Thermal Diffusivity of Plastic

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Abstract

The thermal diffusivity of epoxy resin was obtained by recording how the axial temperature of an epoxy resin rod varied over time. The rod was immersed into boiling water and ice water providing a range of 100 °C. The thermal diffusivity was obtained through two different methods: a sudden change in temperature and a periodic temperature change. A final result for the thermal diffusivity was obtained by taking a weighted mean of all values obtained. We obtained a final result of $D = (1.1 \pm 0.2)10^{-7} m^2 s^{-1}$. for the thermal diffusivity. The values obtained are consistent and agree well within 3 standard deviations.

1   Introduction

Thermal diffusivity is a ratio of how well a material conducts heat to how well it stores heat. Materials with high values of thermal diffusivity are good diffusers of thermal energy [1], this means that they reach thermal equilibrium faster than materials with lower values of thermal diffusivity. Therefore materials with low thermal diffusivities are good insulators, epoxy resin in particular has a very low thermal diffusivity in the order of $10^{-7} m^2 s^{-1}$[2].

The value of thermal diffusivity is important when predicting cooling processes as it indicates the material's ability to transmit heat and store heat energy. We can use this knowledge to determine how the material cools. Thermal diffusivity is also used in simulating temperature fields as the value can be used to determine how the heat will be transferred across a material [3].

Thermal diffusivity is also needed in order to solve Fourier differential equations in the case of unsteady heat flow. For high enough values of thermal diffusivity, we can determine two dimensionless quantities known as the Biot and the Fourier number. The Biot number is a ratio of resistance to internal heat flow against resistance to external heat flow. The Fourier number is a quantity that translates the conduction equation into a completely dimensionless form, making it easier to come up with transient solutions [4].

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2 Theory

Thermal diffusivity, D, is given by the equation below.

$$D = \frac{k}{\rho c_p} \tag{1}$$

Where k is the thermal conductivity, ρ is the density and c_p is the specific heat capacity of the material, this all assumes a constant pressure[5].

2.1 Experiment A

When the surface temperature of a cylinder is suddenly changed to a new temperature θ_1, the axial temperature, θ, approaches the surface temperature, θ_1, exponentially after the initial effects have died away.
If we write the temperature difference between the new surface temperature and the old surface temperature as $\Delta\theta$, then we can obtain the following equation :

$$\theta = \theta_1 + A\Delta\theta e^B \tag{2}$$

Where A and B are arbitrary constants.

As shown by Carslaw and Jaeger[6] for a cylindrically symmetric material the surface temperature through the centre of the cylinder is given by :

$$\theta(0,t) = \theta_0 + (\theta_1 - \theta_0)(1 - \sum_{n=1}^{\infty} a_n e^{\frac{-\lambda_n^2 Dt}{a^2}}) \tag{3}$$

Where $\theta(0,t)$ is the temperature at the centre of the rod as a function of time, θ_1 is the surface temperature, λ_n are the positive roots of the Bessel function $J_0(x) = 0$, a_n are constants of order unity and a is the radius of the cylinder.
If we consider only the first term of the series this simplifies to the following :

$$\theta(0,t) = \theta_1 - (\theta_1 - \theta_0)a_1 e^{\frac{-\lambda_1^2 Dt}{a^2}} \tag{4}$$

We can see that after the initial effects have died away, after some time $t = \tau$, Equation 4 resembles Equation 2.

Substituting A equal to $-a_1$, B equal to $\frac{-\lambda_1^2 Dt}{a^2}$ and $\Delta\theta$ equal to $\theta_1 - \theta_0$ into Equation 2 we obtain Equation 4.

2.2 Experiment B

The temperature of the surface changes periodically from the temperature of the ice-water beaker to the temperature of the boiling water beaker. This creates a damped heat wave which propagates radially in towards the centre of the cylinder. These heat waves are damped quite severely. For short enough time

periods, higher frequency fourier components are damped to insignificance and we are left with a single harmonic. The axial temperature therefore behaves sinusoidally.

The peak-to-peak amplitude is given by the following expression :

$$\frac{4(\theta_2 - \theta_1)}{\pi |M_0(a\sqrt{\frac{2\pi}{TD}})|} \tag{5}$$

Where T is double the time period of switching between beakers, and M_0 is the Kelvin function.

The phase lag ϕ is given by the following expression:

$$\phi = arg M_0(a\sqrt{\frac{2\pi}{TD}}) \tag{6}$$

3 Experimental approach

3.1 Experimental set-up

Figure 1 shows our experimental set-up. Beakers of ice-water and boiling-water were prepared making sure to stir the beakers so that the temperature was evenly distributed. The thermopile was connected to the chart-recorder and we calibrated the chart-recorder so that 1 division on the chart-paper corresponded to $1\,°C$. This allowed for easy interpretation of the data as our range from 0-$100\,°C$ corresponded to 0-100 on the chart paper. A clamp and stand was set up so that the thermopile was fixed in place. It is important that the thermopile is not left loose as this makes it more likely to accidentally spill the contents of the beaker which is dangerous when working with boiling water.

Before taking measurements for Experiment B, it is important to do some preliminary tests to see what time intervals produce recognisable sinusoidal curves so that time is not wasted when taking measurements. A time period which is too large will waste significant time.
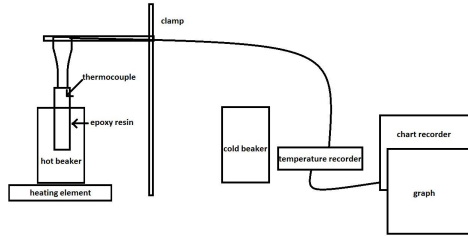

Figure 1: Experimental set-up.

3.2 Experimental method

3.2.1 Experiment A

In Experiment A we determined the thermal diffusivity of the epoxy resin rod by a sudden temperature change. This was achieved by initially leaving the rod in one of the beakers to attain thermal equilibrium and then swapping the rod to the other beaker allowing it to reach thermal equilibrium once again. When doing this it was important to note the time at which the rod swapped beakers. We did this moving the rod to the new beaker when the nib of the plotter reached the end of the section of the plotting paper.

It is also important to make sure to stir the ice water when the rod transitions from boiling water to ice water, as if not stirred the rod will heat the surrounding water and this may not be detected by the thermometer if it is positioned elsewhere. This would lead to a different value of $\Delta\theta$ and would introduce a systematic error.

3.2.2 Experiment B

In Experiment B we determined the thermal diffusivity of the epoxy resin rod by a periodic temperature change. This was achieved by periodically switching the beaker the rod is immersed in on a regular interval. This introduced some further things to take into account when taking measurements.

Whilst it is still important to stir the ice-water it should be noted that when the rod is immersed in boiling water the ice-water is still heating up. Therefore it is important to top-up the ice-water as the rod is immersed in the boiling water. Likewise as the rod is immersed in the ice-water beaker the water is being boiled off in the boiling water beaker. For large periodic intervals this can pose a problem as when it is time to transition the rod from the ice-water beaker to the boiling water beaker, there is not enough content in the boiling water beaker to immerse the rod. This can lead to systematic error, so as the rod is immersed in ice-water the boiling water beaker should be topped up with hot water and given time to reach $100\,°C$.

3.3 Estimation of errors

3.3.1 Experiment A

The error in the thermal diffusivity in Experiment A is dependent on the variables from $D = \frac{-ma^2}{\lambda_1^2}$ where m is the gradient obtained from the plot of log temperature against time. The error in the gradient was determined by the linear fit function on MATLAB. This used errors in the temperature and time. These errors were obtained from the measurement of the results from the graph paper. We determined that a $1\,°C$ error on the Temperature was appropriate, as well as a $2s$ error on the time. We determined that a 1mm error on the radius was appropriate as there was a 2mm error in measuring the diameter which was then halved.

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3.3.2 Experiment B

The error in the thermal diffusivity in Experiment B is evident from $D = \frac{2\pi a^2}{Tx^2}$ for both the phase lag and amplitude lag cases. The error in the radius is the same as discussed before. There is an error in the time period T which we determined to be 0.4s double our reaction time for both starting and stopping the stopwatch. There is also an error in the x value as we do not have knowledge on how the Kelvin function varies outside of the data given. We decided to therefore make the uncertainty in x the size of the division which was 0.5.

4 Results

4.1 Calibration

Before we could do any analysis, we needed to reliably interpolate values for the Kelvin function. We were given a table of values for the modulus and argument of the Kelvin function as a function of x. This table of values is shown in Figure 2. We then plotted these values to get an idea of what the function looked like. This plot is shown in Figure 3.

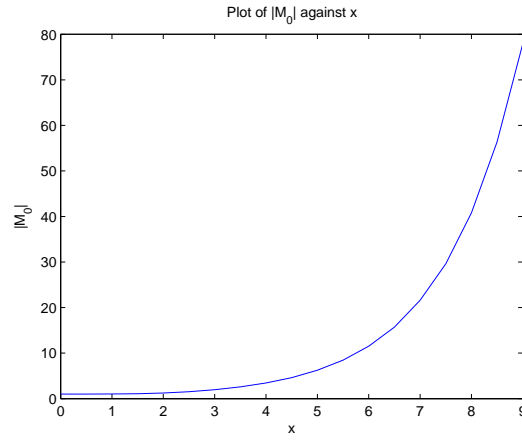| x	$|M_0|$	$argM_0$ (degrees)
0	1.0	0
0.5	1.00	4
1.0	1.02	14
1.5	1.08	31
2.0	1.23	52
2.5	1.51	75
3.0	1.95	96
3.5	2.58	118
4.0	3.44	139
4.5	4.62	159
5.0	6.23	180
5.5	8.45	199
6.0	11.5	220
6.5	15.7	240
7.0	21.6	260
7.5	29.6	281
8.0	40.8	301
8.5	56.4	321
9.0	78.0	342

Figure 2: Values of the Kelvin function for given x

Figure 3: Calibration curve using given data.

Judging by the shape of the calibration curve, we tried to linearise it by plotting
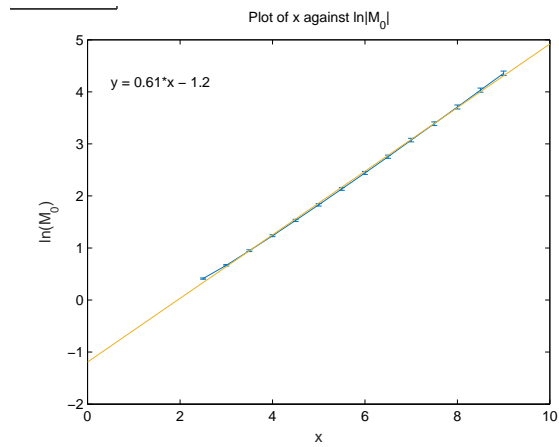x against the $\log |M_0|$ as shown in Figure 4.

Figure 4: log calibration curve using given data.

Using this, we now have an equation with which to calculate our own values for
the modulus of the Kelvin function accurately.

4.2   Experiment A

The thermal diffusivity for experiment A was obtained from the gradient in the plot of log temperature against time. This is shown in Figure 5.
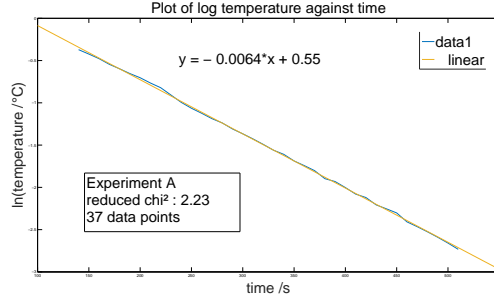

Figure 5: Plot of log temperature against time.

Where the error obtained was given by : $\sigma_D = \sqrt{\frac{a^2 m 2}{\lambda_1^4}\sigma_a^2 + \frac{a^4}{\lambda_1^4}\sigma_m^2}$ where m is the gradient and a is the radius of the cylinder.

4.3   Experiment B

The thermal diffusivity values obtained from the peak-to-peak amplitude are summarised in Figure 6

D	σ_D
$1.07 \times 10^{-7} m^2 s^{-1}$	$4.05 \times 10^{-8} m^2 s^{-1}$
$1.05 \times 10^{-7} m^2 s^{-1}$	$5.14 \times 10^{-8} m^2 s^{-1}$
$1.08 \times 10^{-7} m^2 s^{-1}$	$3.10 \times 10^{-8} m^2 s^{-1}$
$1.06 \times 10^{-7} m^2 s^{-1}$	$2.61 \times 10^{-8} m^2 s^{-1}$

Figure 6: Diffusivity values obtained from peak-to-peak amplitudes

The thermal diffusivity values obtained from the phase lag are summarised in Figure 7. Although these values mostly agreed within 3 standard deviations, we were not satisfied with the inconsistency of the values and so we decided to omit this from the final weighted mean calculation.

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D	σ_D
$1.07 \times 10^{-7} m^2 s^{-1}$	$4.05 \times 10^{-8} m^2 s^{-1}$
$1.05 \times 10^{-7} m^2 s^{-1}$	$5.14 \times 10^{-8} m^2 s^{-1}$
$1.08 \times 10^{-7} m^2 s^{-1}$	$3.10 \times 10^{-8} m^2 s^{-1}$
$1.06 \times 10^{-7} m^2 s^{-1}$	$2.61 \times 10^{-8} m^2 s^{-1}$

Figure 7: Diffusivity values obtained from phase lag

The error obtained was given by : $\sigma_D = \sqrt{\frac{16\pi^2 a^2}{T^2 x^4}\sigma_a^2 + \frac{4\pi^2 a^4}{T^4 x^4}\sigma_T^2 + \frac{16\pi^2 a^4}{T^2 x^6}\sigma_x^2}$ where T is the double the time period of switching beakers and x is the argument of the Kelvin function.

4.4   Final result

Collating all the data together, we took a weighted mean of all the thermal diffusivity values obtained from each experiment except for those from the phase lag. These can be found in Figure 8. This gave us a final value for the thermal diffusivity: $D = (1.1 \pm 0.2)10^{-7} m^2 s^{-1}$.

D	σ_D
$1.11 \times 10^{-7} m^2 s^{-1}$	$5.70 \times 10^{-8} m^2 s^{-1}$
$1.07 \times 10^{-7} m^2 s^{-1}$	$4.05 \times 10^{-8} m^2 s^{-1}$
$1.05 \times 10^{-7} m^2 s^{-1}$	$5.14 \times 10^{-8} m^2 s^{-1}$
$1.08 \times 10^{-7} m^2 s^{-1}$	$3.10 \times 10^{-8} m^2 s^{-1}$
$1.06 \times 10^{-7} m^2 s^{-1}$	$2.61 \times 10^{-8} m^2 s^{-1}$

Figure 8: Final weighted mean values

5   Discussion

5.1   Experiment A

The final value of thermal diffusivity we obtained in Experiment A was consistent was consistent with all the values obtained for the thermal diffusivity in Experiment B to 3 standard deviations. We can also be fairly confident in our linearisation of the data as a χ^2 value of 2.23 with 37 data points is satisfactory. The most significant source of error in the error propagation came from the time as we read off values from the chart produced and there was an uncertainty associated with this. Directly recording the data onto a computer would significantly reduce this and other associated human errors. This would also have the benefit of manipulating all of the data directly with software such as MATLAB.

Another source of error would be due to the ice-water heating up during the experiment. In Experiment A it was important to define $\Delta\theta$ the difference between the temperature of the boiling water and ice-water. It is therefore problematic that this temperature difference is treated as constant when in fact it is changing. However this effect is likely to be small compared to other errors.

As seen in Equation 3): we can express the axial temperature as a series of exponential terms. Using our value for thermal diffusivity, we can substitute values into this equation and observe where the transient effects on our graph died out.

After a long enough time, the terms for $n \geq 2$ will become negligible and we will be left with just the first term. The axial temperature is then an exponential curve and hence the natural logarithm of the axial temperature will be linear after this point.

First we observe that the positive roots of the Bessel function become greater with each successive term. The bigger this root is, the faster it will fall off in time.

Hence the second term in the series will fall off last, and after this point we have only the first term.

We can calculate the time, τ , at which this will occur.

If we take the term to be negligible when the exponent is equal to -3, we get the equation : $\frac{\lambda_n^2 Dt}{a^2} \geq 3$ Substituting in the equation for the root $\lambda_2 \approx 5.520$, $a = 1cm$ and our final value of the Thermal Diffusivity $D = 1.07 \times 10^{-7} m^2 s^{-1}$ gives $\tau = 92.0s$

In Figure 9 we can see that the curve becomes linear at around 100s. Figure 10 is a magnified image of Figure 9 and we can see more clearly that the plot becomes linear closer to 90s.
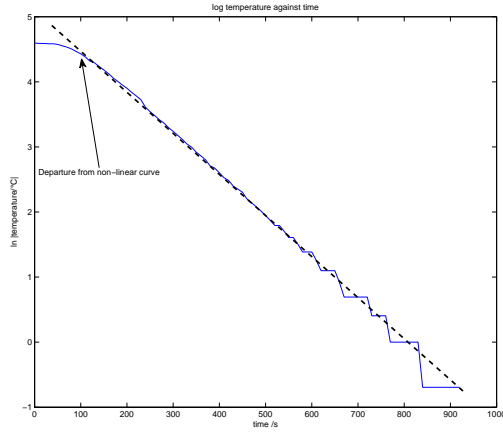
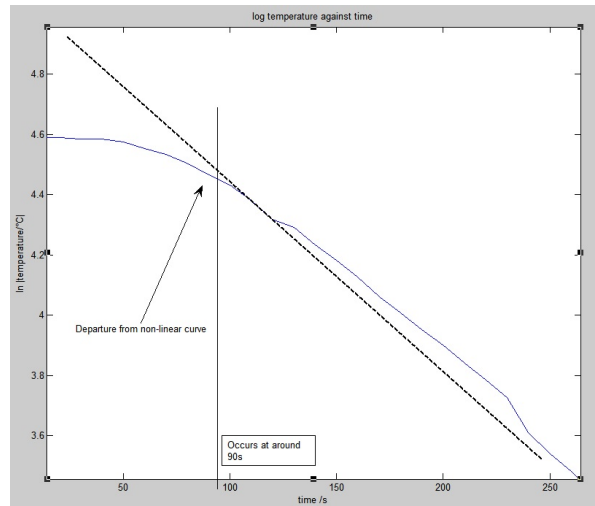Figure 9: Plot of log temperature against time.

Figure 10: Magnified picture of log temperature against time.

5.2 Experiment B

$D = \frac{2\pi a^2}{Tx^2}$ was used to determine the errors. The main contributor for the error in thermal diffusivity is from the error in x. This is because we gave it an error corresponding to the entire division given by the results table Figure 2. We did this because we could not be certain how the Kelvin function behaved between

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these values from the data given.

The values obtained from the phase lag method are inconsistent and vary over two orders of magnitude. For this reason we decided not to include it in our weighted mean.

One possible reason why the phase lag values for thermal diffusivity are not reliable could be due to the calculation of x. The measurement of Δt at small time periods the heat wave had not yet propagated to the centre of the rod by the time the rod switched beakers again. This meant it was difficult to judge which peak was the leading peak.

After omitting the values of diffusivity obtained from the phase lag method, the values obtained from the peak-to-peak amplitude method are all consistent with eachother and agreed well within 3 standard deviations.

6   Conclusions

Our final value for the thermal diffusivity of a epoxy resin cylinder was $D = (1.1 \pm 0.2)10^{-7} m^2 s^{-1}$.

We were satisfied with the measurements obtained from Experiment A and the peak-to-peak amplitude measurements from Experiment B. In future, more time would have been put into longer time period measurements for Experiment B, so that the phase lag measurements would be more clear to see and thus would give us more consistent values.

If the data was recorded directly by a computer, it would reduce the errors introduced by human measurement and would give us many more data points to work with. This is because each data point had to be recorded from the graph by hand which made it laborious to get many data points as well as introduced significant error.

In future it may be useful to use other liquids so that we could have a higher range of temperatures, as in this experiment we were limited to 0-100 °C because of the melting and boiling point of water.

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