81, but it did not payoff because the leader did not locate according to the proposal. In the third period, there was another double swap and failed undercut attempt. In the fourth round, the nonleader tried to undercut relative to the leader's anticipated swapped location as opposed to the proposed location. However, the seller took a more centralized location.

At this point, it appears that the nonleader does not regard the leader's proposals as credible and plays a location of 50 for the rest of the session except one deviation in Period 12. The leader continues to try to coax the nonleader by offering (51, 49), or (50, 48); otherwise the proposal is meaningless. The nonleader may have regarded any inequitable proposals with suspicion, although it did coax the nonleader to play 49 in Period 12.

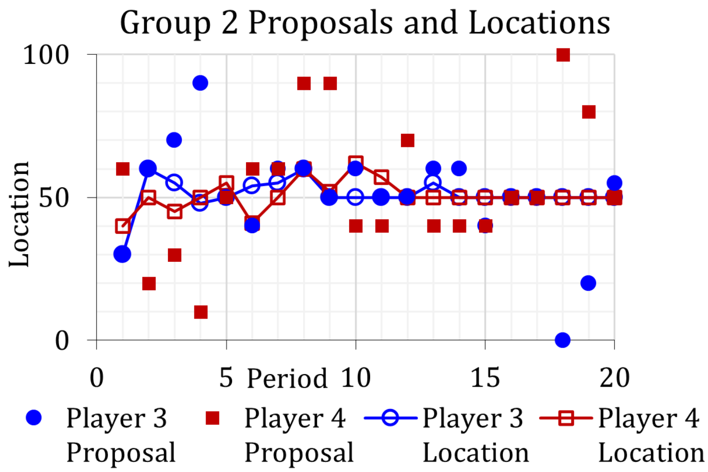

Figure 4.2: Proposals and locations, 2-player, fixed matching, Group 2.

The second group's proposals and locations by period are shown in Figure 4.2. In this group, the nonleader was noncompliant to the leader's proposal (30, 60) even though it was generous to the nonleader. In the second period, the leader proposed a slightly inequitable proposal (60, 20), and the nonleader chose 50. In the third round, the leader proposed an equitable pair (70, 30), but the leader defected for the first time.

After that, the leader had little credibility, and it seemed that the nonleader was not interested in complying anyway, and both players played 50 from Period 14 onwards. The leader never gave up proposing plausible and equitable or near-equitable pairs, but this signal had no bearing on the play.

The third group's proposals and locations by period are shown in Figure 4.3. In the first period, the leader proposed a generous pair (25, 70), and the nonleader played 50. The leader actually backed away further by playing 10. Either the leader was trying to develop trust, or the leader was in pre-learning status. The rest of Group 3 is very dynamic, and it is not straightforward to interpret the strategy by just observing the proposals and locations. Although location choice variation narrowed, Group 3 was the only group that did not completely converge to the NE in the last few periods.

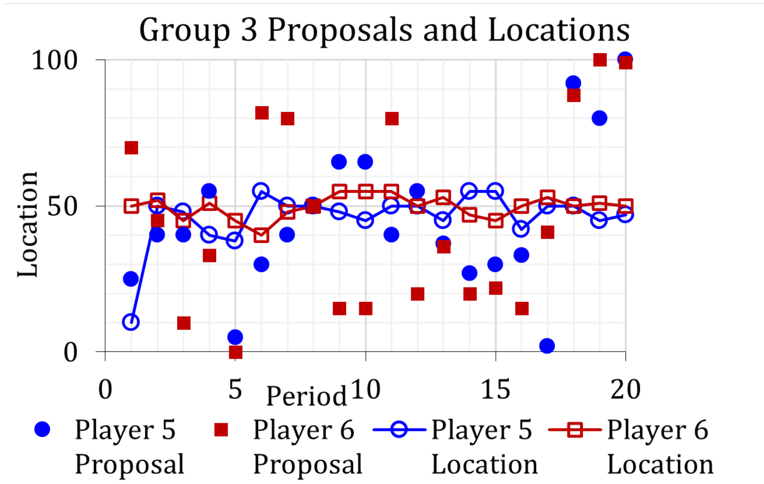

Figure 4.3: Proposals and locations, 2-player, fixed matching, Group 3.

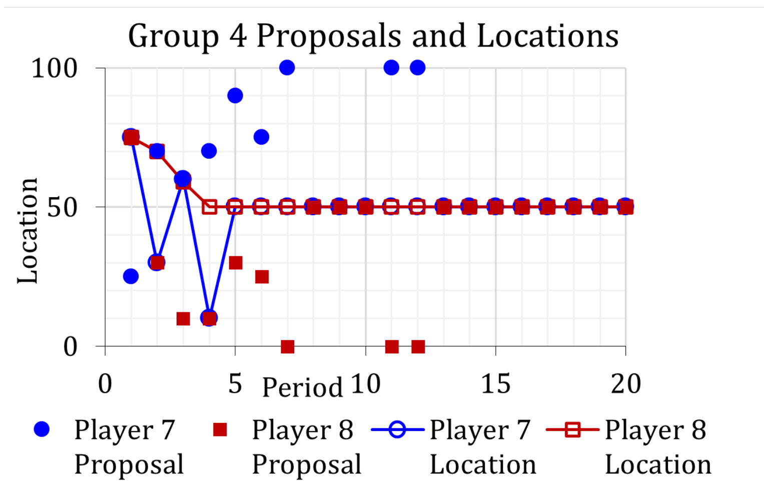

Figure 4.4: Proposals and locations, 2-player, fixed matching, Group 4.

The fourth group's proposals and locations by period are shown in Figure 4.4. In the first period, the leader proposes the socially optimal solution of (25, 75), and the nonleader abides. However, the leader flips to 75, resulting in a 50-50 split. The leader could have made a big gain by choosing the undercutting location of 74. In addition, the seller loses credibility for subsequent rounds by not choosing the location it proposed for itself.

In the second period, the leader proposes a more modest, but still perfectly equitable location pair of (70, 30). The players did a double flip and located at (30, 70). I am not sure what the

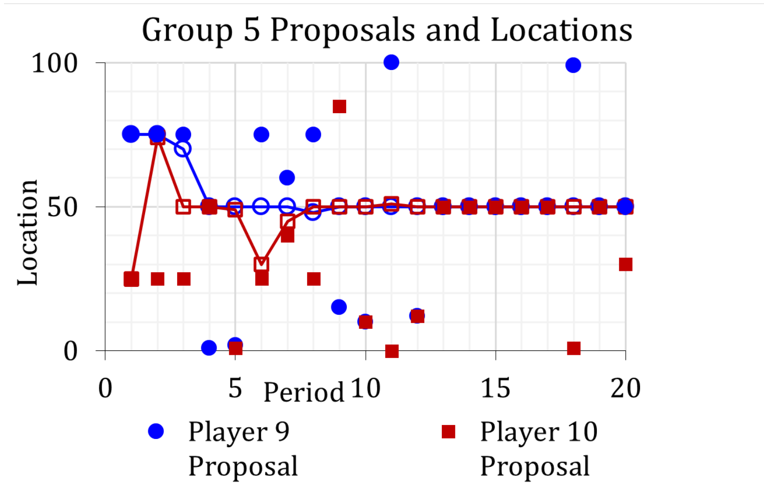

Figure 4.5: Proposals and locations, 2-player, fixed matching, Group 5.

reasoning was behind this. In the third period, the leader chose an inequitable pair (60, 10). The leader followed through, but the nonleader took advantage and chose the undercutting choice of 59. From the fourth period on, the nonleader chose 50, and from the 5th period on, the leader chose 50. The leader made a few attempts to propose the socially optimal (25, 75) and the endpoints (100, 0), but there was no compliance. The fifth group's proposals and locations by period are shown in Figure 4.5. Group 5 got off to a great start with the leader proposing the socially optimal solution, and full compliance was realized, in the first period. The leader repeated the signal in Period 2, but the nonleader took advantage and undercut the leader. Despite the defection, the leader repeats the socially optimal signal in the third period. The leader, however, cheats in from 75 to 70 to protect from a repeat undercutting, but the nonleader chose 50, which will lead to difficulties in the rest of the session. In round 4, the leader proposes an inequitable pair of (1,50), which leads to locations (50, 50). In round 5, the leader proposes the most inequitable pair possible: (2, 1), which was not followed.

In Periods 6 through 9, the leader tried different combinations of equitable pairs; however the leader did not follow it—choosing instead 50 or close to 50. From Period 12 through

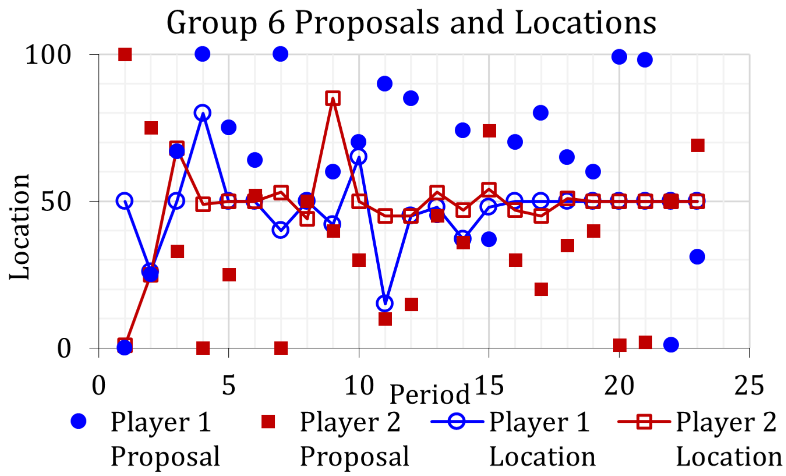

Figure 4.6: Proposals and locations, 2-player, fixed matching, Group 6.

the end of the session, both players chose 50. The sixth group's proposals and locations by period are shown in Figure 4.6. In the first period, the leader proposes (0, 100), which is not better socially than (50, 50), and also poses the most temptation for locating in the center to gain market share and also improving access for the consumers. The nonleader in this group reveals itself as an aggressive competitor when it chooses a location of 1 in an attempt to undercut the leader. The leader reveals itself as noncredible when it chooses a location of 50.

In the second round, the leader proposes the socially optimal pair of (25, 75). The nonleader swaps sides and chooses 25, and the leader boxes the nonleader in by choosing 26. Strategic proposals and locations continue through round 18, after which both players play 50 through the end of the session. The seventh group's proposals and locations by period are shown in Figure 4.7. In the first period, there is full compliance to an inequitable proposal. Then the leader gets even more greedy, and the nonleader defects, resulting in a market split. The leader proposes a similar strategy in Period 3, and again the nonleader defects to split the market. In Period 4, again, the leader proposes a slightly inequitable pair, and the nonleader chooses the location that splits the market given that the leader plays the proposed location.

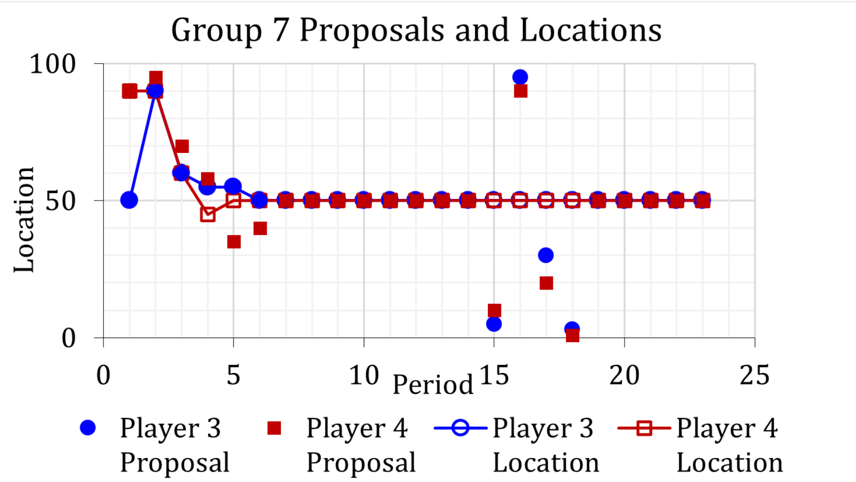

Figure 4.7: Proposals and locations, 2-player, fixed matching, Group 7.

In period 5 through the end of the session, the nonleader plays 50. The leader of this group has been the most honest. Although the leader proposed inequitable pairs, the leader played the proposal for all periods through Period 14. In Period 15, the leader experimented with a proposal favorable to the nonleader (5, 10), attempting to lure the leader left. But it did not make sense unless the leader was planning to play a location other than what was proposed.

The eighth group's proposals and locations by period are shown in Figure 4.8. The group gets off to a great start with full compliance of an equitable and socially considerate pair of (40, 60). The second period's proposal seems suspicious at (70, 50). Why would the leader flip sides, and why would they offer the nonleader more? The location choices were a double flip (50, 70). In the third period, again, the leader offers a generous pair (80, 60), and there was a double flip, plus the leader took an undercutting position(60, 78). This action reveals the manipulative hand of the leader and will not bode well for future periods. Now in Period 4, the leader offers an even more generous pair, (80, 70). That is just not credible. The leader did not follow its own proposal, a pattern of flipping was observed in the last two periods, and the nonleader is learning about the payoffs. Recall the payoff function, assuming the leader plays any location other than 50, the choice of 50 is a dominant strategy and results

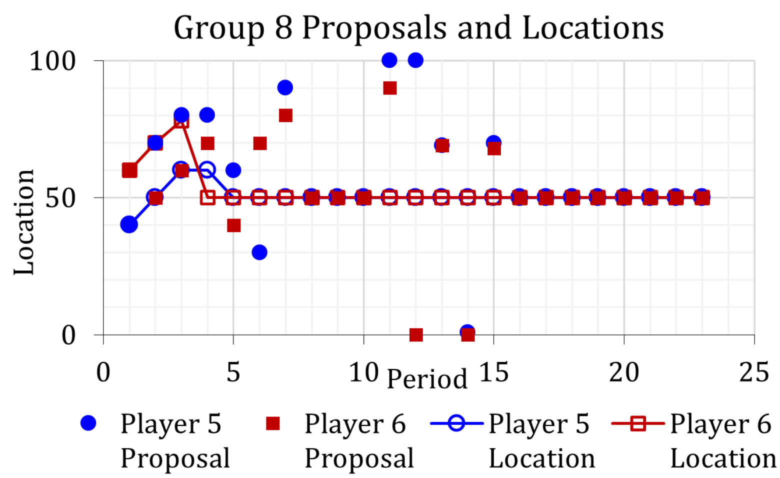

Figure 4.8: Proposals and locations, 2-player, fixed matching, Group 8.

in a payoff strictly greater than 50. From Period 4, the nonleader plays 50, and from Period 5, the leader also plays 50. The leader attempted to lure the nonleader into other locations in Periods 11-15, but the proposals had no weight.

The ninth group's proposals and locations by period are shown in Figure 4.9. Group 9 was a unique case where trust and credibility was recovered and full compliance on a non-NE proposal relatively late in the session. In Period 1, the leader proposes a fair, but asymmetric pair (55, 55). The nonleader might see this proposal as suspicious and may wonder if it is an attempt for the leader to get on the larger side of the market, so the nonleader instead chose 45; however, payouts were even either way. In period 2, the proposal is (60,60). This time, the nonleader tried 55, but the leader chose 53, resulting in a payout of (54, 46).

The leader continues to signal to come over to the right side of the market with a proposal of (80, 75) in period 3. The nonleader did move over somewhat, which the leader successfully exploited, with (55, 60), resulting in payoffs of (57.5, 42.5). At this point, the leader's trustworthiness has taken a hit, and the group settles in to the NE for proposals, outcomes, and payoffs in Periods 7, 8, 11, and 12. Then there was a fluke in Period 13 where there was full compliance on the proposal of (65, 65). How was this possible? The leader was

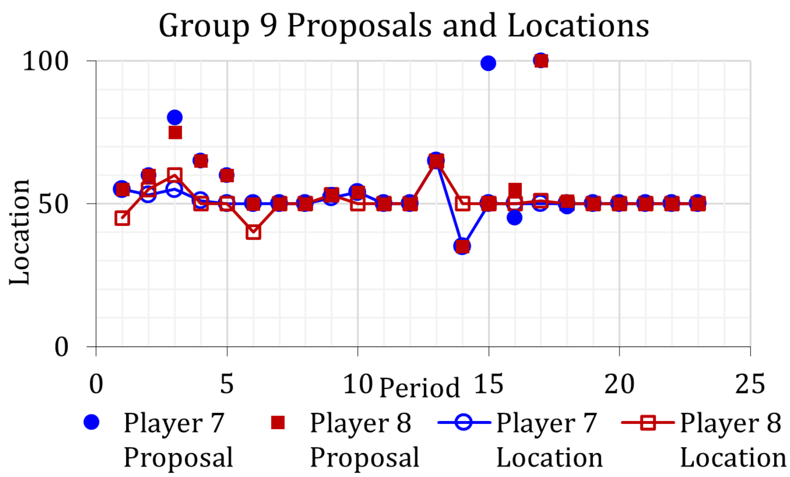

Figure 4.9: Proposals and locations, 2-player, fixed matching, Group 9.

honest in the past nine periods—playing the location consistent with the proposal. Also the nonleader could take a chance, and play the proposal with a payout equal to the NE payout. Compliance would also send a signal back to the leader as a desire to cooperate, which could either be used for the "greater good" (if the leader would select a socially optimizing proposal), or that could be exploited, in subsequent periods.

In the next period, the leader proposes (35, 35). I don't know why the leader decided to flip side of the market, but the nonleader may have thought this was suspicious. The nonleader balked and played 50. Going forward, players essentially played NE through the end of the session although the leader tried some other proposals.

Observing the location frequency distribution, it appears very similar to that with no communication: a strong, single peak at 50. The distribution is almost identical to that without communication: 72.4 peak vs. 72.5 peak. The distribution is shown in Figure 4.10. The frequency distribution of leaders and nonleaders are similar (see Figure 4.11).

Next, I will briefly describe the rematching results. One session of rematching consisted of 5 pairs, one of whom was the leader. Since the pairs were regrouped after each period, there

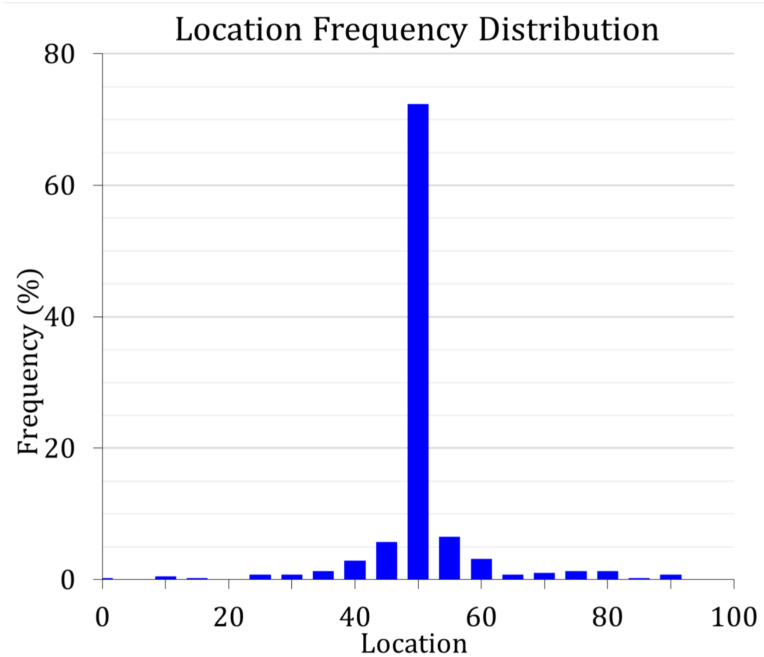

Figure 4.10: Frequency distribution of locations for all 2-player, fixed data.

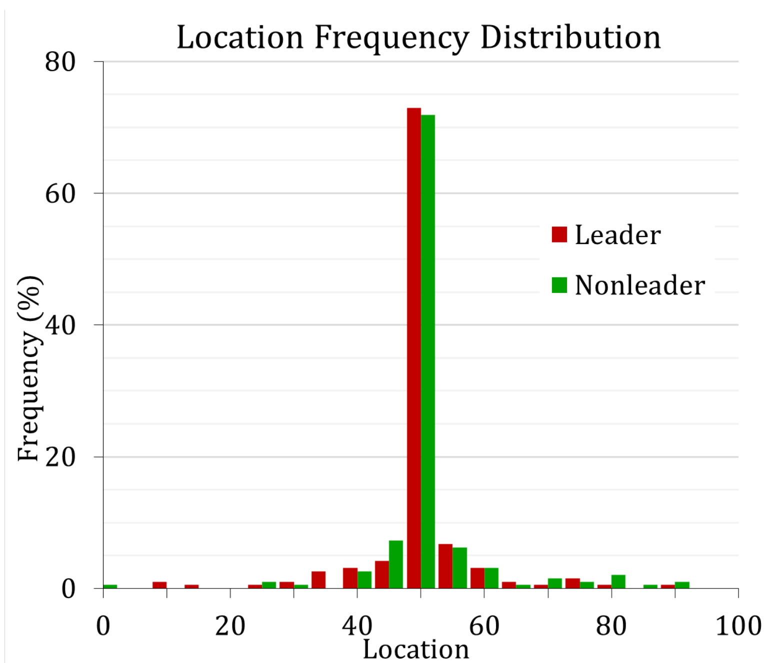

Figure 4.11: Frequency distribution of locations for 2-player, fixed data, disaggregated by leader and nonleader roles.

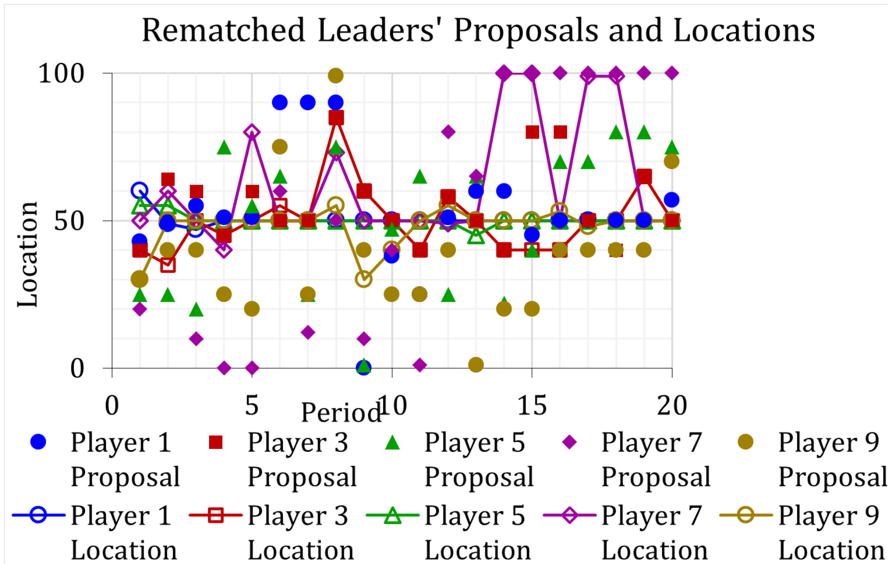

Figure 4.12: Proposals and locations, 2-player, randomly rematched, all leaders.

was not the opportunity for the leader to develop any kind of rapport with the nonleader across periods although the nonleaders will have experience with leaders' behaviors as a class. In this experimental design leaders were selected randomly and remained leaders for all periods (there was no change in role).

The proposals and locations are shown for the leaders in Figure 4.12 and for the nonleaders in Figure 4.13. In general, the proposals are more dispersed than the chosen locations, for both leaders and non-leaders. One nonleader played 50 for all 20 periods. Also more dispersed proposals extend later in the sessions than in the fixed groups.

The frequency distribution of locations has a peak at 50, constituting 72.5% of the distribution in the interval [48, 52]. The frequency distribution is shown in Figure 4.14.

I also looked at the total earnings of each role. In the 4-group session, all four leaders earned slightly more than the nonleaders, but in the 5-group session, four out of five nonleaders earned slightly more. In total five out of nine leader groups earned slightly more. When one individual earned slightly more, it is usually the case that one person got the lead in

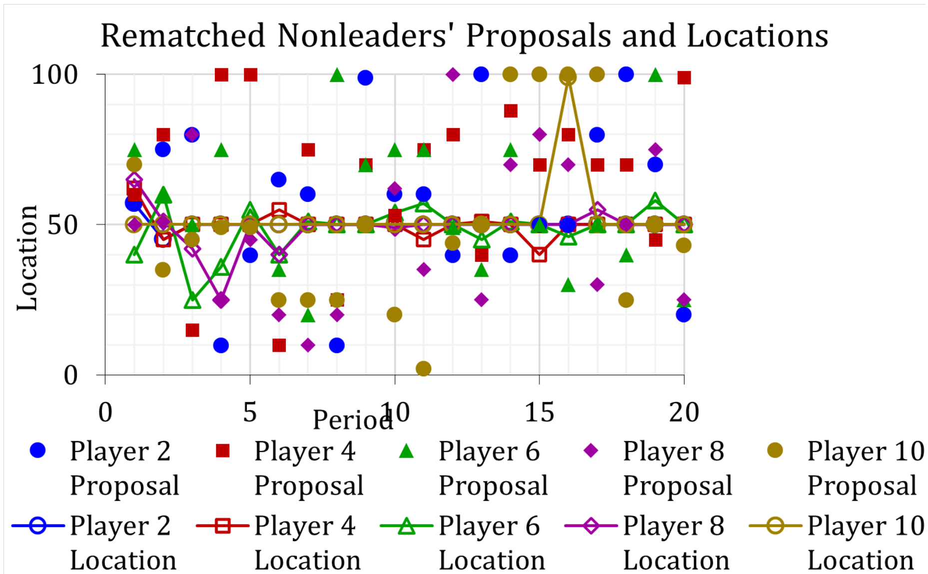

Figure 4.13: Proposals and locations, 2-player, randomly rematched, all nonleaders.

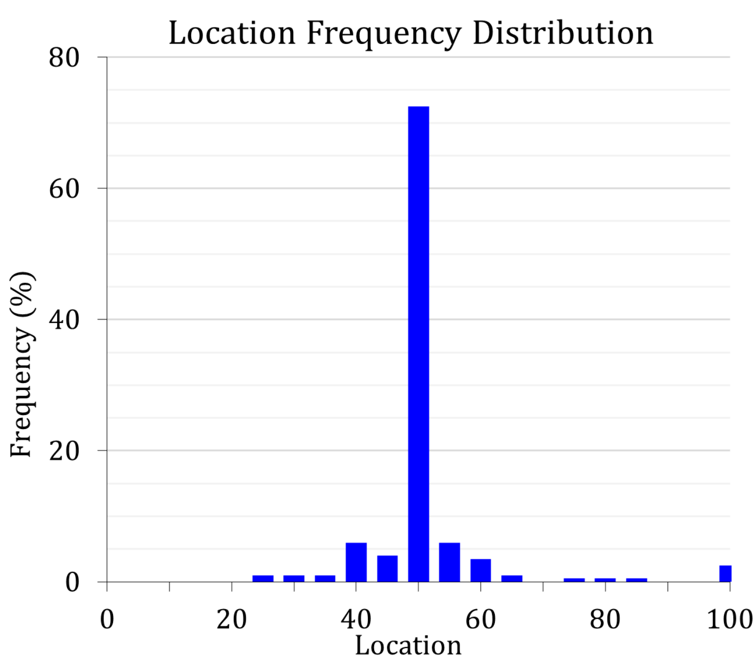

Figure 4.14: Frequency distribution of locations for all 2-player, rematched data.

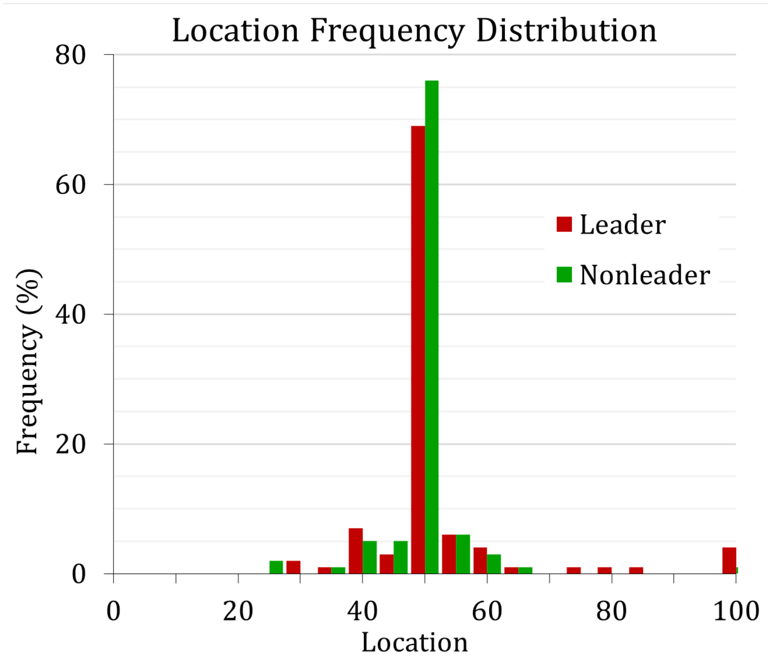

Figure 4.15: Frequency distribution of locations for 2-player, rematched data, disaggregated by leader and nonleader roles.

the early rounds, when individuals were still experimenting. Then when convergence to 50-50 occurred, the same lead was maintained throughout the rest of the session although on average, the nonleaders partially recover some lost ground. The average sales per period for leaders and nonleaders are shown in Table 4.3.

Table 4.3: Average sales per period for leaders and nonleaders.

Treatment	Time range	Leaders	Nonleaders
Fixed	All periods	50.11	49.89
	First 10 periods	50.54	49.46
	Last 10 periods	49.86	50.14
Rematched	All periods	48.73	51.28
	First 10 periods	49.27	50.73
	Last 10 periods	48.18	51.82

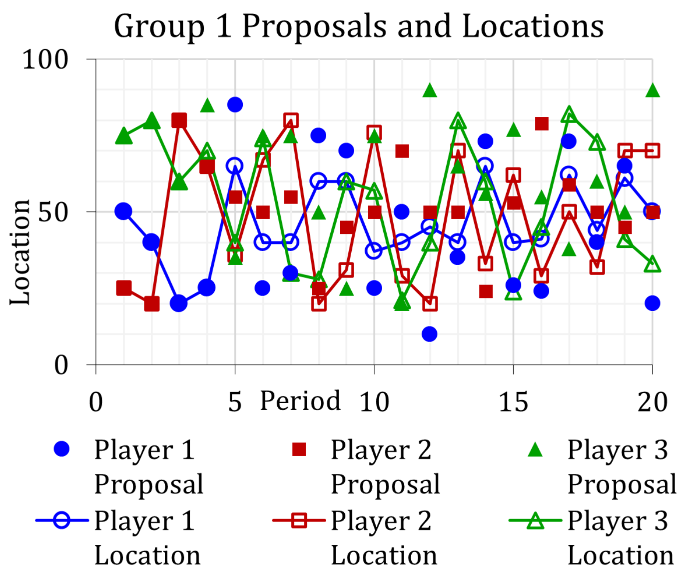

Figure 4.16: Proposals and locations, 3-player, fixed matching, Group 1.

4.3.2 The Three-Player Case

Fourteen groups were conducted across four sessions in this treatment. There were a total of 672 individual observations. The next 14 figures show proposals and locations by group.

The next set of graphs, Figure 4.30 and Figure 4.31, compare the frequency distribution of the locations with communication, with three-player groups without communication from the previous chapter. The location frequency distribution with communication has more concentration in the middle. The total distribution is clearly single-peaked, as opposed to the bimodal distribution observed without communication. The leaders and nonleaders have similar distributions, and both feature a middle peak of over 16

How does one interpret the mechanism that is occurring here that results in this different distribution? Maybe the leader is unfair, suggesting locations that allocate higher payoffs to himself. For example, choosing 0-50-100, with leader at 50. Followers may reject the proposal by moving to the middle. Indeed, the leader's location is more concentrated in the

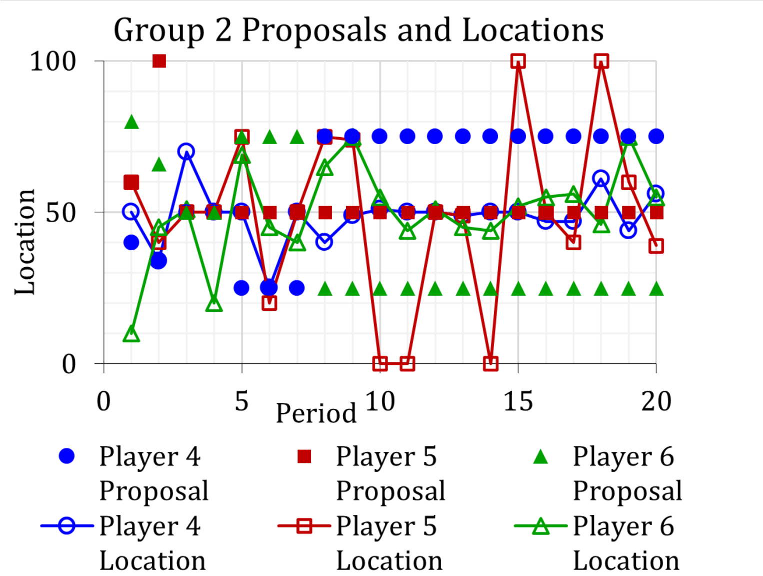

Figure 4.17: Proposals and locations, 3-player, fixed matching, Group 2.

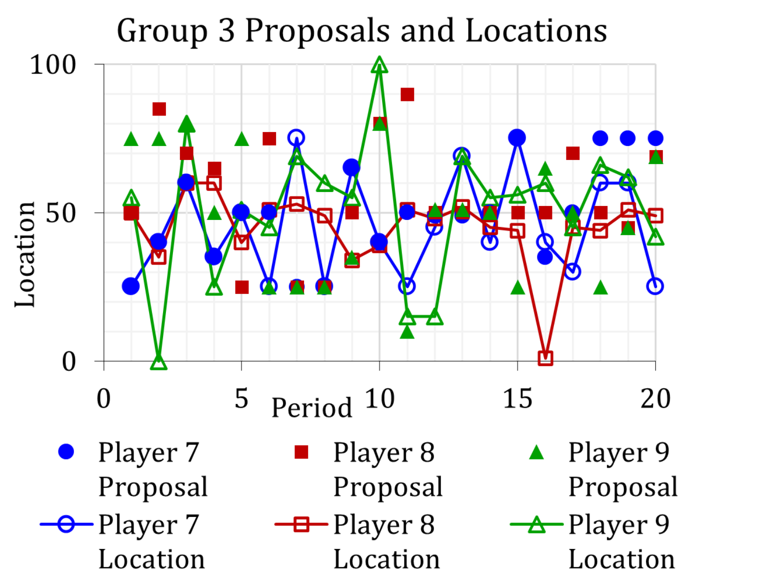

Figure 4.18: Proposals and locations, 3-player, fixed matching, Group 3.

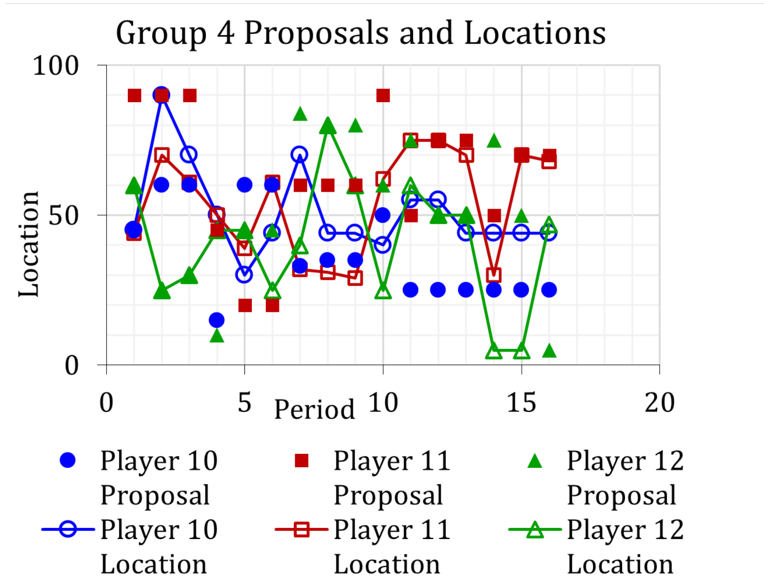

Figure 4.19: Proposals and locations, 3-player, fixed matching, Group 4.

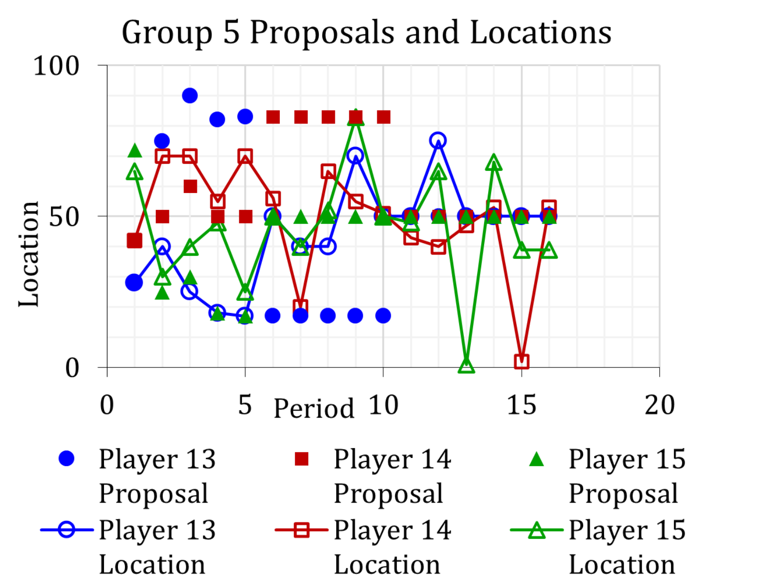

Figure 4.20: Proposals and locations, 3-player, fixed matching, Group 5.

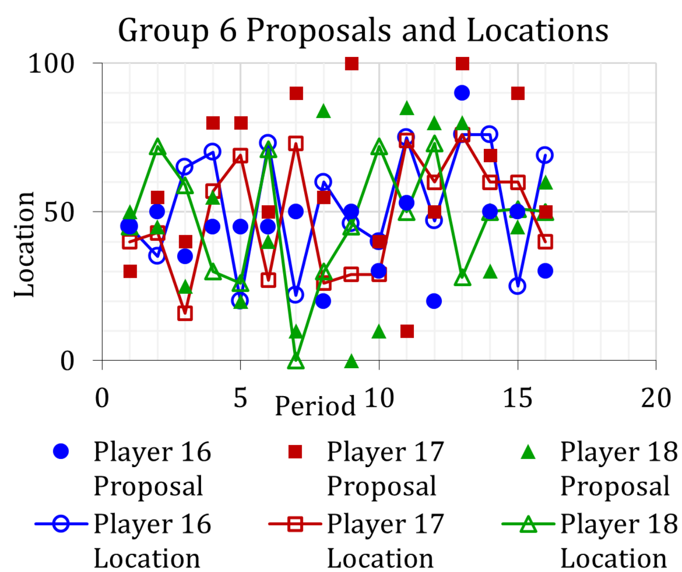

Figure 4.21: Proposals and locations, 3-player, fixed matching, Group 6.

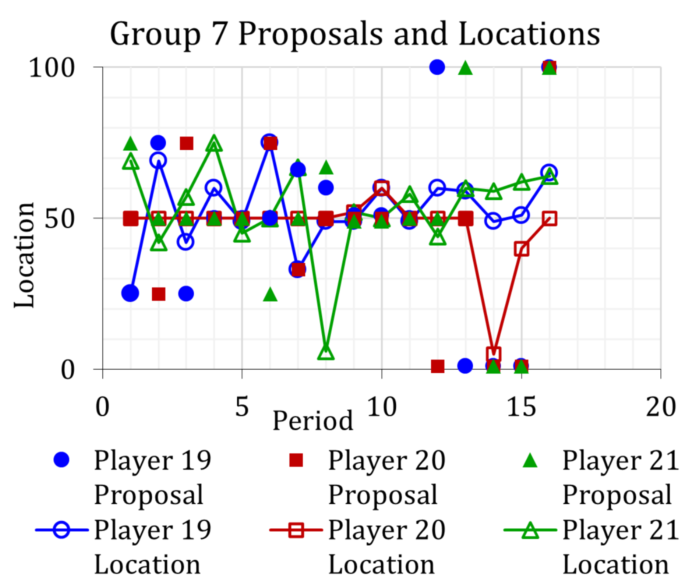

Figure 4.22: Proposals and locations, 3-player, fixed matching, Group 7.

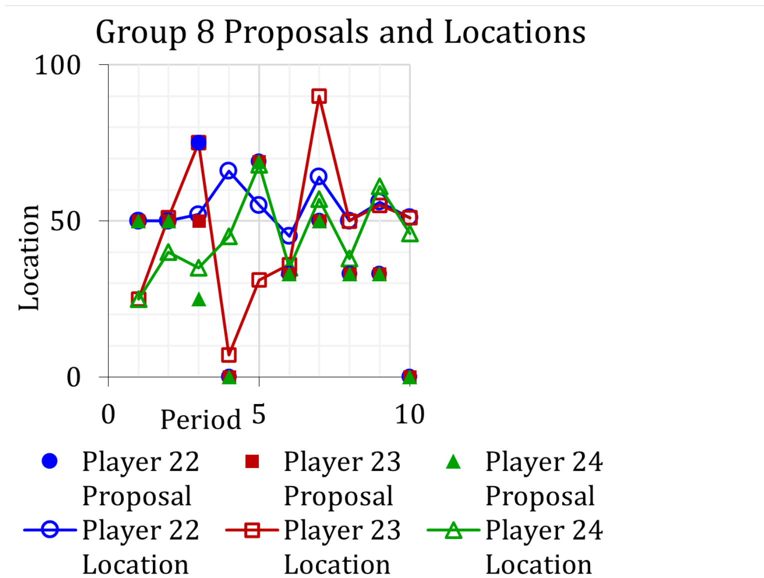

Figure 4.23: Proposals and locations, 3-player, fixed matching, Group 8.

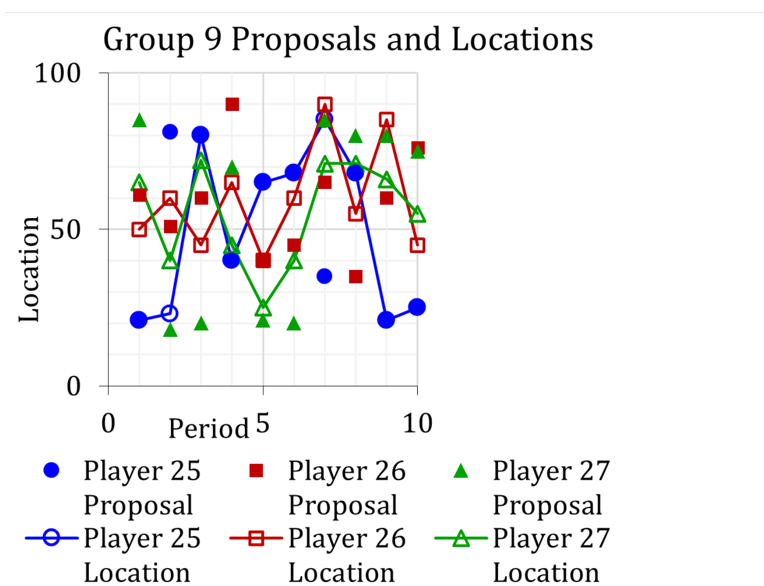

Figure 4.24: Proposals and locations, 3-player, fixed matching, Group 9.

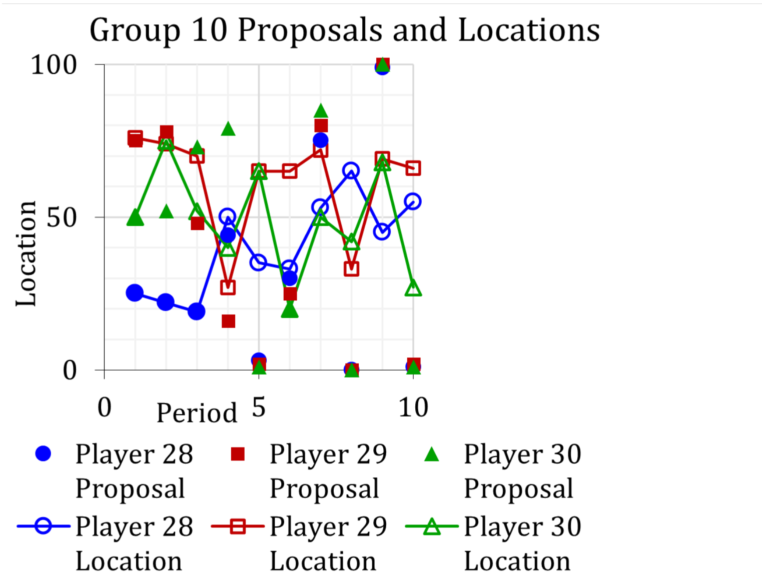

Figure 4.25: Proposals and locations, 3-player, fixed matching, Group 10.

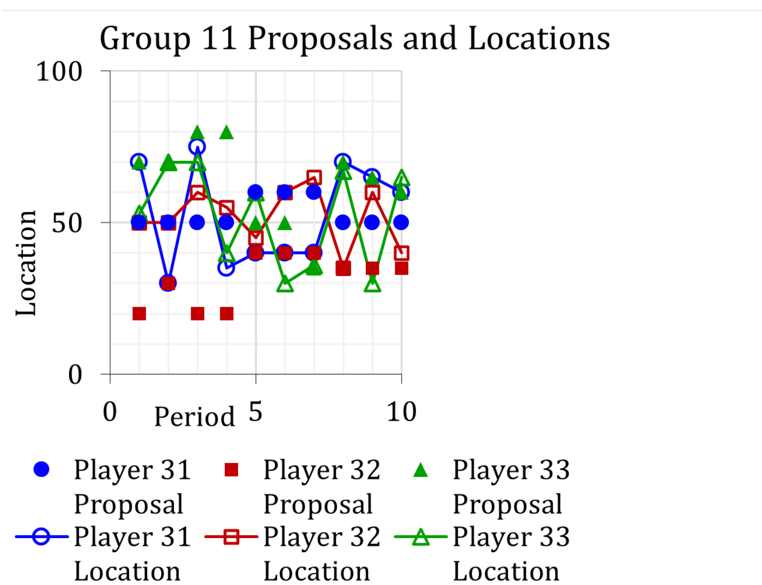

Figure 4.26: Proposals and locations, 3-player, fixed matching, Group 11.

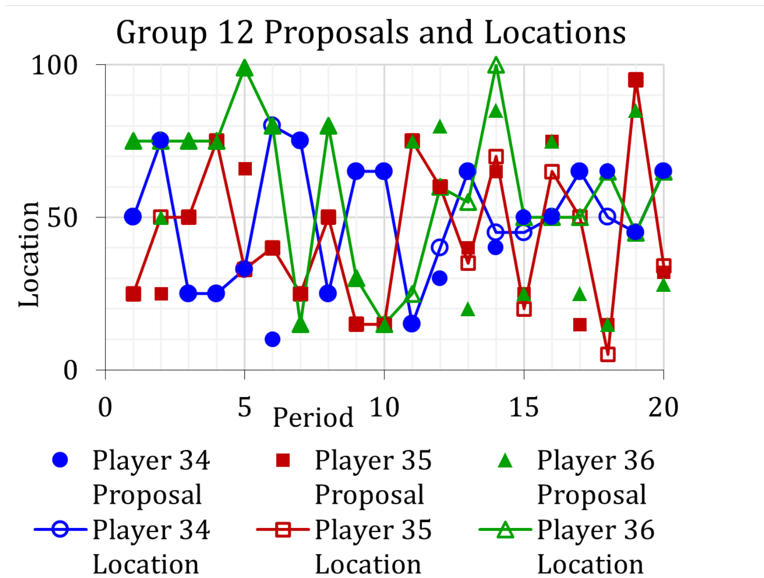

Figure 4.27: Proposals and locations, 3-player, fixed matching, Group 12.

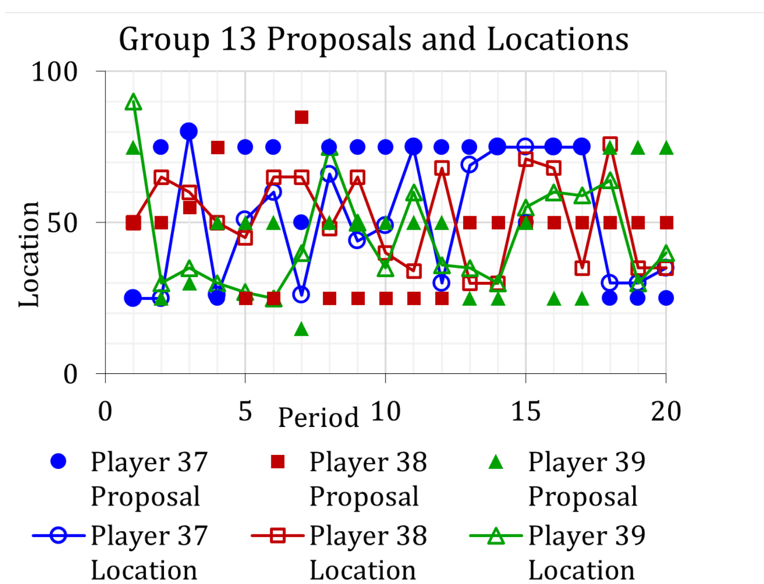

Figure 4.28: Proposals and locations, 3-player, fixed matching, Group 13.

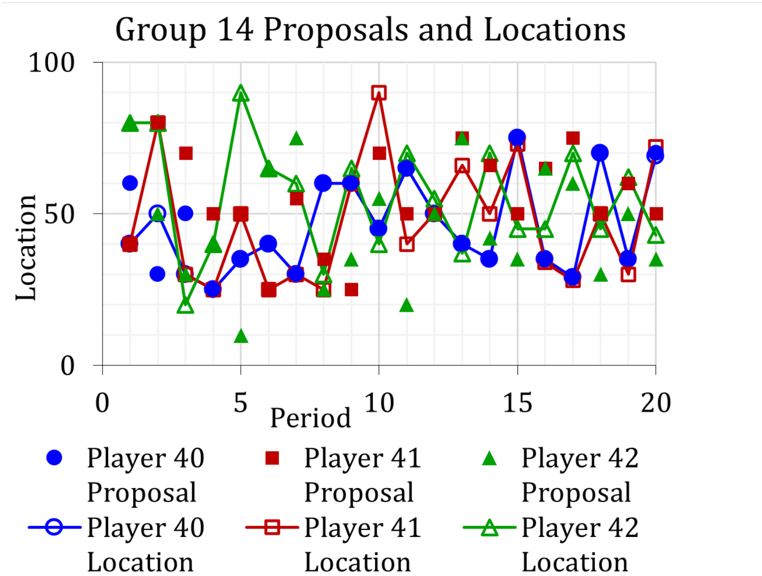

Figure 4.29: Proposals and locations, 3-player, fixed matching, Group 14.

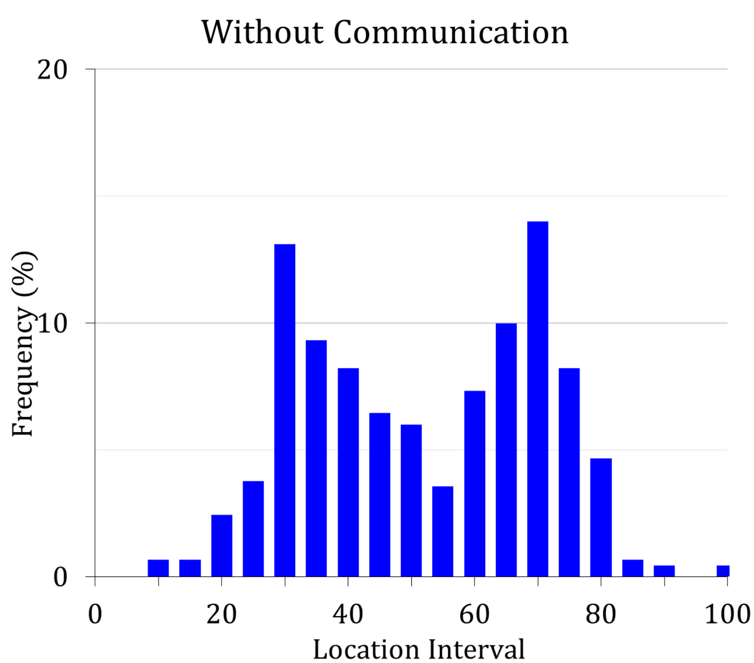

Figure 4.30: Frequency distribution, without communication, 3-player, fixed data.

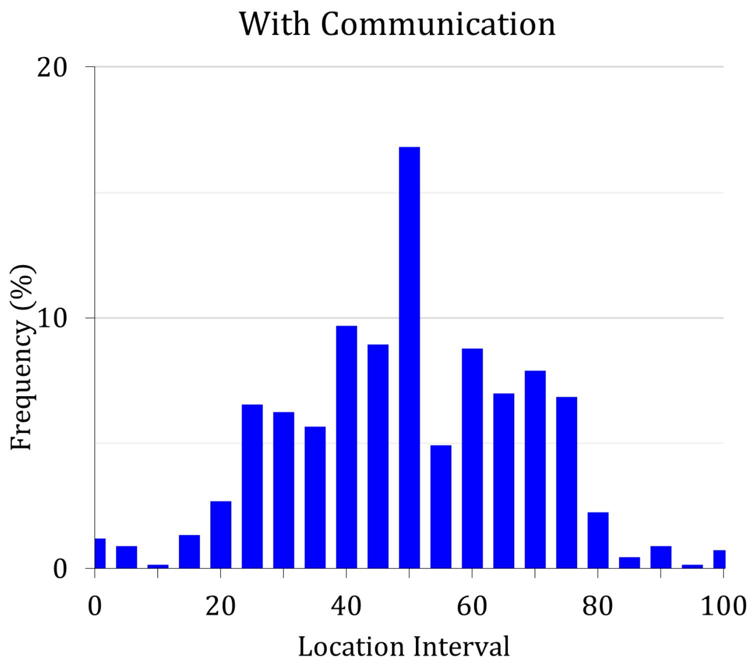

Figure 4.31: Frequency distribution, with communication, 3-player, fixed data.

middle than the nonleaders. But this may not work: a credible leader locates himself in the middle, and followers take advantage by crowding him in the middle.

In noncommunication, I hypothesized that avoidance of the middle was due to risk aversion. However, leader communication supersedes the risk aversion. Cheap talk makes a big difference in the three-player case.

4.3.3 The Four-Player Case

Nine groups were conducted across four sessions, which ran for from 16 to 24 periods. Total individual observations were 704. Only fixed-matching treatment was conducted (no rematched sessions for four sellers). The individual proposals and locations are shown in Figure 4.32 through Figure 4.40.

A visual comparison of the overall location distributions with that of the noncommunication

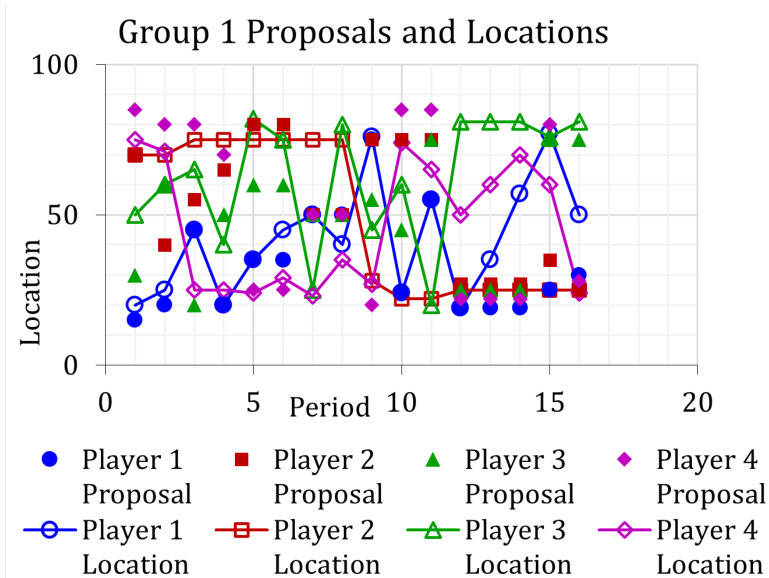

Figure 4.32: Proposals and locations, 4-player, fixed matching, Group 1.

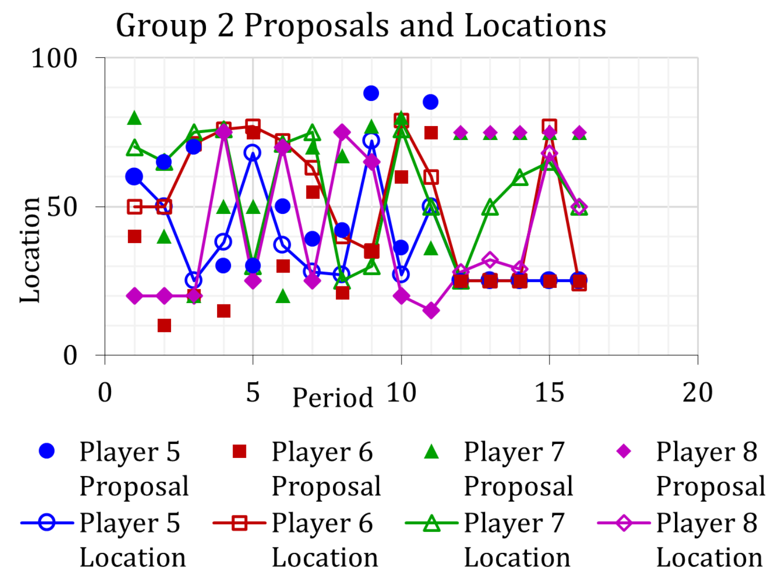

Figure 4.33: Proposals and locations, 4-player, fixed matching, Group 2.

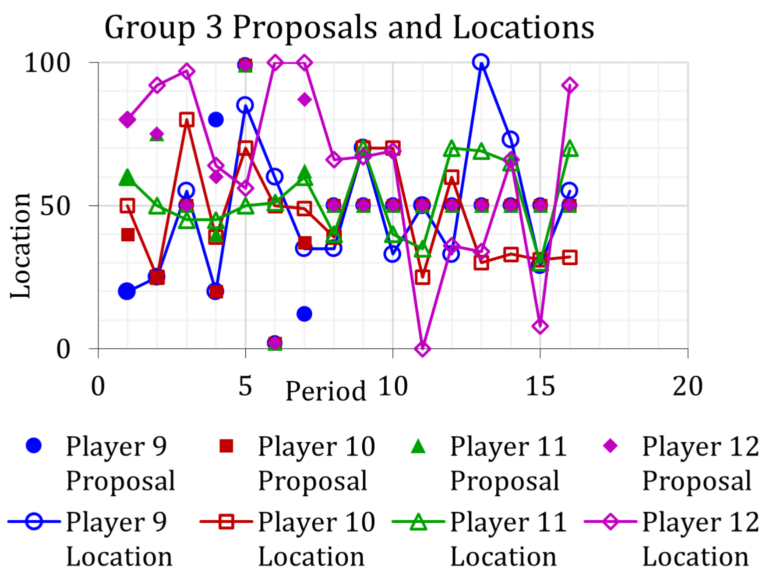

Figure 4.34: Proposals and locations, 4-player, fixed matching, Group 3.

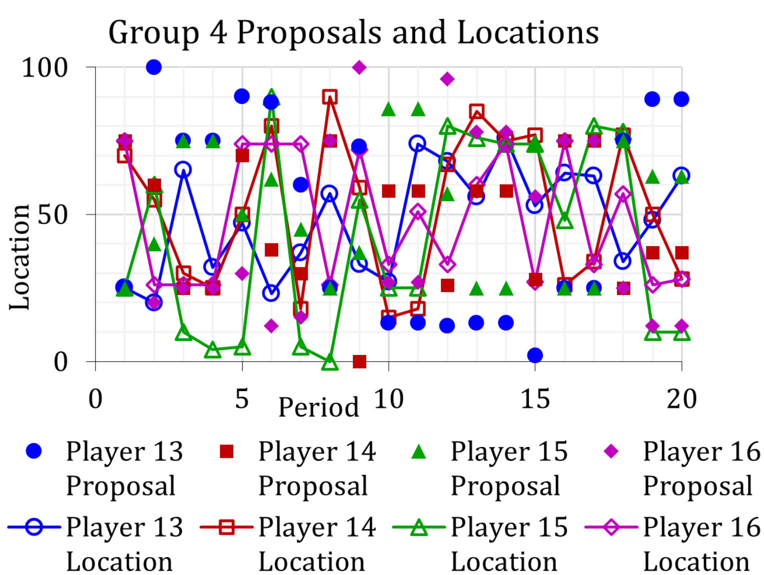

Figure 4.35: Proposals and locations, 4-player, fixed matching, Group 4.

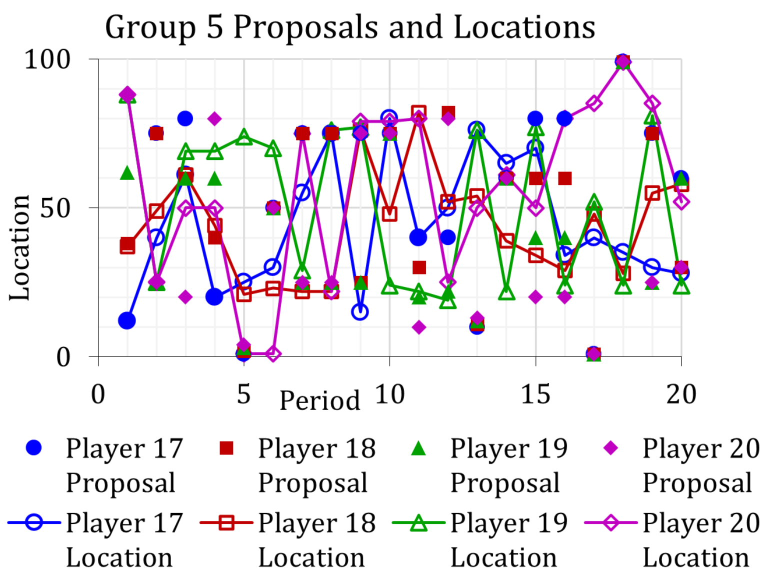

Figure 4.36: Proposals and locations, 4-player, fixed matching, Group 5.

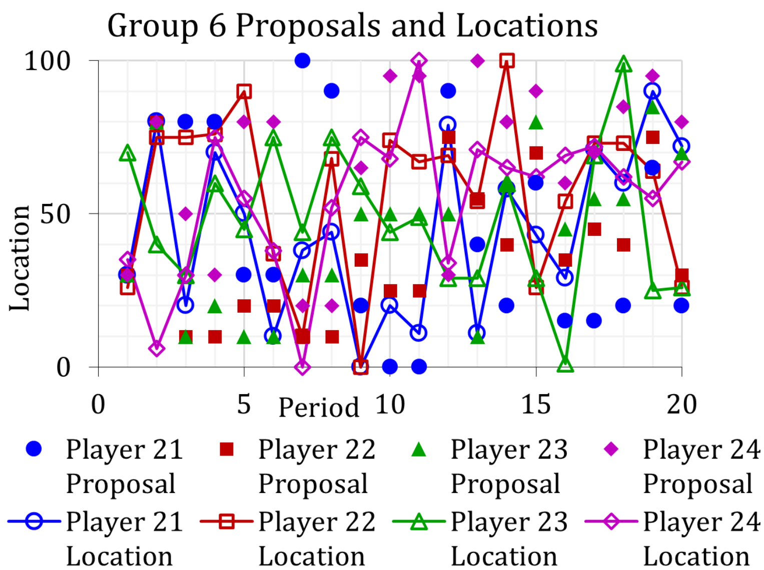

Figure 4.37: Proposals and locations, 4-player, fixed matching, Group 6.

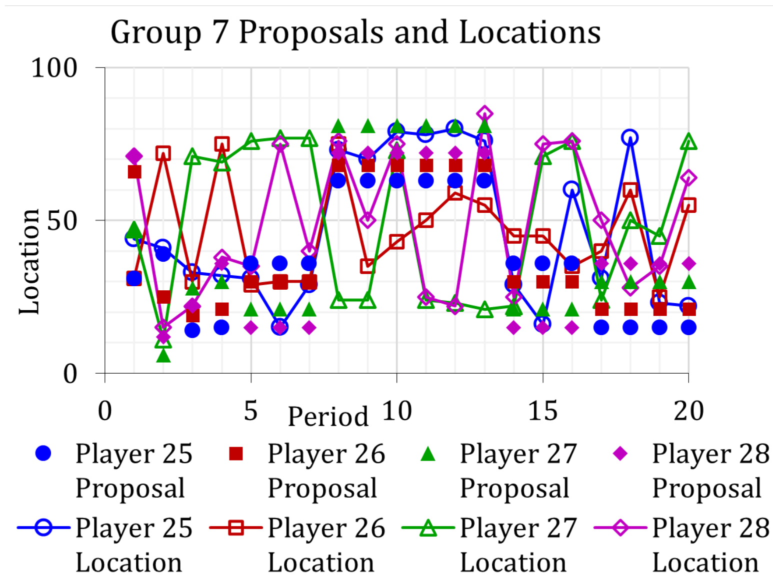

Figure 4.38: Proposals and locations, 4-player, fixed matching, Group 7.

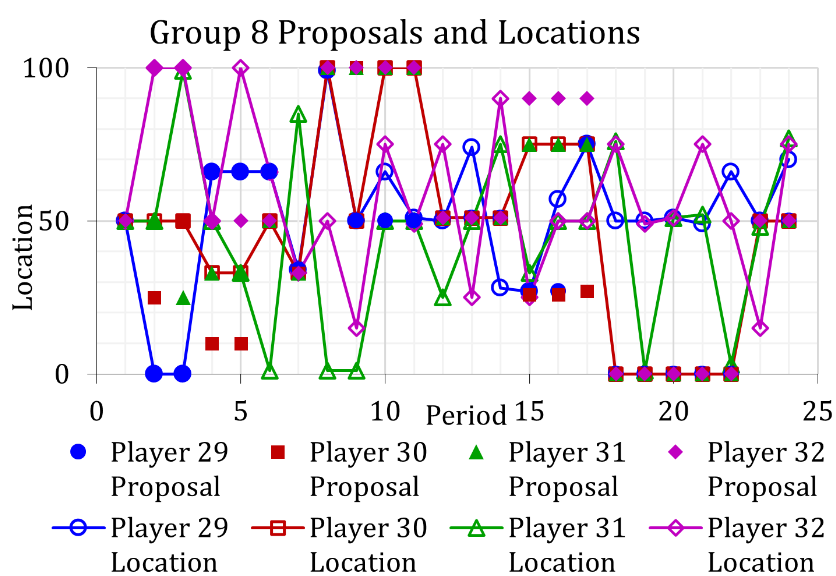

Figure 4.39: Proposals and locations, 4-player, fixed matching, Group 8.

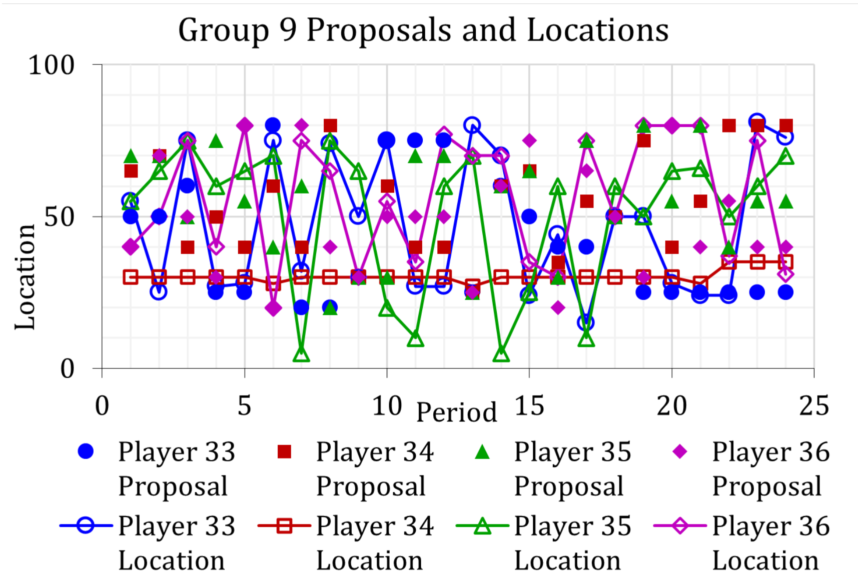

Figure 4.40: Proposals and locations, 4-player, fixed matching, Group 9.

treatment is shown in Figure 4.41 and Figure 4.42. These figures were produced using all fixed-matching groups, first 20 periods. A visual comparison of these figures reveals that the communication treatment has higher and narrower peaks than the noncommunication treatment. The middle peak is especially more prominent with communication than without.

Table 4.4 compares the peak values, which were higher for both leaders and nonleaders, than the noncommunication treatment.

Table 4.4: Comparison of peak values, Periods 1-16.

Treatment and role	Average quartile peak (25+75)/2	Middle peak (50)
No communication	10.79	6.56
With communication, leaders	12.15	8.33
With communication, nonleaders	13.66	11.34
With communication, overall	13.28	10.59

Another observation is that there is a slight ordering effect for leaders vs. nonleaders. This is confirmed in the summary statistics. The mean location is 46.1 for leaders and 49.9 for nonleaders, 49.0 overall. For the noncommunication case, it is 51.0.

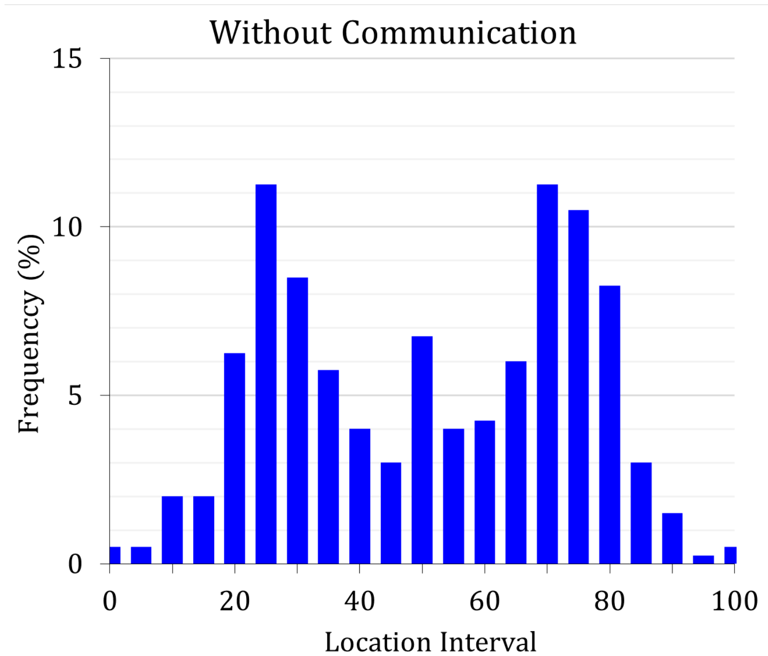

Figure 4.41: Frequency distribution, without communication, 4-player, fixed data.

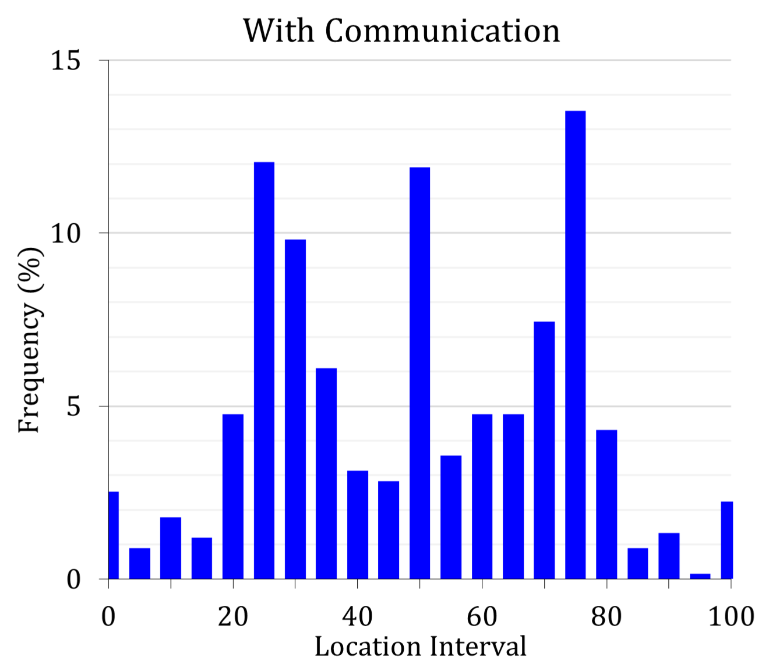

Figure 4.42: Frequency distribution, with communication, 4-player, fixed data.

Next, I compare the distributions using a 2-sample Kolmogorov-Smirnov test, which compares the greatest difference between the cumulative differences between the two distributions.[3] I also ran these tests reversing the data of one distribution to negate any ordering effects.

Table 4.5: Comparison of distributions, Periods 1-16.

Test	P-value (fd./reverse)
Leaders vs. no-communication treatment	0.0127/0.0659
Nonleaders vs. no-communication treatment	0.5700/0.3105
Leader vs. nonleader roles	0.0233/0.0437

The tests indicate a high probability that the leaders' location distribution is different than the noncommunication case, but the tests do not reveal this same difference between the nonleaders and the noncommunication case. Furthermore, the tests seem to indicate a difference in distribution between the leaders' and nonleaders' distributions.

Next, I focused on the nonleaders and conjectured possible combinations of actions that could generate this location distribution. For instance, subjects might be loyal (playing the proposals), total randomizers, playing pure-strategy equilibrium playing mixed-strategy equilibrium, playing best-response to the proposals, playing best-response to the previous period locations, or some other strategy.

Each of these actions will have its own location distribution. For example, a pure randomizer has uniform distribution from 0 to 100. Pure strategy is two spikes at 25 and 75. Mixed strategy is a U-shaped distribution in the support [17, 83]. One can also generate location distributions for the proposals, the best response to the proposals, and the best response to the previous period locations.

[3] All ordinal tests were run using the actual single-unit data, not grouped into 5-unit blocks, as in the graphs.

Randomization, pure strategy, and mixed strategy by themselves are much worse in fitting the nonleader data than the noncommunication data; however, a linear combination gets close. For example, take the distribution produced by .52(mixed strategy)+.34(random)+.14(pure strategy), which appears to be the closest fit (pds 1-16). Running a one-sample Kolmogorov-Smirnov test to compare the cumulative distributions of this combination with that of the nonleader distribution, the P-value is .37421, which is in between that of the forward and reverse comparisons with the noncommunication distribution.

Next, I compare the distribution of proposals to that of the locations. There is a strong peak in the proposal distribution at 50. By itself, the proposals do a fairly well job of predicting locations, with a Kolmogorov-Smirnov 2-sample test P-value of .14298. This is not as good a fit as the random/mixed/pure combination, or the noncommunication, but if the proposals were included in the combination, it very well may improve the fit by adding some weight to the middle peak.

The next step along this line of overall data comparison is to run similar tests comparing the distribution with that of the best-response behaviors. I have not yet generated the best-response locations from the data for the four-firm case. It requires a considerable amount of calculation.

A further investigation of the data involves looking at the closeness of the group location sets to that of a pure-strategy equilibrium. Here is a table of summary statistics:

Table 4.6: Closeness to pure-strategy NE.

Treatment	Min	LQ	Med	Mean	UQ	Max
No communication	6	36.25	51	50.8	63	108
With communication	5	38	54	55.5	71	113
Proposals	0 (18)	40	65	71.2	100	200

The closeness to the pure-strategy Nash equilibrium is measured by taking the difference between the two lower locations from 25 and the two higher locations from 75. A Nash equilibrium will have a value of zero. The closest achieved was 6 in the noncommunication case and 5 in the communication case. The means and other comparisons were similar.

An interesting thing is happening in the proposals. Although the pure-strategy Nash equilibrium was suggested 18 times, the mean proposal is further away from the pure-strategy Nash equilibrium than the mean noncommunication locations. Despite this, players locate similar to the noncommunication case.

Next, I looked for difference in group sales equity between the comm. And noncommunication case—the test indicated no significant difference.

4.4 Behavioral Analysis

4.4.1 Introduction

In this section, I look at the individual player behavior across periods. Players are classified by type according to the strategy that they employ. A strategy is a mapping from information set to action. An action is a mapping from an information set to a specific location. For example, if the leader didn't follow his own proposal in the previous period, and the leader offered me a lousy location, I might choose the action of best response to the proposals. A strategy is the complete mapping of all possible scenarios and their associated actions.

Also for example, the proposal-best-response action to the proposal set $(x, y, z) = (20, 30, 50)$, for seller x in a three-player group is location $= 29$. Now pure proposal-best-response can also be a strategy. This would describe a player that plays proposal-best-response all of the time.

The following are some actions that are directly related to the communication model. Proposal-best-response is to select the location that would yield the player the highest sales in the hypothetical case that the other players choose their suggested location. Compliance is the action of choosing location equal to the suggested location.

There are other actions that do not take into account the proposals. They are lag-location-best-response: choosing the location that would yield the highest sales assuming the other sellers will play their previous location. Another is randomization: choose a random location across the entire location space. Bounded randomization is a randomization across a subset of the location space. In particular, the mixed-strategy equilibrium is a randomization along a support that is a subset of the market space. The pure-strategy equilibrium action is to choose a location in the pure-strategy set. A player may try to punish another player by trying to crowd him.

Action modifications add variations to the actions. Nonleapfrogging respects the suggested ordering. For example, nonleapfrogging proposal best response is a modified action. Selectivity is another action. A seller might identify one other seller as compliant and another as deviant. The seller may take action that may be beneficial to one and/or harmful to another. So a seller might be selectively nonleapfrogging or selectively crowding.

The challenge is to correctly interpret locations as actions. Any location that does not correspond to that resulting in compliance is classified as deviance. One can seek an explanation for the deviance. Sometimes a player might choose the best response location to the proposals. In that case, it is plausible to suggest that the player is choosing the action of proposal best response. However, sometimes the proposal best response location and the compliance location are the same location.

The following are information set factors that will influence the choice of action. First, the action of fellow group members will affect this, the most obvious of which is whether or not fellow group members are compliant to proposals.

The other factors are ones that are related to a merit judgment of the proposals, which include individual factors and group factors. The individual factors include the proposed sales to the individual, and the consideration to deviate, which is the difference between the proposed sales and the proposal best response. Group factors include a measure of the fairness of the proposals, group temptation, how close the proposals are to the pure-strategy equilibrium, and a plausibility/appearance of rationality factor, which judges how credible the proposals appear.

I ran a number of panel data regressions to see what I could discern about the relationship between the information set and actions. The models were designed to predict actual location, compliance, and deviation from proposals. The arguments for location are the proposals, the best response to the proposals, the best response to the previous period's locations, and the previous period's location.

The arguments for both compliance and deviation are the previous period's group compliance, the temptation to deviate, the plausibility of proposals, previous period's sales, the hypothetical full-group-compliance pay, and the fairness of the proposals.

Compliance is a dummy variable with value =1 if location equals proposals; 0 otherwise. In contrast, deviation is the absolute difference between proposal and location. Group compliance a measure of the compliance of other members of the group, normalized to one. Temptation is the difference between hypothetical proposal-best-response sales and hypothetical full group compliance individual pay. Plausibility is the group sum of temptation. Fairness of proposals is measured as the difference between the minimum and maximum hypothetical full-compliance pay.

The regressions are all time series cross-sectional (panel) data analysis, random-effects GLS, using Stata.

4.4.2 The Two-Player Case

Compliance was 46.9% for leaders vs. 33.3% for nonleaders; however, the absolute difference between locations and proposals was 16.25 for nonleaders and 14.40 for leaders. So when nonleaders deviated, they deviated at a greater magnitude.

The data shows a strong correlation between the location and the previous period's location, and best response to the proposals. The actual proposals were not a good predictor of nonleader location. For the leaders, there is a much stronger correlation to the best response to the nonleader's previous period location and much weaker correlation to the leader's own previous period location. When I restrict my set to observations in which leader credibility and plausibility was high, the proposals became a much stronger predictor of locations and price stickiness was eliminated.

To determine nonleader compliance, important factors are group credibility (which corresponds to leader credibility in the 2-player case) in the previous period, the plausibility of the proposals, and the individual's proposed pay based on the proposals. However, in measuring actual deviation from proposals, rather than the compliance dummy variable, previous period sales become more of a factor, as does fairness and the temptation to deviate. Furthermore, leader credibility is less of a factor.

An issue with the two-firm case is that in many cases, the proposals correspond to the equilibrium of (50, 50). So I decided to look at the subset of the data in which the proposals did not equal (50, 50). When I did that, leadership credibility becomes slightly more important.

4.4.3 The Three-Player Case

Three-player groups: leader and nonleader average earnings are not significantly different (33.27 vs. 33.50, favoring nonleaders). Perhaps leaders sacrificed too much sales in failed attempts to lead its group. Credibility rate: the rate at which leaders followed their own proposal: 33.48% overall. Loyalty rate: the rate at which nonleaders followed their leader's proposals: 15.40% overall. Average deviation from proposals when not followed were 23.54 for leaders and 23.35 for nonleaders.

There is a moderately strong, positive correlation between the proposal and the location. The coefficient is slightly higher among leaders. This indicates that the proposals are somewhat reliable in predicting the location of the leaders, and that the leaders have some influence on the nonleaders. The lag location effect is not very strong, and it is slightly negative. This indicates a lot of flip-flopping, but it precludes a regular alternating scheme. A lot of switching around; not much price stickiness.

The two measures of deviation of sellers' chosen locations from the proposals are compliance and deviation. Both of these variables were run with the following arguments: lagged group compliance and lagged sales. The idea behind using lagged sales as an argument is that sellers might have a tendency to be less loyal/credible when results are unfavorable. This factor is insignificant for the nonleaders, but it is significant positive for leaders. This means leaders will tend to be more credible when they experience high sales. However, when I run the absolute difference between locations and proposals, the lag sales argument is significantly negative for leaders (insignificant for nonleaders).

The lag group compliance argument is a feedback mechanism. It measures the tendency of the leaders to comply as a function of its nonleaders' loyalty. And for the nonleaders, it measures their loyalty as a function of the compliance of other members in its group, including its own leader's credibility. These feedback arguments are very strongly correlated for both leader and nonleader with compliance. Nonleader deviation is also significantly correlated with lag group compliance. However, leader deviation is just borderline significantly negatively correlated with their group members' loyalty.

4.4.4 The Four-Player Case

Mean compliance was 15.1%. Location was strongly correlated with lag location for nonleaders, but not for leaders. Proposals were highly correlated with leaders, but nonleaders. When only those observations in which group compliance was nonzero was considered, the coefficient for proposals went up, and lag location became a slightly better predictor, with higher coefficient, for leaders. For nonleaders, both variables became less correlated.

Lag group loyalty is a good predictor for leader credibility and deviance; however, lag group compliance does not seem to correlate well with nonleader loyalty. Lag sales are not very correlated with compliance or magnitude of deviance for either leaders or nonleaders.

4.5 Concousions and Extensions

4.5.1 Conclusions

In this paper, I have attempted to describe the behavior of players in an experimental location game environment with one-way communication. The analysis of the data is ongoing, but it appears that leaders have some influence on the actions of their group, and that leader credibility is an important factor contributing to how much influence can be exerted.

Overall, the leaders in our experimental sessions were not able to influence their groups very well due to implausible and highly unfair proposals and disloyal group members. Group credibility generally deteriorated to a low level after a few periods. Leaders were not able to exploit their position in the form of increased average sales.

4.5.2 Extensions

Some possible ideas for analysis: Bayesian updating of leader credibility or group compliance, or a multilag opinion of this using a discount factor. Characterize deviations more specifically: into crowding and leapfrogging. A crowder would be someone who adjusts location slightly, but respects assigned ordering. A leapfrogger does not respect ordering. Obviously leapfrogging is a more severe deviation, and it creates more disorder in the group. And the rates of leapfrogging vs. crowding might be different with different group sizes.

There are other characteristics that might be used to determine plausibility, such as symmetry.

Extensions beyond this paper incluse simulations with automated sellers with various behavior algorithms. I will group them, see what the outcomes are, and compare them with experimental results: both with and without communication. I will be able to extend this to many more periods than practical or affordable in the lab as well. Beyond simulations with hard-coded behavioral algorithms, AI machine learning agents could be used.

In the four-firm case: the result of the proposals being further away from the PS NE than the average NC locations begs for the following modified session: run groups with proposals from the experimenter (no leaders). Suggest the NE and see if the distribution gets closer to PS NE. Then change the order and see what happens.

The game environment could include alternative play or even continuous play. I think this environment could have impact on the ability to observe other player's actions and to coordinate locations.

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Appendices

Appendix A

Proof of Lemma 1

Lemma A.1. *The distribution of a symmetric mixed-strategy equilibrium is not uniform for* $n = 4$.

Proof. For $n = 4$, the expected payoff for a firm located at a point, z, interior to the mixed-strategy support (not at endpoints):

$E(z)$ =(prob(z is leftmost))(($z+x$)/2) + (prob(z is second leftmost))(($z+y$)/2)-($z+w$)/2)) + (prob(z is third leftmost))(($z+v$)/2)-($z+u$)/2)) + (prob(z is rightmost))(1-(($z+r$)/2)), where x is the expected location of the next leftmost firm conditional that z is the leftmost firm. To determine the expected locations of these adjacent firms, I bifurcate the support at z. The value of x is equivalent to the expected location of the leftmost of $n-1$ firms randomizing on the support subinterval $[z, 1-(1/(2n-2))]$. This value is determined in a similar manner as the part of the proof in the main section. It is s'$/n$ to the right of z, where s' is the length of the support subinterval.

Also, y is the expected location of the 3rd leftmost conditional that z is second leftmost; w is the expected location of the leftmost firm; u is the expected location of the rightmost $|z$ is second rightmost; v is expected location of the 2nd leftmost $|z$ is third leftmost; and r is the expected location of the 2nd rightmost $|z$ is rightmost.

This works for any n. For example, for $n = 3$, suppose one firm locates at ½. For an equilibrium, it must be that the expected payoff is equal to the mixed-strategy expected payoff of 1/3. In this case, the expected payoff is the expected location of the neighboring firms times the expected relative orientation of the firms. Because the locations of the two other firms are independent, the probability that this firm is in the middle is 50 percent. Other times, the two other firms are both to the left or both to the right. The expected location of adjacent sellers conditional that one is a middle seller is 3/8 and 5/8. The expected profit of a middle seller is then 1/8. The expected location of the adjacent firm when one is the leftmost or rightmost is 5/12 or 7/12, respectively. The expected profit is thus 13/24. Therefore the expected profit is $.5(1/8) + .5(13/24) = 1/3$.

In the case of $n = 4$, $x = z + ((5/6) - z)/4$; $y = z + ((5/6) - z)/3$; $w = (z + (1/6))/2$; $u = ((5/6) + z)/2$; $v = (1/6) + 2(z - (1/6))/3$; and $r = (1/6) + 3(z - (1/6))/4$.

Specifically, when $z = 1/2$, $E(z) = (1/4)(13/24) + (3/4)((21 - 16)/36) = .1354167 + .1041667 = .2395834 < .25$. Therefore, a firm among a group playing the proposed mixed strategy support with uniform distribution can improve their payoff by moving some distribution from the middle to the support endpoints. Therefore, the mixed strategy density function is not uniform.

$n = 5$ and $n = 6$ are treated similarly. $\square$

Appendix B

Proof of Lemma 4

Lemma B.1. *If the distribution is uniform, the support for an n-firm symmetric mixed-strategy equilibrium would be* $[1/2(n-1), 1-(1/2(n-1))]$.

Proof. First, note that it has been shown that the mixed strategy support distribution is NOT uniform, but it is useful to calculate the support as an upper bound. Second, note that a deviating firm playing a pure strategy at any one point (and in particular, the left endpoint) in the support will receive the same payoff as when playing the mixed-strategy. Pure strategy played outside this interval will receive a lower payoff.

The expected payoff of a deviating firm playing the lower bound of the support of an n-player mixed-strategy equilibrium is determined by the expected location of the leftmost of $n-1$ firms randomizing within that support. Specifically, the market boundary is the midpoint between the lower bound of the interval and the expected location of the leftmost firm. Let $z =$ the lower (leftmost) of $n-1$ draws and $y_i =$ the location of firm i. Then

$F(z) = 1-\text{prob}(y_1, y_2, ..., y_{n-1} \geq z) = 1-(1-z)^{n-1}$

$f(z) = (n-1)(1-z)^{n-2}$

$E(Z) = \int (n-1)(1-z)^{n-2} z dz$

Integrate by parts:

$$\begin{vmatrix} -(1-z)^{n-1} & (n-1)(1-z)^{n-2}dz \\ z & dz \end{vmatrix}$$

$$E(z) = [-z(1-z)^{n-1} + \int_0^1 (1-z)^{n-1}dz]|_0^1 = [-z(1-z)^{n-1} - ((1-z)^n)/(n+1)]|_0^1 = 1/n.$$

This means that z is s/n to the right of the lowerbound of the support, x, where s is the length of the support, $= 1 - 2x$. Therefore, the actual value for z on the unit line is $x + (1 - 2x)/n$. Also, the payoff of a deviating firm playing pure strategies, $p = 1/n$, and $p = (x + z)/2$. Substituting and solving for z: $1/n = (x + z)/2$, solve for z: $z = 2/n - x$. So $2/n - x = x + (1 - 2x)/n$; $x = 1/(2n - 2)$. □

The following table summarizes the values for z, x, and p for 2-6 firms.

Table B.1: Summary values related to the mixed-strategy support by number of players.

Number of firms, n	Expected location of leftmost adjacent firm, $E(z)$	Infimum of mixed-strategy support, x	Payoff, p
2	1/2	1/2	1/2
3	5/12	1/4	1/3
4	1/3	1/6	1/4
5	11/40	1/8	1/5
6	7/30	1/10	1/6
n	$x + (1 - 2x)/n$	$1/(2n - 2)$	$1/n$

The density functions are illustrated in Figure 0.1. The vertical axis represents the density. The integral of the distribution function equals one, so, for example, the density function of a uniform distribution in a support of length .5 is equal to 2. The support for the mixed strategy grows larger (wider) as the number of firms increases. The case of two firms is trivial, in which the mixed-strategy support has measure zero, and so it coincides with the

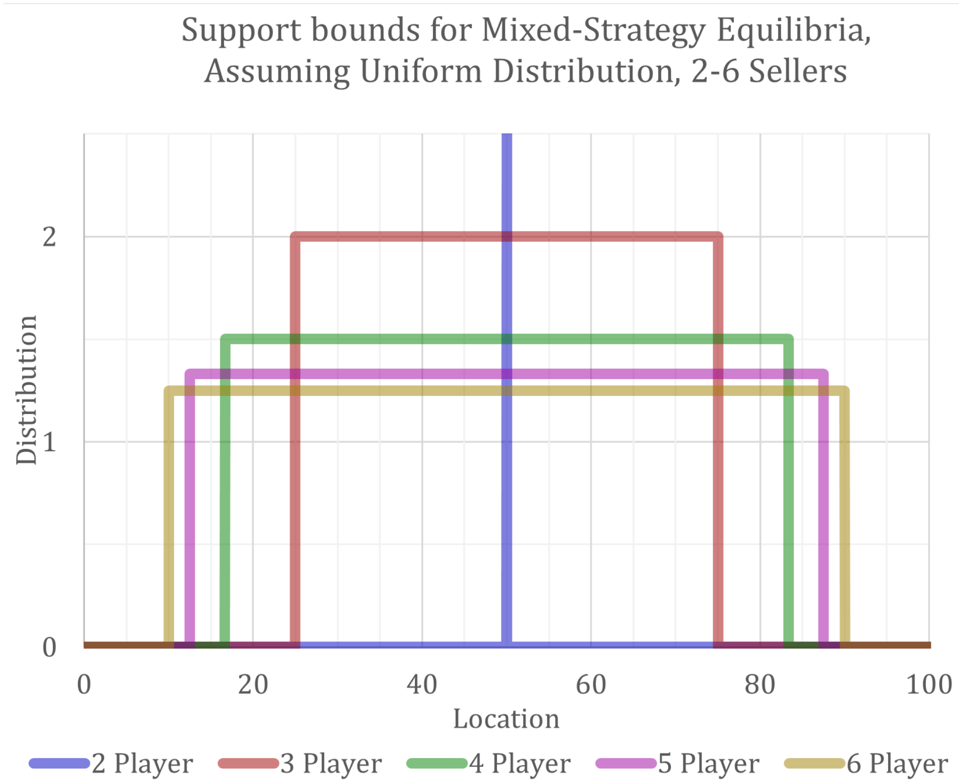


Figure B.1: uniform density

pure-strategy equilibrium of playing 1/2 with probability one. It should be noted that, while the density is depicted as uniform for 4 to 6 players, I have shown that the actual density is likely U-shaped, as described in subsection 3.2.2.

Appendix C

Density Function for the Support of a Four-Player Mixed-Strategies Equilibrium

We know from Appendix A that a uniform distribution is not an equilibrium. The actual equilibrium is U-shaped, continuous, and differentiable in the support. Suppose the solution is quadratic. For simplicity, consider a parabola[1] that is tangent at .5, and consider the following geometric properties.

First, to determine the support that satisfies the condition where endpoints yield a payoff of 1/4, the adjacent firm is now less proximate to the center. Specifically, the expected location of the leftmost firm is equivalent to the location of the center of mass from the infimum of the support and the midpoint. The center of mass will be the cubed root of 1/2 distance (0.7937006) from the center to the infimum.

The condition that the deviating firm playing the endpoint must receive 1/4 is that $(z + s)/2 = 1/4$, where s is the infimum and z is the expected location of the leftmost of three firms distributed on the support according to the density function. This will heretofore be known as the endpoint condition.

[1] The actual function is not quadratic.

Our second condition describes the position of z along the support: $1/2 - z = 0.7937006(1/2–s)$.

From these two equations, we estimate $s = 0.2212467$ and $z = 0.2787533$. The support measure is 0.5575066.

A deviating firm playing the midpoint will receive an expected payoff of 0.5878018 when all three other firms are to one side and an expected payoff of 0.2072617 when two firms are to one side. The total expected payoff is $0.25(0.5878018) + 0.75(0.2072617) = 0.1469505 + 0.1554463 = 0.3023968 > .25$.

This is too high, so a firm could increase its payoff by moving some support to the center. However, a linear combination of a uniform and tangent quadratic function could satisfy both the endpoint and midpoint tests. More formally, it is a quadratic function that has a minimum at 0.5, but has a positive density (not tangent).

Here is a more formal solution to this quadratic function. Find a density function $f(x)$ of the form $ax^2 + bx + c$ such that

(1) Int from s to z $f(x)dx$ = Int from z to ½ $f(x)dx$ (sets location of z),

(2) $z + s = 1/2$ (endpoint condition),

(3) $a = -b$ (sets the critical point of the parabola at $x = .5$),

(4) Int from s to $(1 - s)$ $f(x)dx = 1$ (normalizes the area under the density function), and

(5) The midpoint condition.

To find the midpoint condition, we need two locations. Location j will be the expected location of the leftmost of two firms in the interval $[.5, 1 - s]$.

(5a) Int from .5 to j $f(x)dx = 1/3$.

Also k will be the expected location of the leftmost of three firms in the same interval.

(5b) Int from .5 to k $f(x)dz = 1/4$.

And in the midpoint condition, $(j + .5)/2–((1 - k) + .5)/2 = 1/4$, or

(5c) $j + k = 1.5$.

Aside from these equations, the following inequalities should be met:

(6) $a > 0$ (parabola is increasing with distance from the center) and

(7) $a/4 + b/2 + c > 0$ (the midpoint has positive density).

The inequalities will not help us solve the system of equations. However, we have a system of 7 equations with 7 variables: z, s, a, b, c, j, and k. I need to find some mathematical tools to solve for some cubic roots. My current best guess is $f(x) = 25/3x2 - 25/3x + 10/3$. Testing this function, the endpoints of the support are at 0.1757546 and 0.8242454. The expected location of the leftmost of the three other firms, z, is 0.3242454. However, $j = 0.7370161$ and $k = 0.6857564$, for a total of $1.4227725 < 1.5$. So this function has too much central density.

Appendix D

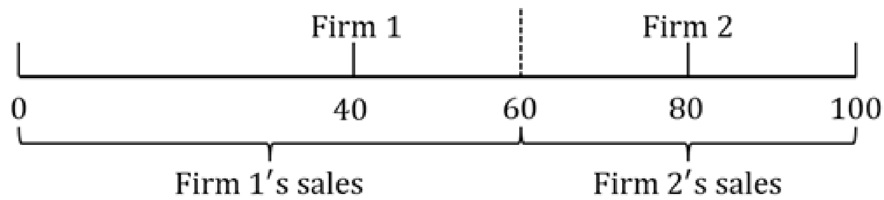
Instructions, Sessions Without Communication, Fixed Matching

This is an experiment in economic decision-making. If you make good decisions, you could make a considerable amount of money that will be paid to you in cash at the end of the session.

In this experiment, you are one of (2-6) sellers of a fictitious product in a market. We will be running many groups simultaneously. You will not be told with whom you are matched, but you will know how many sellers are in your group. There will be many periods, and you will be matched with the same sellers for all periods (different sellers after each period). Each group will be serving a market of 100 simulated customers. These customers are located along a road that is 100 blocks long, with one customer per block.

You and the other seller(s) will each be asked to choose a location on this road. This location must be a whole number from 0 to 100. Depending on the location of your firm, some customers will be closer to your store while others will be closer to the other seller's store(s). Each customer will purchase one unit of product from the nearest seller. In the case that two or more sellers choose the same location, the customers who solicit that location will be divided equally among the sellers at that location.

Here is an example:



In this example, firm 1 locates at 40 and firm 2 locates at 80. The market will be divided at 60, which is midway between the two firms. All the buyers between 0 and 60 will go to firm 1 because firm 1 is the closest. All of the buyers between 60 and 100 will go to firm 2. Thus firm 1 will sell 60 units, and firm 2 will sell 40 units.

You and the other sellers in your market will choose a location each period. In subsequent periods, you may either choose a different location, or you may repeat a previous location choice.

If you have any questions, raise your hand, and an instructor will assist you.

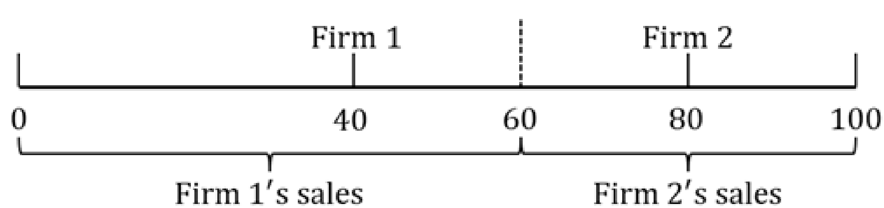
Appendix E

Instructions, Sessions With Communication, Fixed Matching

In this experiment, you are one of (2-6) sellers of a fictitious product in a market. We will be running many groups simultaneously. You will not be told with whom you are matched, but you will know how many sellers are in your group. There will be many periods, and you will be matched with the same group of sellers in all periods.

Each group will be serving a market of 100 simulated customers. These customers are located along a road that is 100 blocks long, with one customer per block. You and the other seller(s) will each be asked to choose a location on this road. This location must be a whole number from 0 to 100. Depending on the location of your firm, some customers will be closer to your store while others will be closer to the other seller's store(s). Each customer will purchase one unit of product from the nearest seller. In the case that two or more sellers choose the same location, the customers who solicit that location will be divided equally among the sellers at that location. Here is an example:

In this example, firm 1 locates at 40 and firm 2 locates at 80. The market will be divided at



60, which is midway between the two firms. All the buyers between 0 and 60 will go to firm 1 because firm 1 is the closest. All of the buyers between 60 and 100 will go to firm 2. Thus firm 1 will sell 60 units, and firm 2 will sell 40 units.

You and the other sellers in your market will choose a location each period. In subsequent periods, you may either choose a different location, or you may repeat a previous location choice.

Each group has a leader. Before you choose your location, the leader will send certain information to other members of his/her group. This information includes a proposed location choice for each of the group members, including the leader him/herself. Each group member (including the leader) must decide whether to choose the recommended location, or some other location. The group members cannot respond to the group leaders; it is one-way communication.

Specific instruction for group leaders: As the leader of your group, you will be able to send location proposals to members of your group before locations are chosen. At the beginning of each period, you will decide what locations to suggest to your group members, and you will record this information on your log sheet. You are free to use the back of your log sheet as scrap paper.

Specific instruction for nonleaders: In the beginning of each period, you will wait for the suggested locations from your leader. When the proposal appears on your screen, you will first write the proposals onto your log sheet, and then you will choose your location.

When everybody has chosen their location, the program will calculate and display the results, which you will record on your log sheet. Then press ”next period”. Group leaders are again able to distribute information. Then group members are again asked to choose a location for the period and so on, until the end of the experiment.

Your objective is to earn as much as you can over all periods. Your earnings will be rounded up to the nearest \$0.25. Your earnings rate will be: number per group/earnings per unit sold: 2/.67 cents, 3/1 cent, 4/1.33 cents, 5/1.67 cents, 6/2 cents. At any time during the experiment, raise your hand if you have any additional questions.

Appendix F

Sample Screen Shots

This appendix shows sample screen shots of the program that delivered the computer-assisted experimental sessions. The players selected their location while viewing a simple number line. The number had to be a whole number between 0 and 100, and when they typed in the number, a yellow circle appeared on the number line. A screen shot of the selection screen is shown in Figure 0.1.

After all players within a group have submitted their location choices, they receive a results screen that displays the location choices and sales of all players in their group (see Figure 0.2).

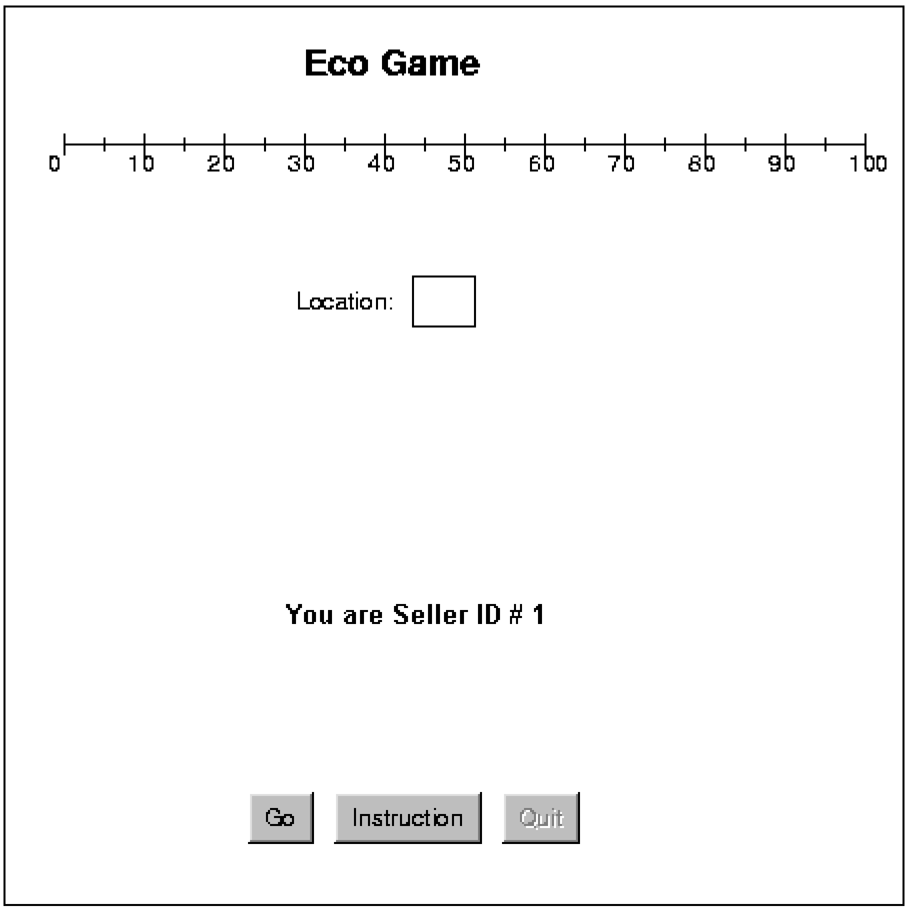


Figure F.1: Screenshot #1: screen to enter a location choice.

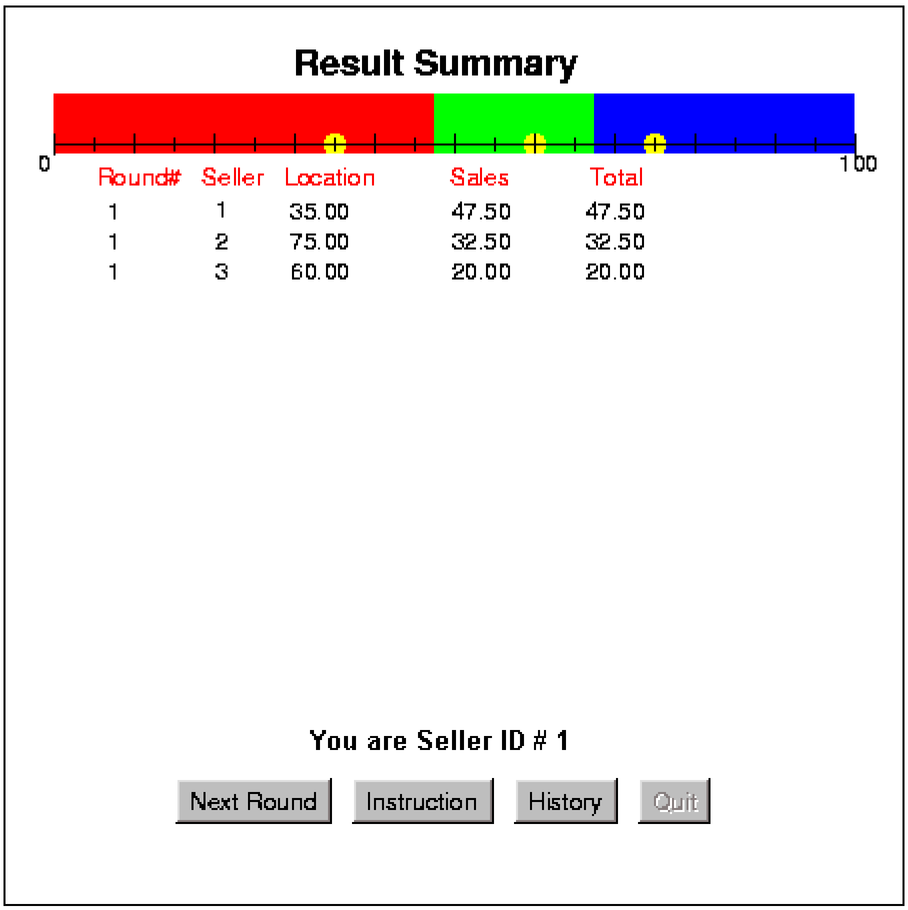


Figure F.2: Screenshot #2: results summary.