Spatial Competition With Incomplete Information

Anthony E. Dziepak*

July 2025†

Abstract

Asymmetric pure-strategy equilibria is found in a location model in which consumers have incomplete information about prices. Two firms are located in a unit line with a uniform distribution of consumers, possessing unitary demand. The consumers must walk to a firm (incurring sunk cost) to obtain full information about prices. After reaching a firm, the consumer decides from which firm to purchase. The following results are obtained: There exists a pure-strategy price equilibrium for location pairs that are neither too close nor too far apart. In addition, all such equilibria exhibit location asymmetry and price dispersion. There is no pure-strategy equilibrium in location and prices; however, conditions are such that mixed-strategies equilibria exist. Also, if firms are given the option to costlessly reveal prices to consumers, the lower-priced firm will reveal its price when firms are sufficiently apart. Finally, in a unit circle market space, a pure-strategy price equilibrium does not exist because there are no asymmetric locations possible for two firms on a unit circle.[1]

*I am grateful to Amoz Kats, Mamoru Kaneko, and Helmuth Cremer for comments.
†An earlier version was shared as VPI&SU Working Paper #E95-50, December 1995.
[1]Keywords: location, Hotelling

1 Introduction

This paper introduces shopping behavior to the traditional Hotelling (1929) location model. Consider two T-shirt vendors. Imagine you can decide to walk North or South, and you walk to nearest one, only to discover that the T-shirts are very expensive at that store. However, the other store is far away, so you pay the high price. However if the stores are closer together, you may balk at the expensive store and walk to the other store.

The scenario just described is similar to Hotelling's (1929) model of where firms should locate in a space with costly transportation. The illustration used is a game between two ice cream vendors along a linear beach, with customers uniformly distributed along the beach. Transportation cost, embodied in the consumer's walking effort, is linear with distance, and the quality of the product is assumed to be identical among both vendors. Assuming location is fixed and firms will choose only price, a unique equilibrium exists in pure strategies if the firms are sufficiently far apart. However, when firms are too close to each other, no price equilibrium exists in pure strategies. Also, with prices fixed and equal among all firms, the two firms will want to move toward each other, until they are both located at the same point.

A 2-stage game of location and prices is introduced by d'Aspremont, Gabszewicz, and Thisse (1979). In stage 1, firms choose location; in stage 2, locations are fixed, and firms choose price. If firms are sufficiently far apart, there is a price equilibrium in pure strategies, but firms want to get closer to each other. Eventually, firms are so close that they are tempted to undercut each other, and thus the price equilibrium no longer exists in pure strategies.

There exists equilibria in mixed strategies, however, as Dasgupta and Maskin (1986) point out. Osborne and Pitchik (1987) characterize the set of price-location equilibria where mixed strategies are allowed in prices, but locations are pure.

The major departure of this paper from these models is the introduction of incomplete information. Here, buyers do not have perfect information on prices prior to incurring some transportation costs. Of course, if buyers had perfect prior information, they would walk directly to the store that minimizes the sum of their purchase price and transportation cost, which could possibly be the further store. However, it is more often the case in the real world that one does not know the price of the goods until one is in the store and reads the price tag.

Suppose consumers do not initially know the prices that the two firms are charging. First, a potential customer may walk to the nearest firm. Upon entering the store, the consumer will learn the price that this firm is charging. Then, depending on the price charged and the distance to the other firm, the consumer must decide whether to purchase at Firm 1, or to walk over to Firm 2. After walking over to Firm 2, the consumer knows both prices and will either purchase at Firm 2, or walk back to Firm 1 and purchase.

Gabszewicz and Garella (1986) and (1987) are two papers that explore incomplete information in location theory. In Gabszewicz and Garella (1986), consumers do not know the prices of the firms with certainty, but form "subjective probabilistic beliefs" about these prices. Assuming that the consumer only knows the price of the closer firm, the buyer can either buy at the known firm's price or incur search cost to learn the price of the distant firm, and then decide to buy at either of the firms. It turns out that if a price equilibrium exists, both firm's prices are equal. Also, a price equilibrium exists only if buyers have prior knowledge of a price range in which the distant firm's price lies, and the firms are located far enough apart. Also, in equilibrium, no consumer searches.

In Gabszewicz and Garella (1987), buyers know the two firms exist, but do not know the prices; however, buyers know the average price. Soliciting is costly, and buyers must solicit a firm to discover price. Once a buyer solicits firm i, the buyer knows the price at both firms, since the buyer has prior knowledge of the average price.[2] The buyer may then purchase at firm i, or solicit the other firm and purchase there (an additional cost). Assuming soliciting cost increases linearly with distance, buyers will always solicit the nearest firm first. Buyers

[2]This is equivalent to saying visiting the closest shop provides perfect price information. See Phlips (1988), 2.4.3.

will buy from this nearest seller if and only if this firm's price is lower than the other firm's price plus the cost of soliciting the other shop.

The results are that unique pure strategies price equilibria exist only if firms are sufficiently close to each other, and these equilibria necessarily display price dispersion and locational asymmetry. Also, in a two-stage game of locations and prices, the larger firm does not care about its location, but the smaller firm wants to move close to the larger firm.

The paper just described assumes that the buyer solicits firms "by phone" (i.e., the cost of soliciting the firms are independent of each other). However, one may change the assumptions so that the transportation costs can be made more like solicitation in person. In this case, when a buyer lives to the left of both firms and first solicits Firm 1, the cost of the second solicitation will only be the transportation cost between the two firms. In Gabszewicz and Garella (1987), this person would be in effect walking to the close firm, then walking back home, then walking past the close firm again to get to the further firm. This key difference in the nature of the solicitations will give new results, and this is the focus of the second part of this paper.

Another phenomenon to consider is price displays as attractions. When is a firm have an incentive to advertise its price so that it is visible from afar, or to advertise its price in media? Perhaps a complete information situation yields higher payoffs. The third part of this paper attempts to explain when firms should reveal prices by advertising.

Finally, some have found a unit circle market space to be more useful than the unit line in models of spatial competition. The unit circle was first introduced by Eaton and Lipsey (1975), in which price equilibria are shown to exist for certain locations. Salop (1977) introduces the two-stage game of locations and prices, in which multiple pure strategies equilibria exist. Kats (1989) has shown that all mixed strategies in prices (for locations in which pure strategies do not exist) are Pareto dominated by pure strategies for locations farther apart.

Unlike the linear model, the unit circle space is homogeneous because there are no boundaries. However, because of this homogeneity, there does not exist the possibility of locational asymmetry necessary for the types of equilibria that will be discussed in this paper.

The remainder of this paper is organized as follows: In Part 2, spatial competition with incomplete information is examined with solicitation in person (rather than by phone) in linear space. This alteration of the solicitation cost assumption changes the results; the number of location combinations that have a price equilibrium in pure strategies is reduced due to the addition of the possibility of undercutting. Also, there is no pure strategies equilibrium in a two-stage game of locations and prices; this is also due to undercutting.

In Part 3, firms are given the option to costlessly reveal their price, and thus revert to the perfect information case. In Part 4, I change the space to a unit circle with in-person solicitation. In the unit circle model, no pure strategy price equilibrium exists. In Part 5, I discuss some additional extensions, including quadratic transportation costs; and Part 5 is a discussion and conclusion.

2 Linear Space, Solicitation in Person

This section is the most similar to Gabszewicz and Garella (1987) in that the area is a unit line market space. The key difference lies in the assumption about the transportation cost. Now, a buyer is allowed to walk from one firm to the other without having to walk home first. With solicitation "in person," there is a difference in transportation costs only for the consumers located either to the left, or to the right, of both firms. For these buyers, the cost of soliciting the further firm is not the distance from their house to the far firm, but rather only the distance between the two firms. The model is as follows: Let x be the location of a buyer, y_1 and y_2 are the locations of firms 1 and 2 respectively, with $y_1 \leq y_2$. Without loss of generality, the length of the line segment shall be one. Also, the transportation cost shall be linear and normalized to one.[3] The

[3]Since the buyer has to walk to the firm and back, transportation costs are actually $c = 1/2$ each way, thus $c = 1$ for a round trip.

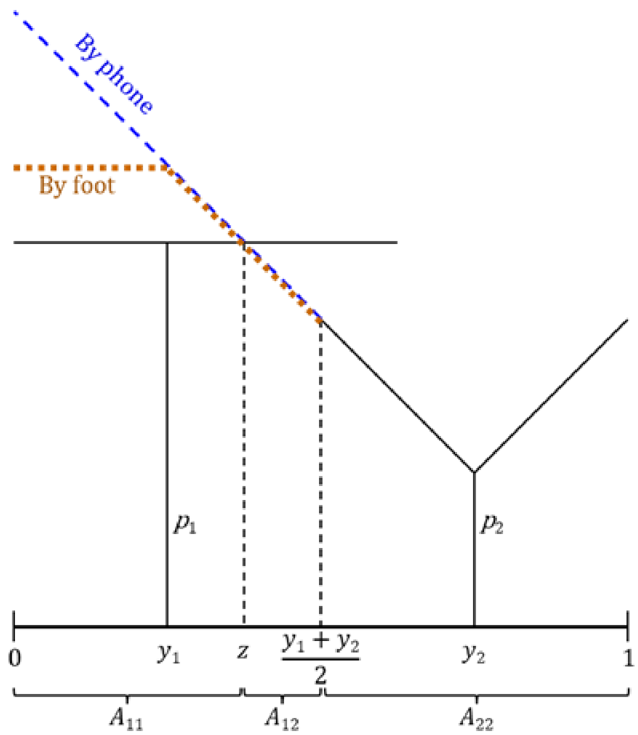

Figure 1: Comparison of phone and in-person solicitation.

cost of soliciting the further shop is:

$$
\begin{aligned}
c(x) &= y_2 - y_1 && \text{if } \ 0 \leq x < y_1, \\
&= y_2 - x && \text{if } \ y_1 \leq x < (y_1 + y_2)/2, \\
&= x - y_1 && \text{if } \ (y_1 + y_2)/2 \leq x \leq y_2, \\
&= y_2 - y_1 && \text{if } \ y_2 < x \leq 1.
\end{aligned}
$$

The buyer's costs in both cases are compared in Figure 1.

As in Gabszewicz and Garella (1987), let A_1 be the set of buyers who solicit Firm 1 first; $A_1 = [0, (y_1 + y_2)/2]$. Similarly, let A_2 be the set of buyers who solicit Firm 2 first; $A_2 = [(y_1 + y_2)/2, 1]$. Note that the buyers always solicit the closest firm first because this strategy minimizes expected search costs. Buyers will have to incur the cost of the first solicitation as a sunk cost. The buyer then compares the price at this (near) firm with the price of the further firm plus the additional solicitation cost necessary for purchase at the further firm. Thus define A_{11} as the set of buyers who solicit Firm 1 first, and then decide to buy at Firm 1: $A_{11} = [0, \min[(y_1 + y_2)/2, z]]$, where z is the location of the buyer who is indifferent between buying at either firm. Let p_1 and p_2 be the prices that Firms 1 and 2 charge respectively. Then when $z \in A_1$, $z = p_2 - p_1 + y_2$; and when $z \in A_2$, $z = p_2 - p_1 + y_1$. Hence, $A_{11} = [0, min(y_1 + y_2)/2, p_2 - p_1 + y_2]$. Similarly, A_{22} is the region of buyers who first solicit Firm 2 and buy at Firm 2; $A_{22} = [max(y_1 + y_2)/2, p_2 - p_1 + y_1, 1]$.

Now define A_{21} (resp. A_{12}) as the set of buyers first soliciting Firm 1 (resp. 2), but then buying at Firm 2 (resp. 1) i.e.,

$$A_{12} = A_1 - A_{11} = \min[(y_1 + y_2)/2, p_2 - p_1 + y_2, (y_1 + y_2)/2],$$

$$(\text{resp. } A_{21} = A_2 - A_{22} = [(y_1 + y_2)/2, \max[(y_1 + y_2)/2, p_2 - p_1 + y_1]]).$$

Note that at least one element of A_{12}, A_{21} is empty.[4] Let $p'1$ be the price below which Firm 1's demand would increase from penetrating Firm 2's "natural market" (i.e., $p'1$ is the solution to the equation $(y_1 + y_2)/2 = p_2 - p_1 + y_1$, or $p'_1 = p_2 - (y_2 - y_1)/2$). Also, let $p"_1$ be the price above which, given p_2, Firm 1's demand would begin to decrease again (i.e., $p"_1$ is the solution to the equation $(y_1 + y_2)/2 = p_2 - p_1 + y_2$, or $p"_1 = p_2 + (y_2 - y_1)/2$). Thus the interval $(p', p")$ is the same demand-inelastic price range as in Gabszewicz and Garella(1987), but now add the segment $[p\hat{}, p\hat{}\hat{}]$ with the following endpoints: Let $p\hat{}_1$ be the highest price at which Firm 1 can undercut Firm 2, given p_2 (i.e., $p\hat{}_1$ is the solution to the equation $y_2 = p_2 - p_1 + y_1$; or $p\hat{}_1 = p_2 - (y_2 - y_1)$). Similarly, let $p\hat{}\hat{}_1$ be the lowest price at which Firm 1 is undercut by Firm 2, given p_2 (i.e., $p\hat{}\hat{}_1$ is the solution to the equation $y_1 = p_2 - p_1 + y_2$; or $p\hat{}\hat{}_1 = p_2 + (y_2 - y_1)$).

Note that there are discontinuities of the demand and revenue (payoff) functions of Firm 1 at $p\hat{}_1$ and at $p\hat{}\hat{}_1$. With a price below $p\hat{}_1$, Firm 1 gets the

[4] That is, it will never be the case where some customers from Firm 1's natural market buy from Firm 2 and simultaneously some customers from Firm 2's natural market buy from Firm 1.

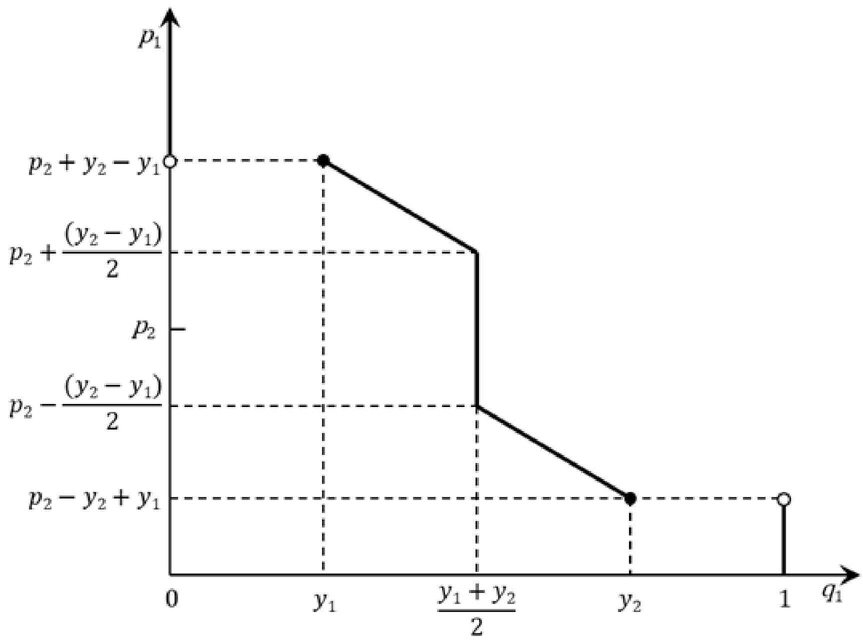

Figure 2: Demand function.

whole market; with a price above $p\hat{}\hat{}{}_1$, Firm 1 gets no demand. Then the contingent demand schedule for Firm 1's t-shirts can be written as:

$$
\begin{aligned}
q_1(p_1, p_2) &= 1 && \text{if } 0 \leq p_1 < p_2 - (y_2 - y_1) \; (= p\hat{}{}_1), \\
&= p_2 - p_1 + y_1 && \text{if } p_2 - (y_2 - y_1) \leq p_1 < p_2 - (y_2 - y_1)/2 \qquad (= p_1'), \\
&= (y_1 + y_2)/2 && \text{if } p_2 - (y_2 - y_1)/2 \leq p_1 \leq p_2 + (y_2 - y_1)/2 \; (= p"_1), \\
&= p_2 - p_1 + y_2 && \text{if } p_2 + (y_2 - y_1)/2 < p_1 \leq p_2 + (y_2 - y_1) \; (= p\hat{}\hat{}{}_1), \\
&= 0 && \text{if } p_2 + (y_2 - y_1) < p_1.
\end{aligned}
$$

The demand (market) function for Firm 1 is illustrated in Figure 2, and the revenue (payoff) function is shown in Figure 3. Notice that this payoff function combines the discontinuous characteristics of d'Aspremont et al. (1979) with the price-inelastic demand region of Gabszewicz and Garella (1987).

The price dispersion characteristics of a price equilibrium from Gabszewicz

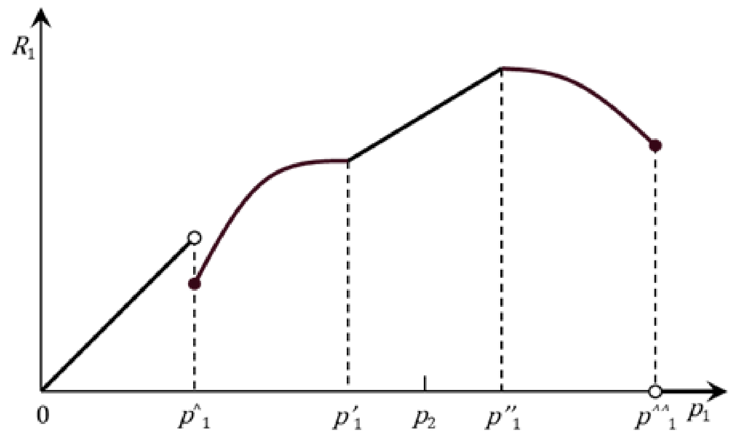

Figure 3: Revenue function.

and Garella (1987) still hold here (i.e., one firm must be pricing on the upper negatively-sloped portion of its demand curve, while the other firm must be pricing on the lower negatively-sloped portion of its demand curve). This gives rise to two possible types of equilibria: a type I equilibrium is defined as one in which Firm 1 is charging the lower price, while a type II equilibrium is defined as one in which Firm 2 has the lower price. Thus in a type I equilibrium, $p\hat{}_1 \le p_1 \le p'_1$; $p"_2 \le p_2 \le p\hat{}\hat{}_2$; $(p_1 < p_2)$. Solving for the equilibrium prices, $R_1 = p_1 q_1 = (p^*_2 - p_1 + y_1)p_1$; $R_2 = p_2 q_2 = (1 - (p_2 - p^*_1 + y_1))p_2$. From first order conditions, $p^*_1 = 1 + y_1/3$; $p^*_2 = 2 - y_1/3$; or for a type II equilibrium, $p^*_1 = 1 + y_2/3$; $p^*_2 = 2 - y_2/3$. (Complete math is in the appendix.)

It suffices to examine the type I equilibrium only, keeping in mind that every type II equilibrium is equivalent to a corresponding type I equilibrium with the y_1's and y_2's switched. A type I equilibrium must satisfy: $p\hat{}_1 \le p^*_1 \le p'_1$. Substituting values for p^*_1 and p^*_2, two conditions are obtained: (see App. 2.2)

$$y_2 \le (y_1 + 1)/3, \tag{1}$$

$$y_2 \le (2 - y_1)/3. \tag{2}$$

Also, a type I equilibrium must satisfy: $p"_2 \le p^*_2 \le p\hat{}\hat{}_2$. From this the same conditions, (1) and (2), are obtained. To make sure there is no incentive for undercutting: For a price equilibrium, it must be that for Firm 1, $R_1(p\hat{}_1, p_2^*) < R_1(p_1^*, p_2^*)$; substituting in values for $p\hat{}_1$ and p_2^*:

$$y_2 > -((y_1^2 - 4y_1 - 5)/9) \text{ (see app. 2.3).} \tag{3}$$

Also for Firm 2, $R_2(p_1^*, p\hat{}_2) < R_2(p_1^*, p_2^*)$ From this:[5]

$$y_2 > -((y_1^2 - 16y_1 + 1)/9). \tag{4}$$

For an equilibrium in both prices and locations, it must be that $dR_1/dy_1 = dR_2/dy_2 = 0$, at some price equilibrium (p_1^*, p_2^*) for some y_1 and y_2 that satisfies conditions (1) through (4). $dR_1/dy_1 = (2y_1 + 6)/9 = 0$; $y_1 = -3$. Thus for all $y_1 > 0$, $dR_1/dy_1 > 0$. Firm 1 always wants to move to the right, or get closer to Firm 2. However, $dR_2/dy_2 = 0$ for all y_2; thus for a type I equilibrium, Firm 2 does not care at all about its own location.

Claim: There are locations that satisfy the conditions necessary for a type I equilibrium.

Proof: Assume $y_1 = 0$, then from the conditions necessary for a type I equilibrium, all locations for firm 2 such that $5/9 < y_2 \leq 2/3$ is a pure strategies type I equilibrium.[6]

Notice that when y_2 is too large, Firm 1's profit-maximizing price is too large to induce defectors from Firm 2's natural market. Also, when y_2 is too small, undercutting becomes a viable option for firm 1.

Claim: There is no location-symmetric equilibrium in prices.

Proof: The situation is illustrated in Figure 4. At a type I equilibrium, any firm raising or lowering its price would result in an equal gain or loss in its market area. Thus, the payoff rectangles of the firms must be squares (i.e., the price must equal the width of the market). Suppose there exists a symmetric

[5] Of course, the lower-priced firm is going to undercut first; this is included for completeness.

[6] The set of locations that support a pure strategies price equilibrium, for both phone solicitation and in-person solicitation, is illustrated in Figure 5, which will be described fully in the next section.

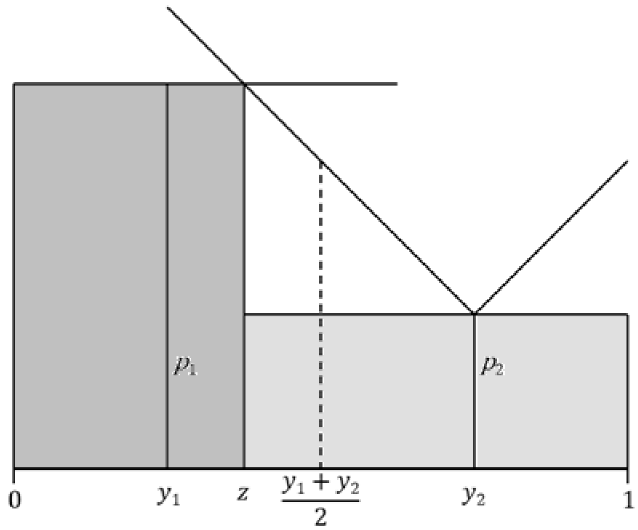

Figure 4: No symmetric equilibrium.

price equilibrium; then $(y_1 + y_2)/2 = 1/2$. Also, A_{21} is nonempty, so $z > 1/2$. We know $q_1 + q_2 = 1$, so $q_2 < 1/2$; thus $p_2 < 1/2$. Also, because $q_1 > 1/2$, $p_1 > 1/2$; hence $p_1 > p_2$. However, a type I equilibrium requires $p_1 < p_2$; a contradiction. //

Intuitively, it makes sense that there must be a locational asymmetry to support this price dispersion among otherwise identical firms. Notice when pure strategies equilibria exist, the firm closer to the center of the market will be the higher-priced firm.[7]

Claim: There exists no equilibrium in both prices and locations, in pure strategies.

Proof: Looking at Firm 1's payoff as a function of firm locations, R_1 is increasing in y_1. Thus Firm 1 always wants to move closer to Firm 2. Eventually, they will be close enough so that Firm 1 has an incentive to undercut Firm 2, and a pure strategy price equilibrium cannot exist when undercutting takes

[7] Of course, an assumption made is that consumers do not recognize this; otherwise, a buyer might decide to solicit the further firm first.

place. //

3 Price Revelation

This section considers the possibility of firms to costlessly advertise their price. If any firm reveals its price to the market, the situation becomes one of perfect information because of the information endowment of average price. A firm will want to reveal its price if its payoff is higher in the perfect information case than in the imperfect information case.

The payoffs for the firms in a type I equilibrium are as follows: $R_1 = (y_1 + 1)2/9$; $R_2 = (2 - y_1)2/9$. As an illustration, take the case where $y_1 = 0$. Then from conditions (1) through (4), any location in which $5/9 < y_2 < 2/3$ is a pure strategies equilibrium. Furthermore, with phone solicitation, any location of Firm 2 in the interval [0, 2/3] is a pure strategies equilibrium.[8] Payoffs, for Firm 1 and Firm 2 are respectively: 1/9 and 4/9, regardless of the location of Firm 2, where a pure strategy exists.

Comparing these payoffs with perfect information mixed strategies payoffs, for Firm 2 locations in the interval $[0, \approx 3/8]$, mixed strategies payoffs are low, and neither firm chooses to reveal price. In the interval $[\approx 3/8, \approx 4/9]$, however, Firm 1 prefers to reveal prices, while firm 2 wished that Firm 1 didn't reveal prices. And finally, when y_2 is greater than 4/9, both firms prefer the perfect information case.

The revelation preferences for all location pairs are illustrated in Figure 4. The regions were generated by comparing the pure strategies incomplete information payoffs with the profit contours in Figure 4 from Osborne and Pitchik (1987). Comparing profits in each case, if firms are close together, neither firm wants to reveal its price. If firms are sufficiently far apart, the lower-priced firm will want to reveal its price. Also, if firms are even farther apart, the higher-priced firm can also do better in the perfect information case. Notice it is never

[8] Condition #2 is the only condition necessary for a pure strategies price equilibrium with phone solicitation.

the case where firm 2 reveals prices when Firm 1 prefers to stay in the imperfect information case.

Now that price revelation is an option, Firm 2's location matters to Firm 2. Firm 2 must not be too far away from Firm 1, or Firm 1 will reveal the prices. Thus Firm 1 would want to locate close enough to Firm 2 to prevent Firm 2 from revealing the prices. Yet if Firm 2 locates sufficiently far away from Firm 1, then Firm 2 also prefers the perfect information case.

Firm 1 would like to get close to Firm 2 in the imperfect information case; however, Firm 1 does not want to do this in the perfect information case.[9] Notice that the perfect information case in the two-stage game of mixed strategy prices and pure strategy locations Pareto dominates all locations of pure strategies price equilibria in the imperfect information case.

4 Circular Space

The unit circle was introduced by Eaton and Lipsey (1975) in an attempt to avoid the location asymmetries associated with the endpoints of the linear segment. However, as will be shown, these very asymmetries of location are necessary for pure strategies price equilibria in the incomplete information environment, and thus no such equilibria exist in circular space.

This section examines in-person solicitation applied to a unit circle market area. The results for a unit circle with phone solicitation will be identical in pure strategies (although more complicated). The difficulties of in-person solicitation in circular space is mentioned in the appendix.

The circular model with incomplete information is as follows: The cost of

[9]For the location best-response function of firms faced with mixed strategies prices, see Figure 4 of Osborne and Pitchik (1987).

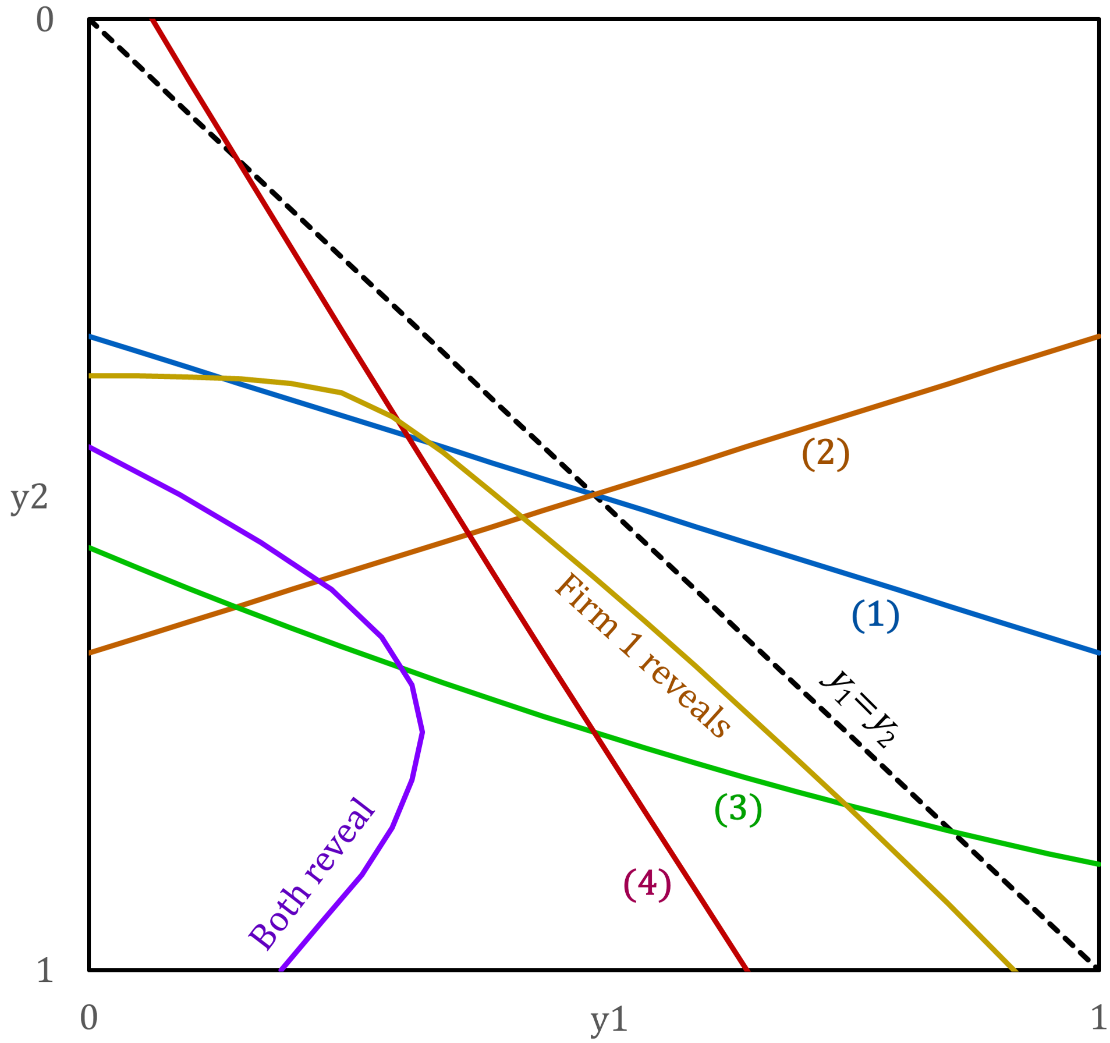

Figure 5: Phase diagram showing pure-strategy equilibria and price revelation.

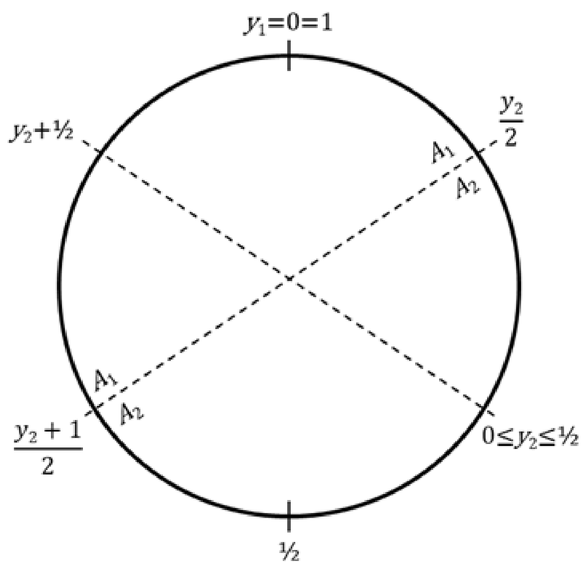

Figure 6: Circular market space.

the second solicitation for an A_1 household,

$$
\begin{aligned}
c(x) &= y_2 - x && \text{if } 0 \leq x \leq y_2/2 \text{ (buyers inbetween firms),}\\
&= y_2 - y_1, \text{ or } y_2 && \text{if } y_2 + 1/2 \leq x \leq 1 \text{ (buyers just to the left of Firm 1),}\\
&= x - 1/2 && \text{if } (y_2+1)/2 \leq x \leq y_2 + 1/2 \text{ (buyers far left of Firm 1).}
\end{aligned}
$$

The last case is when it is cheaper for a buyer to make a whole circuit rather than backtrack. This is the set $x \in A_1 | x \leq y_2 + 1/2$. Also, the last two cases can be combined as $min(y_2, x - 1/2)$.[10] The circular market space is illustrated in Figure 7.

$A_1 = [(1 + y_2)/2, y_2/2]$; $A_2 = [y_2/2, (1 + y_2)/2]$.

Defining $|A_i|$ as the length of A_i around the circle, note that $|A_1| = |A_2| = 1/2$.

[10]The cost of the second solicitation for an A_1 buyer in the case where it is cheapest to make a whole circuit equals the cost of making the circuit minus the sunk cost of the first solicitation $= 1/2 - 1 + x = x - 1/2$.

$A_{11} = [max[y_2 + p_1 - p_2, (y_2 + 1)/2], min[y_2/2, y_2 - (p_1 - p_2)]]$,
$A_{22} = [max[p_2 - p_1, y_2/2], min[(y_2 + 1)/2, y_2 + 1/2 - p_2 + p_1]]$, (see app. 4.1).
$A_{12} = A_1 - A_{11}$; $A_{21} = A_2 - A_{22}$.

Note that at least one element of $[A_{12}, A_{21}]$ is empty. Also, the A_{ij} region may be divided into two subregions: Let $\dot{A}_{ij}, i \neq j$ be the set between the firms where they are closer together (i.e., $\dot{A}_{ij}$ subset [0,1/2]); and define $\ddot{A}_{ij}$ as the other part of A_{ij} (i.e., between the firms where they are farther apart $(\dot{A}_{ij} + \ddot{A}_{ij} = A_{ij})$). Also let $|A_{ij}|$ be the length of set A_{ij} around the circle. As before, let $|A_{ij}|$ be the length of interval A_{ij} on the circle.

Claim: $|\dot{A}_{ij}| = |\ddot{A}_{ij}|$.
Proof: Suppose $x = y_2/2$, then the cost of the second solicitation $= y_2 - y_2/2 = y_2/2$. Now suppose $x = (y_2+1)/2$, then the cost of the second solicitation would also equal $(y_2 + 1)/2 - 1/2 = y_2/2 + 1/2 - 1/2 = y_2/2$. //[11]

Thus, the A_{12} areas will always be the same "size" on either side of the firm. In other words, let p'_{1L} be the lowest price at which buyers from A_2 immediately to the left of A_1 do not solicit Firm 1. p'_{1L} is the solution to the equation $(1+y_2)/2 = p_1 - p_2 + 1$; or $p'_{1L} = p_2 - (1 - y_2)/2$. Similarly, let p'_{1R} be the lowest price at which buyers from A_2 to the right of A_1 do not solicit Firm 1. p'_{1R} is the solution to the equation $y_2/2 = p_2 - p_1$; $p'_{1R} = p_2 - y_2/2$.

Also let $p"_{1R}$ be the solution to the equation $y_2/2 = p_2 - p_1 + y_2$. $p"_{1R} = p_2 + y_2/2$ (i.e., $p"_{1R}$ is the highest price at which no buyer in the right part of A_1 would solicit Firm 2). Similarly, $p"_{1L}$ is the highest price at which no buyer in the left part of A_1 would solicit Firm 2. $p"_{1L}$ is the solution to the equation $(1 + y_2)/2 = p_1 - p_2 + y_2$; $p"_{1L} = p_2 + (1 - y_2)/2$. Then $p'_{1L} = p'_{1R}$;

[11] For phone solicitation, A'_{ij} is nonempty if $A"_{ij}$ is nonempty. Proof: Suppose $x = y_2/2$, then the cost of the second solicitation equals $y_2 - y_2/2 = y_2/2$. Now suppose $x = (y_2 + 1)/2$, then the cost of the second solicitation equals $(y_2+1)/2 - y_2 = y_2/2 + 1/2 - y_2 = 1/2 - y_2/2 = (1-y_2)/2$. Then $(1-y_2)/2 \geq y_2/2$ because $0 \leq y_2 \leq 1/2$. So the A_{12} segment immediately to the right of Firm 1 will become nonempty subsequent to the segment immediately to the left of Firm 1, as p_1 increases relative to p_2. // This leads to a more complicated payoff function.

and $p"_{1L} = p"_{1R}$.[12]

Again, $p"_1$ is the highest price at which A_{12} is empty (i.e., $p"_1$ is the solution to the equation $y_2/2 + p_2 = p_1$; $p"_1 = p_2 + y_2/2$). Similarly, p'_1 is the lowest price at which A_{21} is empty; $p'_1 = p_2 - y_2/2$. Also, $p\hat{}\hat{}_1$ is the lowest price at which Firm 2 does not undercut Firm 1; $p\hat{}\hat{}_1 = p_2 + y_2$. And, $p\hat{}_1 = p_2 - y_2$. Note that Firm 1 will always be undercut from the right, and Firm 2 will always be undercut from the left.

The demand for Firm 1's t-shirts,[13]

$$
\begin{aligned}
q_1(p_1, p_2) &= 1 && \text{if } 0 \leq p_1 < p\hat{}_1 \text{ (Firm 1 undercuts 2),} \\
&= 1/2 - y_2 + 2p_2 - 2p_1 && \text{if } p\hat{}_1 \leq p_1 < p'_1 \text{ (Firm 1 encroaches),} \\
&= 1/2 && \text{if } p'_1 \leq p_1 \leq p"_1 \text{ (inelastic demand),} \\
&= 1/2 + y_2 + 2p_1 - 2p_2 && \text{if } p"_1 < p_1 \leq p\hat{}\hat{}_1 \text{ (Firm 1 concedes),} \\
&= 0 && \text{if } p\hat{}\hat{}_1 < p_1 \text{ (Firm 2 undercuts 1).}
\end{aligned}
$$

The demand and payoff functions for in-person solicitation are the same whether on a unit line or a unit circle. Thus refer to Figure 3 for the demand function and to Figure 4 for the payoff function.

Looking at a type I equilibrium,

$R_1 = p_1 q_1 = p_1(1/2 + y_2 - 2p_1 + 2p_2^*)$.

$R_2 = p_2 q_2 = p_2[1 - (1/2 + y_2 - 2p_1^* + 2p_2)]$.

From first order conditions,

$p_1^* = R - y_2/6; p_2^* = R + y_2/6$ (see app. 4.2).

A type I equilibrium must satisfy (1): $p^1 \leq p_1^* \leq p'_1$; (2): $p"2 \leq p_2^* \leq p\hat{}\hat{}_2$.

[12] In phone solicitation, since $0 \leq y_2 < 1/2$, the range of prices $[p'_{1R}, p"_{1R}]$ is contained within the range of prices $[p'_{1L}, p"_{1L}]$ (i.e., the buyers who solicit the further firm first will lie between the firms where they are closer together, which is contained in the segment $[0, y_2]$).

[13] In the second case, demand equals $1 - (z' - z) = 1 - (y_2 + 1/2 - p_2 + p_1) + (p_2 - p_1) = 1/2 - y_2 + 2p_2 - p_1$.

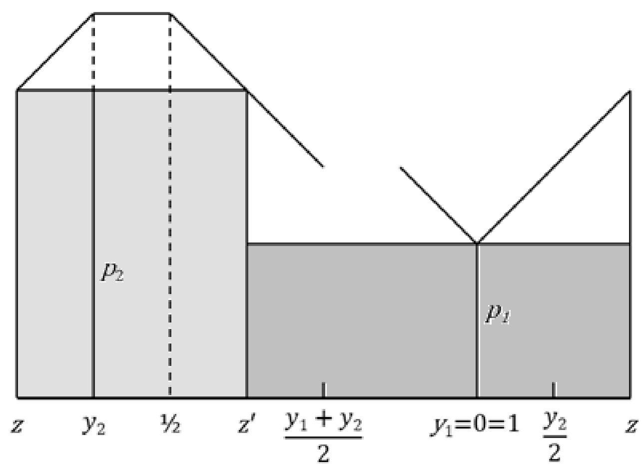

Figure 7: No pure-strategy equilibrium.

Claim: No price equilibrium exists in pure strategies.

Proof: The problem is illustrated in Figure 8. In a type I equilibrium, an increase [resp. decrease] in a firm's price by a factor of one will result in a reduction [increase] in that firm's market area by a factor of two (due to defecting from both sides). Therefore, for payoff rectangles to be maximized, their width must be twice their height. Also, in a type I equilibrium, A_{21} is nonempty; thus, $z > y_2/2$. Hence, $q_1 > 1/2$, and $p_1 > R$. Since $q_1 + q_2 = 1$, we know that $q_2 < 1/2$; also $p_2 < R$. Hence $p_1 > p_2$, but a type I equilibrium requires $p_1 < p_2$; a contradiction. //

That no pure-strategy price equilibria exist is to be expected because circular space does not allow for location asymmetries.

Algebraically, the condition that is not met is: $p_1^* \leq p_1'$ (see App. 4.3).

5 Extensions

This section contains extensions of the information considerations. It includes welfare considerations, quadratic transportation costs, mixed strategies, and the shape of the information curve.

Welfare Considerations

There are additional transportation costs associated with switching in the imperfect information case. Thus it is difficult to compare the welfare implications of a perfect information case with an imperfect information case. The socially optimum locations are 1/4 and 3/4, with prices close enough that everyone shops at the nearest firm. However, this is not an equilibrium in the imperfect information case.

It may then be in the public interest for the government to require the firms to reveal their prices. However, there is no pure strategies equilibrium in the perfect information case, either.

Quadratic Transportation Costs

D'Aspremont et al. (1979) noted that maximum differentiation (or dispersion) occurs in linear space when transportation costs are quadratic. If we are to incorporate quadratic transportation costs in this model, first with phone solicitation in linear space, the cost of the second solicitation (at the further shop) would be: $(y_2 - x)^2$, if x is in A_1; or $(x - y_1)^2$, if x is in A_2. $A_{11} = [0, min(y_1 + y_2)/2, z]$, where z is indifferent between p_1 and $p_2 + (y_2 - x)^2$. When A_{11} is nonempty, $z = y_2 - \sqrt{p_1 - p_2}$. When A_{12} is nonempty, $z = y_1 - \sqrt{p_2 - p_1}$. p'_1 is defined as: $(y_1 + y_2)/2 = y_1 + \sqrt{p_2 - p'_1}$; or $p'_1 = p_2 + (y_2 - z)^2$, where $z = (y_2 + y_1)/2$; $p'_1 = p_2 + y_1^2/4 - y_1 y_2/2 + y_2^2/4$; $p'_1 = p_2 + (y_1 - y_2)^2/4$.

The demand for Firm 1:

$$
\begin{aligned}
q_1(p_1, p_2) &= 1 && \text{if } \ 0 \le p_1 < p_2 - y_2^2, \\
&= && \text{if } \ p_2 - y_2^2 \le p_1 < p'_{,} \\
&= (y_1 + y_2)/2 && \text{if } \ p'_1 \le p_1 \le p"_1, \\
&= && \text{if } \ p"_1 < p_1 \le p_2 + y_2^2, \\
&= 0 && \text{if } \ p_1 > p_2 + y_2^2.
\end{aligned}
$$

With foot solicitation, however, the cost of the second solicitation would be $(y_2 - x)^2$ for $x \ge y_1$. There are two possible ways of treating the case where the consumer is to the left of both firms. One is: $(y_2 - x)^2$, minus the cost of the first solicitation; or $(y_2 - x)^2 - (y_1 - x)^2$. An alternative way of measuring the second solicitation is: $(y_2 - y_1)^2$.

In Case 1, the cost of the second solicitation equals the cost of the entire solicitation activity minus the cost of the first solicitation; whereas in Case 2, the cost of the second solicitation equals the squared distance between firms. Intuitively, with quadratic costs, Case 1 makes more sense. There would be undercutting in Case 2, but not in Case 1 (see Figure 9).

Cost of the second solicitation:

$$
\begin{aligned}
c(y_1, y_2) &= (y_2 - x)^2 - (y_1 - x)^2 && \text{if } \ 0 \le x < y_1, \\
&= (y_2 - x)^2 && \text{if } \ y_1 \le x < (y_1 + y_2)/2, \\
&= (x - y_1)^2 && \text{if } \ (y_1 + y_2)/2 < x < y_2, \\
&= (x - y_1)^2 - (x - y_2)^2 && \text{if } \ y_2 < x \le 1.
\end{aligned}
$$

6 Discussion and Conclusion

The linear market with in-person solicitation in Part 2 combines the features of Gabszewicz and Garella (1987) and d'Aspremont et al. (1979) into one model. The price-inelastic demand leads to equilibria with price dispersion, but the discontinuous payoff functions lead to undercutting if firms are too close. In

Gabszewicz and Garella (1987), the nature of phone solicitation eliminates the discontinuities of the payoff function associated with the possibilities of undercutting, allowing pure strategies equilibria in both locations and prices.

Comparing the results with the perfect information case of d'Aspremont et al. (1979), the incomplete information allows the firms to be closer without undercutting. For example, in the case where $y_1 = 0$, with perfect information, y_2 must be greater than $7/10$ for pure strategies, whereas in the incomplete information case, y_2 need only be greater than $5/9$.

When firms are far apart, the lower-priced firm wants to reveal its price because there are a greater number of customers in Firm 2's natural market that find it is not worth driving all the way to Firm 1; even though these same customers would have gone immediately to Firm 1 if they had prior perfect information.

That giving information to the buyers will lead to higher profits for the sellers seems to be counterintuitive. However this may be explained by two theories. One possibility is that the lower-priced firm is not just revealing its own price, but also the price of the high-priced firm. So actually, the buyers learn which is the lower-priced firm. Another possibility is that the welfare loss due to switching in the imperfect information case is great enough to more than offset any advantages the firms have with the asymmetric information. It would also be interesting to examine the welfare implications of the incomplete information.

The price revelation analysis is incomplete, however, because mixed strategies in the incomplete information case must be considered. For instance, it is not clear that the perfect information case Pareto dominates all mixed strategies equilibria in the imperfect case. This will require some work, as many cases are involved here. Another extension of price revelation may be that revelation is made costly, perhaps increasing linearly with extent of market targeted. It might be interesting to see how the advertising cost relative to the transportation cost affects revelation decisions.

Finally, average price is perhaps the simplest possible price information endowment assumption. Further research could include incorporating a type of search as in Stigler (1961), or a sequential search from a known distribution of prices as in Rothschild (1974). This would, however, require a model with many firms.

There are many options for the future. First, there is a need to examine part 3 more closely. Also, there are many combined possibilities to be examined. For instance, in considering mixed strategies involving undercutting part 2, the lower-priced firm may find undercutting feasible before the higher-priced firm does; this may lead to two cases. There may also be mixtures between type I and type II equilibria, where firms switch between charging high and low prices. These ideas require the consideration of mixed strategies.

In addition, one might consider cases with more than two firms. Gabszewicz et al. (1989) extends the model of Gabszewicz and Garella (1986) to more than two firms. When symmetry of locations is assumed, all prices are equal, and no consumer searches. However, with asymmetry and three firms, an equilibrium with search and price dispersion may also exist. Also in a unit circle market space, while two firms are always symmetric in location, three firms may be asymmetric.

Finally, the price information endowment assumption of average price is very basic. Perhaps one could think of other information structures that would be more like search.

References

d'Aspremont, C., J.J. Gabszewicz, and J.F. Thisse (1979) "On Hotelling's 'Stability in Competition'" Econometrica 47: 1145-50.

Dasgupta, P. and E. Maskin (1986) "The Existence of Equilibrium in Discontinuous Economic Games, I and II" Review of Economic Studies 53: 1-41.

Eaton, B.C. and R.G. Lipsey (1975) "The Principle of Minimum Differentiation

Reconsidered: Some New Developments in the Theory of Spatial Competition" Review of Economic Studies 42: 27-49.

Gabszewicz, J.J. and P. Garella (1986) "'Subjective' Price Search and Price Competition" International Journal of Industrial Organization 4: 305-316.

—– and —– (1987) "Price Search and Spatial Competition" European Economic Review 31: 827-42.

Hotelling, H. (1929) "Stability in Competition" Economic Journal 39: 41-57.

Kats, A. (1989) "Equilibria in a Circular Spatial Oligopoly" mimeo, Virginia Tech, Blacksburg, Va.

Osborne, M.J. and C. Pitchik (1987) "Equilibrium in Hotelling's Model of Spatial Competition" Econometrica 55: 911-22.

Phlips, L. (1988) The Economics of Imperfect Information. Cambridge: Cambridge University Press.

Rothschild, M. (1974) "Searching for the Lowest Price when the Distribution of Prices is Unknown" Journal of Political Economy 82: 689-711.

Salop, S. (1977) "Monopolistic Competition Reconstituted or – Circular Fashions in Economic Thought" mimeo, Federal Reserve Board, Washington, D.C.

Stigler, G.J. (1961) "The Economics of Information" Journal of Political Economy 69: 213-85.

Appendix

2.1: For a type I equilibrium, $R_1 = p_1 q_1 = p_1(p_2^* - p_1 + y_1)$.

Firm 1 chooses p_1 to maximize: $R_1 = p_1 p_2^* - p_1^2 + y_1 p_1$;

Firm 2 chooses p_2 to maximize: $R_1 = p_2 - p_2^2 + p_1^* p_2 - y_1 p_2$.

$p_1^* = (1 + y_1)/3$; $p_2 = (2 - y_1)/3$.

Type II equilibrium prices are calculated similarly.

2.2: A type I equilibrium must satisfy: $p\hat{}_1 \leq p_1^* \leq p_1'$;

$p_2^* - (y_2 - y_1) \leq (1 + y_1)/3 \leq p_2^* - ((y_2 - y_1)/2)$;

$(2 - y_1)/3 - (y_2 - y_1) \leq (1 + y_1)/3 \leq (2 - y_1)/3 - ((y_2 - y_1)/2)$;

$2 - y_1 - 3y_2 + 3y_1 \leq 1 + y_1 \leq 2 + y_1 - 3y_2/2 + 3y_1/2$.

From this two conditions are obtained:

$$y_1 \leq 3y_2 - 1, \text{ or } y_2 \leq (y_1 + 1)/3; \text{ and} \tag{1}$$

$$y_2 \leq (2 - y_1)/3. \tag{2}$$

A type I equilibrium must also satisfy: $p"_2 \leq p_2^* \leq -p\hat{}\hat{}_2$;

$p_1^* + (y_2 - y_1)/2 \leq (2 - y_1)/3 \leq p_1^* + (y_2 - y_1)$;

$(1 + y_1)/3 + (y_2 - y_1)/2 \leq (2 - y_1)/3 \leq (1 + y_1)/3 + y_2 - y_1$;

$1 + y_1 + 3y_2/2 - 3y_1/2 \leq 2 - y_1 \leq 1 + y_1 + 3y_2 - 3y_1$.

From this equation, the same 2 conditions, (1) and (2), are obtained.

2.3: For a type I equilibrium, the conditions for which neither firm is tempted to undercut are (1): $R_1(p\hat{}_1, p_2^*) < R_1(p_1^*, p_2^*)$, and (2): $R_2(p_1^*, p\hat{}_2) < R_2(p_1^*, p_2^*)$.

First, solving for the Nash equilibrium payoffs:

$R_1(p_1^*, p_2^*) = p_1 * q_1(p_1^*, p_2^*) = p_1^*(p_2^* - p_1^* + y_1)$

$= (1 + y_1)/3(2 - y_1/3 - 1 + y_1/3 + y_1) = (y_1 + 1)2/9$.

$R_2(p_1^*, P_2^*) = p_2 * q_2(P_1^*, P_2^*) = p_2 * [1 - (p_2^* - p_1^* + y_1)] = p_2^*(1p_2^* + P_1^* y_1)$

$= (2 - y_1/3)(12 - y_1/3 + 1 + y_1/3y_1) = (2 - y_1)2/9$. $R_1(p\hat{}_1, P_2^*) < R_1(P_1^*, P_2^*)$.

$p\hat{}_1 < (y_1 + 1)2/9$, $p_2^* - (y_2 - y_1) < y_1^2 + 2y_1 + 1/9$, $2 - y_1/3y_2 + y_1 < y_1^2 + 2y_1 + 1/9$;

$6 - 3y_1 9y_2 + 9y_1 < y_1^2 + 2y_1 + 1$; $-9y_2 < y_1^2 - 4y_1 - 5$;

$$y_2 > -(y_1^2 - 4y_1 - 5)/9. \tag{3}$$

$R_2(p_1^*, p^2) < R_2(p_1^*, p_2^*)$: $p^2 < (y_1 - 2)2/9$, $p_1^* - (y_2 - y_1) < (y_1 - 2)2/9$, $1 + y_1/3 - y_2 + y_1 < y_1^2 - 4y_1 + 4/9$, $3 + 3y_1 - 9y_2 + 9y_1 < y_1^2 - 4y_1 + 4$, $-9y_2 < y_1^2 - 16y_1 + 1$,

$$y_2 > -(y_1^2 - 16y_1 + 1)/9. \tag{4}$$

4.1: Phone Solicitation in Circular Space: Solicitation cost of the further firm for an A_1-type buyer, $c(x) =$

$y_2 - x$ for buyers inbetween firms

$y_2 + (1 - x)$ for buyers just to the left of Firm 1

$x - y_2$ for buyers far to the left of Firm 1.[14]

Note that buyers that are sufficiently far enough to the left of Firm 1 are going to solicit Firm 2 using phone lines that are strung around the bottom of the circle.

$A_1 = [(1 + y_2)/2, y_2/2]$; $A_2 = [1/2y_2, (1 + y_2)/2]$.

Defining $|A_i|$ as the length of A_i around the circle, note that $|A_1| = |A_2| = 1/2$.

$A_{11} = [max(1 + y_2)/2, p_1 - p_2 + y_2, min1/2y_2, p_2 - p_1 + y_2]$; $A_{22} = [max1/2y_2, p_2 - p_1, min(1 + y_2)/2, p_1 -$
$A_{12} = A_1 - A_{11}; A_{21} = A_2 - A_{22}$.

Note that at least one element of (A_{12}, A_{21}) is empty. Also, the A_{ij} region may be divided into two subregions: Let $\dot{A}_{ij}, i \neq j$ be the set between the firms where they are closer together (i.e. $\dot{A}_{ij} \subseteq [0, 1/2]$); and define $\ddot{A}_{ij}$ be the other part of A_{ij} (i.e. between the firms where they are farther apart). Note: $\dot{A}_{ij} + \ddot{(}A_{ij} = A_{ij}$. Also let $|A_{ij}|$ be the length of set A_{ij} around the circle.

Claim: $\dot{A}_{ij}$ is nonempty if $\ddot{A}_{ij}$ is nonempty.

[14]These include the set $[x \in A_1 | x \leq y_2 + 1/2]$.

Proof: Suppose $x = y_2/2$, then the cost of the second solicitation is equal to $y_2 - y_2/2 = 1y_2/2$. Now suppose $x = (y_2+1)/2$, then the cost of the second solicitation equals $(y_2+1)/2 - y_2 = y_2/2 + 1/2 - y_2 = 1/2 - y_2/2 = (1-y_2)/2$. $(1-y_2)/2 \geq y_2/2$ because $0 \leq y_2 \leq 1/2$. So the A_{12} region immediately to the right of Firm 1 will become nonempty first (before the region immediately to the left of Firm 1) as p_1 increases relative to p_2. //

Let p'_{1L} be the lowest price at which buyers from A_2 immediately to the left of A_1 do not solicit Firm 1. p'_{1L} is the solution to the equation $(1+y_2)/2 = p_1 - p_2 + 1$. $p'1L = p_2 - (1-y_2)/2$. Similarly, let p'_{1R} be the lowest price at which buyers from A_2 to the right of A_1 do not solicit Firm 1. p'_{1R} is the solution to the equation $y_2/2 = p_2 - p_1$. $p'_{1R} = p_2 - y_2/2$.

Now, let p'_{1R} be the solution to the equation $1/2y_2 = p_2 - p_1 + y_2$. $p"_{1R} = p_2 + 1/2y_2$ (i.e., p'_{1R} is the highest price at which no buyer in the right part of A_1 would solicit Firm 2). Similarly, p'_{1L} is the highest price at which no buyer in the left part of A_1 would solicit Firm 2. p'_{1L} is the solution to the equation $(1+y_2)/2 = p_1 - p_2 + y_2$; $p"_{1L} = p_2 + (1-y_2)/2$.

Since $0 \leq y_2 \leq 1/2$, the range of prices $[p'_{1R}, p"_{1R}]$ is contained within the range of prices $[p'_{1L}, p'_{1L}]$ (i.e., the buyers who solicit the further firm first will lie between the firms where they are closer together, somewhere in the segment $[0, y_2]$). Therefore, the demand for Firm 1's t-shirts,

$$\begin{aligned}
q_1(p_1,p_2) &= 1 - (p_1 - p_2 + 1) + (p_2 - p_1) && \text{if } 0 \leq p_1 < p'_{1L}, \\
&= 1 - [(1+y_2)/2] + (p_2 - p_1) && \text{if } p'_{1L} \leq p_1 < p'_{1R}, \\
&= 1 - [(1+y_2)/2] + 1/2y_2 = 1/2 && \text{if } p'_{1R} \leq p_1 \leq p"_{1R}, \\
&= 1 - [(1+y_2)/2] + (p_2 - p_1 + y_2) && \text{if } p"_{1R} < p_1 \leq p"_{1L}, \\
&= 1 - [(p_1 - p_2 + y_2)] + (p_2 - p_1 + y_2) && \text{if } p"_{1L} < p_1.
\end{aligned}$$

4.2: In a type I equilibrium, $A_{22} = [z, z']$, where $z = p_2 - p_1$ and $z' = (y_2+1)/2(z - 1/2y_2) = (y_2+1)/2 - [(p_2 - p_1) - 1/2y_2]$
$= 1/2y_2 + 1/2 - p_2 + p_1 + 1/2y_2 = y_2 + 1/2p_2 + p_1$.

4.3: Looking at a type I equilibrium, $R_1 = p_1q_1 = p_1(A_1+A_{21}) = p_1(1-A_{22}) = p_1 1 - [(y_2+1/2-p_2+p_1)-(p_2-p_1)]$
$= p_1[1-(y_2+1/2-p_2+p_1-p_2+p_1)] = p_1(1-y_2-1/2+2p_2-2p_1) = p_1(1/2-y_2+2p_2^* 2p_1)$.

Firm 1 chooses p_1 to maximize $R_1 = p_1/2 - y_2p_1 + 2p_1P_2^* 2p_1^2$. $d-1/dp_1 = 1/2y_2 - 4p_1 + 2P_2^* = 0; 4p_1 = 1/2y_2 + 2P_2^*$;

$$2p_1 = R - 1/2y_2 + P_2^*.$$

$R_2 = p_2q_2 = p_2A_{22} = p_2[(y_2+1/2-p_2+p_1)-(p_2-p_1)] = p_2(y_2+1/2-2p_2+2p_1)$.

Firm 2 chooses p_2 to maximize $R_2 = p_2y_2 + 1/2p_2 + 2p_1 * p_2 - 2p_2^2$.

$$dR_2/dp_2 = y_2 + 1/2 + 2P_1^* - 4p_2 = 0.$$

Substituting (1) into (2): $y_2 + 1/2 - 4p_2 + R - 1/2y_2 + p_2 = 0$; $3p_2 = 3/4 + 1/2y_2$;

$$P_2^* = R + y_2/6.$$

Substituting (3) into (1): $2p_1 = R - 1/2y_2 + R + y_2/6 = 1/2 - y_2/3$;

$$P_1^* = R - y_2/6.$$

4.4: A type I equilibrium must satisfy (1): $p\hat{}_1 \leq p_1^* \leq p_1'$; and (2): $p"_2 \leq p_2^* \leq$

$p\hat{\ }\hat{\ }_2$.

Substituting for (1): $p_2 - y_2 \leq 1/4 - y_2/6 \leq p_2 - y_2/2$;

$1/4 + y_2/6 - y_2 \leq 1/4 - y_2/6 \leq 1/4 + y_2/6 - y_2/2$.

From the first inequality of (1), we get: $y_2/6 - y_2 \leq -y_2/6$; or $-5y_2/6 \leq -y_2/6$. This condition is not troublesome. However, from the second inequality of (1), we get $-y_2/6 \leq y_2/6 - y_2/2$; or $-y_2/6 \leq -y_2/3$. This condition is violated; hence pure strategies equilibria do not exist.