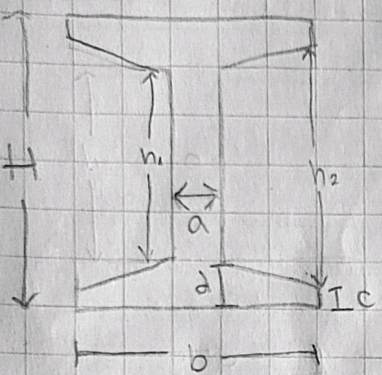
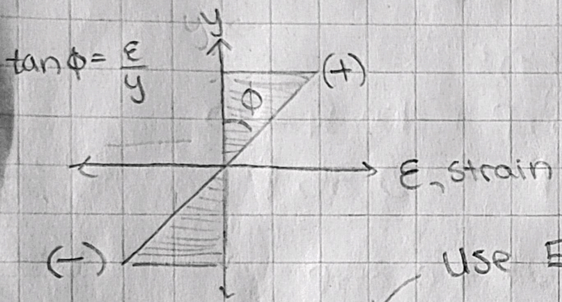
- Theoretical Predictions of the Strain @ the locations of the strain gauges



$a = 9.02$
$h_1 = 59.93$
$H = 76.05$
$h_2 = 65.36$
$c = 5$
$d = 12.16$
$b = \left(\frac{63.95 + 64.20 + 64.03}{3}\right) = 64.06\,mm$



ϕ = curvature (rad/mm)
y = vertical distance from centroidal axis (mm)
$y = \phi\varepsilon$

Use Euler Bernoulli Beam Theory to find M

$$\varepsilon = \phi = \frac{M}{EI} \qquad \sigma(y) = \frac{My}{I}$$

could approximate w triangles or Trapazoid

Approximate the I beam to be:

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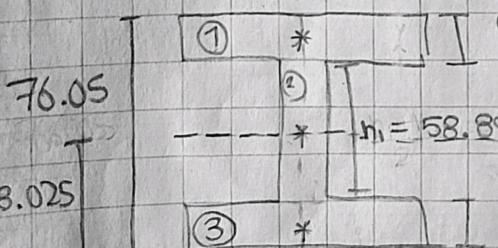
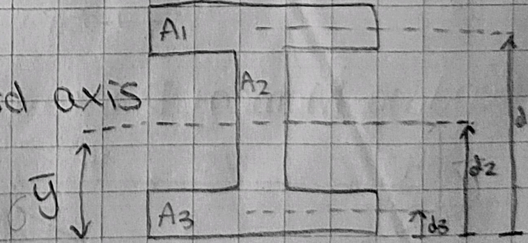


$\frac{c+d}{2} = 8.58$

Neutral/centroid axis



#	A	d	Ad	$\bar{d}$	$A\bar{d}^2$	I
1	549.6	71.76	39439.3	+33.74	625 657.8	421.5
2	531.2	37.98	20175	0	0	153515
3	549.6	4.29	2357.8	−33.74	625 657.8	421.5
Σ	1630.4		61972.1		1 251 315.65	154357.7

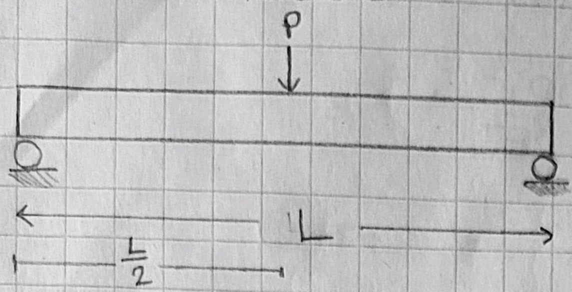
$$\bar{Y} = \frac{\sum A_i d_i}{\sum A_i} = \frac{61\,972.1}{1630.4} = 38.03$$

$\bar{d} = d - \bar{y}$
$I = \frac{bh^3}{12}$

$$I_{total} = \sum(A\bar{d}^2 + I) = 1251315.65 + 154357.7 = 1405\,673.35\ mm^4$$

Moment of Inertia

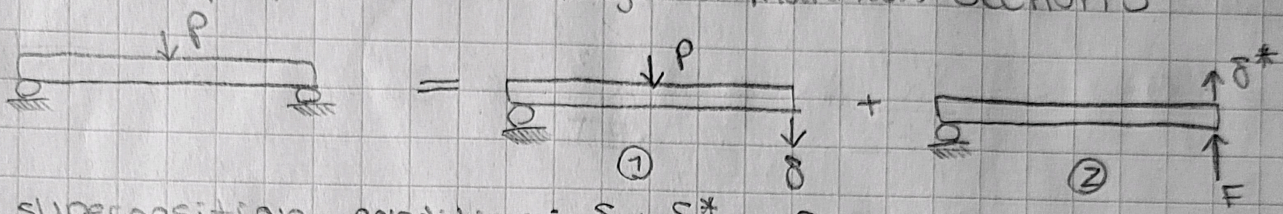
3-Point Load:



$L = 400\,mm$
P = total load (N)

$\mapsto x$

- Use Superposition → Strength of Materials Section 5



superposition condition: $\delta + \delta^* = 0$

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1. Look @ the section $0 \leq x \leq \frac{L}{2}$

Boundary Conditions:

$u(0) = 0$
$u'(0) = 0$ — zero distributed Load
$u''(0) = 0$ ← is this BC correct? → OR is it $u''(\frac{L}{2}) = 0$
$u'''(\frac{L}{2}) = \frac{-P}{EI}$

- Integrate the distributed load to get the shear force distribution

$V = \int 0 \, dx = C_1$ $\qquad C_1 = -EI\, u'''$

$u'''(\frac{L}{2}) = \frac{-P}{EI}$ $\qquad C_1 = -EI\left(\frac{-P}{EI}\right) = -P$

$V = -P$

- Integrate the shear force to obtain the moment in the beam

$M = EI\left(-\int V dx\right) = -EI \int \frac{(-P)}{EI} dx = Px + C_2$

$u''(0) = 0$

$u''(0) = P(0) + C_2 \rightarrow C_2 = 0$

curvature: $\frac{d^2u}{dx^2} = \frac{M}{EI} = \frac{Px}{EI}$

- Slope of the Beam is the integral of the curvature

$$\theta = \frac{du}{dx} = \int \frac{d^2u}{dx^2}\,dx = \int \frac{Px}{EI}\,dx = \frac{Px^2}{2EI} + C_3$$

$$u'(0) = 0 \Rightarrow 0 = \frac{P(0)^2}{EI} + C_3 \Rightarrow C_3 = 0$$

- The deflection of the Beam is the integral of the slope

$$u = \int \theta\,dx = \int \frac{Px^2}{2EI}\,dx = \frac{Px^3}{6EI} + C_4$$

$$u(0) = 0 \Rightarrow C_4 = 0$$

$$u_{①} = \frac{Px^3}{6EI} \quad \text{for} \quad 0 \leq x \leq \frac{L}{2}$$

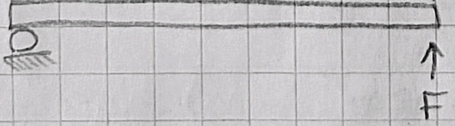
- The deflection @ L is the deflection @ $\frac{L}{2}$ plus the slope @ $\frac{L}{2}$ times the distance $\frac{L}{2}$

$$\delta = u\left(\tfrac{L}{2}\right) + u'\left(\tfrac{L}{2}\right)\underbrace{\left(x - \tfrac{L}{2}\right)}_{\text{when } x = L}$$

$$\delta = \frac{P\left(\frac{L}{2}\right)^3}{6EI} + \frac{P\left(\frac{L}{2}\right)^2\left(L - \frac{L}{2}\right)}{EI} = \frac{PL^3}{48EI} + \frac{PL^3}{8EI} = \boxed{\frac{7PL^3}{48EI}}$$

$$\delta(x) = \frac{PL^3}{48EI} - \frac{PL^3}{8EI} + \frac{PL^2x}{4EI} = \frac{PL^2}{4EI}\left(-\frac{5PL}{12} + Px\right)$$ in terms of X

②



Boundary conditions:

$u(0) = 0$

$u'(0) = 0$ $\quad u'''(L) = \frac{F}{EI}$

$u''(0) = 0$

- Integrate Distributed Load to get Shear:

$$V = \int 0\,dx = C_1 \qquad V = -EI\frac{d^3u}{dx^3} \rightarrow \frac{-V}{EI} = \frac{d^3u}{dx^3}$$

$$u'''(L) = \frac{-C_1}{EI} = \frac{-F}{EI} \Rightarrow C_1 = +F$$

- Integrate shear force to obtain the moment in the beam

$$M = EI\frac{d^2u}{dx^2} = -\int V\,dx = -\int +F\,dx = -Fx + C_2$$

$$\frac{M}{EI} = \frac{d^2u}{dx^2} \rightarrow u''(0) = \frac{-F(0)+C_2}{EI} = 0 \Rightarrow C_2 = 0$$

- Slope of the Beam is the integral of the curvature

$$\theta = \frac{du}{dx} = \int \frac{M}{EI}\,dx = \int \frac{-Fx}{EI}\,dx = \frac{-Fx^2}{2EI} + C_3$$

$$u'(0) = \frac{-F(0)^2}{2EI} + C_3 = 0 \Rightarrow C_3 = 0$$

- Deflection is the integral of the slope

$$u = \int \theta\,dx = \int -\frac{Fx^2}{2EI}\,dx = \frac{-Fx^3}{6EI} + C_4$$

$$u(0) = -\frac{F(0)^3}{6EI} + C_4 = 0 \Rightarrow C_4 = 0$$

Deflection @ L

$$u(L) = \frac{-F(L)^3}{6EI} = \delta^*$$

Apply the superposition condition:

$$\delta + \delta^* = 0$$

$$\frac{7PL^3}{48EI} - \frac{FL^3}{6EI} = 0 \rightarrow \frac{7P\cancel{L^3}}{48\cancel{EI}} = \frac{F\cancel{L^3}}{6\cancel{EI}}$$

$$\frac{42P}{48} = F$$

$$\frac{7}{8}P = F$$

$\star$ Can I just Add the moments?

$$M = M_1 + M_2$$

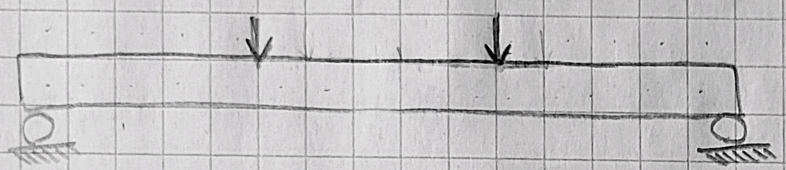
$$M = Px - Fx = (P - \tfrac{7}{8}P)x = \frac{P}{8}x$$

$$E = \frac{\sigma}{\varepsilon} \rightarrow \varepsilon = \frac{\sigma}{E}$$

$$\sigma(y) = \frac{My}{I}$$

$$\varepsilon(y) = \frac{My}{IE} = \frac{\left(\frac{P}{8}x\right)y}{IE}$$

4-point Load



superposition: