Computational Economics 2020

Assignment 1

(due: Monday, March 30, 2020, 11:59 PM (GMT + 1))

The assignment is split into part A and part B which are weighted equally with 15% each. You are allowed to form groups of 2 (if there is a odd number of people, groups of 3 are allowed too). Each group has to present at least one of their solutions.

Please follow these instructions for handing in your assignment:

1. Submit the assignment via Email to Philipp (philipp.mueller@business.uzh.ch). The Email should contain:
   - A single PDF-file with the names of all group members, and assignment number on page 1. The file should contain all your answers and results.
   - The source code in a separate zip archive. The code should be well documented and readable.
2. Only the students taking the course for credits are getting feedback to their solution; feel free to send your solution anyway. A sample solution will be published after the deadline. If you have any specific question, reach out on GitHub `https://github.com/KennethJudd/CompEcon2020/issues`.

Refrain from sharing complete solutions.

For the content of the PDF we expect the following:

1. Provide a brief introduction/ motivation for the problem.
2. Explain how you solved the exercise and show the most relevant calculations (formulas and essential parts of the code) with brief comments.
3. Concisely interpret the results of the exercise.
4. For each exercise the floating text should not exceed 2 pages (this does not include formulas, codes, graphs, and tables).

Exercise B-1

Reproduce Table 4 and Table 5 in Section 3.1 of Judd, Kübler and Schmedders (2011) for $J = 3, 4, 5$. Report all results with at least six significant digits. Also report the condition numbers for the matrices in the linear systems of equations.

Hint: The consumption allocations are given in Section 3.1 and so you do not need to calculate them. Put differently, you do not need to perform the first step of the algorithm described in Appendix A.1. Instead you need to manually enter the consumption allocations. Using the given allocations, compute the price of consumption in state y, denoted by P_y, as the marginal utilities of consumption. With these prices at hand, you can then solve the remaining linear systems of equations.

Bonus: As discussed in the lecture, the linear system of equations is ill-conditioned. Try solving it by either relying on extended precision or regularization strategies.

Exercise B-2

Consider the life-cycle optimization problem

$$\max_{c_t, a_t} \sum_{t=1}^{T} \beta^t \log(c_t) \tag{1}$$

$$\text{s.t. } a_{t+1} \leq (1+r)a_t + w_t - c_t \tag{2}$$

$$a_T \geq 0 \tag{3}$$

$$c_t > 0 \tag{4}$$

and earns

$$w_t = \begin{cases} \max\left(1.5, (0.5 + t(1 - t/T))\right)/16 & t < T_{retirement} \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

(A) Solve the constrained optimization problem with $\beta = 0.96$, $r = 0.1$, $T = 50$, $T_{retirement} = 0.9T$ and $a_1 = 1$.

(B) Derive and implement the analytic gradient and Hessian. Recall that for constrained problems, the Hessian equals the bordered Hessian with

$$\nabla^2_{xx} L(x, \lambda, \mu) = \nabla^2 f(x) + \sum_i \lambda_i \nabla^2 g_i(x) + \sum_i \mu_i \nabla^2 h_i(x) \tag{6}$$

with $g(\cdot)$ and $h(\cdot)$ being the equality and inequality constraints, respectively. Note: A good way to double check the analytic derivatives is to compare them to the finite difference approximations.

(C) Compare the wall time, number of iterations, first-order optimality between the optimizer using the finite-differences, and your analytic gradient as well as Hessian.

(D) Run the optimization for $T \in \{10, 20, 40, 80, 160, 320, 640, 1280, \dots\}$ for FD and analytic derivates and compare their wall times, number of iterations, and first-order optimality. As discussed in the lecture, keep β^T and r^T constant by adjusting β and r.

References

[1] Judd, Kenneth L., Felix Kubler, and Karl Schmedders, "Bond Ladders and Optimal Portfolios," *Review of Financial Studies* (2011) 24 (12): 4123–4166.