Geometrical Optics Experiment

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Abstract

This report is an introduction into the basics of geometrical optics. Experiments were conducted using a *Thorlabs LB1630* convex lens of focal length 100.0mm and Thorlabs LC1315 concave lens of focal length -75.0mm; these are the respective convex and concave lenses referred to throughout. In Experiment 1, the focal length of the convex lens is determined by two methods. The first method is by plotting the reciprocal of the object distance against the reciprocal of the image distance. This gave a focal length of (12.36±0.06)cm. The second method involved plotting magnification against image distance. This gave a focal length of (9.96±0.11)cm.

Experiment 2 used the displacement method to determine the focal length of the convex lens. The focal length obtained was (10.37±0.03)cm.

In Experiment 3 the distance between the object and image was plotted in order to determine the focal length of the convex lens. This is (10.50±0.01) cm.

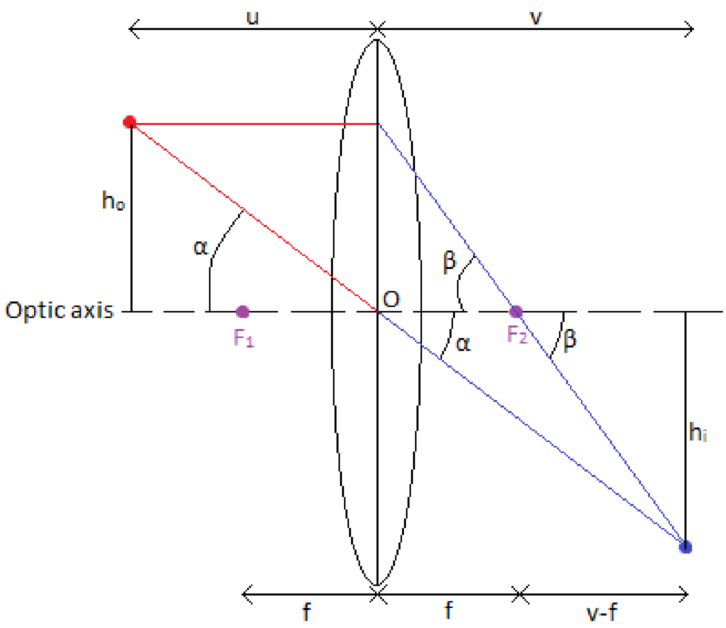
Experiment 4 uses an auxiliary weaker convex lens to determine the focal length of the concave lens. We obtained a focal length of (-7.68±0.16)cm and (-13.87±1.52)cm via two methods.

1. Introduction

Lenses play a vital role in photography. Technology is developing to produce clearer and more focused images. In a digital camera, the sensing plate absorbs light like a film but, unlike a film, it produces an electrical signal. As we will show in the following report, the image distance (the distance from the lens to the screen), varies with the object distance. When taking a picture, the object distance varies throughout. This is known to photographers as the "*depth of field*". Therefore it is important to determine focal lengths for camera lenses as accurately as possible. In the following experiments the focal length, *f*, will be investigated.

2. Theory

The thickness of all lenses used in this experiment is assumed to be negligible compared to the radii of curvature of the lenses, thus we will use the thin lens equations.



u-object distance
v-image distance
o-centre of lens
F_1-first focal point
F_2-second focal point
H_o-object height
H_i-image height
α,β-the angles shown

Figure 1 Convex lens diagram

The following derivation is based upon *University physics [1]*.

Since the two angles α are equal $\frac{h_o}{u} = -\frac{h_i}{v}$. The negative sign is due to the image being formed below the optic axis. Since the two angles β are equal $\frac{h_o}{f} = -\frac{h_i}{v-f}$. Rearranging to make h_o the subject gives : $h_o = -\frac{uh_i}{v}$. Substituting this expression for h_o into the equation:

$$\frac{h_o}{f} = -\frac{h_i}{v-f} \quad (1)$$

$$\frac{u}{vf} = \frac{1}{v-f} \quad (2)$$

$$f = \frac{uv}{u+v} \quad (3)$$

Thus we get the Gaussian form of the thin lens equation[2]: $\frac{1}{f} = \frac{u+v}{uv} = \frac{1}{v} + \frac{1}{u}$ (4). We can also get the equation for magnification from $\frac{h_o}{u} = -\frac{h_i}{v}$. Thus $\frac{h_i}{h_o} = -\frac{v}{u} = m$.

Figure 1 shows the refrection of light through the convex lens. *u* and *v* used in the equations above are represented in the Figure.

3. Experimental Method

3.1 Experiment 1 Method 1

"Determination of the focal length of a convex lens by plotting the reciprocal of the object distance against the reciprocal of the image distance"- Laboratory Specification

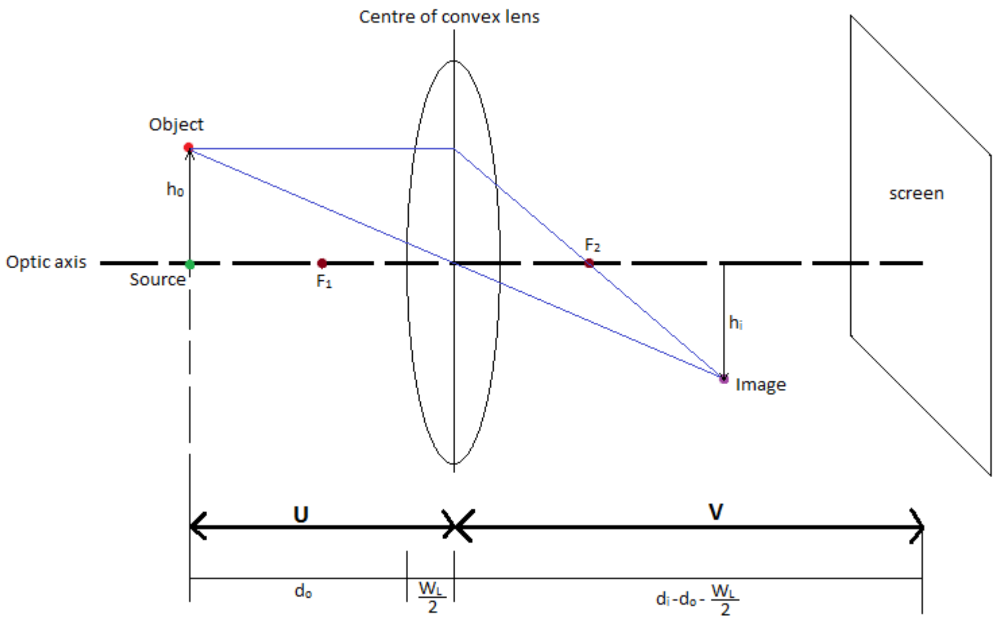


Figure 2: Experimental setup for experiment 1 and 3

h_o is the height of the object

h_i is the height of the image

F_1 is the first focal point

F_2 is the second focal point

d_o is the distance from the source to the outside surface of the lens

d_i is the distance from the source to the screen

W_L is the thickness of the lens

u is the object distance

v is the image distance

The lens was held against sunlight (assuming the rays to be parallel due to great separation) in order to determine a rough range in which to obtain values. The lens was brought closer to the screen until a focused image was formed. In Figure 2 this is a decrease in v.

Acetate with marked scales was placed directly in front of the light source. The screen was brought closer until the image was in focus. This was repeated five times for each *u*.

Care was taken to ensure the lens was parallel to the light source and screen, as well as ensuring parallax error was avoided. The width of the lens was measured using Vernier callipers and the associated uncertainty in this was assumed to be negligible in comparison to the error in the measurements from the ruler. The base of the lens was marked corresponding to the front of the lens and half the width of the lens was added to each value.

The data for this experiment is tabulated in Figure 3 below, where σ_v is the uncertainty in *v*.

<U>/cm	<V>/cm	σ_v /cm
10.48	54.02	0.26
11.48	43.66	0.81
12.48	35.25	0.19
13.48	31.04	0.22
14.48	27.98	0.08
15.48	25.74	0.19
16.48	24.21	0.06

[Figure 3] Tabulated data

3.2 Experiment 1 Method 2

"*Determination of the focal length of a convex lens by plotting magnification against image distance"- Laboratory Specification*

The image was formed by the acetate as in *3.1.* A given separation on the acetate was taken using a marked scale; this was considered to have negligible error since no measurements were taken. The object distance remained fixed and the image distance varied until the image was in focus (as in section *3.1*), but this time values of D were obtained, which represents the distance between the two chosen markings on the image which were

40mm apart. Far enough to minimise error but not far enough for the image to stretch beyond the screen.

Five measurements of D were taken for each value of u to reduce uncertainty due to the "problem of definition". Measurements were taken by displacing the screen from the focal point and bringing it back to apparent focus.

This was repeated for a range of u from $2f$ to close to f. The image is at infinity when $u=f$ so it was not possible to take measurements as u tends towards f.

3.3 Experiment 2

"Determination of the focal length of a convex lens by the displacement method"- Laboratory Specification

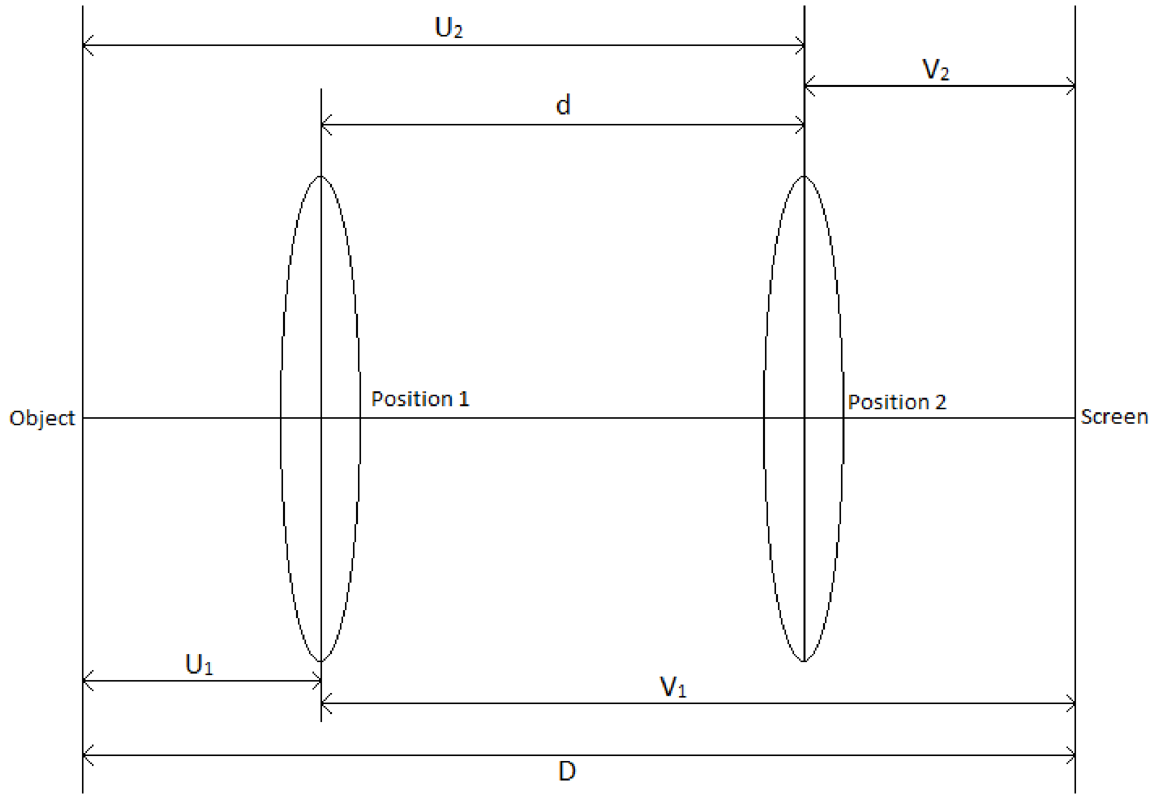


Figure 4 Displacement method experimental setup

u_1-object distance at position 1
u_2-object distance at position 2
v_1-image distance at position 1
v_2-image distance at position 2
d-distance between positions 1 and 2
D-distance between object and screen

The image distance is fixed and the distance between the light box and the convex lens is varied. This setup can be seen in Figure 4. Analysis of the data collected allowed a value for the focal length of the lens to be calculated.

A mean value of five repeat measurements was taken.

D and *d* are the independent and dependent variables respectively.

3.4 Experiment 3

"Determination of the focal length of a convex lens by plotting the distance between an object and image distance against the object distance"-Laboratory Specification

The screen was positioned to obtain a focused image, with the lens just beyond one focal length from the object as shown in Figure 2. A series of values corresponding to image and object distances are obtained. We recycled our data from experiment 1, because the data overlapped. A graph of (u+v) against u was then plotted.

Starting with the Gaussian form of the thin lens equation: $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$. We can multiply through by uv to get $u+v=\frac{uv}{f}$. Hence $v=\frac{uf}{u-f}$. Substituting this value for v into $u+v=\frac{uv}{f}$ gives us the equation: $u+v=\frac{u^2}{u-f}$. Letting y = u+v it follows that y=$\frac{u^2}{u-f}$ which is in the form of a hyperbola.

3.5 Experiment 4

"Determination of the focal length of a concave lens using an auxiliary weaker convex lens"- Laboratory Specification

The convex lens was initially placed at a distance of 25.00cm and the screen was then adjusted to a distance at which it produced a focused image. The concave lens was interposed between the convex lens and a screen, at varying positions. The screen is then moved until a focused image is formed.

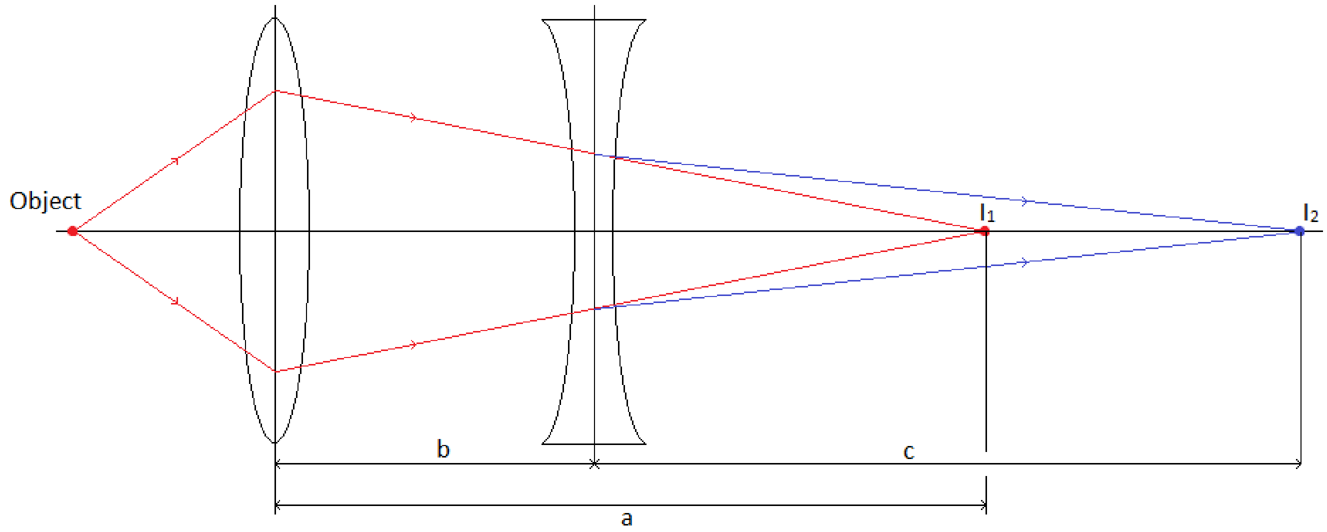


Figure 5 Experimental setup for experiment 4

b-distance between convex and concave lens
a-distance between convex lens and I_1
c-distance between concave lens and I_2
I_1-first image
I_2-second image

For an independent variable of object distance, varying dependent values of *a*, *b* and *c*, see Figure 5, were recorded. Because the light from the object is already converging due to the convex lens, I_1 acts as a virtual object for the concave lens. Therefore I_2 is the real image of the virtual object I_1. Therefore for the concave lens, $u = -(a-b)$ and $v = c$.

4. Results

<u>4.1 Experiment 1 Method 1</u>

The mean was taken, along with its uncertainty. The uncertainty was given by the larger of either half the smallest division on the ruler i.e. ±0.05cm and the standard deviation using $\sqrt{\frac{\sum(v-<v>)^2}{N(N-1)}}$ (5).

σ_v was used to calculate χ^2 using $\sum\frac{(y-ymodel(x))\text{^}2}{sigma\text{^}2}$. This was divided by the number of degrees of freedom to give the reduced χ^2.

An initial value for χ^2 of 5.24 was obtained that was not between 2 and 0.5. This suggests that the errors assigned to data could be grossly optimistic.

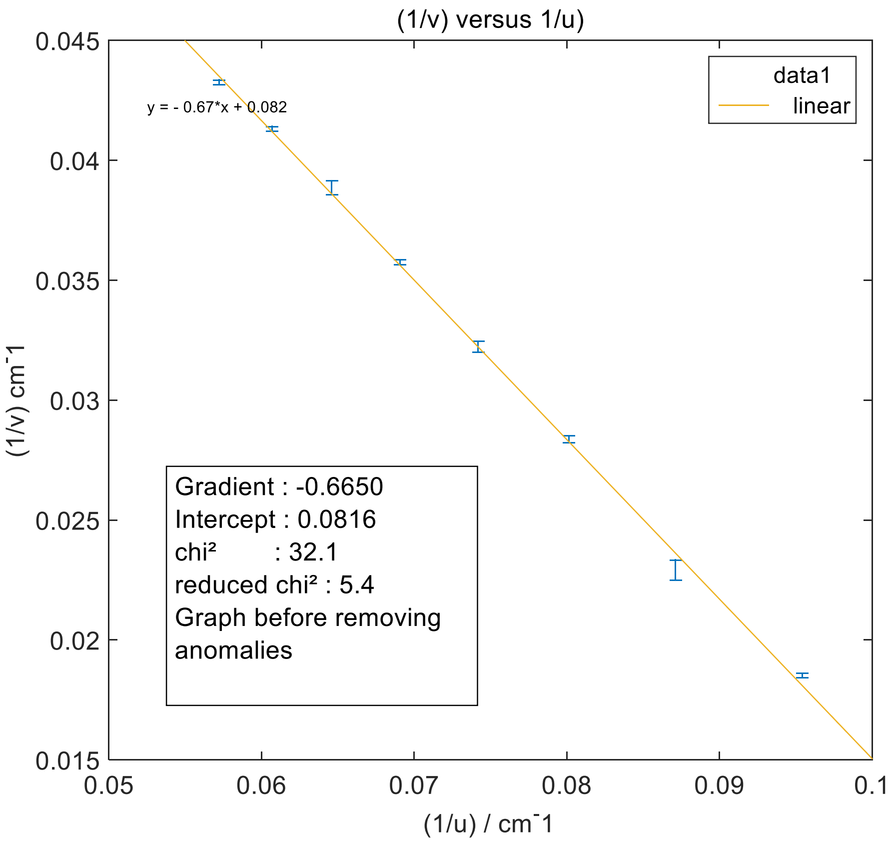


Figure 6: Before removal of data

Data point 2 Figure 6 was identified as an anomaly, thus giving a value of reduced chi squared of 1.7 that was a better match to the number of degrees of freedom.

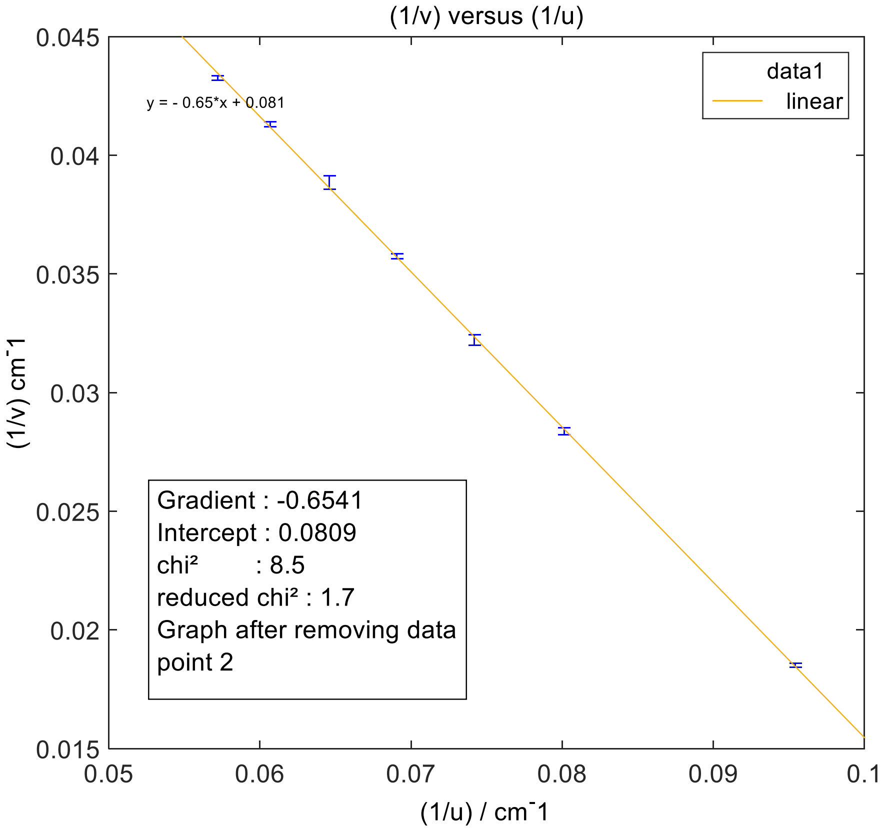


Figure 7: After removal of data

On comparison of our line of best fit Figure 7 with $\frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$, the intercept of the graph corresponds to $\frac{1}{f}$ and the error in the intercept given by linearfit as $4\text{x}10^{-4}$cm. Error propagation gives the uncertainty in *f* and this gave us a final value of (12.36±0.06) cm.

4.2 Experiment 1 Method 2

The uncertainty was given by the larger value of either half of the smallest value, ±0.05cm or the value for the standard deviation, as in method one. The error propagation proceeded as follows:

$$m = -\frac{D}{d}$$

$$\left(\frac{\sigma_m}{m}\right)^2 = \left(\frac{\sigma_D}{D}\right)^2$$

Hence,

$$\sigma_m = \sqrt{\frac{(\sigma_D)^2}{(D)^2} . (m)^2}$$

A line of best fit and errorbars were added to the plot. The gradient was -0.1009±0.0008cm^{-1} and the y-intercept was 1.2892±0.0289. With inclusion of all data points a value for χ^2 of 22.7 was found. This is too high in comparison with the number of degrees of freedom since the value for the reduced χ^2 was calculated to be 3.8. The deviation from the line of best fit can be seen in Figure 8. This could be a result of grossly over-optimistic error allocation.

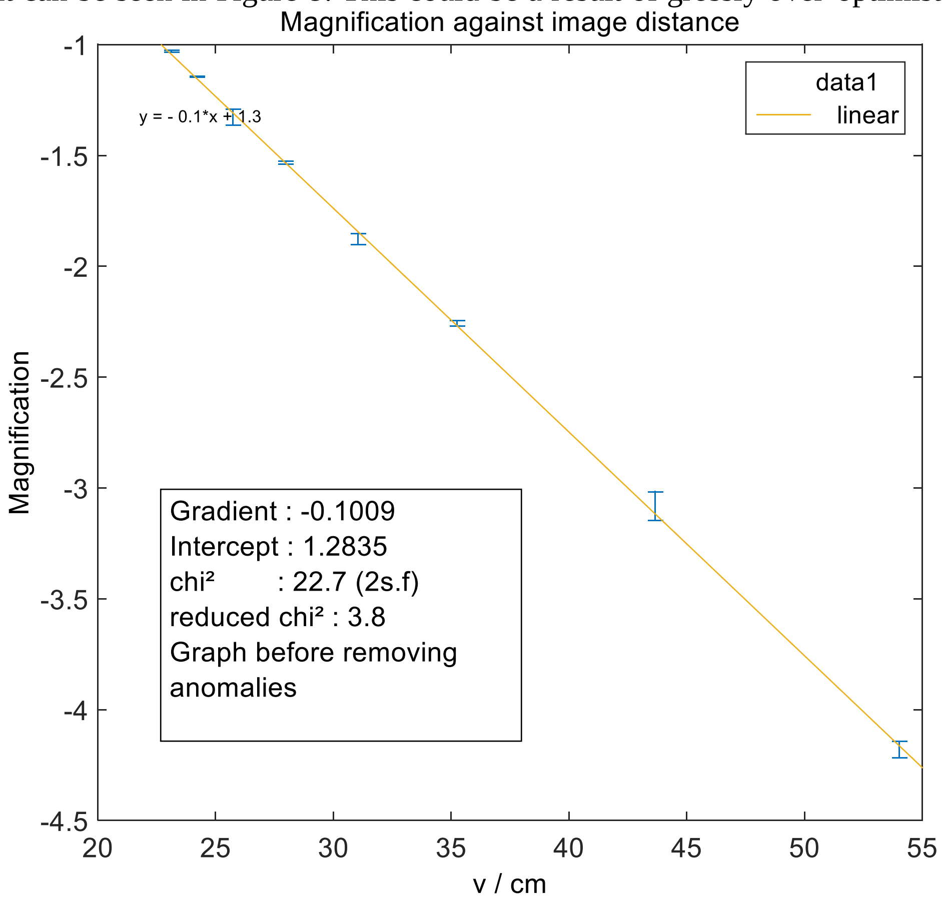


Figure 8: Before data removal

By removing data points 7 and 8, which were anomalous, the reduced χ^2 reached was 1.3, this data is graphed in Figure 9. This gave us a gradient of $-(1.064\pm0.0011)\text{cm}^{-1}$ and a *y*-intercept of (1.2667±0.0421).

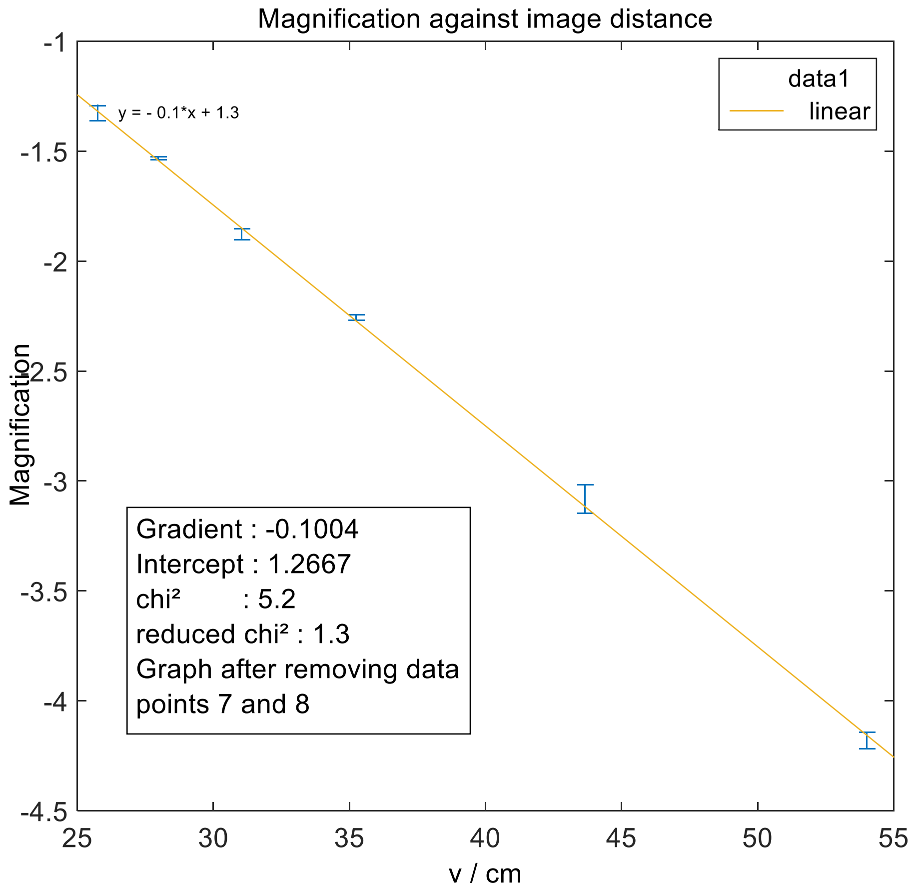


Figure 9: After data removal

With reference to Equation (4) it can be seen that the gradient is equal to $-\frac{1}{f}$ and therefore we calculated a value for the focal length of the lens to be (9.96±0.11)cm.

The error was propagated using $\left(\frac{\sigma f}{f}\right) = \left(\frac{\sigma g}{g}\right)$ where g is the gradient of the line of best fit. Hence, $\sigma f = \left|\frac{f\sigma g}{g}\right|$; giving the uncertainty in the focal length of the lens to be ±0.11cm.

4.3 Experiment 2

Errors in experiment 2 were calculated using the formula for standard deviation: $\sqrt{\frac{\sum(v-<v>)^2}{N(N-1)}}$ and by using the greater value either of this or of ±0.05cm. "*f*" was calculated using the formula: $f = \frac{D^2-d^2}{4D}$ for each individual value of *D* and *d* and took a mean of these. This gave us a final value for *f* to be (10.37±0.03)cm.

4.4 Experiment 3

Refer to section 2 for the derivation of the standard lens equation, equations (1), (2) and (3). Equation (3) can be manipulated to find the minimum of the hyperbola in Figure 10, as follows:

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad (6)$$

$$u+v=\frac{u^2}{u-f} \quad (7)$$

$$y=u+v, y=\frac{u^2}{u-f} \quad (8)$$

Differentiating with respect to *u* gives *u=2f* at the minimum.

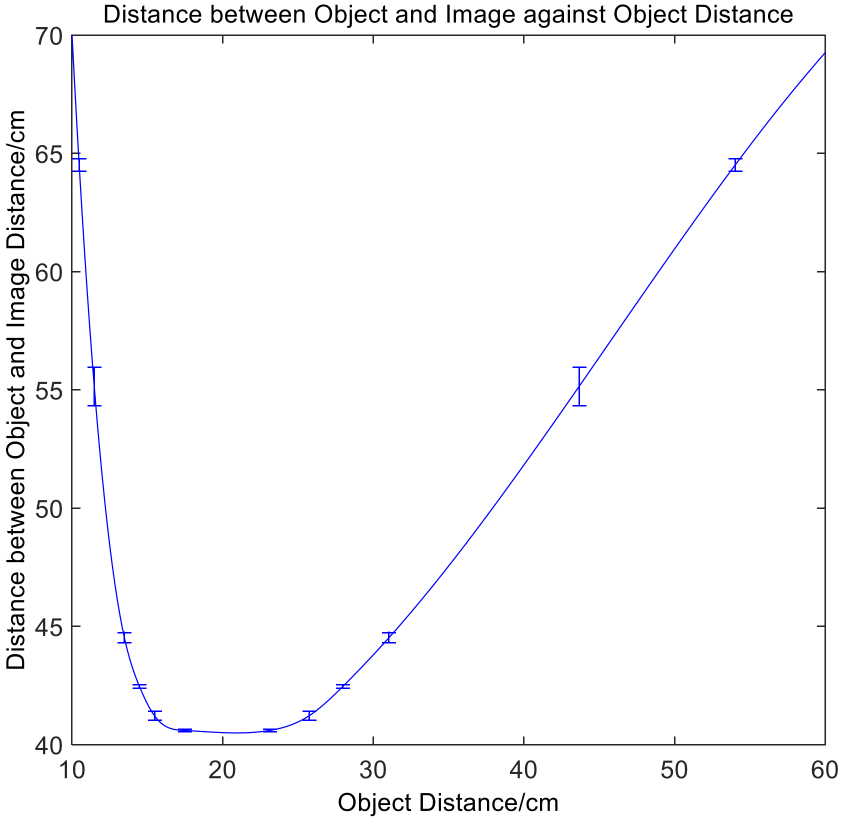


Figure 10: Plot of (*u*+*v*) against *u*

The focal length obtained by this method was (10.5±0.5)cm. The measurement was taken by eye and the uncertainty on this measurement was given by the inability to resolve exactly where the minimum of the curve was. We assigned an uncertainty of 0.5cm as this was half of what we judged to be the uncertainty in the minimum of the hyperbola.

4.5 Experiment 4

Figure 11 below is the table of data obtained for this experiment. The data spread begins at 25.00cm because the image appeared to come into focus at a point beyond the end of the ruler at values of u of 20.00cm or smaller. The value obtained for f using this data is (-13.87±1.52)cm.

A graph of $\frac{1}{-(a-b)}$ against $\frac{1}{c}$, where *a*,*b* and *c* are described in Figure 4, was plotted. This graph is shown in Figure 12, where *–(a-b)* is the object distance and *c* is the image distance.

u/cm	**Mean a/cm**	**b/cm**	**Mean c/cm**
55.00	12.4	6.25	25.8
50.00	12.8	7.45	22.3
45.00	13.2	7.45	26.7
40.00	13.8	7.50	32.5
35.00	14.2	8.00	37.1
30.00	15.6	8.70	50.8
25.00	17.2	10.05	61.2

Figure 11: Data collected for experiment 4

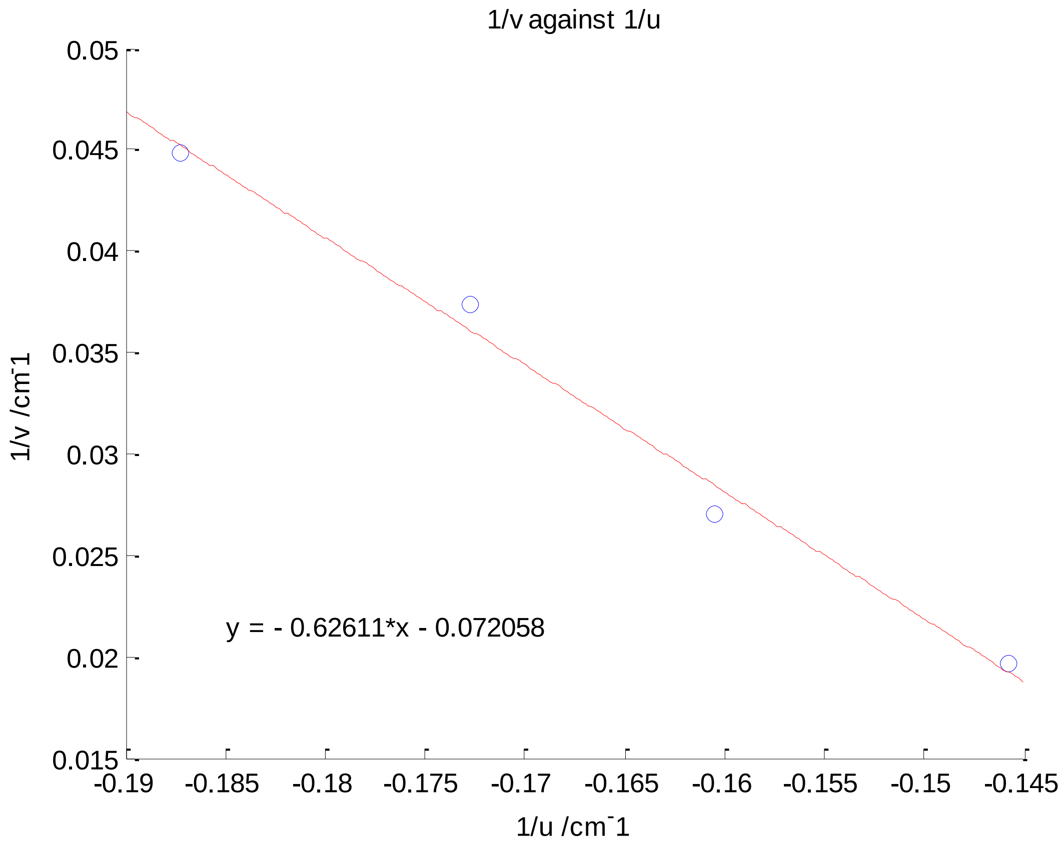


Figure 12: *1/v* against *1/u*

A second method for focal length of the concave lens determination was used to estimate the focal length. This involved using the same raw data values for *u* and *v*, shown graphically in Figure 12, and calculating individual values for f using Equation (6) and took a mean. We determined the error on these using the standard deviation calculation Equation (5). The value for f obtained by this method was (-7.68±0.16)cm.

5. Discussion and Conclusion

5.1 Experiment 1 Method 1

The obtained focal length of the convex lens appears too large. A value of 10.00cm was given to be the accurate focal length of the convex lens which is not within three standard deviations of (12.36±0.06)cm. Errors have affected the data. We experienced great difficulty trying to determine the point at which the image was in focus by eye, this is called the *problem of definition*. This could also be attributed to chromatic aberrations [4]. This is where deviations from the ideal conditions of Gaussian optics can be seen due to the fact that the refractive index is actually a function of colour (frequency). This is relevant because all experiments used white light. Repeats on each individual experiment varied by as much as a centimetre. To account for this we took five repeat measurements, reliability could have been improved with more than five. Furthermore, the image on the screen was more focused at the centre than the edges. This is due to the difference in thickness of the lens at the centre than at the edges. We assume the lens is thin and take the focus to be the same point on the image each time, but error is not eliminated. The height at which the lens sits is not adjustable to a high enough degree of precision. The lenses were approximated to be at the same height due to manual adjustment.

5.2 Experiment 2 Method 2

This final value of f does not seem to deviate greatly from 10.00cm. Therefore the experiment appears to be successful since 10.00cm (true value) lies within one standard deviation of (9.96±0.11)cm. Similar errors to the errors mentioned in *5.1* have influenced the results, in particular that the image was more focused at the centre than at the edges because a horizontal image distance was taken. A unique error that occurred when the position between the light box and the lens was fixed, its position was assumed to have no error. We know the error of the ruler was ±0.05cm, but this was not accounted for in our calculations. This would mean that the independent variable had an error as well as the dependent variable, which may not be graphically feasible and so it was a reasonable assumption to assume no error here.

5.3 Experiment 2

This value of f calculated was accurate to the true value of 10.00cm to within 13 standard deviations. This is too great a deviation to consider the experiment to be a success. 99.7% accuracy is within 3 standard deviations. The errors throughout *5.1*, *5.2* and *5.3* are relevant to Experiment 2.

5.4 Experiment 3

The uncertainty in this experiment is large due to the "judgement by eye" method. This has led to the accurate value of the focal length to lie within one standard deviation of (10.5±0.5)cm. The determination of the minimum of the hyperbola proved a difficult method. The curve appeared almost stationary over a wide range of values for *u* see [Figure 10].

5.5 Experiment 4

This experiment involved taking lots of data measurements. This introduces a high uncertainty into the value of f since $\frac{1}{f} = -\frac{1}{a-b} + \frac{1}{c}$. The value obtained for f is less negative than the accurate value. This could be due to the misalignment of our points with the line of best fit, this means that it is most probably due to a systematic error, causing the entire graph to shift. The second method of determination seems to back this idea up since the systematic error has been removed when the method of analysis is changed.

6. References

[1] Page 1248 Young Freedman, University Physics, Pearson Education Limited Thirteenth Edition

[2] http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/lenseq.html#c2 accessed 10/11/14

[3] http://www.physicsclassroom.com/class/refrn/Lesson-5/The-Mathematics-of-Lenses accessed 10/11/14

[4] Page 253 E.Hecht, Optics, Addison Wesley Fourth Edition

7. Length and Date

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